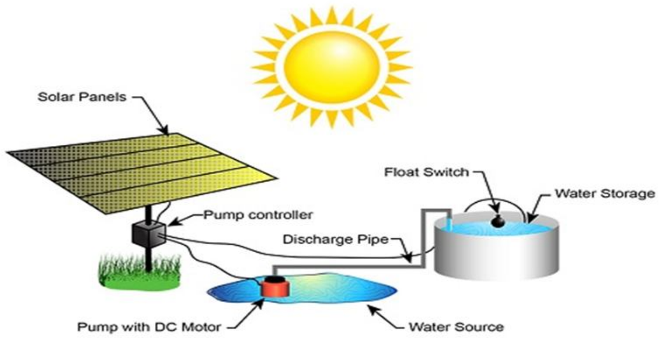


Control System

Time Domain Analysis

Content

- Transient and Steady State Response.
- Significance of standard test inputs.
- Introduction to S-plane representation.
- First order control system.
- Second order control system.
- Effect of Damping.

Learning Objectives

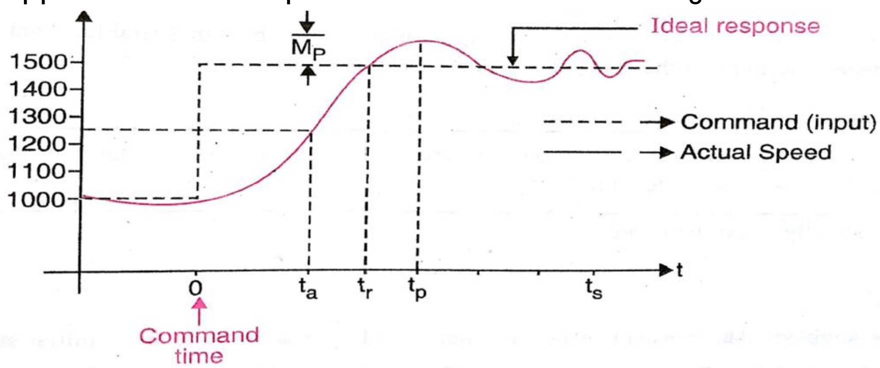
- Understand the basics of transient and steady state response.
- Importance of standard test signal.
- Differentiate between poles & zeros.
- Analyze first & second order control system for step input.
- Understand the effect of damping.

Time Response

- In time domain analysis, time is independent variable. When system is given an excitation (input), there is a response(output).
- Response of system to an applied excitation is called **<u>Time Response</u>** and it is a function of time, denoted by c(t).

Time Response Example

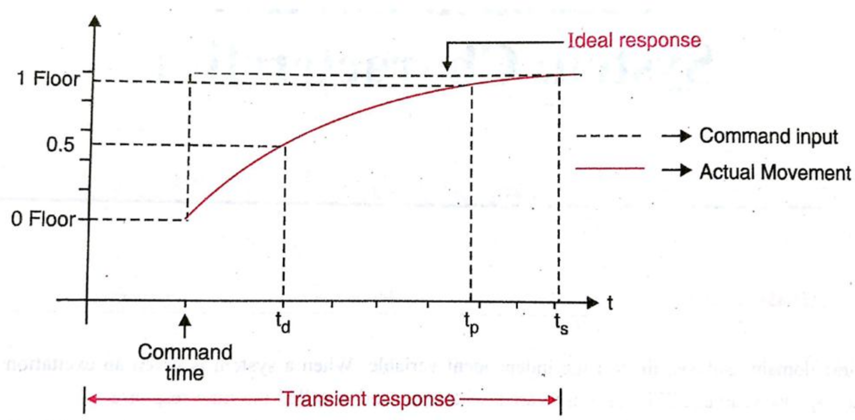
- The response of motor speed when input(command) signal is applied to increase speed of a motor is shown in figure.



- Speed of motor gradually picks up from 1000 rpm and move towards 1500 rpm. It overshoots around that value and again corrects itself and finally settles down at last value.

Transient Response

- Consider an example of a lift moving in upward direction.



Transient Response

- The variation of output response during the time; it takes to reach its final value is called **transient response.**
- Also, part of time response which goes to zero as time becomes very large is called as **Transient Response.**

$$\underset{t\to\infty}{L}\; c(t) = 0$$

- As name suggests transient response remains only for some time from initial state to final state.

Transient Response

- From Transient response we can know;

- When the system begins to respond after input is given.
- How much time it takes to reach output for first time.
- Whether output shoots beyond the desired value and how much.
- Whether the output oscillates about its final value.
- When does it settle to final value.

Steady State Response

- Part of response that remains after transients have died out is called "**Steady State Response**".

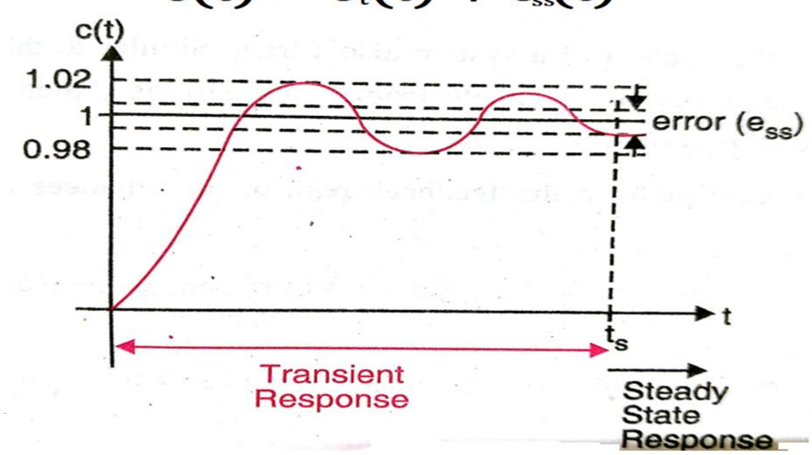
-From the steady state we can know;

- How long it took before steady state was reached.
- Whether there is any error between desired and actual values.
- Whether this error is constant , zero or infinite i.e. unable too track input.

Total Response

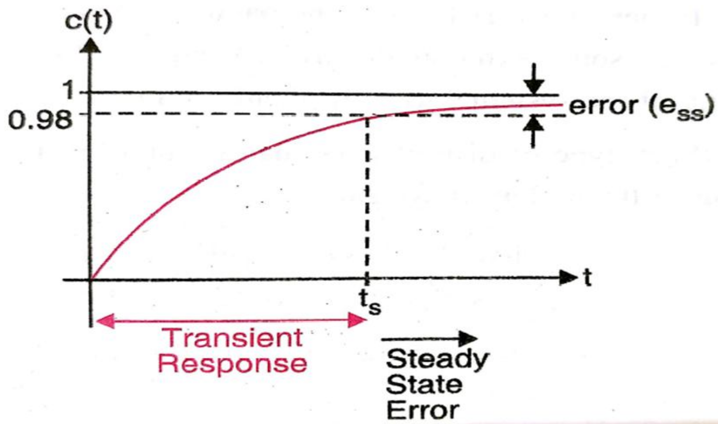
- Total response of system is addition of transient response and steady state response.

$$c(t) = c_t(t) + c_{ss}(t)$$



Steady state error

- Difference between desired output and actual output of system is called as steady state error.



Need of Standard Test Signals

- It is very interesting to know that most control systems do not know what their inputs are going to be.
- System design cannot be done from input point of view as we are unable to know in advance type of input.
- **For example:**

1. When a radar tracks an enemy plane.
2. The terrain, curves on the road.
3. Loading on shearing machine.

Standard Test Signals

- **Thus from such types of inputs we can expect a system in general to get an input which may be:**

1. A sudden change.
2. A momentary shock.
3. A constant velocity.
4. A constant acceleration.

- **Hence these signals form standard test signals. Response to these signals is analyzed. Above inputs are called:**

1. Step input :- Signifies sudden change.
2. Impulse input :- Signifies momentary shock.
3. Ramp input :- Signifies constant velocity.
4. Parabolic input :- Signifies constant acceleration.

Step Input

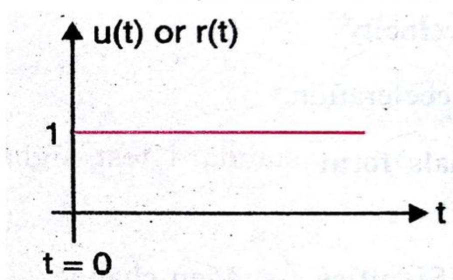
- Unit step input:-

Mathematical Representations

$r(t) = 1.u(t) = 1 \quad t>0$

$\quad = 0 \quad t<0$

Graphical Representations



This signal signifies a sudden change in the reference input r(t) at time t=0

Laplace Representations $\quad L\{u(t)\} = \frac{1}{s}$

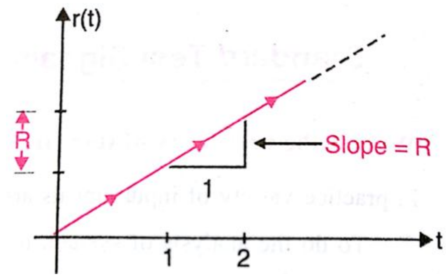
Ramp Input

- **Unit ramp input:-**

Mathematical Representations

r(t) = 1.t   t>0
= 0   t<0

Graphical Representations



Signal have constant velocity i.e. constant change in it's value w.r.t. time

Laplace Representations $L\{1\,t\} = \dfrac{1}{s^2}$

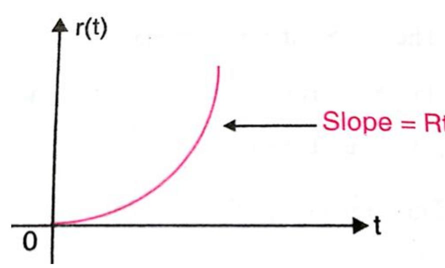
Parabolic Input

Mathematical Representations

r(t) = $\dfrac{Rt^2}{2}$ t>0

= 0   t<0

Graphical Representations



Laplace Representations $L\{\mathrm{Rt}\} = \dfrac{R}{s^3}$

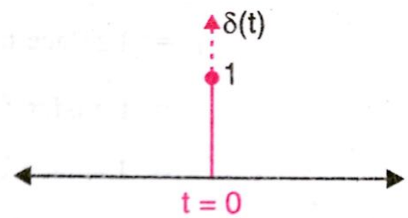
Impulse Input

Mathematical Representations

$r(t) = \delta(t) = 1 \qquad t>0$

$\quad = 0 \qquad t<0$

Graphical Representations



The function has a unit value only for t=0. In practical cases, a pulse whose time approaches zero is taken as an impulse function.

Laplace Representations $\quad L\{\delta(t)\} = 1$

Poles & Zeros of Transfer Function

The transfer function is given by,

$$G(s) = \frac{C(s)}{R(s)}$$

Both C(s) and R(s) are polynomials in s

$$\therefore G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots\ldots\ldots\ldots + b_o}{s^n + a_{n-1} s^{n-1} + \ldots\ldots\ldots\ldots\ldots\ldots + a_n}$$

$$= \frac{K(s-b_1)(s-b_2)(s-b_3)\ldots\ldots\ldots\ldots(s-b_m)}{(s-a_1)(s-a_2)(s-a_3)\ldots\ldots\ldots\ldots(s-a_n)}$$

Where, K= system gain
n= Type of system

Poles of Transfer Function

The values of 's', for which the transfer function magnitude |G(s)| becomes infinite after substitution in the denominator of the system are called as **"Poles"** of transfer function.

Example

Determine the poles of given transfer function.

$$G(s) = \frac{s(s+2)(s+4)}{s(s+3)(s+4)}$$

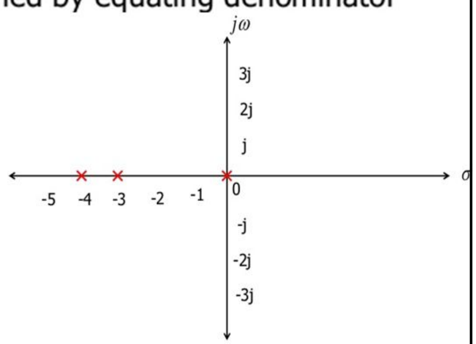
Solution: The poles can be obtained by equating denominator with zero

$$s(s+3)(s+4) = 0$$

$$\therefore s = 0$$

$$\therefore s+3 = 0 \qquad \therefore s = -3$$

$$\therefore s+4 = 0 \qquad \therefore s = -4$$



The poles are s=0, -3, -4

Zeros of Transfer Function

The values of 's', for which the transfer function magnitude |G(s)| becomes zero after substitution in the numerator of the system are called as "**Zeros**" of transfer function.

Example

Determine the zeros of given transfer function.

$$G(s) = \frac{s(s+2)(s+4)}{s(s+3)(s+4)}$$

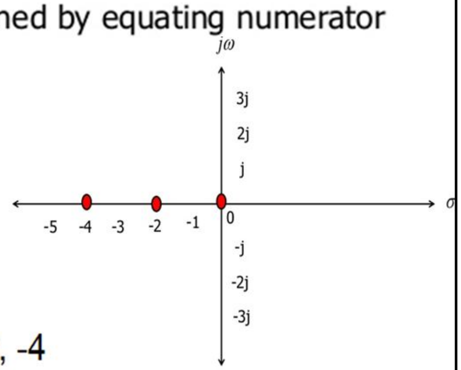
Solution: The zeros can be obtained by equating numerator with zero

$$s(s+2)(s+4) = 0$$

$$\therefore s = 0$$

$$\therefore s+2 = 0 \qquad \therefore s = -2$$

$$\therefore s+4 = 0 \qquad \therefore s = -4$$



The zeros are s=0, -2, -4

Characteristics Equation

Definition: The equation obtained by equating the denominator polynomial of a transfer function to zero is called as the "**Characteristics Equation**"

$$s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots\ldots\ldots\ldots\ldots + a_n$$

Example 1:

For the given transfer function,

$$T.F. = \frac{K(s+6)}{s(s+2)(s+5)(s^2+7s+12)}$$

Find: (i) Poles (ii)Zeros
(iii) Pole-zero Plot (iv) Characteristics Equation

Solution: (i)Poles

The poles can be obtained by equating denominator with zero

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

$$\therefore s = 0$$

$$\therefore s+2 = 0 \qquad \therefore s = -2$$

$$\therefore s+5 = 0 \qquad \therefore s = -5$$

Example 1:

$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$(s^2+7s+12)=(s+3)(s+4)$$

$$\therefore s+3=0 \qquad \therefore s=-3$$

$$\therefore s+4=0 \qquad \therefore s=-4$$

The poles are s=0, -2, -3, -4, -5

(ii) Zeros:

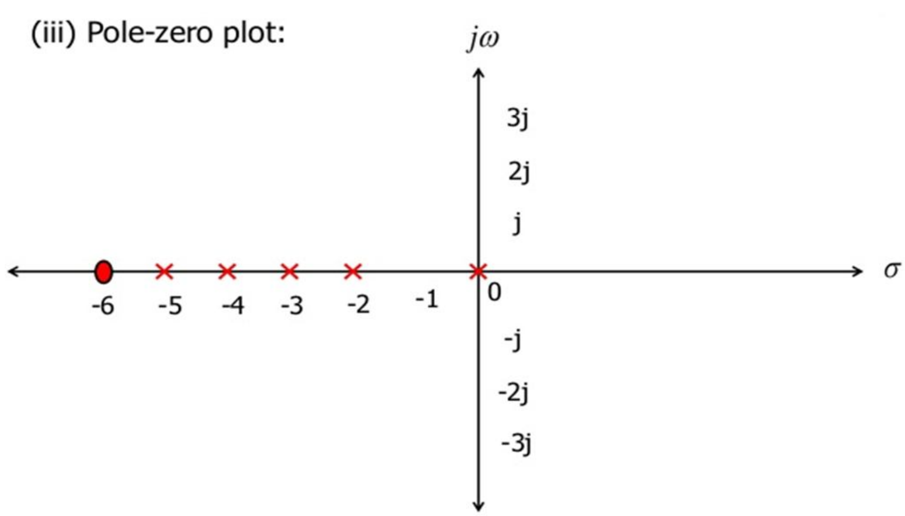
The zeros can be obtained by equating numerator with zero

$$s+6=0 \qquad \therefore s=-6$$

The zeros are s=-6

Example 1:

(iii) Pole-zero plot:



Example 1:

(iv) Characteristics Equation:

$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$s(s^2+7s+10)(s^2+7s+12)=0$$

$$\therefore (s^3+7s^2+10s)(s^2+7s+12)=0$$

$$\therefore s^5+7s^4+12s^3+7s^4+49s^3+84s^2+10s^3+70s^2+120s=0$$

$$\therefore s^5+14s^4+71s^3+154s^2+120s=0$$

Example 2:

For the given transfer function,

$$\frac{C(s)}{R(s)}=\frac{(s+2)}{s(s^2+2s+2)(s^2+7s+12)}$$

Find: (i) Poles (ii)Zeros
(iii) Pole-zero Plot (iv) Characteristics Equation

Solution: (i)Poles

The poles can be obtained by equating denominator with zero

$$\underline{s}(s^2+2s+2)\underline{(s^2+7s+12)}=0$$

$$\therefore s=0$$

$$\therefore s+3=0 \qquad \therefore s=-3$$

$$\therefore s+4=0 \qquad \therefore s=-4$$

Example 2:

$$s(s^2+2s+2)(s^2+7s+12)=0$$

$$roots=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\therefore s=-1+j$$

$$\therefore s=-1-j$$

The poles are s=0, -3, -4, -1+j,-1-j

(ii) Zeros:

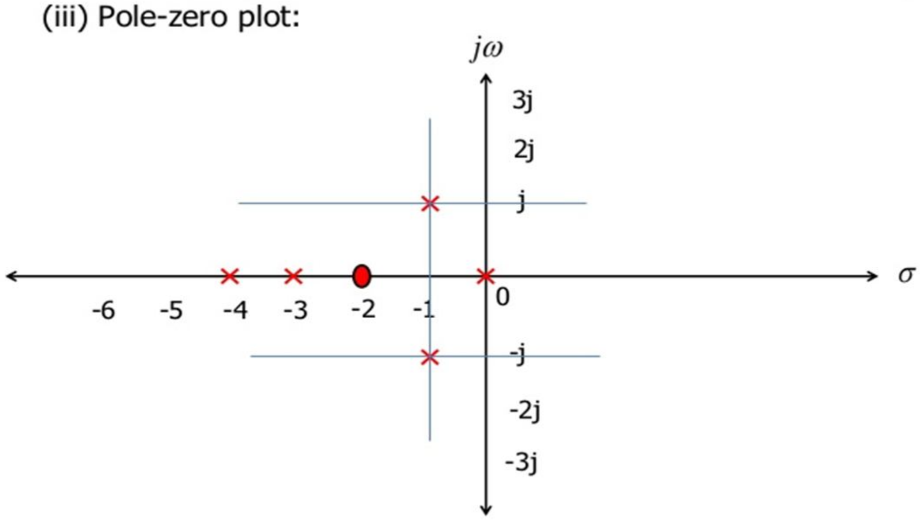
The zeros can be obtained by equating numerator with zero

$s+2=0 \qquad \therefore s=-2$

The zeros are s=-2

Example 2:

(iii) Pole-zero plot:



Example 2:

(iv) Characteristics Equation:

$$s(s^2+2s+2)(s^2+7s+12)=0$$

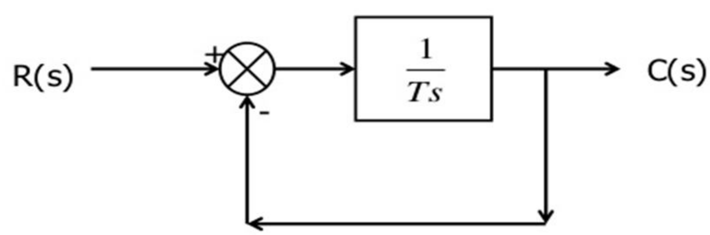
$$\therefore (s^3+2s^2+2s)(s^2+7s+12)=0$$

$$\therefore s^5+7s^4+12s^3+2s^4+14s^3+24s^2+2s^3+14s^2+24s=0$$

$$\therefore s^5+9s^4+28s^3+38s^2+24s=0$$

Analysis of First Order System for Step input

Consider a first order system as shown;



Here $G(s)=\dfrac{1}{Ts}$ and $H(s)=1$

$$\therefore \frac{C(s)}{R(s)}=\frac{G}{1+GH}=\frac{\frac{1}{Ts}}{1+\frac{1}{Ts}}=\frac{1}{1+Ts}$$

Analysis of First Order System for Step input

For step input;

r(t) = u(t) t>0
= 0 t<0

Taking Laplace transform;

$$R(s) = L\{\mathrm{Ru(t)}\} = \frac{1}{s}$$

but

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

$$\therefore C(s) = \frac{1}{1+Ts} \times R(s)$$

Analysis of First Order System for Step input

$$\therefore C(s) = \frac{1}{1+Ts} \times \frac{1}{s}$$

Using partial fraction;

$$\therefore C(s) = \frac{A}{s} + \frac{B}{s+\frac{1}{T}}$$

Solving;

$$\therefore A = s.C(s)\,|_{s=0} = 1$$

$$\therefore B = (s+\frac{1}{T})C(s)\,|_{s=-\frac{1}{T}} = -1$$

Analysis of First Order System for Step input

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

Taking Inverse Laplace transform;

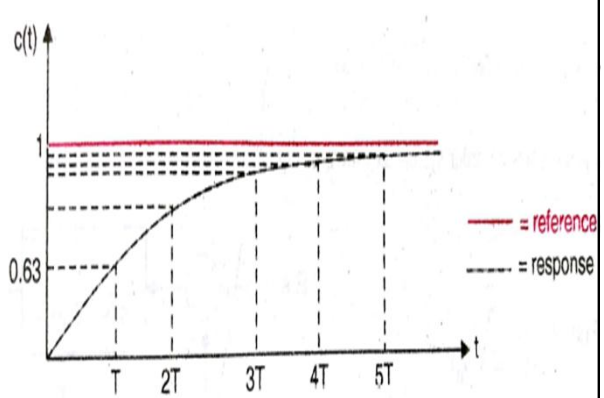
$$\therefore c(t) = L^{-1}\{C(s)\} = L^{-1}\{\frac{1}{s}\} - L^{-1}\{\frac{1}{s + \frac{1}{T}}\}$$

$$\therefore c(t) = 1 - e^{-\frac{1}{T}t}$$

Analysis of First Order System for Step input

Plot c(t) vs t;

Sr. No.	t	C(t)
1	T	0.632
2	2T	0.86
3	3T	0.95
4	4T	0.982
5	5T	0.993
6	∞	1



Time Constant

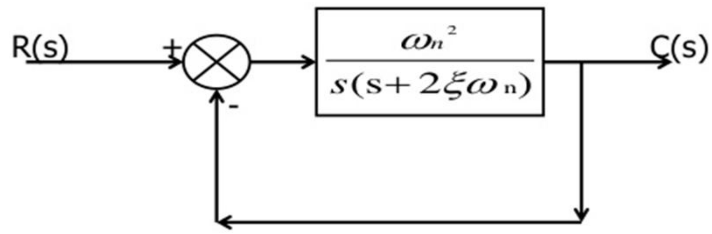
- ✓ The value of $c(t)=1$ only at $t=\infty$.
- ✓ Practically the value of $c(t)$ is within 5% of final value at $t=3T$ and within 2% at $t=4T$.
- ✓ In practice $t=3T$ or $4T$ may be taken as steady state.
- ✓ How quickly the value reaches steady state is a function of the time constant of the system.
- ✓ Hence smaller T indicates quicker response.

Damping & Damping Factor (ξ)

- Every system has a tendency to oppose the oscillatory behavior of the system which is known as "**Damping**"
- The damping in any system is measured by a factor or ratio which is known as **damping ratio.**
- It is denoted by ξ (Zeta).

Analysis of Second Order System for Step input

Consider a second order system as shown;



Here $G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$ and $H(s) = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Analysis of Second Order System for Step input

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

This is the standard form of the closed loop transfer function

These poles of transfer function are given by;

$$s^2+2\xi\omega_n s+\omega_n^2 = 0$$

$$\therefore s = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2-4(\omega_n)^2}}{2}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2-\omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

Analysis of Second Order System for Step input

The poles are;

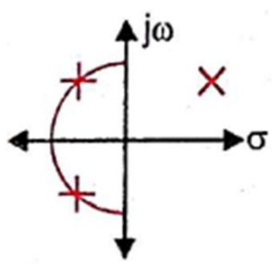
(i) Real and Unequal if $\sqrt{\xi^2 - 1} > 0$
i.e. $\xi > 1$ They lie on real axis and distinct

(ii) Real and equal if $\sqrt{\xi^2 - 1} = 0$
i.e. $\xi = 1$ They are repeated on real axis

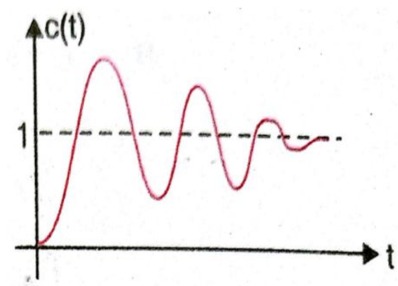
(iii) Complex if $\sqrt{\xi^2 - 1} < 0$
i.e. $\xi < 1$ Poles are in second and third quadrant

Relation between ξ and pole locations

(i) $0 < \xi < 1$ Under damped



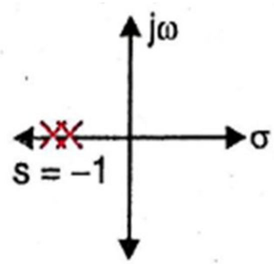
Pole Location

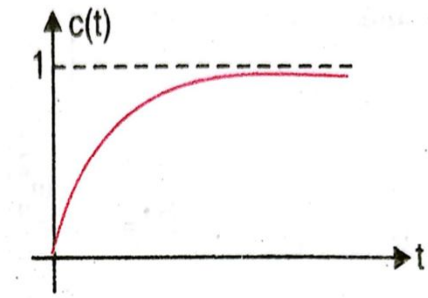


Step Response c(t)

Relation between ξ and pole locations

(ii) $\xi = 1$ Critically damped

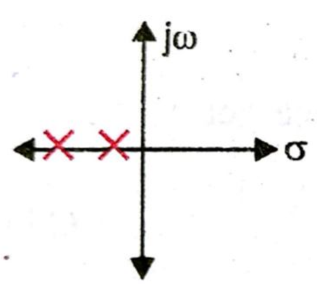


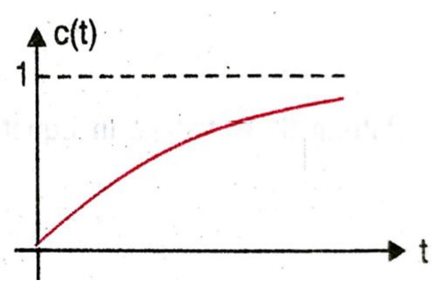


Pole Location **Step Response c(t)**

Relation between ξ and pole locations

(iii) $\xi > 1$ over damped

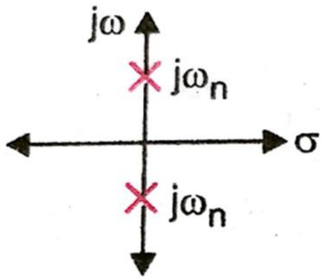


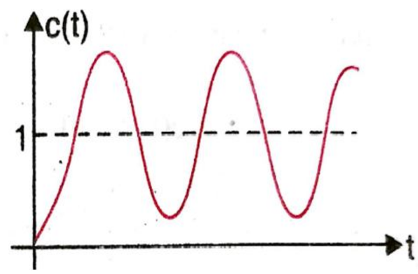


Pole Location **Step Response c(t)**

Relation between ξ and pole locations

(iv) $\xi = 0$

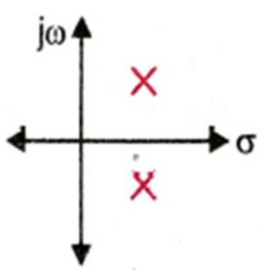


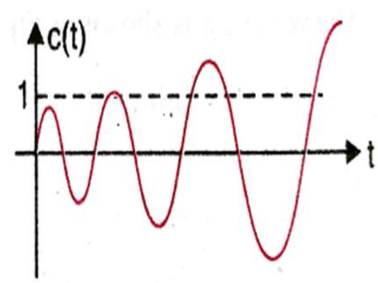


Pole Location **Step Response c(t)**

Relation between ξ and pole locations

(v) $0 > \xi > -1$

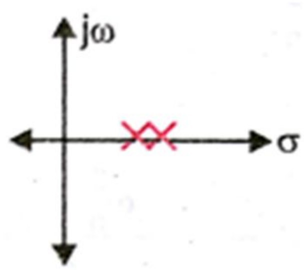


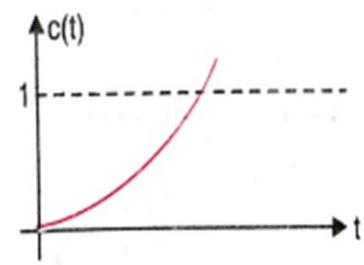


Pole Location **Step Response c(t)**

Relation between ξ and pole locations

(vi) $\xi = -1$

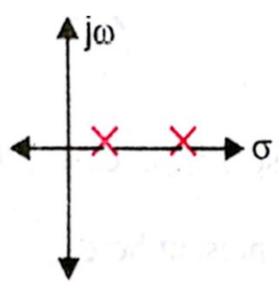


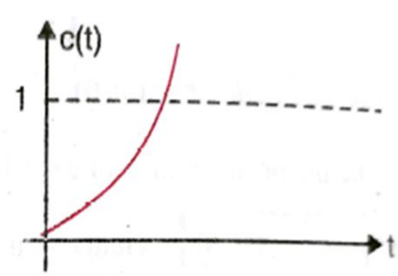


Pole Location **Step Response c(t)**

Relation between ξ and pole locations

(vii) $\xi < -1$





Pole Location **Step Response c(t)**

Summary

- Significance of standard test inputs in control system.
- Significance of poles and zeros in system.
- Analysis of first and second order system
- Effect of damping on system response.