

Control System

Block Diagram Reduction Techniques

Content

- ▶ Review of Last Lecture.
- ▶ Need of Block Diagram.
- ▶ Block Diagram Reduction Technique.
- ▶ Examples of Block Diagram Reduction.

Learning Objectives

- ▶ Able to understand the need of Block diagram in Control System.
- ▶ Able to apply various rule of block diagram reduction techniques.

Block Diagram Algebra

- ▶ Block diagram is a **shorthand, graphical representation** of a physical system, illustrating the functional relationships among its components.

OR

- ▶ A Block Diagram is a shorthand pictorial representation of the **cause-and-effect** relationship of a system.

Block Diagram Algebra

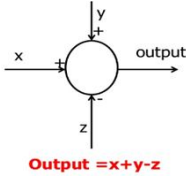
- ▶ Entire complicated system can be completely represented by connecting different blocks.
- ▶ One block corresponds to one element of system.
- ▶ A corresponding block is drawn by inserting a transfer function of that element inside the block.

Components of a Block Diagram for a LTI System

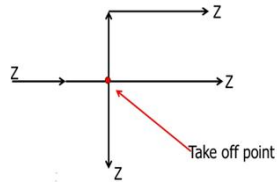
- ▶ System components are alternatively called elements of the system.
- ▶ Block diagram has four components:
 - ❖ **Signals**
 - ❖ **System/block**
 - ❖ **Summing junction**
 - ❖ **Pick-off/ Take-off point**

Element of Block Diagram

▶ **Summing Point**:- Two or more signals can be added or subtracted at summing point.



▶ **Take-off Point**:- The output signal can be applied to two or more points from a take-off point.



Block Diagram Reduction Techniques

▶ **Rule 1 :- For Blocks in Series/Cascade**

-Gain of blocks connected in series gets multiplied with each other.



$$R1(s) = G1R(s)$$

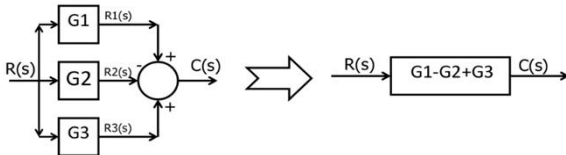
$$C(s) = G2R1(s) = G1G2R(s)$$

$$C(s) = G1G2R(s)$$

Block Diagram Reduction Techniques

▶ **Rule 2 :- For Blocks in Parallel**

-Gain of blocks connected in parallel gets added algebraically.



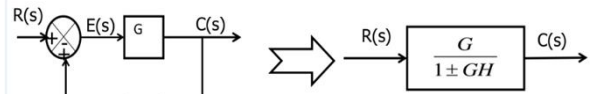
$$C(s) = R1(s) - R2(s) + R3(s) = G1R(s) - G2R(s) + G3R(s)$$

$$C(s) = (G1 - G2 + G3) R(s)$$

$$C(s) = (G1 - G2 + G3) R(s)$$

Block Diagram Reduction Techniques

▶ **Rule 3:- Elimination of Feedback loop**



$$\frac{C(s)}{R(s)} = \frac{G}{1 \pm GH}$$

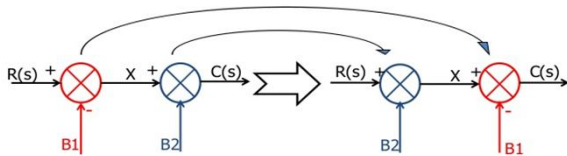
In General

For -ve f/b: +ve sign in above expression & vice-versa.

Block Diagram Reduction Techniques

▶ **Rule 4:- Associative law for summing point**

- Order of summing point can be changed if two or more summing points are in series.



$$X = R(s) - B1$$

$$C(s) = X - B2$$

$$C(s) = R(s) - B1 - B2$$

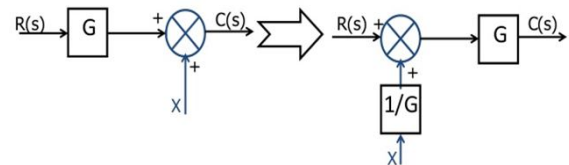
$$X = R(s) - B2$$

$$C(s) = X - B1$$

$$C(s) = R(s) - B2 - B1$$

Block Diagram Reduction Techniques

▶ **Rule 5:- Shift summing point before block**

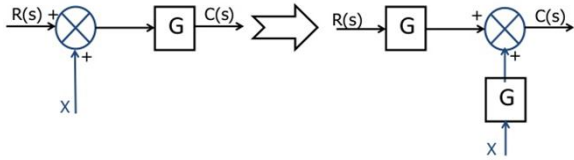


$$C(s) = R(s)G + X$$

$$C(s) = G\{R(s) + X/G\} = GR(s) + X$$

Block Diagram Reduction Techniques

Rule 6:- Shift summing point after block



$$C(s) = G\{R(s) + X\}$$

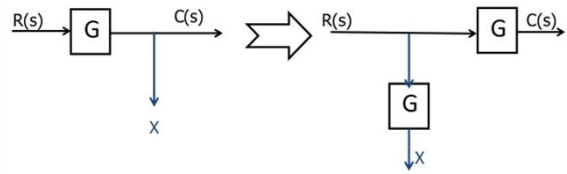
$$= GR(s) + GX$$

$$C(s) = GR(s) + XG$$

$$= GR(s) + XG$$

Block Diagram Reduction Techniques

Rule 7:- Shift a take-off point before block



$$C(s) = GR(s)$$

$$\text{and}$$

$$X = C(s) = GR(s)$$

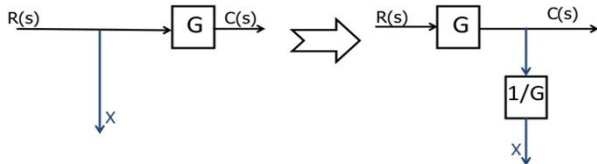
$$C(s) = GR(s)$$

$$\text{and}$$

$$X = GR(s)$$

Block Diagram Reduction Techniques

Rule 8:- Shift a take-off point after block



$$C(s) = GR(s)$$

$$\text{and}$$

$$X = R(s)$$

$$C(s) = GR(s)$$

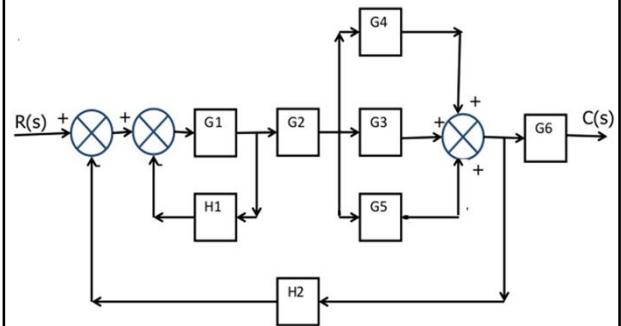
$$\text{and}$$

$$X = C(s) \cdot \{1/G\}$$

$$= GR(s) \cdot \{1/G\}$$

$$= R(s)$$

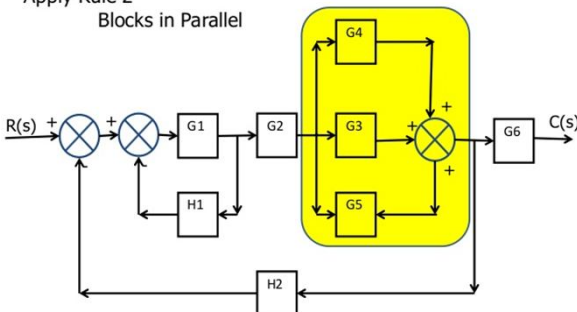
Example 1:



Example 1:

Cont....

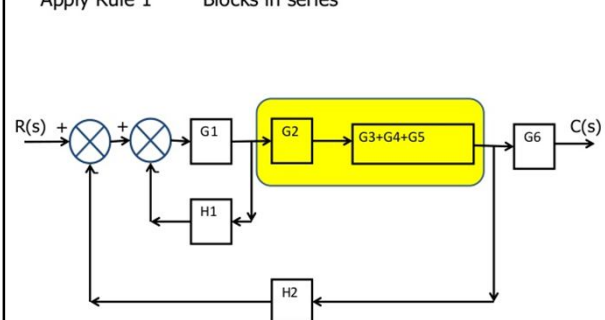
Apply Rule 2
Blocks in Parallel



Example 1:

Cont....

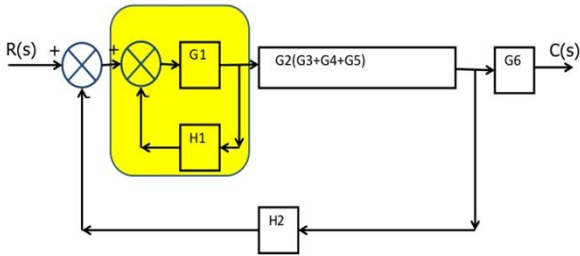
Apply Rule 1
Blocks in series



Example 1:

Cont....

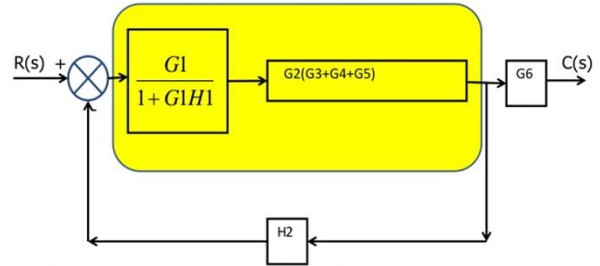
Apply Rule 3 Elimination of feedback loop



Example 1:

Cont....

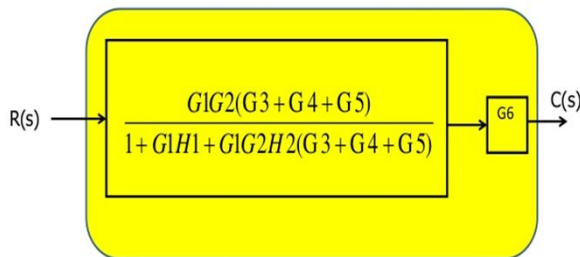
Apply Rule 1 Blocks in series



Example 1:

Cont....

Apply Rule 1 Blocks in series



Example 1:

Cont....

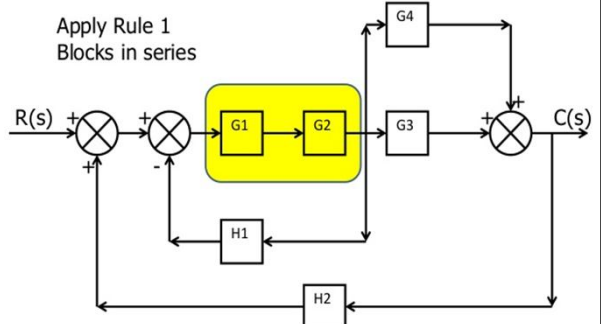
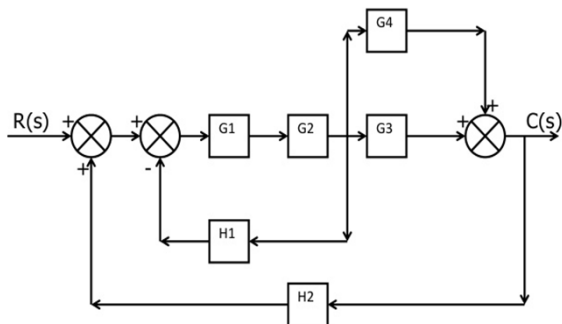
$$\frac{C(s)}{R(s)} = \frac{G1G2G6(G3+G4+G5)}{1+G1H1+G1G2H2(G3+G4+G5)}$$

Example 2:

Example 2:

Cont....

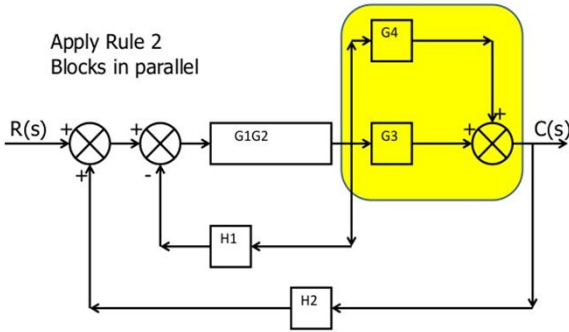
Apply Rule 1
Blocks in series



Example 2:

Cont....

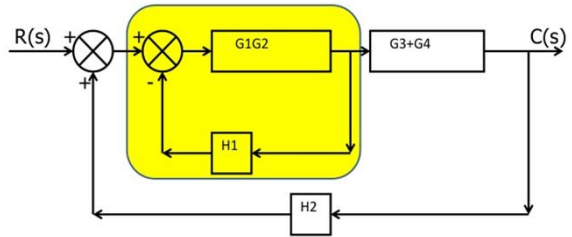
Apply Rule 2
Blocks in parallel



Example 2:

Cont....

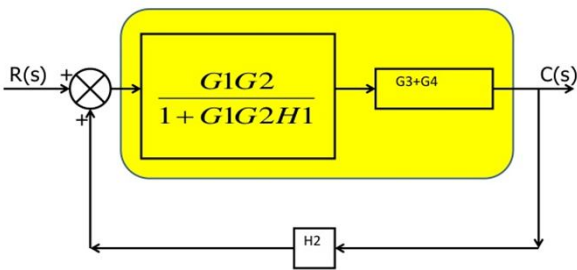
Apply Rule 3
Elimination of feedback loop



Example 2:

Cont....

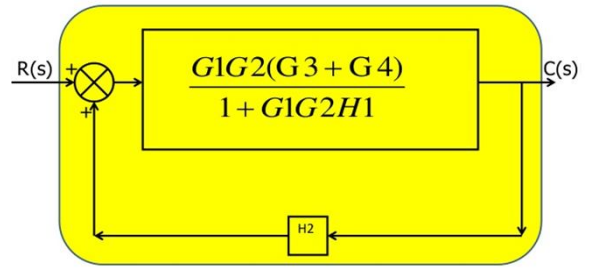
Apply Rule 2 Blocks in series



Example 2:

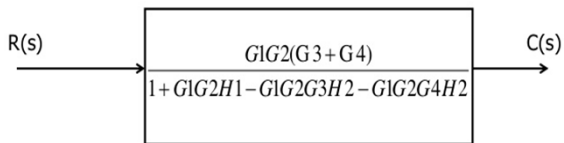
Cont....

Apply Rule 3 Elimination of feedback loop



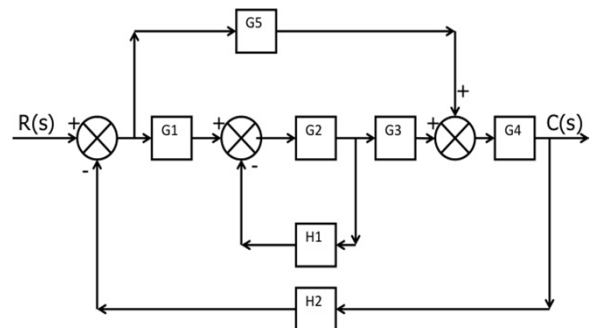
Example 2:

Cont....



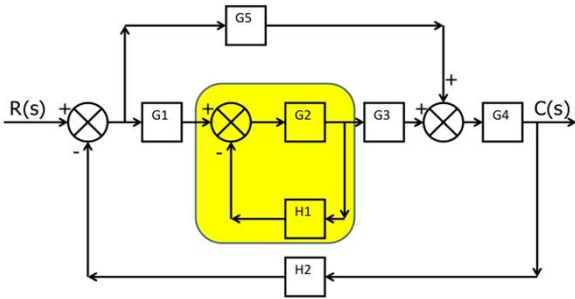
$$\frac{C(s)}{R(s)} = \frac{G1G2(G3 + G4)}{1 + G1G2H1 - G1G2G3H2 - G1G2G4H2}$$

Example 3:



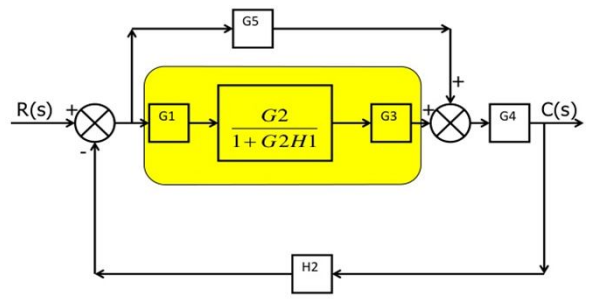
Example 3: Cont....

Apply Rule 3 Elimination of feedback loop



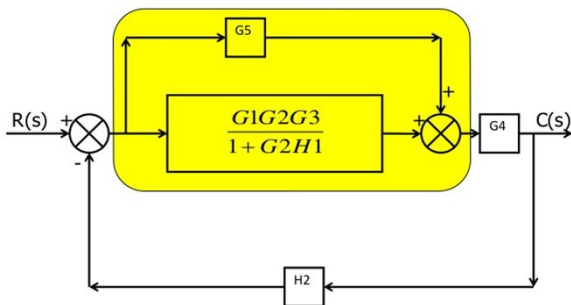
Example 3: Cont....

Apply Rule 1 Blocks in series



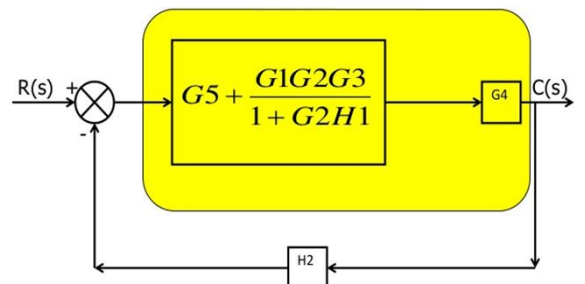
Example 3: Cont....

Apply Rule 2 Blocks in parallel



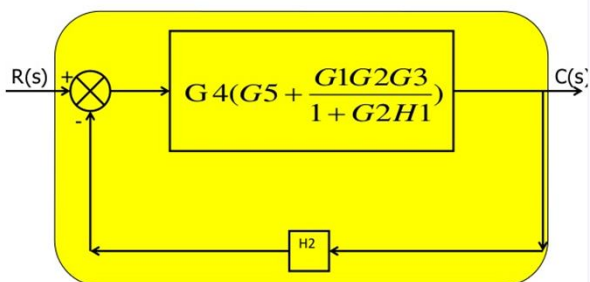
Example 3: Cont....

Apply Rule 1 Blocks in series



Example 3: Cont....

Apply Rule 3 Elimination of feedback loop

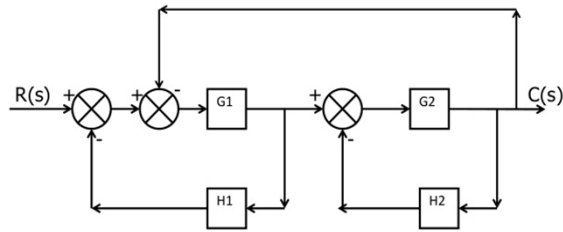


Example 3: Cont....

$$R(s) \rightarrow \frac{G4G5 + G2G4G5H1 + G1G2G3G4}{1 + G2H1 + G4G5H2 + G2G4G5H1H2 + G1G2G3G4H2} C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G4G5 + G2G4G5H1 + G1G2G3G4}{1 + G2H1 + G4G5H2 + G2G4G5H1H2 + G1G2G3G4H2}$$

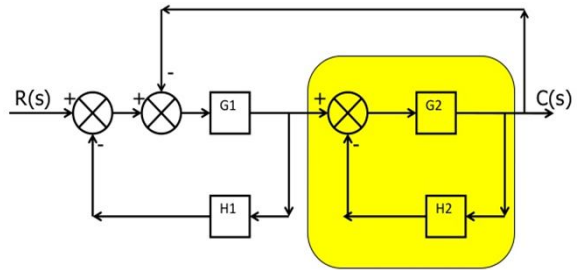
Example 4:



Example 4:

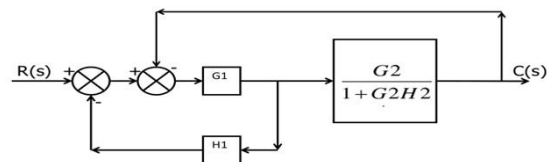
Cont....

Apply Rule 3 Elimination of feedback loop



Example 4:

Cont....



➤ Now Rule 1, 2 or 3 cannot be used directly.

➤ There are possible ways of going ahead.

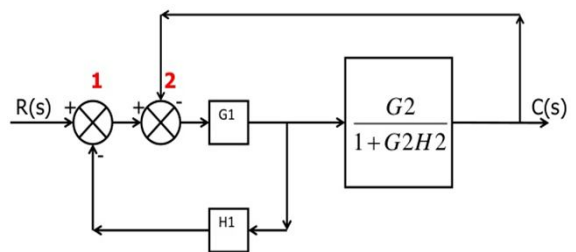
- Use Rule 4 & interchange order of summing so that Rule 3 can be used on G.H1 loop.
- Shift take off point after $\frac{G2}{1+G2H2}$ block reduce by Rule 1, followed by Rule 3.

Example 4:

Cont....

Apply Rule 4

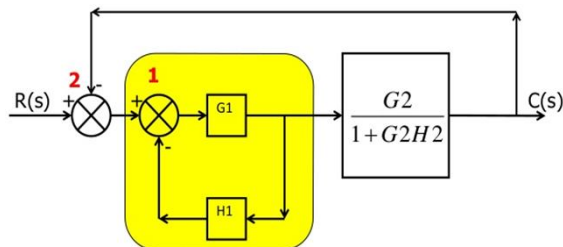
Exchange summing order



Example 4:

Cont....

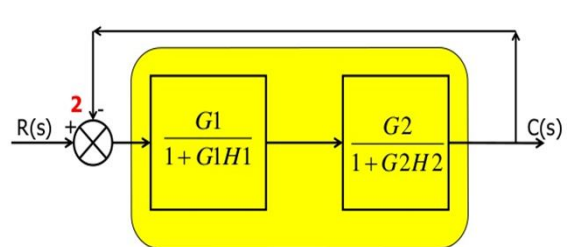
Apply Rule 3 Elimination feedback loop



Example 4:

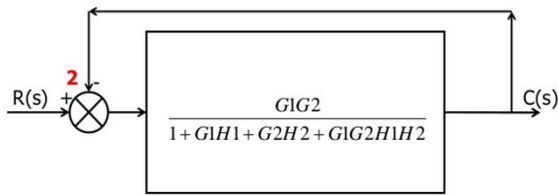
Cont....

Apply Rule 1 Blocks in series



Example 4:

Cont....



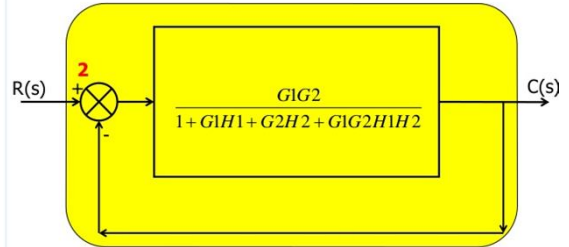
Now which Rule will be applied
 -----It is blocks in parallel
 -----It is feed back loop

OR

Example 4:

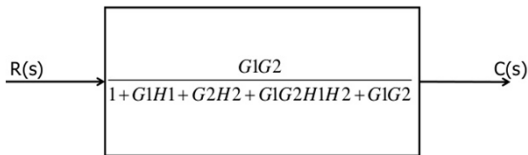
Cont....

Let us rearrange the block diagram to understand
 Apply Rule 3 Elimination of feed back loop



Example 4:

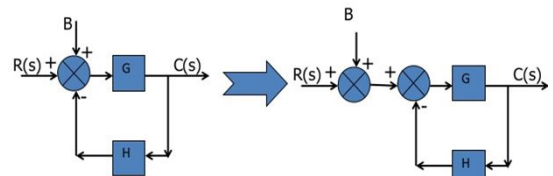
Cont....



$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1 + G_1H_1 + G_2H_2 + G_1G_2H_1H_2 + G_1G_2}$$

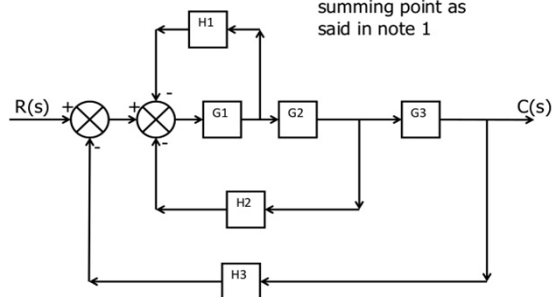
According to Rule 4

➤ By corollary, one can split a summing point to two summing point and sum in any order



Example 5:

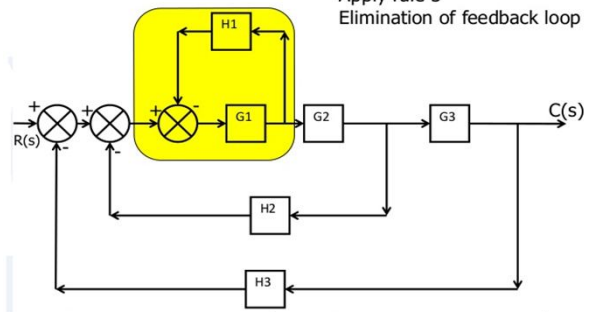
Simplify, by splitting second summing point as said in note 1



Example 5:

Cont....

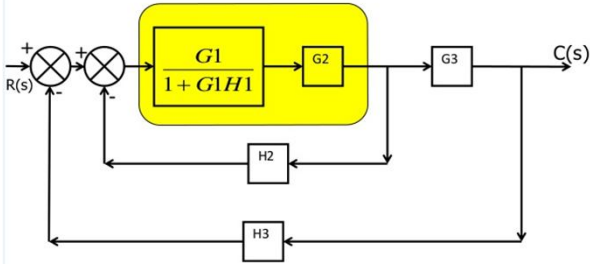
Apply rule 3 Elimination of feedback loop



Example 5:

Cont....

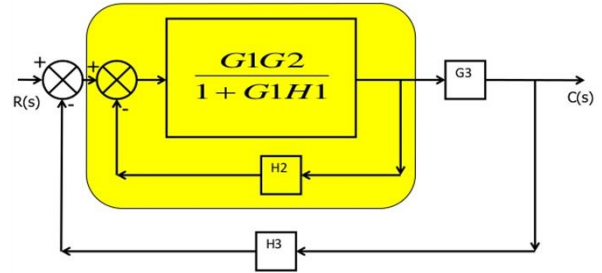
Apply rule 1 Blocks in series



Example 5:

Cont....

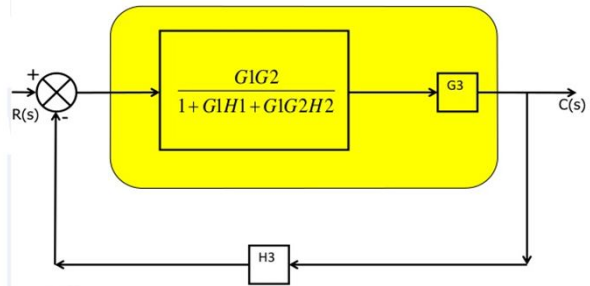
Apply rule 3 Elimination of feedback loop



Example 5:

Cont....

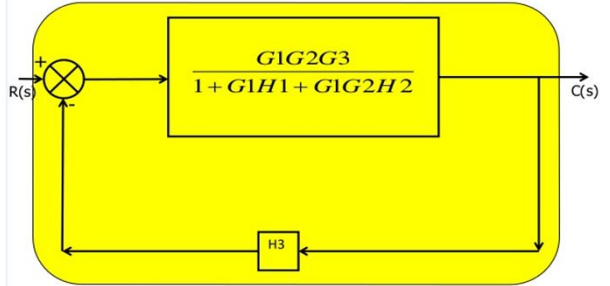
Apply rule 1 Blocks in series



Example 5:

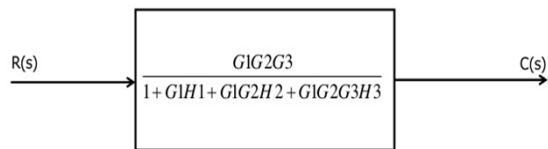
Cont....

Apply rule 3 Elimination of feedback loop



Example 5:

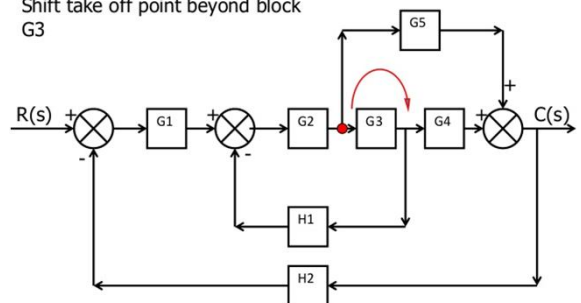
Cont....



$$\frac{C(s)}{R(s)} = \frac{G1G2G3}{1 + G1H1 + G1G2H2 + G1G2G3H3}$$

Example 6:

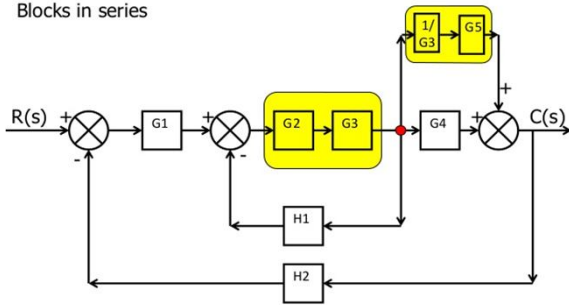
Apply rule 8
Shift take off point beyond block
G3



Example 6:

Cont....

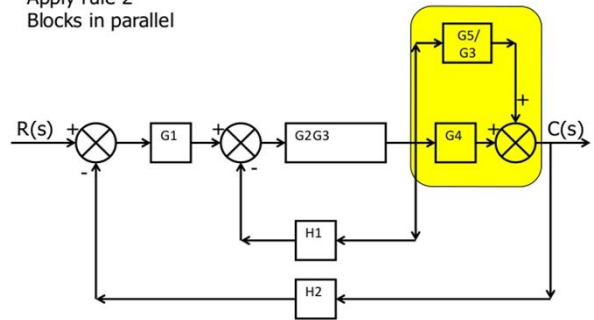
Apply rule 1
Blocks in series



Example 6:

Cont....

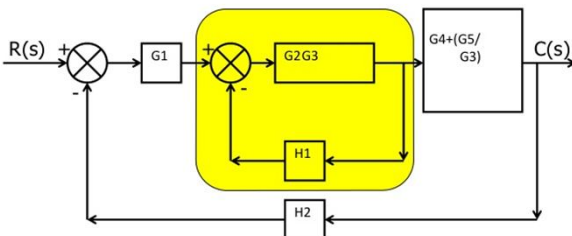
Apply rule 2
Blocks in parallel



Example 6:

Cont....

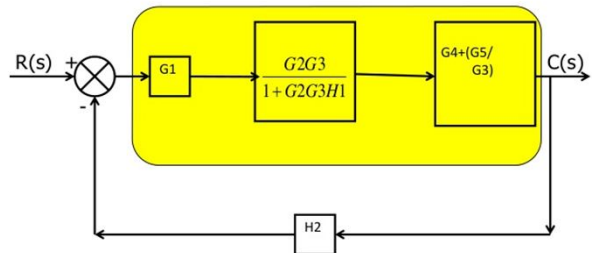
Apply rule 3
Feedback loop



Example 6:

Cont....

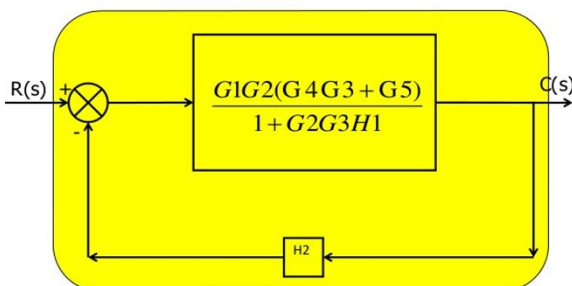
Apply rule 1
Blocks in series



Example 6:

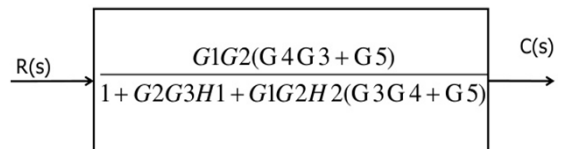
Cont....

Apply rule 3
Feedback loop



Example 6:

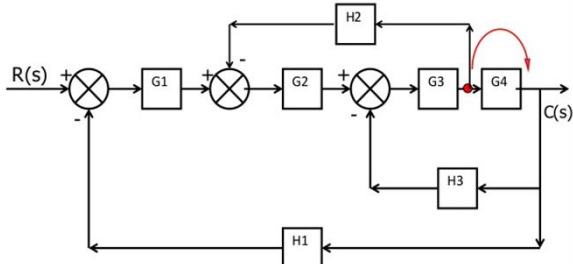
Cont....



$$\frac{C(S)}{R(S)} = \frac{G1G2(G4G3 + G5)}{1 + G2G3H1 + G1G2H2(G3G4 + G5)}$$

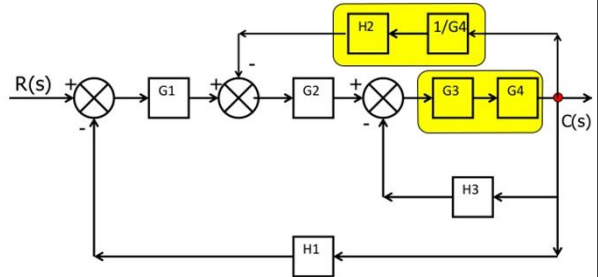
Example 7:

Apply rule 8 Shift take off point after block G4



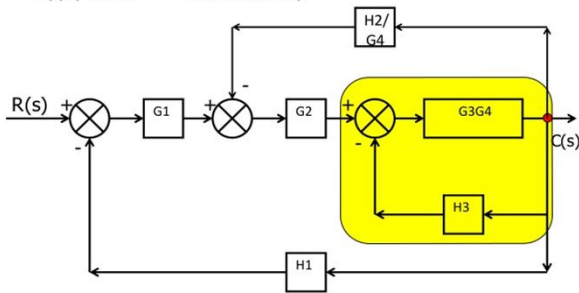
Example 7: Cont....

Apply rule 1 Blocks in series



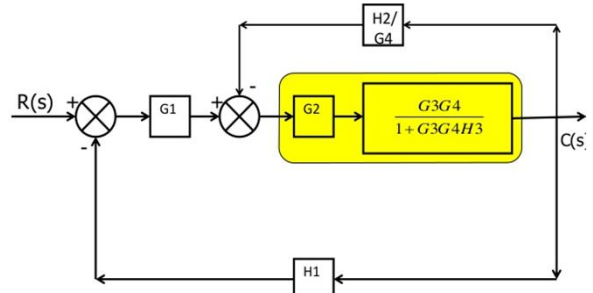
Example 7: Cont....

Apply rule 3 Feedback loop



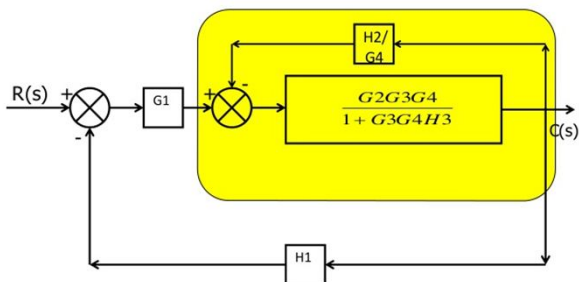
Example 7: Cont....

Apply rule 1 Blocks in series



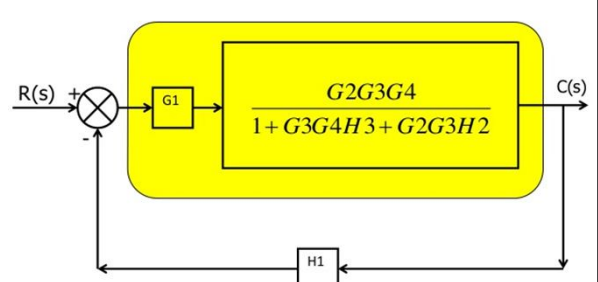
Example 7: Cont....

Apply rule 3 Feedback loop



Example 7: Cont....

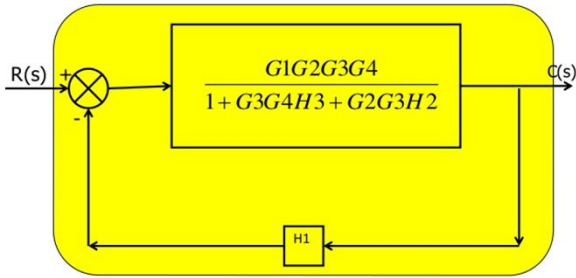
Apply rule 1 Blocks in series



Example 7:

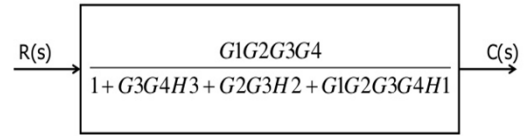
Cont....

Apply rule 3 Feedback loop



Example 7:

Cont....

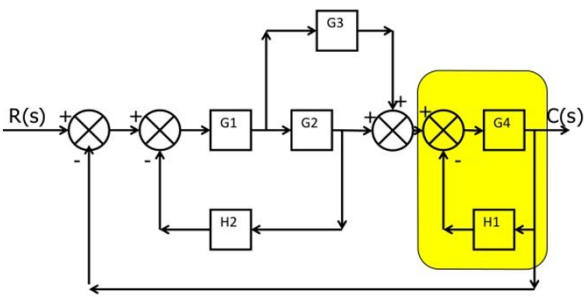


$$\frac{C(S)}{R(S)} = \frac{G1G2G3G4}{1 + G3G4H3 + G2G3H2 + G1G2G3G4H1}$$

Example 8:

Cont....

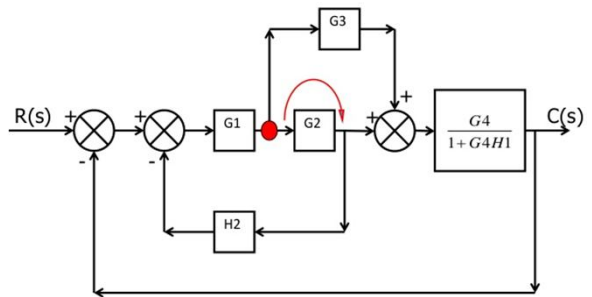
Apply Rule 3 Elimination of Feedback loop



Example 8:

Cont....

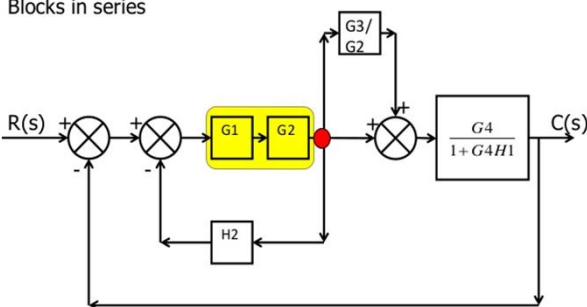
Apply Rule 8 Shift take off point after block



Example 8:

Cont....

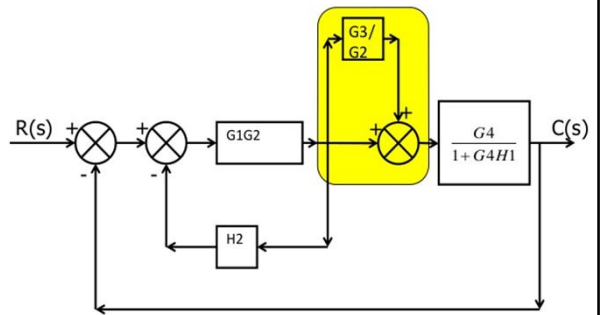
Apply Rule 1 Blocks in series



Example 8:

Cont....

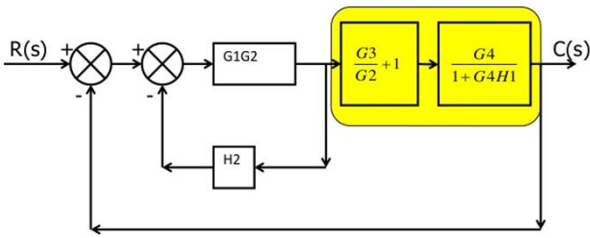
Now which rule we have to use?



Example 8:

Cont....

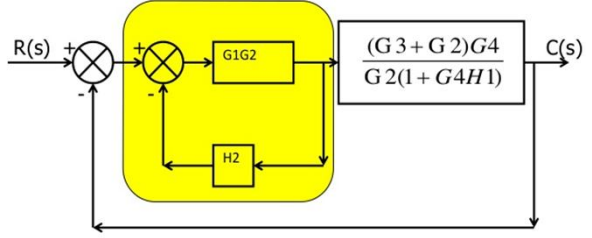
Apply Rule 1 Blocks in series



Example 8:

Cont....

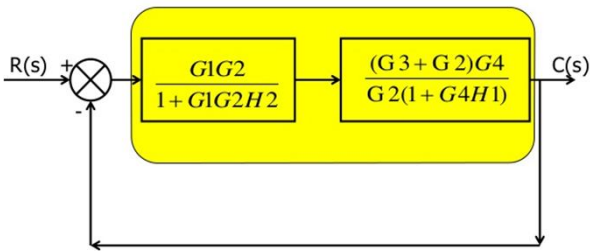
Apply Rule 3 Elimination of Feedback Loop



Example 8:

Cont....

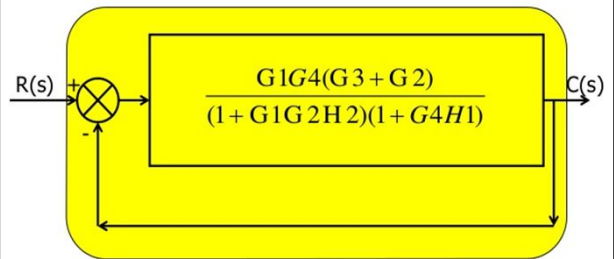
Apply Rule 1 Blocks in series



Example 8:

Cont....

Apply Rule 3 Elimination of Feedback loop



Example 8:

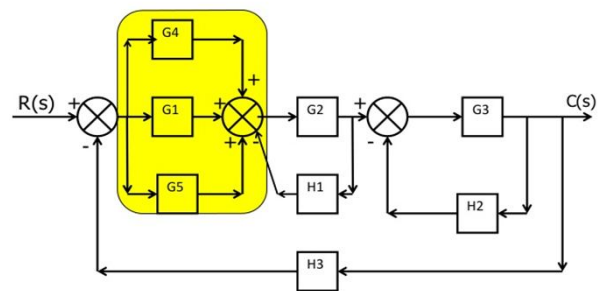
Cont....

$$R(s) \rightarrow \frac{G_1G_4(G_3 + G_2)}{1 + G_4H_1 + G_1G_2H_2 + G_1G_2G_4H_1H_2 + G_1G_4(G_2 + G_3)} C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_4(G_3 + G_2)}{1 + G_4H_1 + G_1G_2H_2 + G_1G_2G_4H_1H_2 + G_1G_4(G_2 + G_3)}$$

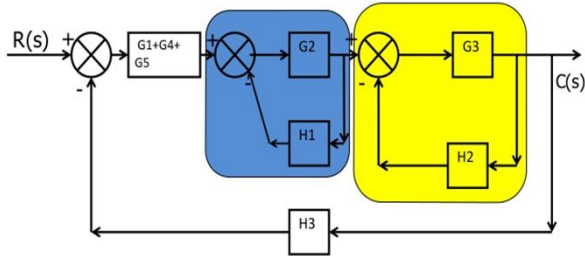
Example 9:

Apply rule 2 Blocks in Parallel



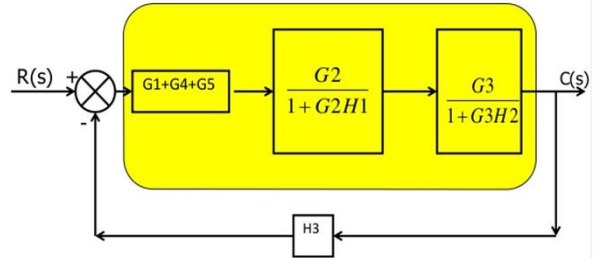
Example 9: Cont....

Apply rule 3 Elimination of Feedback Loop



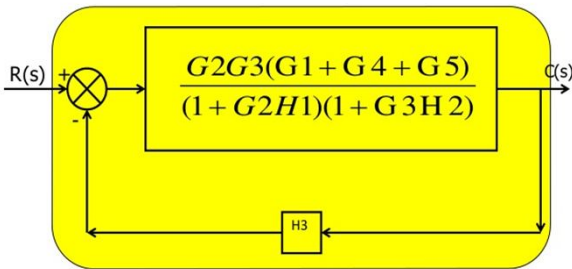
Example 9: Cont....

Apply rule 1 Blocks in Series



Example 9: Cont....

Apply rule 3 Elimination of Feedback loop



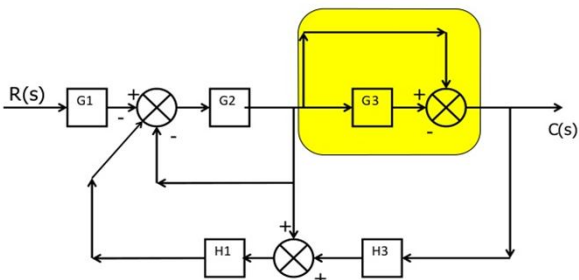
Example 9: Cont....

$$R(s) \rightarrow \frac{G2G3(G1+G4+G5)}{1+G2H1+G3H2+G2G3H1H2+G2G3H3(G1+G4+G5)} \rightarrow C(s)$$

$$\frac{C(s)}{R(s)} = \frac{G2G3(G1+G4+G5)}{1+G2H1+G3H2+G2G3H1H2+G2G3H3(G1+G4+G5)}$$

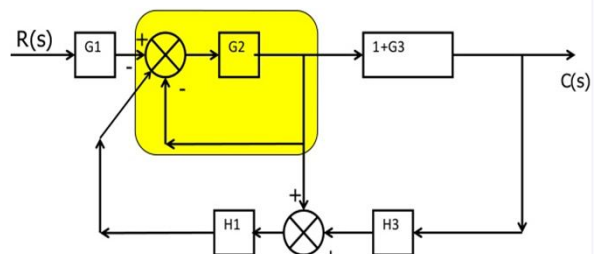
Example 10:

Apply rule 2 Blocks in Parallel



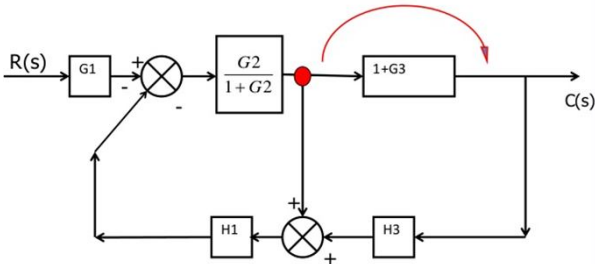
Example 10: Cont....

Apply rule 3 Elimination of Feedback Loop



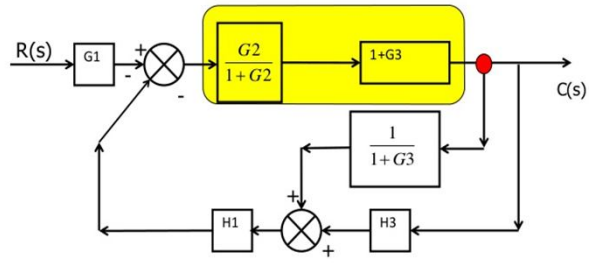
Example 10: Cont....

Apply rule 8 Shift take off point after block



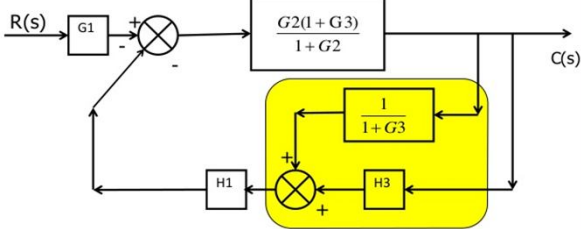
Example 10: Cont....

Apply rule 1 Blocks in series



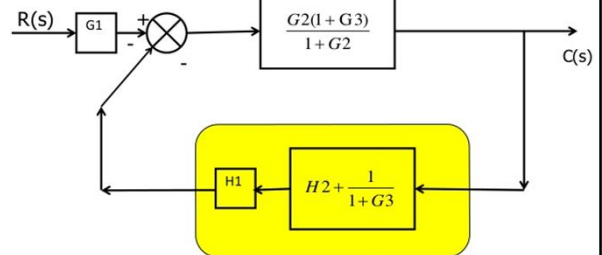
Example 10: Cont....

Apply rule 2 Blocks in Parallel



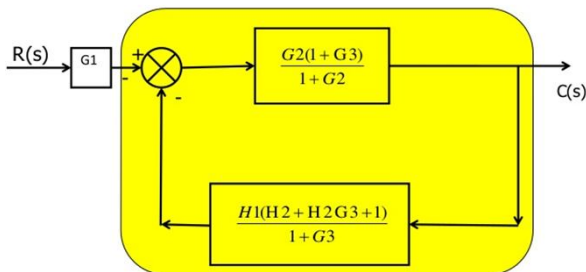
Example 10: Cont....

Apply rule 1 Blocks in Series



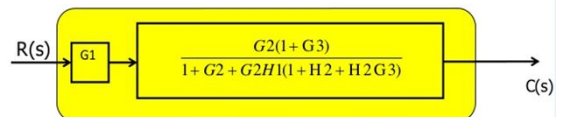
Example 10: Cont....

Apply rule 3 Elimination of Feedback loop



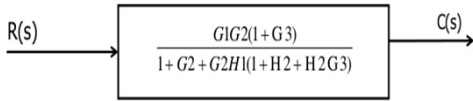
Example 10: Cont....

Apply rule 1 Blocks in series



Example 10:

Cont....



$$\frac{C(s)}{R(s)} = \frac{G1G2(1+G3)}{1+G2+G2H1(1+H2+H2G3)}$$

Summary

- ▶ Significance of Block Diagram.
- ▶ Block Diagram Reduction Techniques.
- ▶ Examples of Block Diagram Reduction.