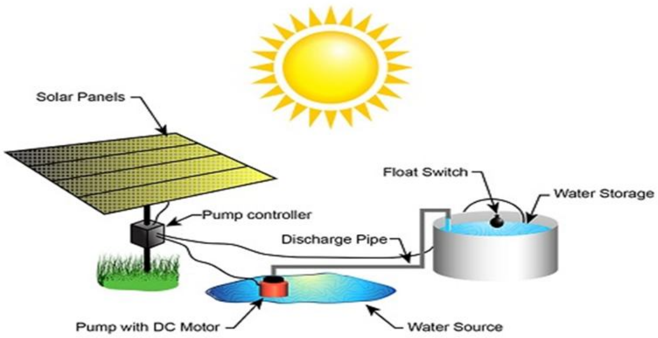


Control System

Time Response Analysis

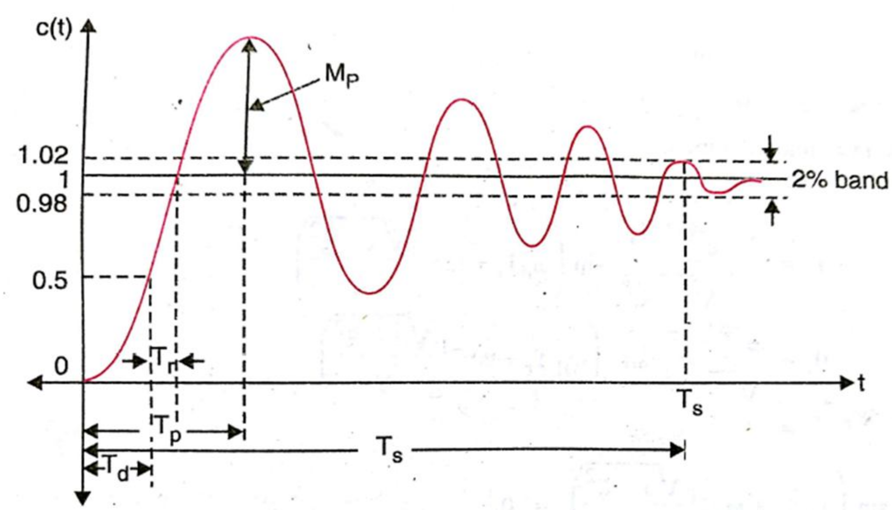
Content

- Review of Last Lecture.
- Time response specification.
- Steady state analysis.
- Steady state error constant.

Learning Objectives

- Know the various time response specifications.
- Steady state error for various signals.
- Relation between steady state error and type of system.

Time Response Specifications



Time Response Specifications

Delay Time (t_d):

- It is time required for the response to reach 50% of the final value in the first attempt.

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

Peak Time (t_p):

- It is time required for the response of system to reach to first peak.

$$T_p = \frac{\pi}{\omega_d}$$

Time Response Specifications

Rise Time (t_r):

- It is time required for the response to rise from 10% to 90% of the final value for overdamped system.
- It is 0 to 100% for under damped systems.

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where, $$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

and $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Time Response Specifications

Peak Overshoot (M_p):

- Maximum overshoot is the maximum peak value of the response curve measured from unity. It is therefore largest error between input and output during transient period.

$$\%M_p = e^{-\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\}} \times 100$$

Settling Time (t_s):

- It is time required for the response curve to reach and stay within a specified percentage (usually 2% or 5%) of the final value.

$$T_s = 4T = \frac{4}{\xi\omega_n}$$

Example 1

A unity feedback system has

$$G(s) = \frac{16}{s(s+5)}$$

If a step input is given calculate

1. Damping Ratio
2. Overshoot
3. Settling Time

Solution: $G(s) = \frac{16}{s(s+5)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{16}{s(s+5)}}{1+\frac{16}{s(s+5)}} = \frac{16}{s^2+5s+16}$$

Example 1 Cont..

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{16}{s^2 + 5s + 16}$$

Compare denominators of both

Natural Frequency;

$$\omega_n^2 = 16 \qquad \therefore \omega_n = 4 \quad rad/\sec$$

Damping Ratio;

$$2\xi\omega_n s = 5s \qquad \therefore \xi = \frac{5}{2\times\omega_n} = \frac{5}{2\times 4} = 0.625$$

Settling Time;

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.625)\times(4)} = 1.6 \text{ sec}$$

Example 1 Cont..

Overshoot

$$\%M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\%M_p = e^{-\left\{\frac{(0.625)\pi}{\sqrt{1-(0.625)^2}}\right\}} \times 100$$

$$\%M_p = 8.08\%$$

Example 2

A unity feedback system has

$$G(s) = \frac{100}{s(s+5)}$$

If it is subjected to unit step input determine;

1. Damped frequency of oscillations
2. Time for first overshoot
3. Settling Time
4. Maximum Peak Overshoot

Example 2 Cont..

Solution: $G(s) = \frac{100}{s(s+5)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{100}{s(s+5)}}{1+\frac{100}{s(s+5)}} = \frac{100}{s^2 + 5s + 100}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{100}{s^2 + 5s + 100}$$

Compare denominators of both

Natural Frequency;

$$\omega_n^2 = 100 \qquad \therefore \omega_n = 10 \quad rad/\sec$$

Example 2 Cont..

Damping Ratio;

$$2\xi\omega_n s = 5s \qquad \therefore \xi = \frac{5}{2\times\omega_n} = \frac{5}{2\times 10} = 0.25$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 10\sqrt{1-(0.25)^2} = 9.68\ rad/\sec$$

Time for first overshoot (Peak Time);

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{9.68} = 0.324\ \sec$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.25)\times(10)} = 1.6\ \sec$$

Example 2 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\}} \times 100$$

$$\%M_p = e^{-\{\frac{(0.25)\pi}{\sqrt{1-(0.25)^2}}\}} \times 100$$

$$\%M_p = 44.48\%$$

Example 3

For the unity feedback control system having open loop transfer function

$$G(s) = \frac{10}{s(s+4)}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 3 Cont..

Solution: $G(s) = \frac{10}{s(s+4)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{10}{s(s+4)}}{1+\frac{10}{s(s+4)}} = \frac{10}{s^2+4s+10}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{10}{s^2+4s+10}$$

Compare denominators of both

Natural Frequency;

$$\omega_n^2 = 10 \qquad \therefore \omega_n = 3.16 \ \ rad/\sec$$

Example 3 Cont..

Damping Ratio;

$$2\xi\omega_n s = 4s \qquad \therefore \xi = \frac{4}{2\times\omega_n} = \frac{4}{2\times 3.16} = 0.633$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 3.16\sqrt{1-(0.633)^2} = 2.44\ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.633)}{3.16} = 0.457\ \text{sec}$$

Example 3 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.633)^2}}{(0.633)} = 0.885\ \text{rad}$$

$$T_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-0.885}{(0.244)} = 0.92\ \text{sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.44} = 1.273\ \text{sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.633)\times(3.16)} = 1.997\ \text{sec}$$

Example 3 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\%M_p = e^{-\left\{\frac{(0.633)\pi}{\sqrt{1-(0.633)^2}}\right\}} \times 100$$

$$\%M_p = 7.66\%$$

Example 4

A second order servo system has a unity feedback,

$$G(s) = \frac{500}{s(s+15)}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 4 Cont..

Solution: $G(s) = \frac{500}{s(s+15)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{500}{s(s+15)}}{1+\frac{500}{s(s+15)}} = \frac{500}{s^2+15s+500}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{500}{s^2+15s+500}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 500 \qquad \therefore \omega_n = 22.36 \; rad/\sec$$

Example 4 Cont..

Damping Ratio;

$$2\xi\omega_n s = 15s \qquad \therefore \xi = \frac{15}{2\times\omega_n} = \frac{15}{2\times 22.36} = 0.335$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 22.36\sqrt{1-(0.335)^2} = 21.06 \; rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.335)}{22.36} = 0.055 \sec$$

Example 4 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.335)^2}}{(0.335)} = 1.229 \text{ rad}$$

$$T_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-1.229}{(21.06)} = 0.091 \text{ sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{21.06} = 32.73 \text{ sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.335)\times(22.36)} = 0.534 \text{ sec}$$

Example 4 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\}} \times 100$$

$$\%M_p = e^{-\{\frac{(0.335)\pi}{\sqrt{1-(0.335)^2}}\}} \times 100$$

$$\%M_p = 32.75\%$$

Example 5

The open loop transfer function of a unity feedback system is,

$$G(s) = \frac{4}{s(s+1)}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 5 Cont..

Solution: $G(s) = \frac{4}{s(s+1)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{4}{s(s+1)}}{1+\frac{4}{s(s+1)}} = \frac{4}{s^2+s+4}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{4}{s^2+s+4}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 4 \qquad \therefore \omega_n = 2 \quad rad/\sec$$

Example 5 Cont..

Damping Ratio;

$$2\xi\omega_n s = s \qquad \therefore \xi = \frac{1}{2\times\omega_n} = \frac{1}{2\times 2} = 0.25$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 2\sqrt{1-(0.25)^2} = 1.936\ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.25)}{2} = 0.587\ \sec$$

Example 5 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.25)^2}}{(0.25)} = 1.310\ \text{rad}$$

$$T_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-1.310}{(1.936)} = 0.945\ \sec$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1.936} = 1.622\ \sec$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.25)\times(2)} = 8\ \sec$$

Example 5 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\%M_p = e^{-\left\{\frac{(0.25)\pi}{\sqrt{1-(0.25)^2}}\right\}} \times 100$$

$$\%M_p = 43.26\%$$

Example 6

The open loop transfer function of a unity feedback system is,

$$G(s) = \frac{25}{s(s+5)}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 6 Cont..

Solution: $G(s) = \frac{25}{s(s+5)}$ $H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{25}{s(s+5)}}{1+\frac{25}{s(s+5)}} = \frac{25}{s^2+5s+25}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{25}{s^2+5s+25}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 25 \qquad \therefore \omega_n = 5 \ rad/\sec$$

Example 6 Cont..

Damping Ratio;

$$2\xi\omega_n s = 5s \qquad \therefore \xi = \frac{5}{2\times\omega_n} = \frac{5}{2\times 5} = 0.5$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 5\sqrt{1-(0.5)^2} = 4.33 \ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.5)}{5} = 0.27 \ \sec$$

Example 6 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}[\frac{\sqrt{1-(0.5)^2}}{(0.5)}] = 1.24 \text{ rad}$$

$$T_r = \frac{\pi - \beta}{\omega_d} = \frac{\pi - 1.04}{(4.330)} = 0.485 \text{ sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{4.330} = 0.725 \text{ sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.5)\times(5)} = 1.6 \text{ sec}$$

Example 6 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\}} \times 100$$

$$\%M_p = e^{-\{\frac{(0.5)\pi}{\sqrt{1-(0.5)^2}}\}} \times 100$$

$$\%M_p = 16.30\%$$

Example 7

The closed loop transfer function of a unity feedback system is,

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 4s + 5}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 7 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 4s + 5}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{10}{s^2 + 4s + 5}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 5 \qquad \therefore \omega_n = 2.23 \quad rad/\sec$$

Example 7 Cont..

Damping Ratio;

$$2\xi\omega_n s = 4s \qquad \therefore \xi = \frac{4}{2\times\omega_n} = \frac{4}{2\times 2.23} = 0.896$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 2.23\sqrt{1-(0.896)^2} = 0.99 \ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.896)}{2.23} = 0.72 \text{ sec}$$

Example 7 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}[\frac{\sqrt{1-(0.896)^2}}{(0.896)}] = 0.46 \text{ rad}$$

$$T_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-0.46}{(0.99)} = 2.70 \text{ sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{0.99} = 2.1515 \text{ sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.896)\times(2.23)} = 2.00 \text{ sec}$$

Example 7 Cont..

Maximum Peak Overshoot

$$\% M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\% M_p = e^{-\left\{\frac{(0.896)\pi}{\sqrt{1-(0.896)^2}}\right\}} \times 100$$

$$\% M_p = 0.17\%$$

Example 8

The closed loop transfer function of a unity feedback system is,

$$\frac{C(s)}{R(s)} = \frac{100}{s^2 + 15s + 100}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 8 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{100}{s^2 + 15s + 100}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{100}{s^2 + 15s + 100}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 100 \qquad \therefore \omega_n = 10 \quad rad/\sec$$

Example 8 Cont..

Damping Ratio;

$$2\xi\omega_n s = 15s \qquad \therefore \xi = \frac{15}{2 \times \omega_n} = \frac{15}{2 \times 10} = 0.75$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 10\sqrt{1-(0.75)^2} = 6.61 \ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.75)}{10} = 0.135 \text{ sec}$$

Example 8 Cont..

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}[\frac{\sqrt{1-(0.75)^2}}{(0.75)}] = 0.722 \text{ rad}$$

$$T_r = \frac{\pi - \beta}{\omega_d} = \frac{\pi - 0.722}{(6.61)} = 0.365 \text{ sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{6.61} = 0.474 \text{ sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.75)\times(10)} = 0.533 \text{ sec}$$

Example 8 Cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\}} \times 100$$

$$\%M_p = e^{-\{\frac{(0.75)\pi}{\sqrt{1-(0.75)^2}}\}} \times 100$$

$$\%M_p = 2.83\%$$

Example 9

The closed loop transfer functions of certain second order unity feedback control system is,

$$\frac{C(s)}{R(s)} = \frac{8}{s^2 + 3s + 8}$$

Determine the type of damping in the system.

Example 9 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{8}{s^2 + 3s + 8}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{8}{s^2 + 3s + 8}$$

By comparing denominators of both

Natural Frequency; $\omega_n^2 = 8$ $\quad \therefore \omega_n = 2.82 \quad rad/\sec$

Damping Ratio; $2\xi\omega_n s = 3s$ $\quad \therefore \xi = \frac{3}{2 \times \omega_n} = \frac{3}{2 \times 2.82} = 0.53$

Since $\xi < 1$, it is an underdamped system.

Example 10

The closed loop transfer functions of certain second order unity feedback control system is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 4s + 2}$$

Determine the type of damping in the system.

Example 10 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 4s + 2}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{2}{s^2 + 4s + 2}$$

By comparing denominators of both

Natural Frequency; $\omega_n^2 = 2$ $\quad \therefore \omega_n = 1.414 \ \ rad/\sec$

Damping Ratio; $2\xi\omega_n s = 4s$ $\quad \therefore \xi = \frac{4}{2\times\omega_n} = \frac{4}{2\times 1.414} = 1.41$

Since $\xi > 1$, it is an overdamped system.

Example 11

The closed loop transfer functions of certain second order unity feedback control system is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 2s + 1}$$

Determine the type of damping in the system.

Example 11 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 2s + 1}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{2}{s^2 + 2s + 1}$$

By comparing denominators of both

Natural Frequency; $\omega_n^2 = 1 \quad \therefore \omega_n = 1 \ rad/\sec$

Damping Ratio; $2\xi\omega_n s = 2s \quad \therefore \xi = \frac{2}{2 \times \omega_n} = \frac{2}{2 \times 1} = 1$

Since $\xi = 1$, it is an critically damped system.

Example 12

The closed loop transfer functions of certain second order unity feedback control system is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 4}$$

Determine the type of damping in the system.

Example 12 Cont..

Solution:

The given closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{2}{s^2 + 4}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{2}{s^2 + 4}$$

By comparing denominators of both

Natural Frequency; $\omega_n^2 = 4 \quad \therefore \omega_n = 2 \ \ rad/\sec$

Damping Ratio; $2\xi\omega_n s = 0 \quad \therefore \xi = 0$

Since $\xi = 0$, it is an undamped system.

Type of System

The open loop transfer function of unity feedback system can be written in two standard forms: the time constant form and the pole-zero form.

$$G(s) = \frac{K(s+z1)(s+z2)\ldots\ldots\ldots\ldots\ldots}{s^n(s+p1)(s+p2)\ldots\ldots\ldots\ldots\ldots} \quad \text{(Pole-zero form)}$$

$$G(s) = \frac{K(1+Tz1s)(1+Tz2s)\ldots\ldots\ldots\ldots\ldots}{s^n(1+Tp1s)(1+Tp2s)\ldots\ldots\ldots\ldots\ldots} \quad \text{(Time constant form)}$$

Type-0 (Zero) System

Definition: A control system with no integration in the open loop transfer function and no pole of transfer function G(s) at the origin of s-plane is designated as "**Type-0** " system.

$$G(s) = \frac{K(1+Tz1s)(1+Tz2s)\ldots\ldots\ldots\ldots\ldots}{(1+Tp1s)(1+Tp2s)\ldots\ldots\ldots\ldots\ldots} \quad \text{(Standard form)}$$

An amplifier type control system is a practical example of Type-0 system

Type-1 (One) System

Definition: A control system with one integration in the open loop transfer function and one pole of transfer function G(s) at the origin of s-plane is designated as "**Type-1** " system.

$$G(s) = \frac{K(1+T\,z1s)(1+T\,z\,2s)..................}{s(1+T\,p\,1s)(1+T\,p\,2s)................}$$ (Standard form)

An pneumatic type control system is a practical example of Type-1 system

Type-2 (Two) System

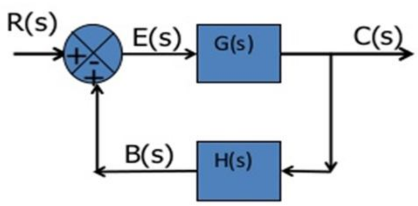
Definition: A control system with two integration in the open loop transfer function and two pole of transfer function G(s) at the origin of s-plane is designated as "**Type-2** " system.

$$G(s) = \frac{K(1+T\,z1s)(1+T\,z\,2s)..................}{s^2(1+T\,p1s)(1+T\,p\,2s)................}$$ (Standard form)

A mechanical displacement system is a practical example of Type-2 system

Derivation of steady state error

The steady state response is important to judge the accuracy of the output. The difference between the steady state response and desired reference gives the steady state error.



For given figure,

$$E(s) = R(s) - B(s)$$

But

$$B(s) = C(s).H(s)$$

$$E(s) = R(s) - C(s).H(s)$$

But

$$C(s) = G(s).E(s)$$

$$E(s) = R(s) - G(s).E(s).H(s)$$

$$R(s) = E(s) + G(s).E(s).H(s)$$

$$R(s) = E(s)\{1 + G(s).H(s)\}$$

$$E(s) = \frac{R(s)}{1 \pm G(s).H(s)}$$

Derivation of steady state error

In time domain,

$$e(t) = L^{-1}\, E(s)$$

and is the expression of error valid for all time. Steady state error is defined as,

$$e_{ss}(t) = \lim_{t \to \infty} e(t)$$

From the final value theorem in Laplace transform,

$$e_{ss}(t) = \lim_{s \to 0} sE(s)$$

Steady state error,

$$e_{ss}(t) = \lim_{s \to 0} sE(s)$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\,H(s)}$$

Steady state error & Standard test signals

Steady state error for step input:

A step input of magnitude A is applied,

r(t) = A. u(t)   t>0
= 0   t<0

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{A\} = \frac{A}{s}$$

Steady state error,

$$e_{ss}(t) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Steady state error & Standard test signals

Steady state error for step input:

$$e_{ss}(t) = \lim_{s \to 0} \frac{s\frac{A}{s}}{1 + G(s)H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{A}{1 + G(s)H(s)}$$

$$e_{ss}(t) = \frac{A}{1 + \lim_{s \to 0} G(s)H(s)}$$

$$e_{ss}(t) = \frac{A}{1 + K_p}$$

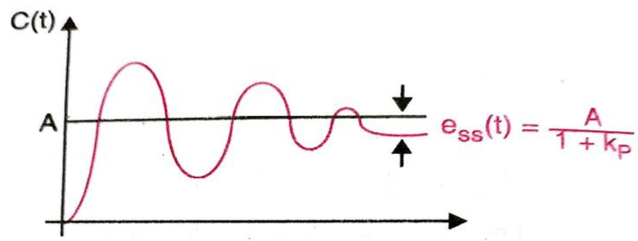
Steady state error & Standard test signals

Steady state error for step input:

The position error constant Kp of a system is defined as,

$$K_p = \lim_{s \to 0} G(s).H(s)$$

When a step input of magnitude A is given, in response to this gives $e_{ss}(t) = \frac{A}{1+K_p}$ steady state error



Steady state error & Standard test signals

Steady state error for ramp input:

A ramp input of slope A is applied,

$$r(t) = A.t \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{At\} = \frac{A}{s^2}$$

Steady state error,

$$e_{ss}(t) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}$$

Steady state error & Standard test signals

Steady state error for ramp input:

$$e_{ss}(t) = \lim_{s \to 0} \frac{s \frac{A}{s^2}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{\frac{A}{s}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{A}{s + sG(s)\,H(s)}$$

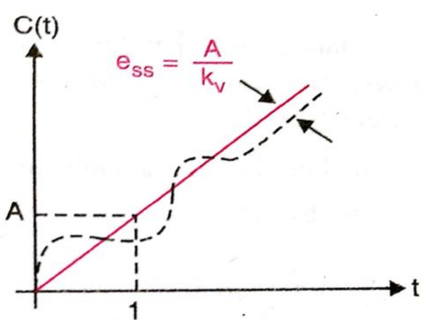
$$e_{ss}(t) = \frac{A}{0 + \lim_{s \to 0} sG(s)\,H(s)} \qquad e_{ss}(t) = \frac{A}{K_v}$$

Steady state error & Standard test signals

Steady state error for ramp input:

The velocity error constant Kv of a system is defined as,

$$K_v = \lim_{s \to 0} sG(s).H(s)$$



Steady state error & Standard test signals

Steady state error for parabolic input:

A parabolic input of slope coefficient A/2 is applied,

$$r(t) = \frac{At^2}{2} \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{\frac{A}{2}t^2\} = \frac{A}{s^3}$$

Steady state error,

$$e_{ss}(t) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

Steady state error & Standard test signals

Steady state error for parabolic input:

$$e_{ss}(t) = \lim_{s\to 0} \frac{s\frac{A}{s^3}}{1+G(s)H(s)}$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{\frac{A}{s^2}}{1+G(s)H(s)}$$

$$e_{ss}(t) = \lim_{s\to 0} \frac{A}{s^2 + s^2 G(s)H(s)}$$

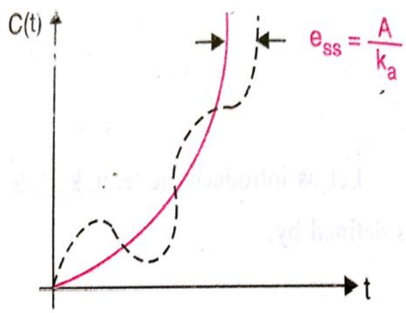
$$e_{ss}(t) = \frac{A}{0+\lim_{s\to 0} s^2 G(s)H(s)} \qquad e_{ss}(t) = \frac{A}{K_a}$$

Steady state error & Standard test signals

Steady state error for parabolic input:

The acceleration error constant Ka of a system is defined as,

$$K_a = \lim_{s \to 0} s^2 G(s).H(s)$$



Steady state error & Standard test signals

Summary:

Sr. No.	Input Signal	Steady State Error	Constant	Constant Expression
1	Step Input	$e_{ss}(t) = \frac{A}{1+K_p}$	Position Error Constant	$K_p = \lim_{s \to 0} G(s).H(s)$
2	Ramp Input	$e_{ss}(t) = \frac{A}{K_v}$	Velocity Error Constant	$K_v = \lim_{s \to 0} sG(s).H(s)$
3	Parabolic Input	$e_{ss}(t) = \frac{A}{K_a}$	Acceleration Error Constant	$K_a = \lim_{s \to 0} s^2 G(s).H(s)$

Relation between steady state error & Type of system

The type of system means the number of poles G(s)H(s) at s=0. Consider the general form,

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s^n (1+T_a s)(1+T_b s)................(1+T_n s)}$$

Here there are n poles at s=0. Hence the type of system is n.

Steady state error for step input & Type 0 system

For type zero system,   n=0

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s\to 0} G(s).H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for step input & Type 0 system

$$K_p = \frac{K(1)(1)...............(1)}{(1)(1)...............(1)}$$

$$K_p = K$$

The steady state error is given by,

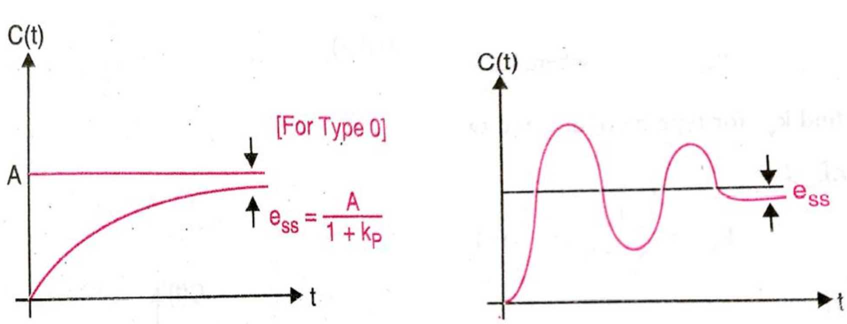
$$e_{ss}(t) = \frac{A}{1+K_p}$$

$$e_{ss}(t) = \frac{A}{1+K}$$

Steady state error for step input & Type 0 system

$$e_{ss}(t) = \frac{A}{1+K}$$

A type zero system has a finite steady state error to a step input ,



Steady state error for step input & Type 1 system

For type one system, n=1

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s\to 0} G(s).H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{0(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for step input & Type 1 system

$$K_p = \frac{K(1)(1)................(1)}{0}$$

$$K_p = \infty$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{1+K_p}$$

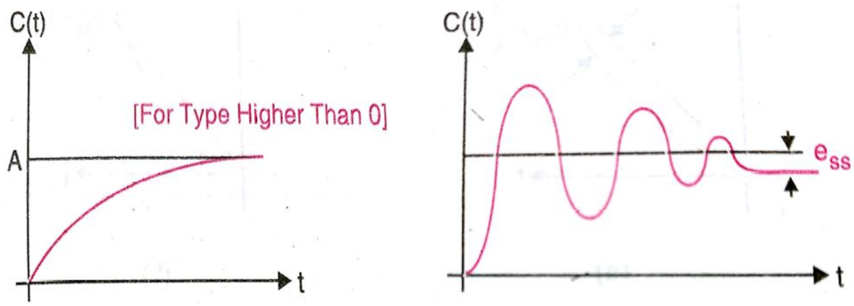
$$e_{ss}(t) = \frac{A}{1+\infty}$$

$$e_{ss}(t) = 0$$

Steady state error for step input & Type 1 system

$$e_{ss}(t) = 0$$

A type one system has a zero steady state error to a step input ,



Steady state error for step input & Type 2 system

For type two system, n=2

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s^2(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s\to 0} G(s).H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s^2(1+T_a s)(1+T_b s)................(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{0(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for step input & Type 2 system

$$K_p = \frac{K(1)(1)\ldots\ldots\ldots\ldots\ldots(1)}{0}$$

$$K_p = \infty$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{1 + K_p}$$

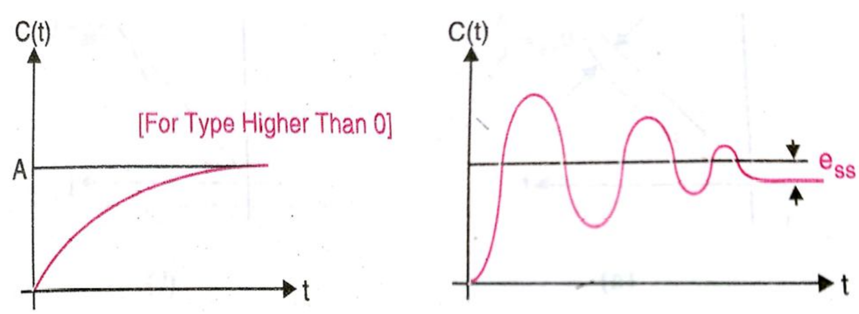
$$e_{ss}(t) = \frac{A}{1 + \infty}$$

$$e_{ss}(t) = 0$$

Steady state error for step input & Type 2 system

$$e_{ss}(t) = 0$$

A type two system has a zero steady state error to a step input ,



Steady state error for ramp input & Type 0 system

For type zero system, n=0

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}$$

The velocity error constant is given by,

$$K_v = \lim_{s\to 0} sG(s).H(s)$$

$$K_v = \lim_{s\to 0} s\{\frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}\}$$

$$K_v = 0\times\{\frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}\}$$

Steady state error for ramp input & Type 0 system

$$K_v = 0$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_v}$$

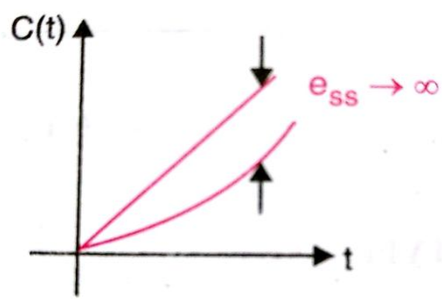
$$e_{ss}(t) = \frac{A}{0}$$

$$e_{ss}(t) = \infty$$

Steady state error for ramp input & Type 0 system

$$e_{ss}(t) = \infty$$

The error increase continuously hence type zero system fails to track a ramp input successfully.



Steady state error for ramp input & Type 1 system

For type one system, n=1

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The velocity error constant is given by,

$$K_v = \lim_{s \to 0} sG(s).H(s)$$

$$K_v = \lim_{s \to 0} s\{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

$$K_v = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for ramp input & Type 1 system

$$K_v = K$$

The steady state error is given by,

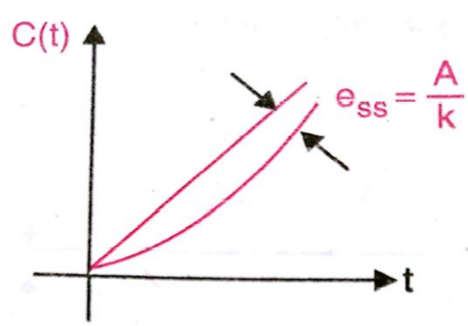
$$e_{ss}(t) = \frac{A}{K_v}$$

$$e_{ss}(t) = \frac{A}{K}$$

Steady state error for ramp input & Type 1 system

$$e_{ss}(t) = \frac{A}{K}$$

This indicates finite steady state error for type one system for ramp input



Steady state error for ramp input & Type 2 system

For type two system, n=2

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{s^2(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}$$

The velocity error constant is given by,

$$K_v = \lim_{s\to 0} sG(s).H(s)$$

$$K_v = \lim_{s\to 0} s\{\frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{s^2(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}\}$$

$$K_v = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots(1+T_m s)}{s(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots(1+T_n s)}$$

Steady state error for ramp input & Type 2 system

$$K_v = \infty$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_v}$$

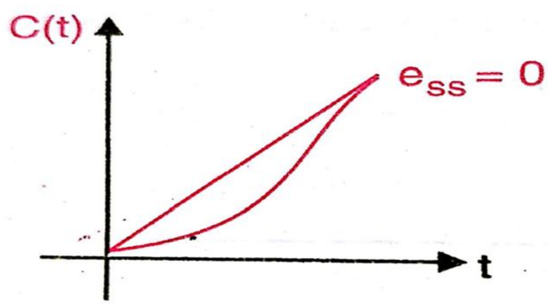
$$e_{ss}(t) = \frac{A}{\infty}$$

$$e_{ss}(t) = 0$$

Steady state error for ramp input & Type 2 system

$$e_{ss}(t) = 0$$

There is no steady state error for a ramp input for type two system



Steady state error for parabolic input & Type 0 system

For type zero system, n=0

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The acceleration error constant is given by,

$$K_a = \lim_{s\to 0} s^2 G(s).H(s)$$

$$K_a = \lim_{s\to 0} s^2 \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

$$K_a = 0 \times \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

Steady state error for parabolic input & Type 0 system

$$K_a = 0$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_a}$$

$$e_{ss}(t) = \frac{A}{0}$$

$$e_{ss}(t) = \infty$$

There is infinite steady state error indicating failure to track a parabolic input in type zero system

Steady state error for parabolic input & Type 1 system

For type one system, n=1

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The acceleration error constant is given by,

$$K_a = \lim_{s \to 0} s^2 G(s).H(s)$$

$$K_a = \lim_{s \to 0} s^2 \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

$$K_a = 0 \times \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

Steady state error for parabolic input & Type 1 system

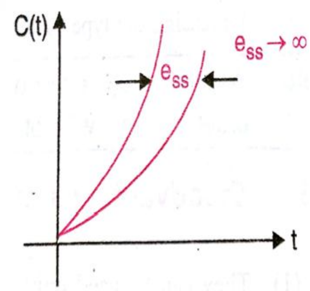
$$K_a = 0$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_a}$$

$$e_{ss}(t) = \frac{A}{0}$$

$$e_{ss}(t) = \infty$$



There is infinite steady state error indicating failure to track a parabolic input in type one system

Steady state error for parabolic input & Type 2 system

For type two system, n=2

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s^2(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The acceleration error constant is given by,

$$K_a = \lim_{s \to 0} s^2 G(s).H(s)$$

$$K_a = \lim_{s \to 0} s^2 \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s^2(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

$$K_a = \{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

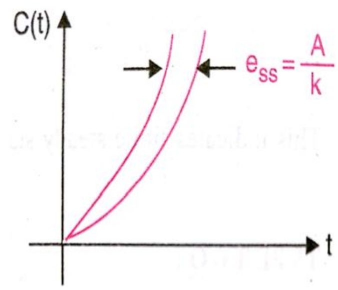
Steady state error for parabolic input & Type 2 system

$$K_a = K$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_a}$$

$$e_{ss}(t) = \frac{A}{K}$$



There is finite steady state error for type two system

Relation between steady state error & Type of system

Summary:

Sr. No.	Type of System	Step Input		Ramp Input		Parabolic Input	
		K_p	e_{ss}	K_v	e_{ss}	K_a	e_{ss}
1	Zero	K	$\frac{A}{1+K}$	0	∞	0	∞
2	One	∞	0	K	$\frac{A}{K}$	0	∞
3	Two	∞	0	∞	0	K	$\frac{A}{K}$

Example 13

The control system having unity feedback has,

$$G(s) = \frac{20}{s(1+4s)(1+s)}$$

Determine

1. Different static error coefficients.
2. Steady State error if input $r(t) = 2 + 4t + \frac{t^2}{2}$

Example 13 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{20}{s(1+4s)(1+s)}$$

$$K_p = \frac{20}{0(1+4s)(1+s)}$$

$$K_p = \infty$$

Example 13 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{20}{s(1+4s)(1+s)}]$$

$$K_v = \frac{20}{(1+4s)(1+s)}$$

$$K_v = 20$$

Example 13 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s \to 0} s^2[\frac{20}{s(1+4s)(1+s)}]$$

$$K_a = 0[\frac{20}{s(1+4s)(1+s)}]$$

$$K_a = 0$$

Example 13 Cont..

Steady state error, for $r(t) = 2 + 4t + \frac{t^2}{2}$

$$R(s) = L\{r(t)\} = \frac{2}{s} + \frac{4}{s^2} + \frac{1}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s[\frac{2}{s} + \frac{4}{s^2} + \frac{1}{s^3}]}{1 + \frac{20}{s(1+4s)(1+s)}}$$

$$e_{ss} = \infty$$

Example 14

The control system having unity feedback has,

$$G(s) = \frac{50(s+5)}{s^2}$$

Determine

1. Different static error coefficients.

Example 14 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{50(s+5)}{s^2}$$

$$K_p = \frac{50(s+5)}{(0)^2}$$

$$K_p = \infty$$

Example 14 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{50(s+5)}{s^2}]$$

$$K_v = \lim_{s \to 0} \frac{50(s+5)}{s}$$

$$K_v = \frac{50(s+5)}{0}$$

$$K_v = \infty$$

Example 14 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$K_a = \lim_{s \to 0} s^2 [\frac{50(s+5)}{s^2}]$$

$$K_a = \lim_{s \to 0} 50(s+5)$$

$$K_a = 250$$

Example 15

The control system having,

$$G(s) = \frac{20}{s(s^2 + 2s + 5)} \qquad H(s) = \frac{10}{(s+4)}$$

Determine

1. Different static error coefficients.
2. Steady State error if input $r(t) = 5 + 10t + \frac{t^2}{2}$

Example 15 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{20}{s(s^2+2s+5)} \times \frac{10}{(s+4)}$$

$$K_p = \infty$$

Example 15 Cont..

Velocity error constant,

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$K_v = \lim_{s\to 0} s[\frac{20}{s(s^2+2s+5)} \times \frac{10}{(s+4)}]$$

$$K_v = \frac{200}{20}$$

$$K_v = 10$$

Example 15 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s \to 0} s^2 [\frac{20}{s(s^2 + 2s + 5)} \times \frac{10}{(s+4)}]$$

$$K_a = 0[\frac{20}{(s^2 + 2s + 5)} \times \frac{10}{(s+4)}]$$

$$K_a = 0$$

Example 15 Cont..

Steady state error, for $r(t) = 5 + 10t + \frac{t^2}{2}$

$$R(s) = L\{r(t)\} = \frac{5}{s} + \frac{10}{s^2} + \frac{1}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s[\frac{5}{s} + \frac{10}{s^2} + \frac{1}{s^3}]}{1 + [\frac{20}{s(s^2 + 2s + 5)} \times \frac{10}{(s+4)}]}$$

$$e_{ss} = \infty$$

Example 16

The control system having unity feedback has,

$$G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$$

Determine

1. Different static error coefficients.

2. Steady State error if input $r(t) = 40 + 2t + 5t^2$

Example 16 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{20(1+s)}{s^2(2+s)(4+s)}$$

$$K_p = \frac{20(1+s)}{0^2(2+s)(4+s)}$$

$$K_p = \infty$$

Example 16 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{20(1+s)}{s^2(2+s)(4+s)}]$$

$$K_v = \lim_{s \to 0}[\frac{20(1+s)}{s(2+s)(4+s)}]$$

$$K_v = \infty$$

Example 16 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2G(s)H(s)$$

$$K_a = \lim_{s \to 0} s^2[\frac{20(1+s)}{s^2(2+s)(4+s)}]$$

$$K_a = \lim_{s \to 0}[\frac{20(1+s)}{(2+s)(4+s)}]$$

$$K_a = \frac{5}{2}$$

Example 16 Cont..

Steady state error, for $r(t) = 40 + 2t + 5t^2$

$$R(s) = L\{r(t)\} = \frac{40}{s} + \frac{2}{s^2} + \frac{10}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s[\frac{40}{s} + \frac{2}{s^2} + \frac{10}{s^3}]}{1 + \frac{20(1+s)}{s^2(2+s)(4+s)}}$$

$$e_{ss} = 4$$

Example 17

The control system having unity feedback has,

$$G(s) = \frac{20(s+1)}{s(s+2)(s^2+2s+2)}$$

Determine

1. Different static error coefficients.
2. Steady State error if input $r(t) = 10 + 20t$

Example 17 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{20(s+1)}{s(s+2)(s^2+2s+2)}$$

$$K_p = \frac{20(s+1)}{0(s+2)(s^2+2s+2)}$$

$$K_p = \infty$$

Example 17 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{20(s+1)}{s(s+2)(s^2+2s+2)}]$$

$$K_v = \lim_{s \to 0} [\frac{20(s+1)}{(s+2)(s^2+2s+2)}]$$

$$K_v = 5$$

Example 17 Cont..

Acceleration error constant,

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s\to 0} s^2 [\frac{20(s+1)}{s(s+2)(s^2+2s+2)}]$$

$$K_a = \lim_{s\to 0} s[\frac{20(s+1)}{(s+2)(s^2+2s+2)}]$$

$$K_a = 0$$

Example 17 Cont..

Steady state error, for $r(t) = 10 + 20t$

$$R(s) = L\{r(t)\} = \frac{10}{s} + \frac{20}{s^2}$$

Steady state error is given by,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s[\frac{10}{s} + \frac{20}{s^2}]}{1 + \frac{20(s+1)}{s(s+2)(s^2+2s+2)}}$$

$$e_{ss} = 4$$

Example 18

The control system having unity feedback has,

$$G(s) = \frac{20(s+4)}{s(s+2)(s^2+2s+2)}$$

Determine

1. Different static error coefficients.
2. Steady State error if input $r(t) = 6t + \frac{3}{2}t^2$

Example 18 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{20(s+4)}{s(s+2)(s^2+2s+2)}$$

$$K_p = \frac{20(s+4)}{0(s+2)(s^2+2s+2)}$$

$$K_p = \infty$$

Example 18 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{20(s+4)}{s(s+2)(s^2+2s+2)}]$$

$$K_v = \lim_{s \to 0}[\frac{20(s+4)}{(s+2)(s^2+2s+2)}]$$

$$K_v = 20$$

Example 18 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s \to 0} s^2[\frac{20(s+4)}{s(s+2)(s^2+2s+2)}]$$

$$K_a = \lim_{s \to 0} s[\frac{20(s+4)}{(s+2)(s^2+2s+2)}]$$

$$K_a = 0$$

Example 18 Cont..

Steady state error, for $r(t) = 6t + \frac{3}{2}t^2$

$$R(s) = L\{r(t)\} = \frac{6}{s^2} + \frac{3}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s[\frac{6}{s^2} + \frac{3}{s^3}]}{1 + \frac{20(s+4)}{s(s+2)(s^2+2s+2)}}$$

$$e_{ss} = \infty$$

Example 19

The open loop transfer function of servo system with unity feedback is,

$$G(s) = \frac{10}{s(0.1s+1)}$$

Determine

1. Different static error coefficients.
2. Steady State error if input $r(t) = a_0 + a_1 t + \frac{a_2}{2}t^2$

Example 19 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{10}{s(0.1s+1)}$$

$$K_p = \frac{10}{0(0.1s+1)}$$

$$K_p = \infty$$

Example 19 Cont..

Velocity error constant,

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$K_v = \lim_{s\to 0} s[\frac{10}{s(0.1s+1)}]$$

$$K_v = \lim_{s\to 0}[\frac{10}{(0.1s+1)}]$$

$$K_v = 10$$

Example 19 Cont..

Acceleration error constant,

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s\to 0} s^2 [\frac{10}{s(0.1s+1)}]$$

$$K_a = \lim_{s\to 0} s[\frac{10}{(0.1s+1)}]$$

$$K_a = 0$$

Example 19 Cont..

Steady state error, for $r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$

$$R(s) = L\{r(t)\} = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s[\frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}]}{1+\frac{10}{s(0.1s+1)}}$$

$$e_{ss} = \infty$$

Example 20

The open loop transfer function of servo system with unity feedback is,

$$G(s) = \frac{10}{s^2(1+s)}$$

Determine

1. Different static error coefficients.

2. Steady State error if input $r(t) = a_0 + a_1 t + a_2 t^2$

Example 20 Cont..

Solution:

Position error constant,

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{10}{s^2(1+s)}$$

$$K_p = \frac{10}{0^2(1+s)}$$

$$K_p = \infty$$

Example 20 Cont..

Velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$K_v = \lim_{s \to 0} s[\frac{10}{s^2(1+s)}]$$

$$K_v = \lim_{s \to 0}[\frac{10}{s(1+s)}]$$

$$K_v = \infty$$

Example 20 Cont..

Acceleration error constant,

$$K_a = \lim_{s \to 0} s^2G(s)H(s)$$

$$K_a = \lim_{s \to 0} s^2[\frac{10}{s^2(1+s)}]$$

$$K_a = \lim_{s \to 0}[\frac{10}{(1+s)}]$$

$$K_a = 10$$

Example 20 Cont..

Steady state error, for $r(t) = a_0 + a_1 t + a_2 t^2$

$$R(s) = L\{r(t)\} = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{2a_2}{s^3}$$

Steady state error is given by,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s[\frac{a_0}{s} + \frac{a_1}{s^2} + \frac{2a_2}{s^3}]}{1 + \frac{10}{s^2(1+s)}}$$

$$e_{ss} = \frac{a_2}{5}$$

Example 21

A unity feedback system is characterized by the open loop transfer function is,

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

Determine steady state errors for unit step input, unit ramp input and unit acceleration input.

Example 21 Cont..

Solution:

Position error constant, $K_p = \lim_{s \to 0} G(s)H(s)$

$$K_p = \lim_{s \to 0} \frac{1}{s(0.5s+1)(0.2s+1)}$$

$$K_p = \infty$$

Steady state error for unit step input is given by,

$$e_{ss}(t) = \frac{1}{1+K_p}$$

$$e_{ss}(t) = \frac{1}{1+\infty}$$

$$e_{ss}(t) = 0$$

Example 21 Cont..

Velocity error constant, $K_v = \lim_{s \to 0} sG(s)H(s)$

$$K_v = \lim_{s \to 0} s[\frac{1}{s(0.5s+1)(0.2s+1)}]$$

$$K_v = 1$$

Steady state error for unit ramp input is given by,

$$e_{ss}(t) = \frac{1}{K_v}$$

$$e_{ss}(t) = \frac{1}{1}$$

$$e_{ss}(t) = 1$$

Example 21 Cont..

Acceleration error constant, $K_a = \lim_{s \to 0} s^2 G(s) H(s)$

$$K_a = \lim_{s \to 0} s^2 [\frac{1}{s(0.5s+1)(0.2s+1)}]$$

$$K_a = 0$$

Steady state error for unit parabolic input is given by,

$$e_{ss}(t) = \frac{1}{K_a}$$

$$e_{ss}(t) = \frac{1}{0}$$

$$e_{ss}(t) = \infty$$

Summary

- Time response specifications
- Steady state error
- Relation between steady state error and type of system.