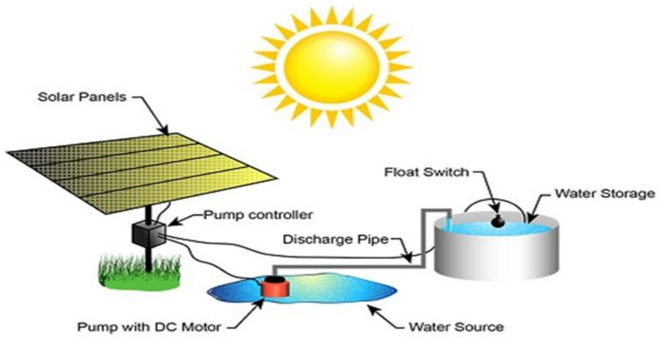


Control System

Transfer Function & Order of System

Content

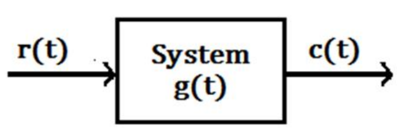
- Review of Last Lecture.
- Transfer Function.
- Transfer function of RC and RLC electrical circuit.
- Examples of Transfer function
- Order of System & its type

Learning Objectives

- Able to define use of TF in control system.
- Able to derive expression for open loop and closed loop system.
- Transfer function of RC and RLC electrical circuits.

Transfer Function

- **Ratio of Laplace transform of *output (or response)* to Laplace transform of *input (or excitation)* under the assumption of zero initial conditions is defined as the transfer function of given system.**



c(t) – output
r(t) – input
g(t) – system function

L{c(t)} = C(s)
L{r(t)} = R(s)
L{g(t)} = G(s)

Therefore,
G(s) = C(s) / R(s)

Properties of Transfer Function

- TF is defined only for linear time invariant systems.it is not defined for non-linear systems.
- Transfer function is independent of inputs to system.
- Systems poles/zeros can be found out from Transfer function.
- Once TF is known, any output for any given input, can be known.
- System differential equation can be obtained by replacement of variable 's' by 'd/dt'.

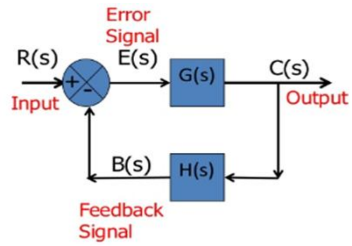
Laplace Transform of Electrical N/W

Element	Time-Domain	S-Domain
R	$i(t) * R$	I(s) * R
L	$L \, . \, di(t)/dt$	Ls * I(s)
C	$1/C \, . \int i(t)dt$	1/sC * I(s)

R- Resistance
L- Inductance

C-Capacitance
I- Current

Transfer Function of Closed Loop System



Error signal is given by,

E(s)=R(s) - B(s) ----------(1)

Thus,

R(s)= E(s)+B(s)

Gain of feedback n/w is given by,

H(s)= B(s)/C(s)

Thus,

B(s)=H(s).C(s)--------(2)

Gain for CL system is given by,

G(s)=C(s)/E(s)

Thus, C(s)=G(s).E(s)---------------(3)

Substitute the value of E(s) from Eq. 1 to 3

C(s)=G(s).[R(s) - B(s)]

Thus,

C(s)= G(s).R(s) – G(s).B(s)------(4)

Substitute the value of B(s) from Eq. 2 to 4

C(s)= G(s).R(s) – G(s).H(s). C(s)

G(s).R(s)=C(s)+G(s).H(s). C(s)

G(s).R(s)=C(s)[1+G(s).H(s)]

Transfer function is given by,

T.F. = C(s)/R(s) = G(s)/ 1+G(s).H(s)

Laplace Transform of ‘ R ’

- Resistor are time and frequency invariant. Therefore, transform of a resistor is same as the resistance of resistor.

L{Resistor}=R(s)

Laplace Transform of 'C'

- Let us look at relationship between voltage, current and capacitance in time domain.

$$i(t) = C\frac{dv(t)}{dt}$$

- Solving for voltage, we get the following integral;

$$v(t) = \frac{1}{C}\int_{to}^{\infty} i(t)\,dt$$

- Then transforming this equation into Laplace domain, we get the following;

$$V(s) = \frac{1}{C}\frac{1}{s}I(s)$$

- Again, solving for ratio V(s)/I(s), we get the following;

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

- Therefore, transform for a capacitor with capacitance C is given by,

$$L\{\text{capacitor}\} = \frac{1}{sC}$$

Laplace Transform of 'L'

- Let us look at relationship between voltage, current and inductance in time domain.

$$v(t) = L\frac{di(t)}{dt}$$

- Then transforming above equation into Laplace domain, we get the following;

$$V(s) = sLI(s)$$

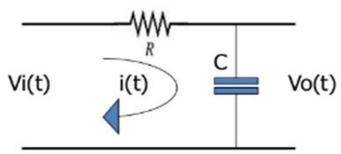
- Again, solving for ratio V(s)/I(s), we get the following;

$$\frac{V(s)}{I(s)} = sL$$

- Therefore, transform for an inductance with inductance L is given by,

$$L\{\text{inductor}\} = sL$$

Transfer Function of RC Network



- Apply KVL for input loop,

$$vi(t) = Ri(t) + \frac{1}{C}\int_0^t i(t)dt$$

- Taking Laplace transform of above equation, we get;

$$Vi(s) = RI(s) + \frac{1}{sC}I(s) ------(1)$$

- Apply KVL for output loop,

$$vo(t) = \frac{1}{C}\int_0^t i(t)dt$$

- Taking Laplace transform of above equation, we get;

$$Vo(s) = \frac{1}{sC}I(s) ------(2)$$

$$\therefore I(s) = sC.Vo(s) ------(3)$$

- From equation 1,

$$Vi(s) = I(s)(R + \frac{1}{sC}) ------(4)$$

- From equation 3 & 4,

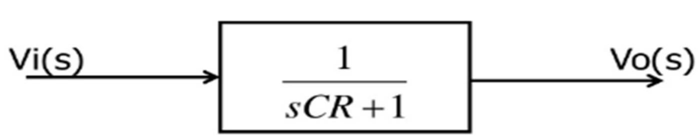
$$Vi(s) = Vo(s).sC.(R + \frac{1}{sC})$$

Transfer Function of RC Network

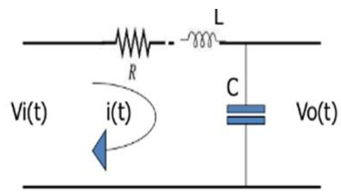
Transfer Function= G(s)= $\frac{Vo(s)}{Vi(s)} = \frac{1}{sC.(R + \frac{1}{sC})}$

$$\frac{Vo(s)}{Vi(s)} = \frac{1}{sC.(\frac{sCR+1}{sC})}$$

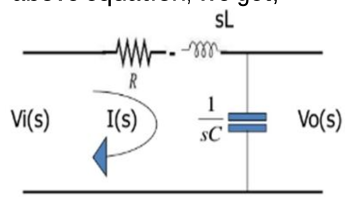
$$\frac{Vo(s)}{Vi(s)} = \frac{1}{sCR+1}$$



Transfer Function of RLC Network



- Taking Laplace transform of above equation, we get;



- Apply KVL for input loop,

$$Vi(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s)$$

$$\therefore Vi(s) = [R + sL + \frac{1}{sC}]I(s) - - - - - - - (1)$$

- Apply KVL for output loop,

$$Vo(s) = \frac{1}{sC}I(s) - - - - - - (2)$$

Transfer Function of RLC Network

From equation 1 & 2,

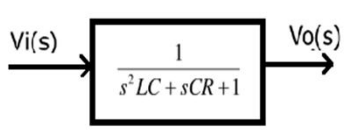
$$\text{Transfer Function} = \frac{Vo(s)}{Vi(s)} = \frac{\frac{1}{sC}I(s)}{[R + sL + \frac{1}{sC}]I(s)}$$

$$= \frac{\frac{1}{sC}}{[R + sL + \frac{1}{sC}]}$$

$$= \frac{\frac{1}{sC}}{\frac{sCR + s^2LC + 1}{sC}}$$

$$= \frac{1}{sCR + s^2LC + 1}$$

$$= \frac{1}{s^2LC + sCR + 1}$$



Order of System

- Order of control system is defined as the highest power of s present in denominator of closed loop transfer function G(s) of unity feedback system.

- A **proper system** is a system where the degree of denominator is larger than or equal to the degree of numerator polynomial.

$$G(s)= s+1/(s^2+2s+1)$$

Order of System

Q. Define the order of given system

$$G(s) = \frac{(S+1)}{S^2+2S+1}$$

Answer: The highest power of equation in denominator of given transfer function is '2'.
Hence, the order of given system is two.

Find the order of given system

$$G(s) = \frac{S(S+2)}{S(S+2)(S+1)}$$

Solution: To obtain highest power of denominator ,
Simplify denominator polynomial.

$$S(S+2)(S+1)=0$$
$$S(S^2+3S+2)=0$$
$$S^3+3S^2+2S=0$$

The highest power of equation in denominator of given transfer function is '3'. Hence, given system is '**Third Order System**'

The degree of denominator is larger than the numerator. Hence, system is '**Proper System** '

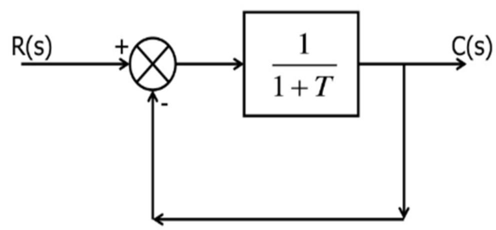
Types of System

- Depending on highest power of denominator, system are classified as:

- Zero Order System
- First Order System
- Second Order System

Zero Order System

- If highest power of complex variable ‘s’ present in characteristic equation is zero, then it is called as **“Zero Order System”**



Zero Order System

- Consider a unity feedback system with transfer function

$$G(s) = \frac{1}{1+T}$$

Hence, characteristic equation is given by ,

$$1+T=0$$

or

$$1+s^{0}T=0$$

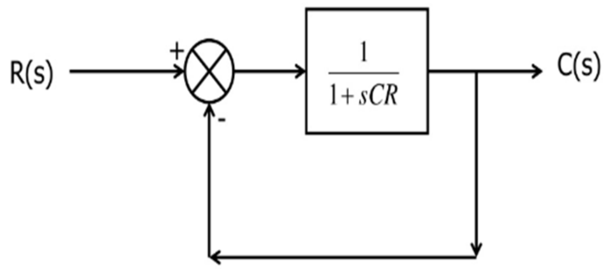
Here, highest power of s is equal to 0,

Hence, system given above is Zero Order System.

Practical Example : Amplifier type control system.

First Order System

- If highest power of complex variable ‘s’ present in characteristic equation is one, then it is called as **<u>“First Order System”</u>**



First Order System

- Consider a unity feedback system with transfer function

$$G(s) = \frac{1}{1+sCR}$$

Hence, characteristic equation is given by ,

$$1 + sCR = 0$$

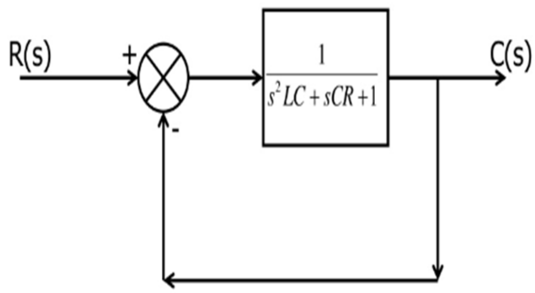
Here, highest power of s is equal to 1,

Hence, system given above is First order system

Practical Example : RC circuits, thermal type systems .

Second Order System

- If highest power of complex variable 's' present in characteristic equation is two, then it is called as **<u>"Second Order System"</u>**



Second Order System

- Consider a unity feedback system with transfer function

$$G(s)=\frac{1}{s^2LC+sCR+1}$$

Hence, characteristic equation is given by ,

$$s^2LC+sCR+1=0$$

Here, highest power of s is equal to 2,

Hence, system given above is First order system

Practical Example : RLC circuits, Robotic control systems .

Summary

- Transfer Function.
- Transfer Function of RC and RLC circuits.
- Order of system.
- Types of system.