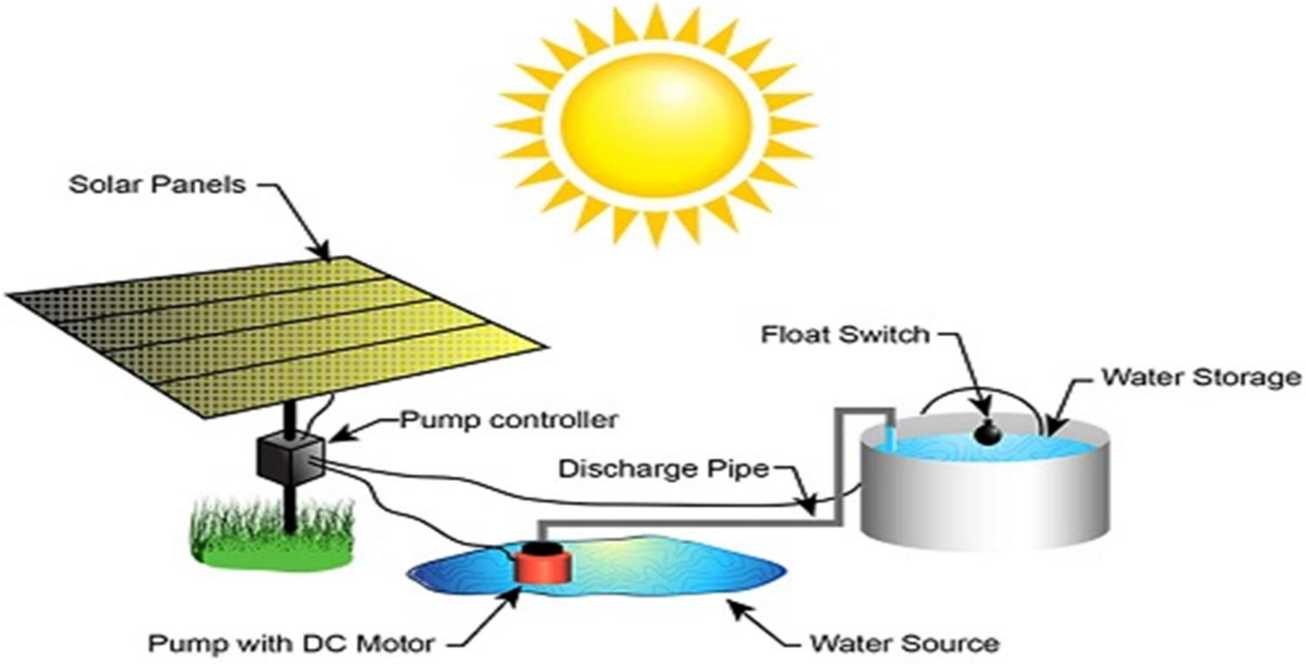


Control System

Stability & Routh's Stability Criterion

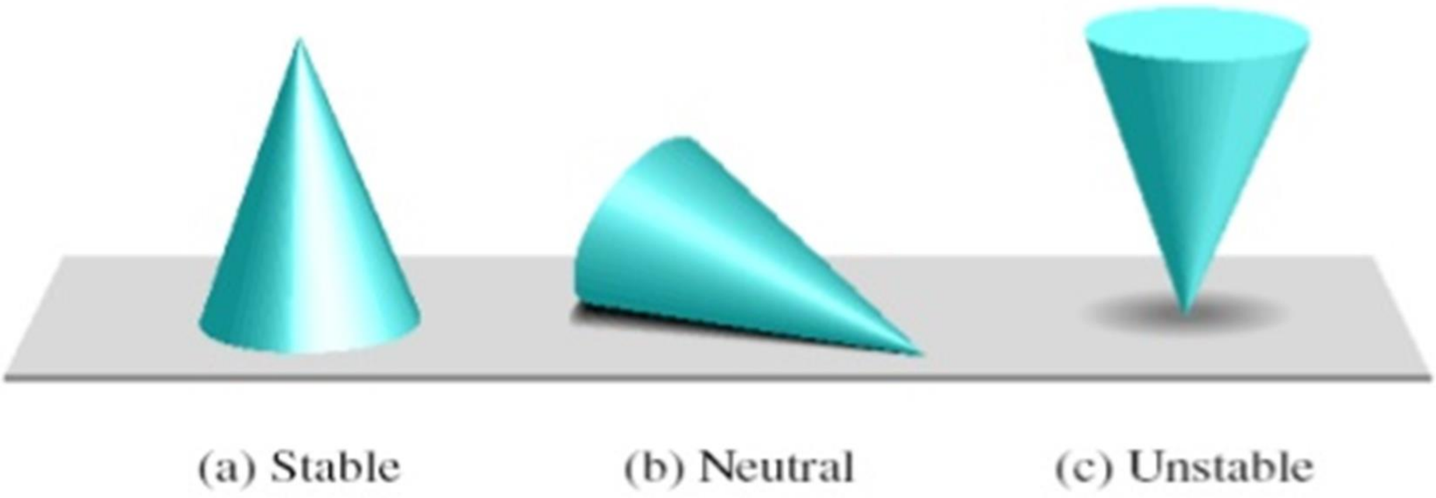
Content

- Review of Last Lecture.
- Transfer Function.
- Transfer function of RC and RLC electrical circuit.
- Examples of Transfer function
- Order of System & its type

Learning Objectives

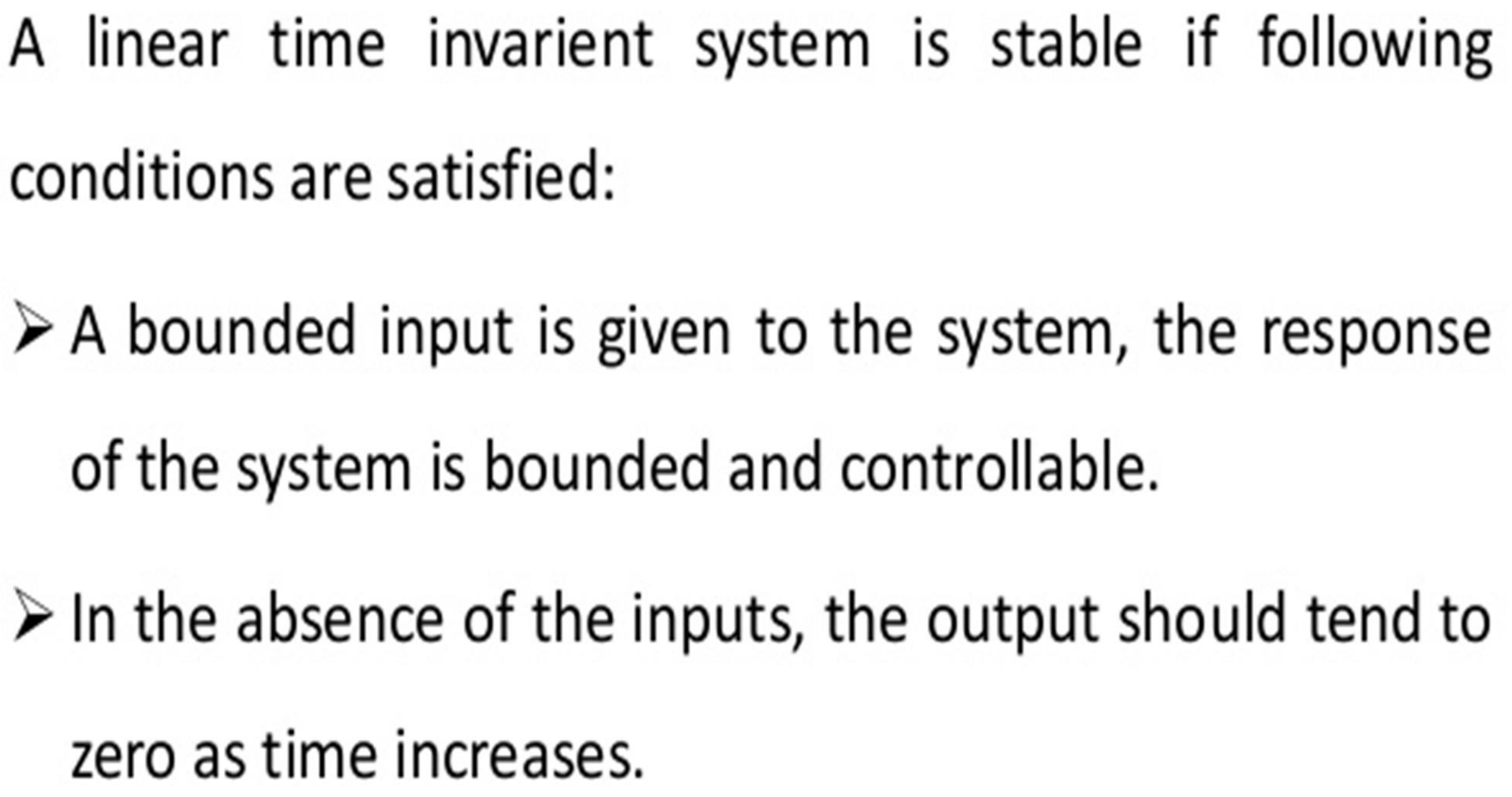
- Able to define use of TF in control system.
- Able to derive expression for open loop and closed loop system.
- Transfer function of RC and RLC electrical circuits.

Concept of Stabilty



The concept of stability can be illustrated by a cone placed on a plane horizontal surface.

Stable System



A linear time invarient system is stable if following conditions are satisfied:

- A bounded input is given to the system, the response of the system is bounded and controllable.
- In the absence of the inputs, the output should tend to zero as time increases.

Unstable System

- A linear time invarient system comes under the class of unstable system if the system is excited by a bounded input, response is unbounded.
- This means once any input is given system output goes on increasing & designer does not have any control on it

Critically Stable System

- When the input is given to a linear time invarient system, for critically stable systems the output does not go on increasing infinitely nor does it go to zero as time increases.
- The output usually oscillates in a finite range or remains steady at some value.
- Such systems are not stable as their response does not decay to zero. Neither they are defined as unstable because their output does not go on increasing infinitely.

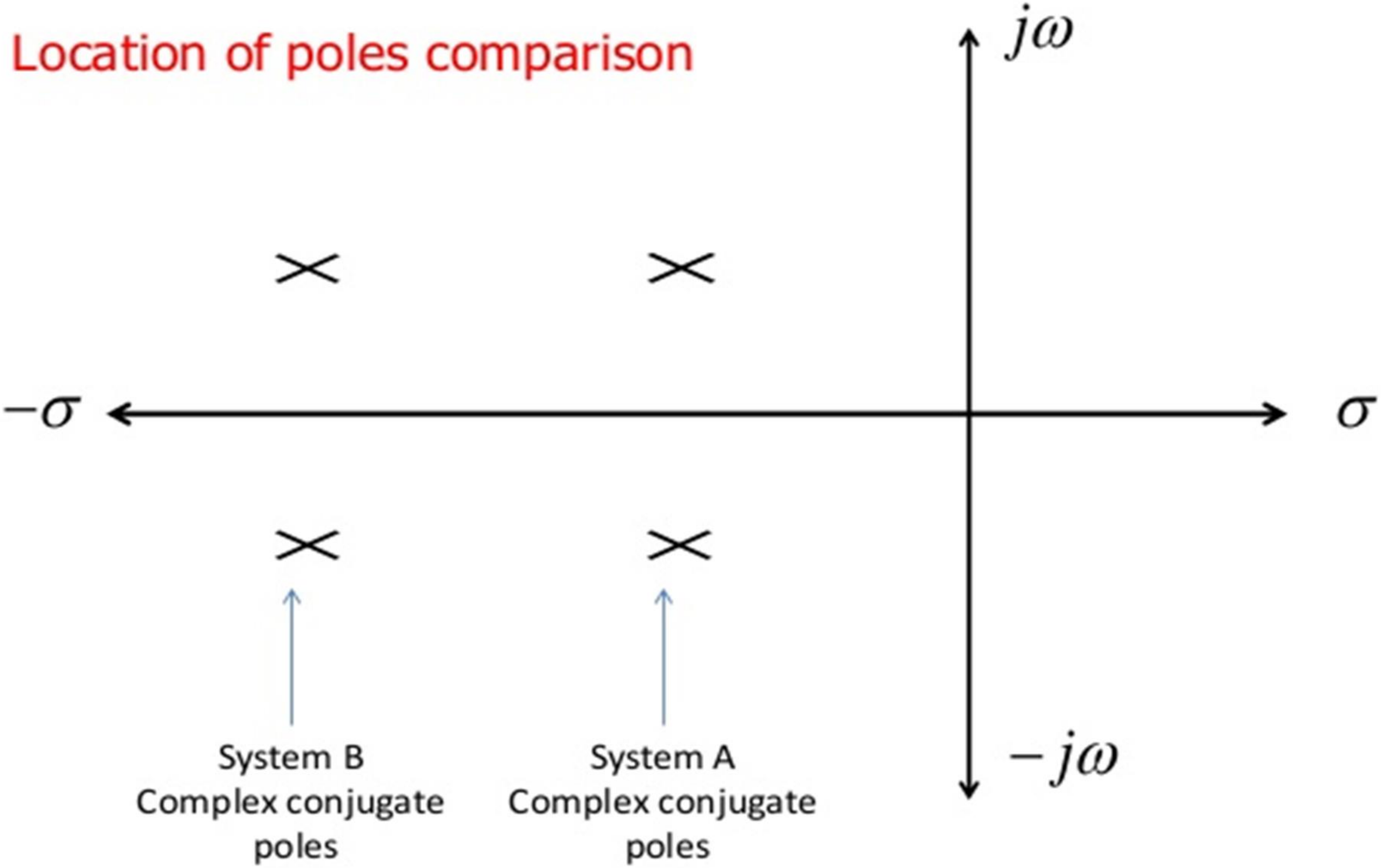
Relative Stability

- A system may be absolutely stable i.e. it may have passed the Routh stability test.
- As a result their response decays to zero under zero input conditions.
- The ratio at which these decay to zero is important to check the concept of “Relative stability”

Relative Stability

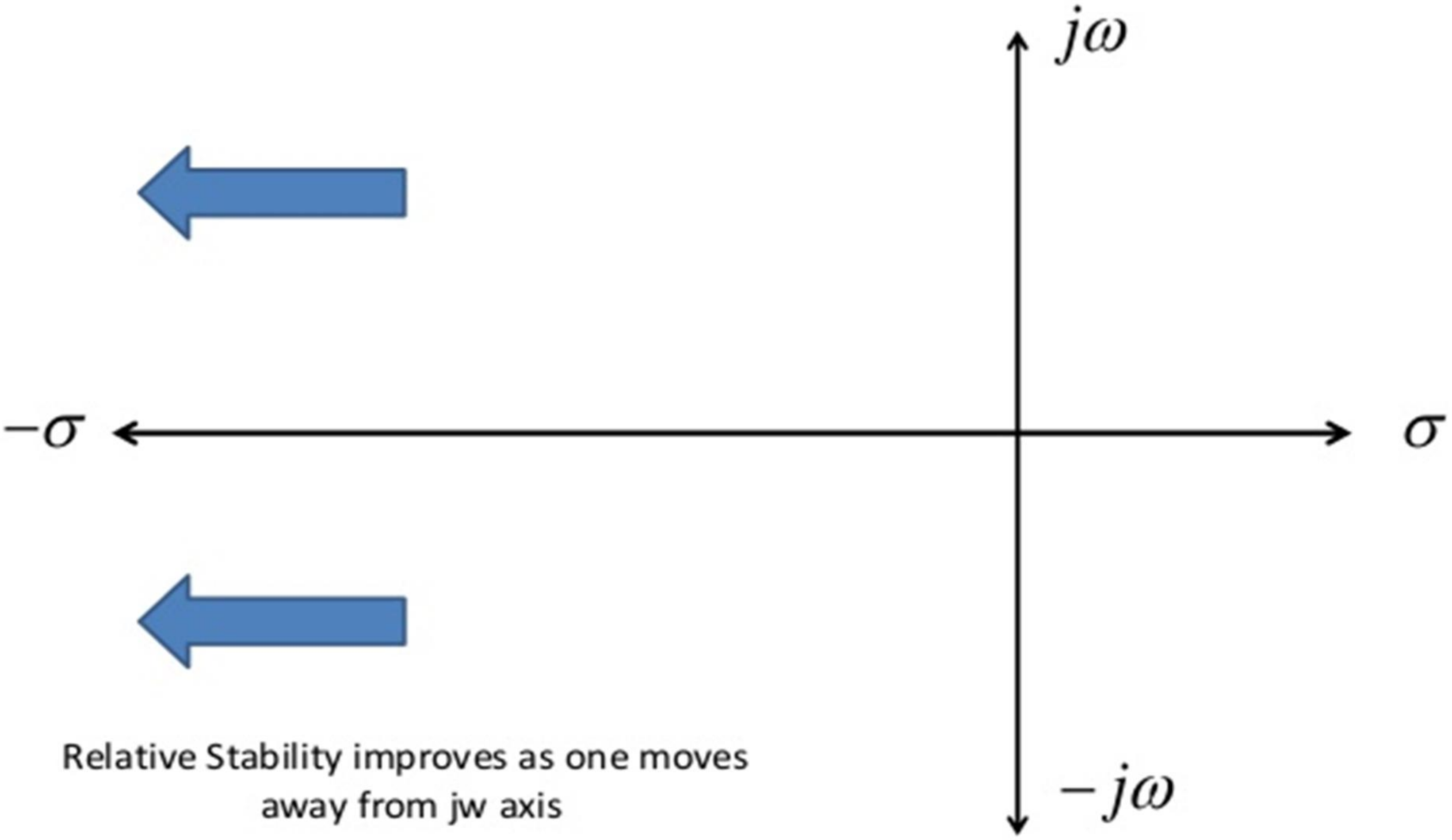
- When the poles are located far away from jw axis in LHP of s-plane, the response decays to zero much faster, as compared to the poles close to jw-axis.
- The more the poles are located far away from jw-axis the more is the system relatively stable.

Relative Stability



Location of poles comparison

Relative Stability



Relative Stability improves as one moves away from jw axis

Routh's Stability Criterion

For the transfer function;

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots\ldots\ldots\ldots\ldots + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n}$$

In this criterion, the coefficients of denominator are arranged in an Array called "Routh's Array";

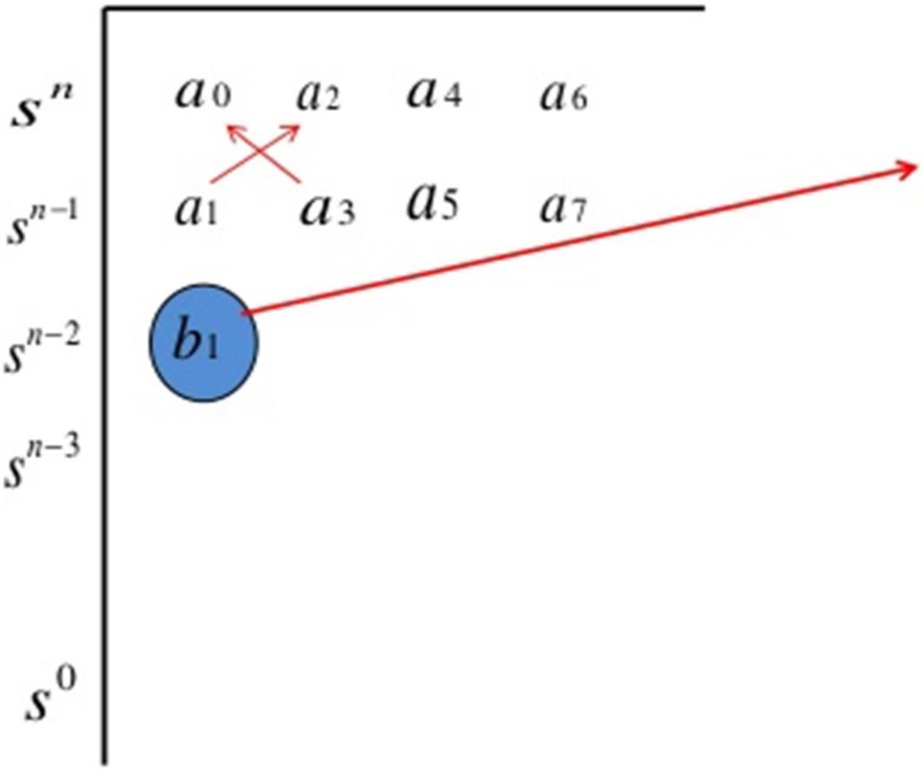
$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

Coefficient of s^n and s^{n-1} row are directly written from the given equation

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;

For next row i.e. s^{n-2} ;



$$b_1 = \frac{a_1 . a_2 - a_3 a_0}{a_1}$$

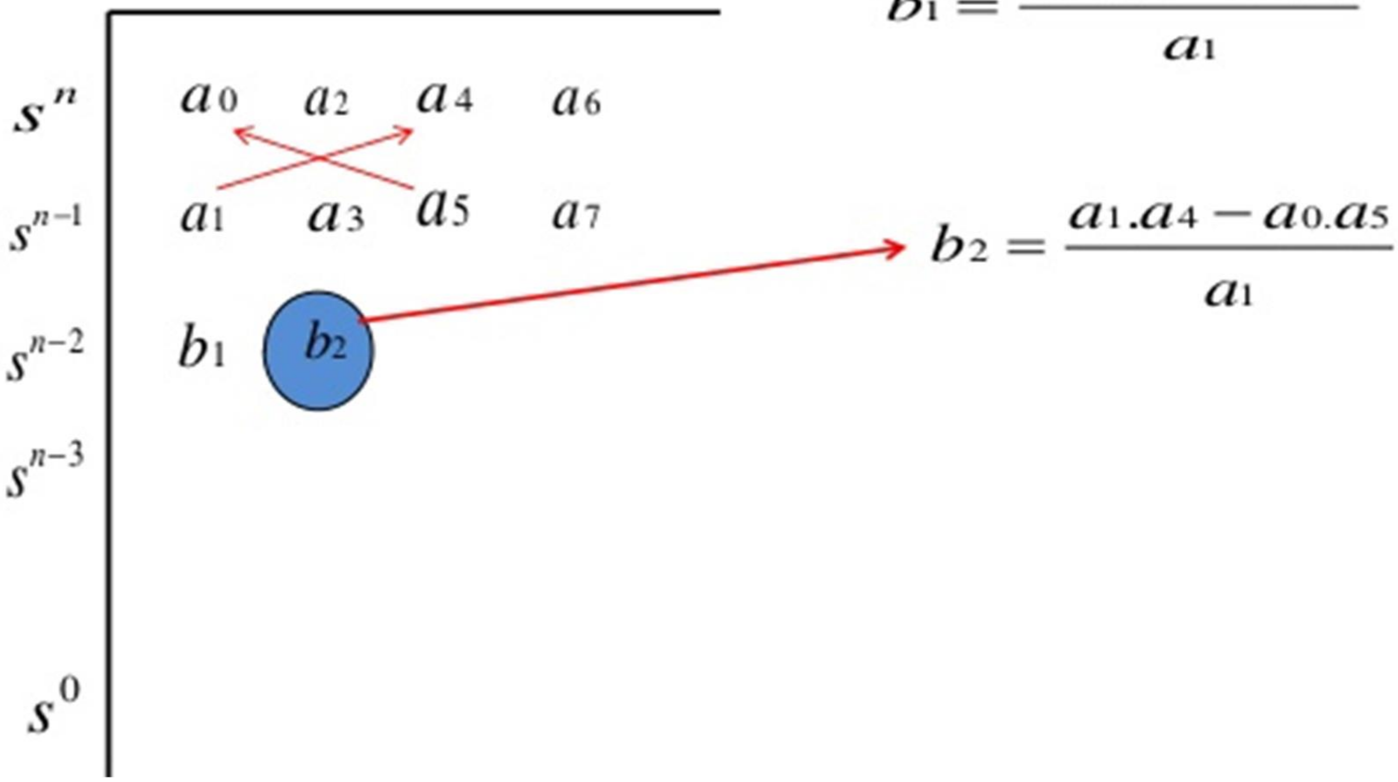
Coefficient of s^n and s^{n-1} row are directly written from the given equation

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$



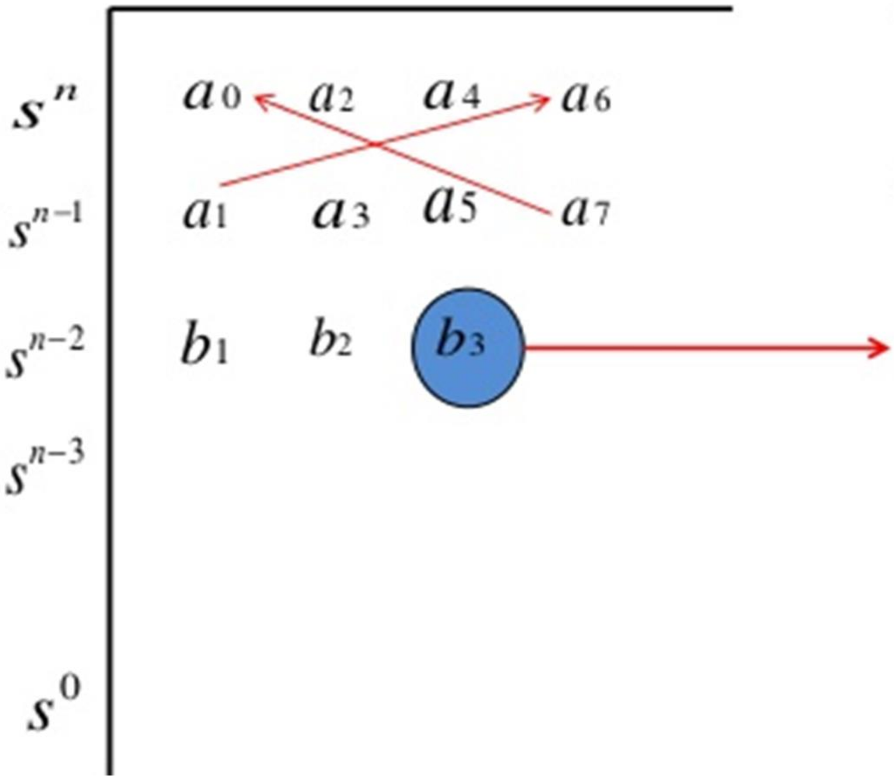
$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & a_6 \\ s^{n-1} & a_1 & a_3 & a_5 & a_7 \\ s^{n-2} & b_1 & b_2 & & \\ s^{n-3} & & & & \\ & & & & \\ s^0 & & & & \end{array}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

Coefficient of s^n and s^{n-1} row are directly written from the given equation

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots\ldots\ldots + a_n = 0$$

The Routh's array as below;



$$\begin{array}{c|cccc} s^n & a_0 & a_2 & a_4 & a_6 \\ s^{n-1} & a_1 & a_3 & a_5 & a_7 \\ s^{n-2} & b_1 & b_2 & b_3 & \\ s^{n-3} & & & & \\ s^0 & & & & \end{array}$$

For next row i.e. s^{n-2} ;

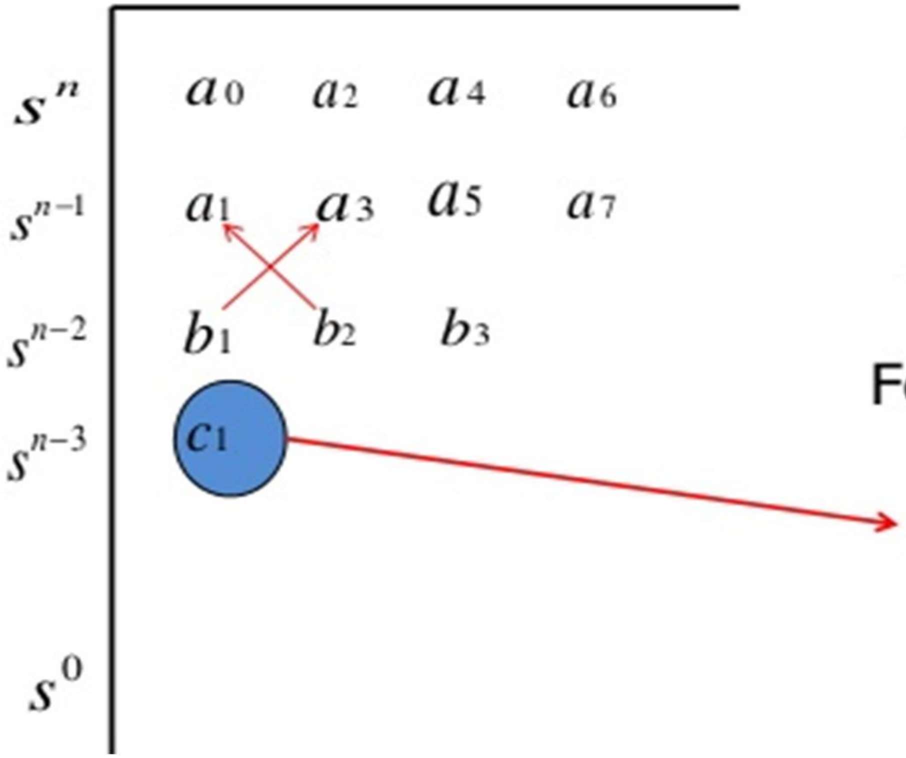
$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

$$b_3 = \frac{a_1.a_6 - a_0.a_7}{a_1}$$

Now the same technique is used, for next row i.e. s^{n-3}, but only previous two rows are used i.e. s^{n-1} & s^n

The Routh's array as below;



s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	
s^{n-3}	c_1			
s^0				

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3 a_0}{a_1}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

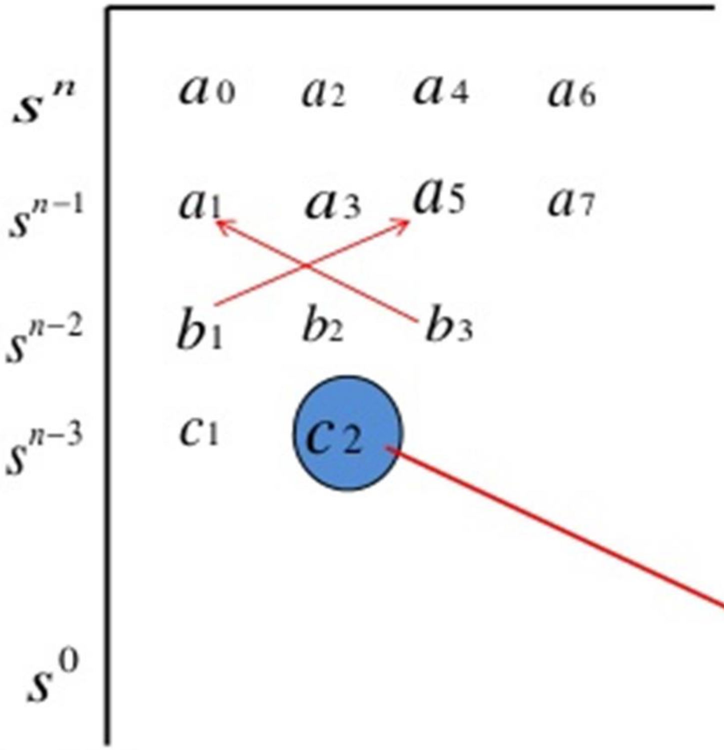
$$b_3 = \frac{a_1.a_6 - a_0.a_7}{a_1}$$

For next row i.e. s^{n-3} ;

$$c_1 = \frac{b_1.a_3 - b_2 a_1}{b_1}$$

Now the same technique is used, for next row i.e. s^{n-3}, but only previous two rows are used i.e. s^{n-1} & s^{n}

The Routh's array as below;



s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	
s^{n-3}	c_1	c_2		
s^0				

For next row i.e. s^{n-2} ;

$$b_1 = \frac{a_1.a_2 - a_3a_0}{a_1}$$

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

$$b_3 = \frac{a_1.a_6 - a_0.a_7}{a_1}$$

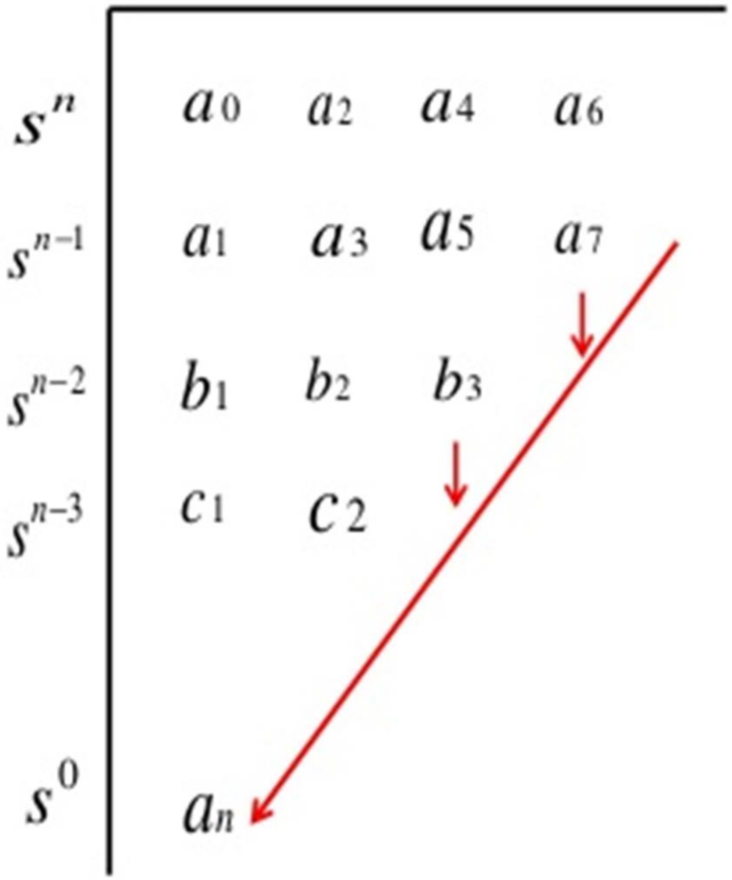
For next row i.e. s^{n-3} ;

$$c_1 = \frac{b_1.a_3 - b_2a_1}{b_1}$$

$$c_2 = \frac{b_1.a_5 - b_3a_1}{b_1}$$

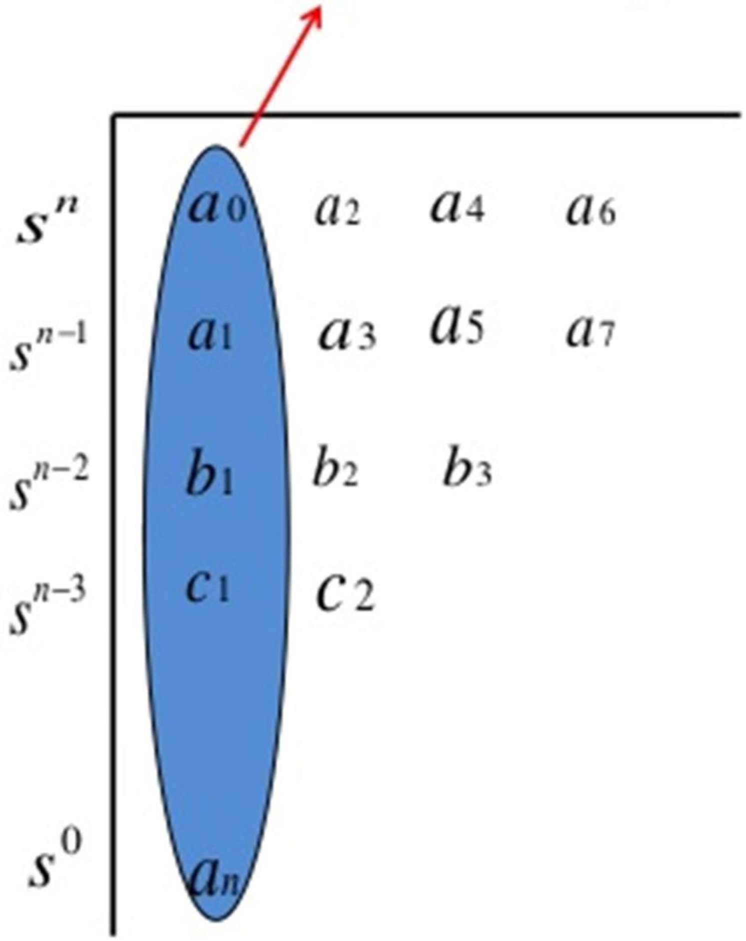
Now the same technique is used, for next row i.e. s^{n-3}, but only previous two rows are used i.e. s^{n-1} & s^n

- Each column will reduce by one as we move down the array.
- This process is obtained till last row is obtained.



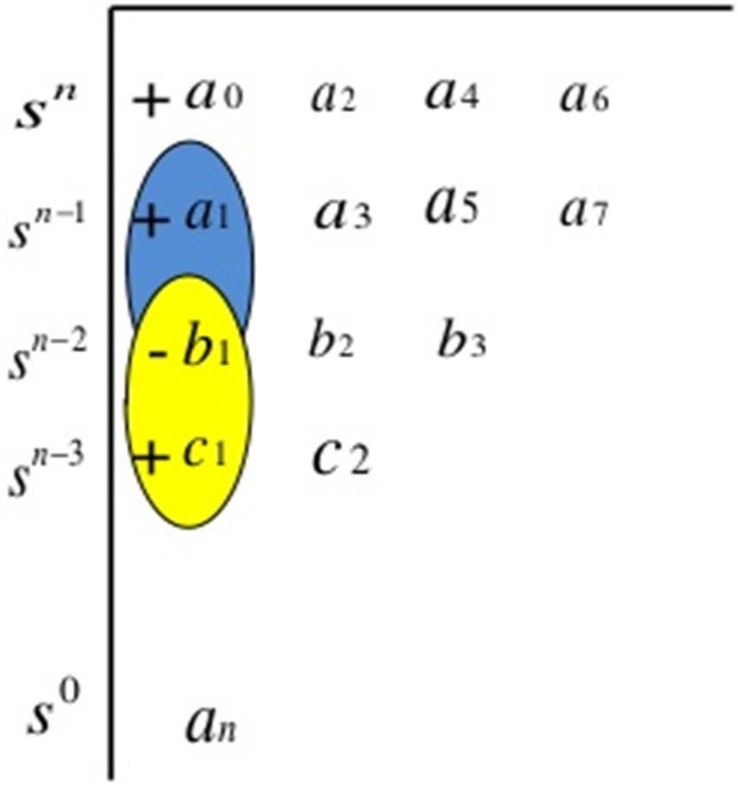
Routh's Criterion

- The necessary & sufficient conditions for a system to be stable is all terms in the first column at Routh's Array should have same sign.
- There should not be any sign change in first column.


Must have same sign

Routh's Criterion

- When there are sign changes in the first column of Routh's array then the system is unstable.
- There are roots in RHP.
- The number of sign changes equal the number of roots in RHP.



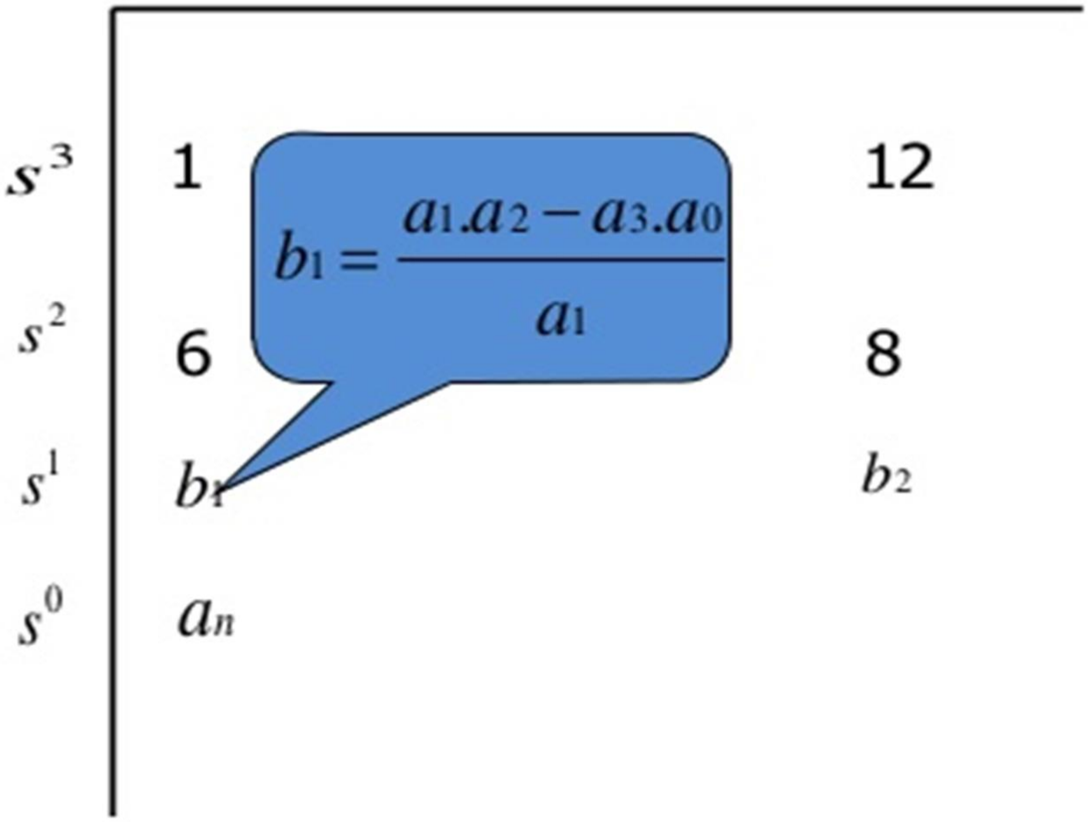
s^n	$+a_0$	a_2	a_4	a_6
s^{n-1}	$+a_1$	a_3	a_5	a_7
s^{n-2}	$-b_1$	b_2	b_3	
s^{n-3}	$+c_1$	c_2		
s^0	a_n			

Example 1

$$\underset{\substack{\downarrow \\ a_0}}{s^3} + \underset{\substack{\downarrow \\ a_1}}{6}s^2 + \underset{\substack{\downarrow \\ a_2}}{12}s + \underset{\substack{\downarrow \\ a_3}}{8} = 0$$

s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

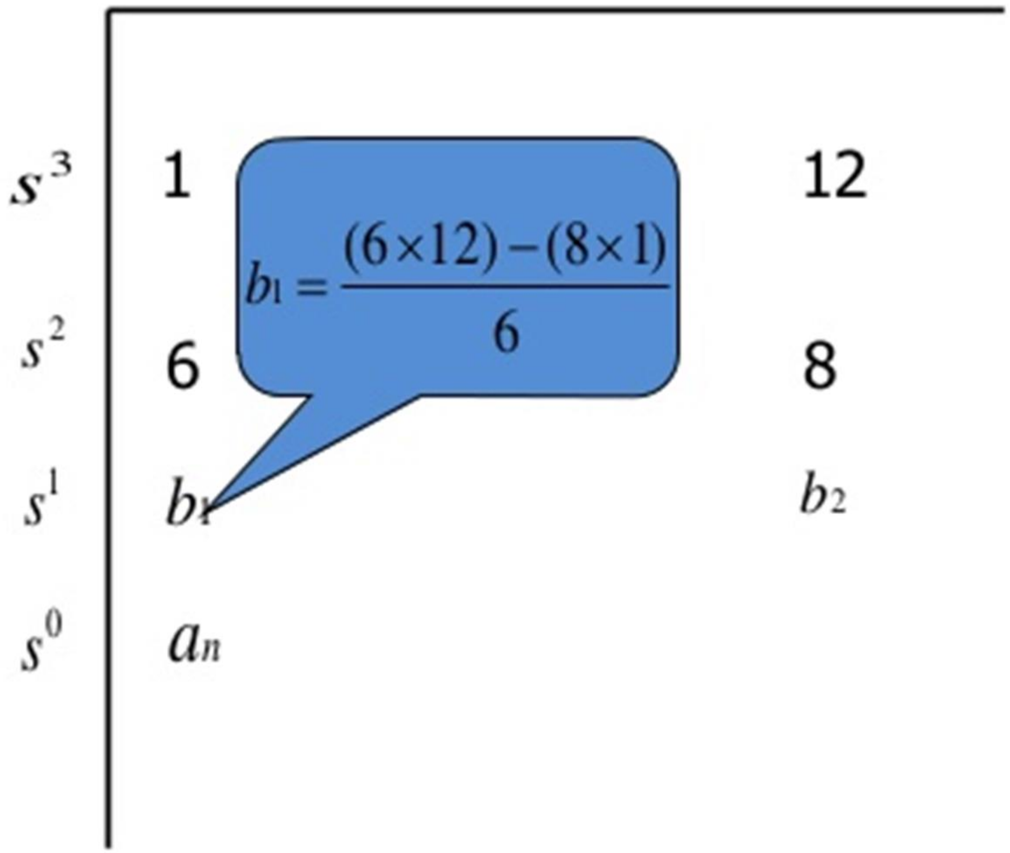
Example 1 Cont...



s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

Example 1 Cont...



s^3	1	12
s^2	6	8
s^1	b_1	b_2
s^0	a_n	

$$b_1 = \frac{(6\times12)-(8\times1)}{6}$$

Example 1 Cont...

s^3	1	12	0
s^2	6	8	0
s^1	10.67	b_2	
s^0	a_n		

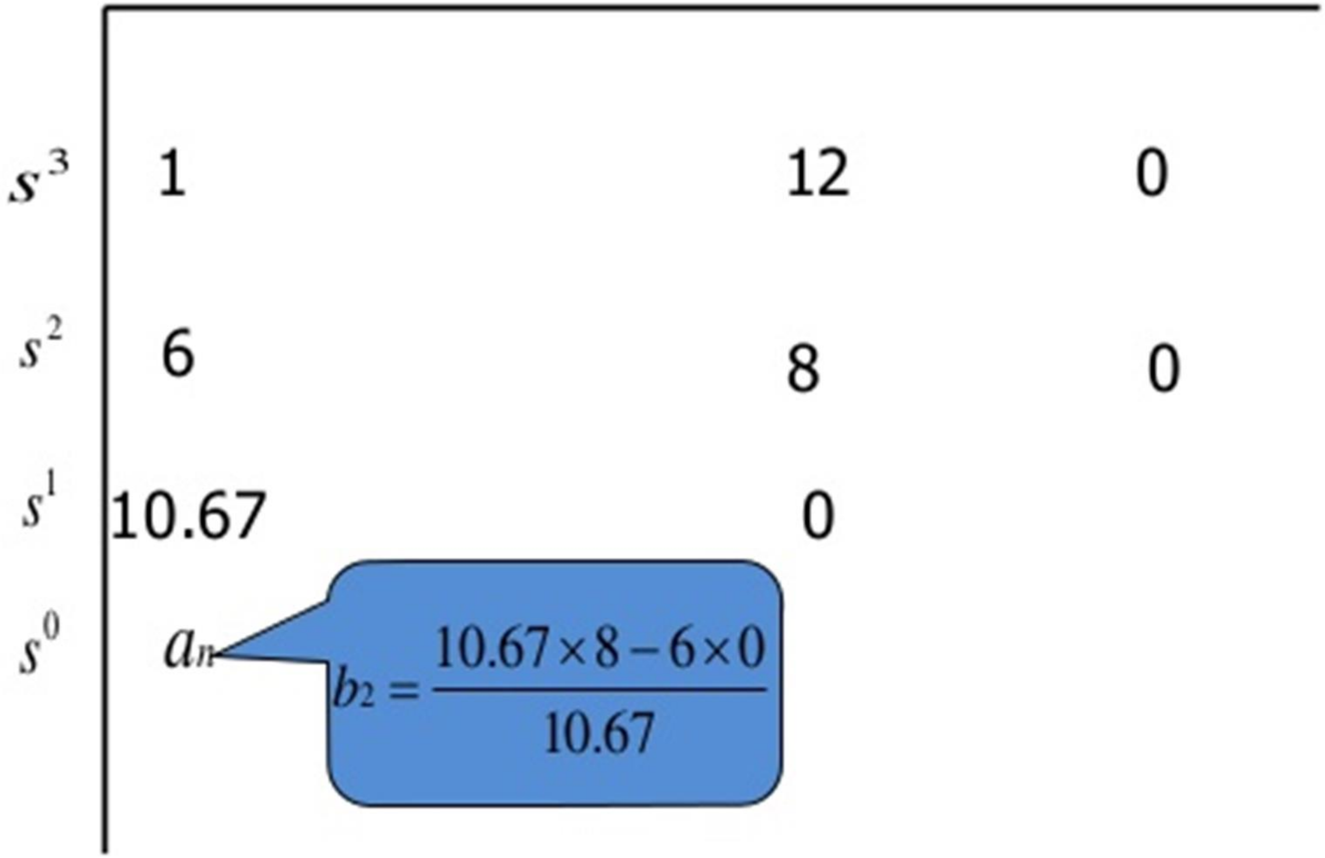
$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

Example 1 Cont...

s^3	1	12	0
s^2	6	8	0
s^1	10.67	b_2	
s^0	a_n		

$$b_2 = \frac{1\times 0 - 6\times 0}{6}$$

Example 1 Cont...



s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	a_n		

$$b_2 = \frac{10.67 \times 8 - 6 \times 0}{10.67}$$

Example 1 Cont...

$$s^3 + 6s^2 + 12s + 8 = 0$$

s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	8		

Example 1 Cont...

Elements in first column are of same sign

s^3	1	12	0
s^2	6	8	0
s^1	10.67	0	
s^0	8		

As all elements in first column are of same sign. Hence system is stable

Example 2

Comment on stability. $s^4 + 2s^3 + 6s^2 + 10s + 3 = 0$

$$\begin{array}{c|ccc}
\hline
s^4 & 1 & 6 & 3 \\
s^3 & 2 & 10 & 0 \\
s^2 & b_1 & b_2 & \\
s^1 & c_1 & & \\
s^0 & a_n & &
\end{array}$$

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

$$b_1 = \frac{(2 \times 6) - (1 \times 10)}{2}$$

$$b_1 = 1$$

Example 2 Cont...

s^4	1	6	3
s^3	2	10	0
s^2	1	b_2	
s^1	c_1		
s^0	a_n		

$$b_2 = \frac{a_1.a_4 - a_0.a_5}{a_1}$$

$$b_2 = \frac{2\times 3 - 1\times 0}{2}$$

$$b_2 = 3$$

Example 2 Cont…

s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	c_1		
s^0	a_n		

$$c_1 = \frac{b_1.a_3 - b_2 a_1}{b_1}$$

$$c_1 = \frac{1 \times 10 - 2 \times 3}{1}$$

$$c_1 = 4$$

Example 2 Cont...

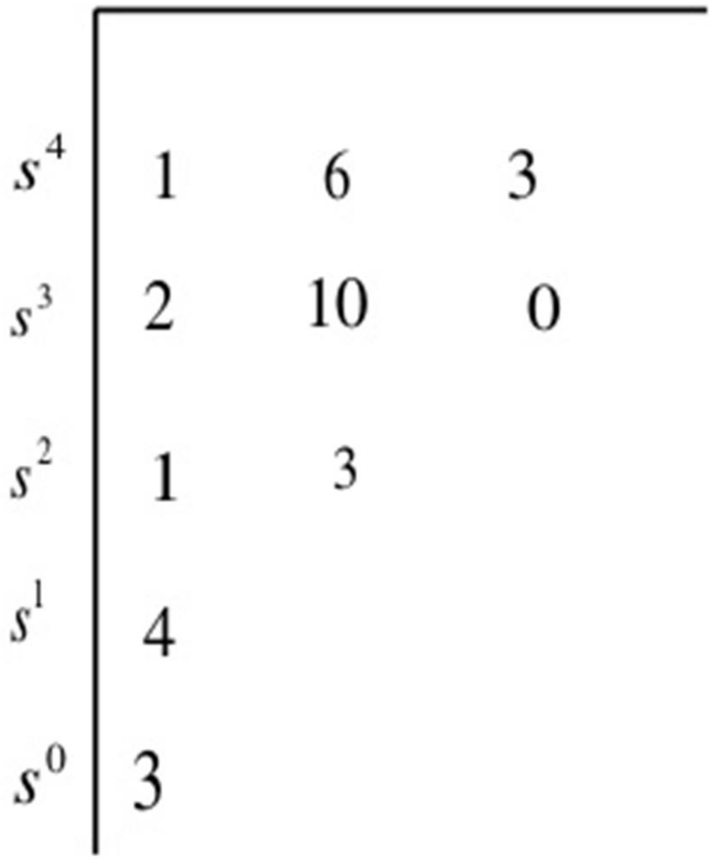
s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	4		
s^0	a_n		

$$a_n = \frac{c_1.\mathrm{b}_2 - b_1 c_2}{c_1}$$

$$a_n = \frac{4 \times 3 - 1 \times 0}{4}$$

$$a_n = 3$$

Example 2 Cont...



s^4	1	6	3
s^3	2	10	0
s^2	1	3	
s^1	4		
s^0	3		

As no sign change in first column; system is stable

Example 3

Comment on stability. $2s^3 + 4s^2 + 4s + 12 = 0$

$$\begin{array}{c|ccc} s^3 & 2 & 4 & 0 \\ s^2 & 4 & 12 & \\ s^1 & b_1 & b_2 & \\ s^0 & a_n & & \end{array}$$

$$b_1 = \frac{a_1.a_2 - a_3.a_0}{a_1}$$

$$b_1 = \frac{(4\times 4) - (2\times 12)}{4}$$

$$b_1 = -2$$

Example 3 Cont...

s^3	2	4	0
s^2	4	12	
s^1	-2	b_2	
s^0	a_n		

$$b_2 = \frac{(4\times 0)-(2\times 0)}{4}$$

$$b_2 = 0$$

Example 3 Cont...

s^3	2	4	0
s^2	4	12	
s^1	-2	0	
s^0	a_n		

$$a_n = \frac{(-2\times 12)-(4\times 0)}{-2}$$

$$a_n = 12$$

Example 3 Cont...

s^3	2	4	0
s^2	4	12	
s^1	-2	0	
s^0	12		

There are two sign changes
+4 to -2 and -2 to +12.
Hence two roots are in RHP
S-plane and system is unstable

Summary

- Transfer Function.
- Transfer Function of RC and RLC circuits.
- Order of system.
- Types of system.