Syllabus

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VIBRATION MEASURING INSTRUMENTS

Introduction

- The instruments which are used to measure the displacement, velocity or acceleration of a vibrating body are called vibration measuring instruments.
- Vibration measuring devices having mass, spring, dash pot, etc. are known as seismic instruments.
- The quantities to be measured are displayed on a screen in the form of electric signal which can be readily amplified and recorded.
- The output of electric signal of the instrument will be proportional to the quantity which is to be measured.

VIBRATION MEASURING INSTRUMENTS (Contd..)

Introduction

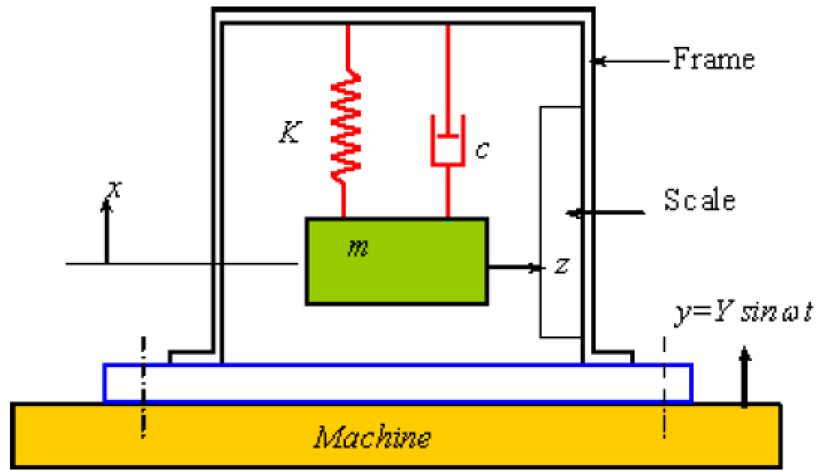
- Two types of seismic transducers known as vibrometer and accelerometer are widely used.
- A vibrometer or a seismometer is a device to measure the displacement of a vibrating body.
- Accelerometer is an instrument to measure the acceleration of a vibrating body.
- Vibrometer is designed with low natural frequency and accelerometer with high natural frequency.
- So vibrometer is known as low frequency transducer and accelerometer as high frequency transducer.

Seismic instruments

Vibrating body is assumed to have a Harmonic Motion given by

$$y = Y\sin\omega t \quad ---1$$

Eq. of motion for the mass m can be written as



$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad ---2$$

Defining relative displacement as

$$z = x - y \quad --3$$

$$or, \quad x = y + z$$

Substituting this value of x in equation 2

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y\sin\omega t \;---4$$

Steady state solution of eq. 4 is given by

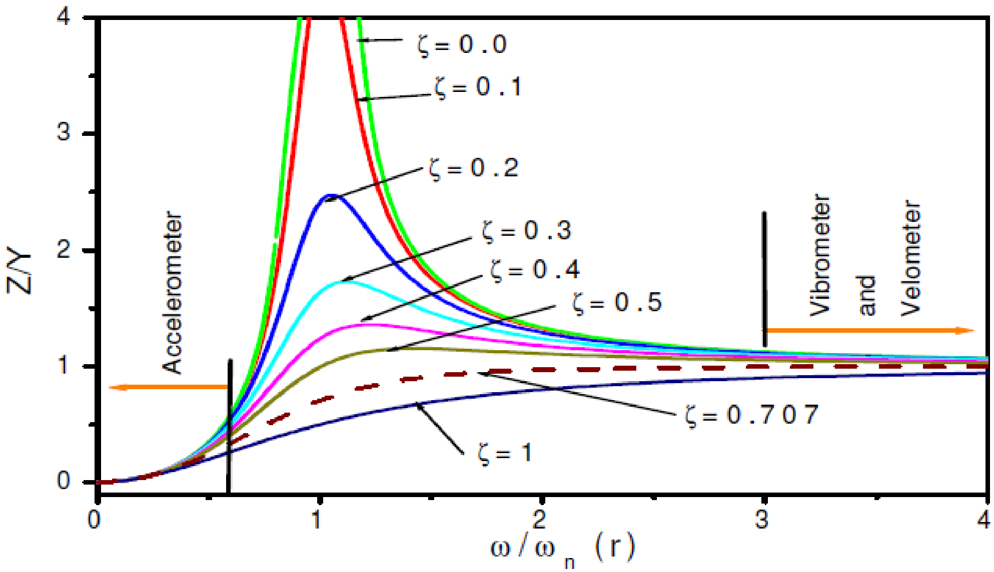
$$z = Z\sin(\omega t - \varphi)$$

$$\frac{Z}{Y}=\frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}=\frac{r^2}{\sqrt{\left[1-r^2\right]^2+\left(2\xi\ r\right)^2}}\quad \text{where}\quad r=\frac{\omega}{\omega_n}$$

$$\varphi=\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]=\tan^{-1}\left[\frac{2\xi\ r}{1-r^2}\right]$$

Vibrometer

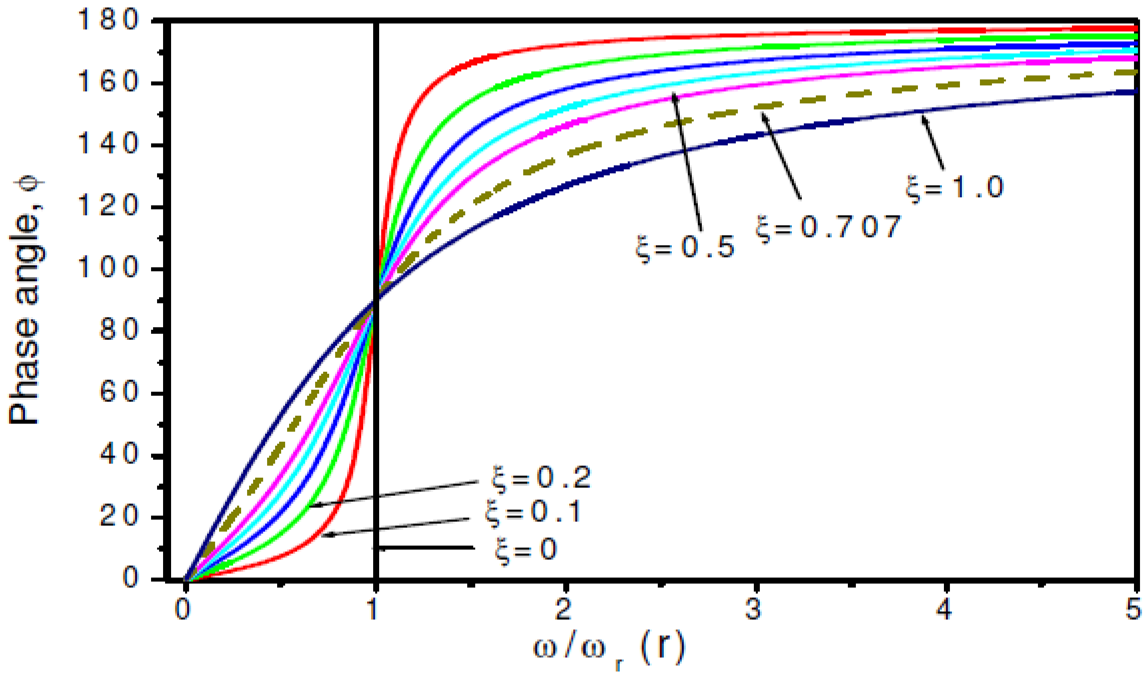
- Let us consider equation $\frac{Z}{B} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\varepsilon\,\omega/\omega_n]^2}}$
- Let us assume $\omega/\omega_n = r$ in the above eqn $\frac{Z}{B} = \frac{r^2}{\sqrt{[1-r^2]^2 + [2\varepsilon r]^2}}$
- Characteristics of this equation is plotted as shown in fig.



From figure, it can be seen that for large values of $\omega/\omega_n = r$, the ratio Z/B approaches unity for every value of damping.

The phase angle plot shown below indicates the phase lag of the seismic mass with respect to vibrating base of machine

$$\varphi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{2\xi\, r}{1-r^2}\right]$$



Vibrometer (Contd..)

Let us consider equation $$\frac{Z}{B} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\varepsilon\,\omega/\omega_n]^2}}$$

Let us assume $\omega/\omega_n = r$ *in the above eqn*

$$\frac{Z}{B} = \frac{r^2}{\sqrt{[1-r^2]^2 + [2\varepsilon r]^2}}$$

When the value of r is very high (more than 3), the above equation can be written as

$$\frac{Z}{B} = \frac{r^2}{\sqrt{[(1-r^2)]^2}} = 1$$

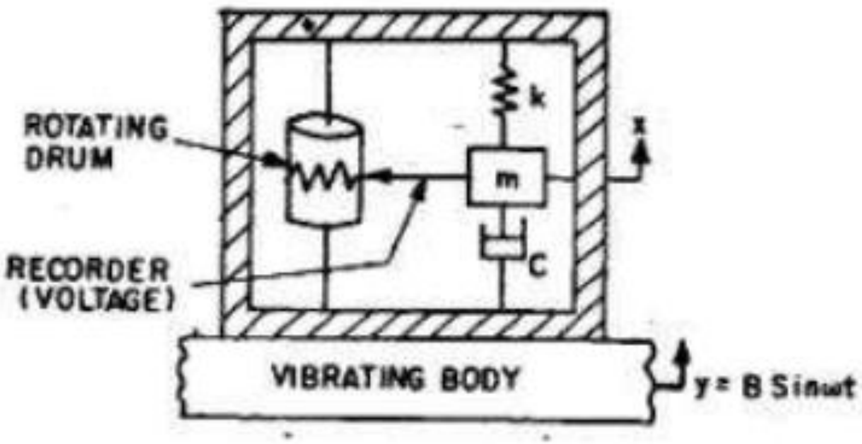
$$Z = B$$

(as 2εr is very small term, so it is neglected for a wide range of damping factors)

- So the relative amplitude Z is shown equal to the amplitude of vibrating body B on the screen.
- Though Z and B are not in the same phase but B being in single harmonic, will result in the output signal as true reproduction of input quantity.

Vibrometer (Contd..)

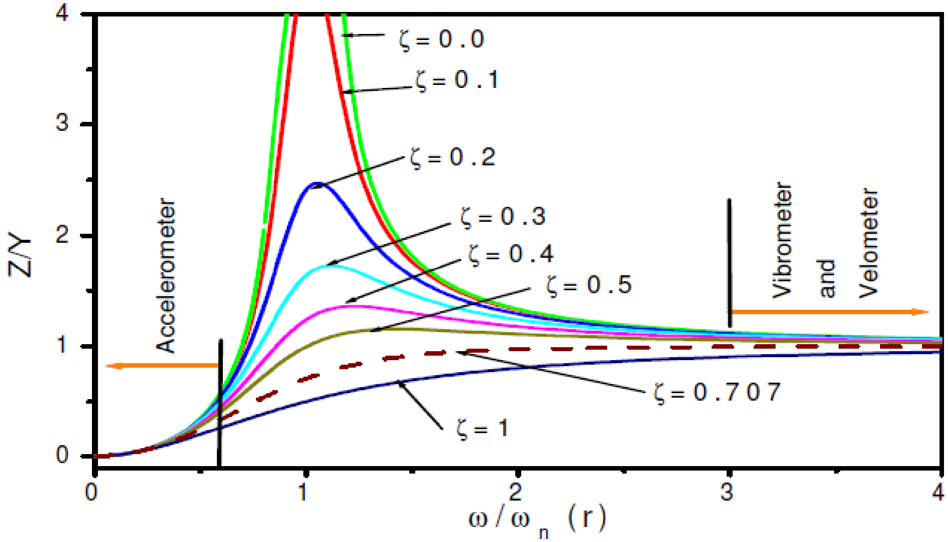
- The instrument shown in figure works as a vibrometer for very large value of r.
- Vibrometer (also known as low frequency transducer) is used to measure the high frequency ω of a vibrating body.
- Since the ratio r is very high, so the natural frequency of the instrument is low.



- Low natural frequency means heavy mass of the body of the instrument which makes its rare application in practice specially in systems which require much sophistication.
- The frequency range of a vibrometer depends upon several factors such as damping, its natural frequency, etc.
- It may have natural frequency 1 Hz to 5 Hz.

Accelerometer

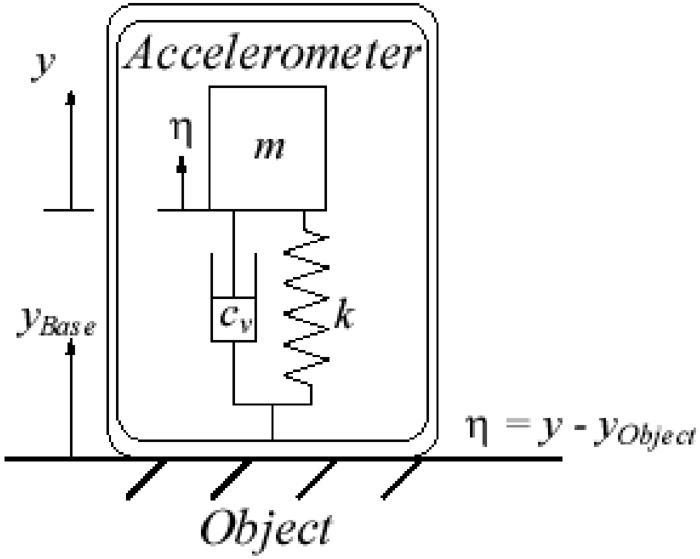
- An accelerometer is used to measure the acceleration of a vibrating body.
- If the natural frequency ω_n of the instrument is very high compared to the frequency ω which is to be measured, the ratio *ω/ω_n is very small. i.e., ω/ω_n=<< 1*
- The range of frequency measurement is shown in figure. Since the natural frequency of the instrument is high so it is very light in construction.



From figure, it can be seen that for large values of ω/ω_n=r, the ratio Z/B approaches unity for every value of damping.

Accelerometer (Contd..)

- With the help of electronics integration devices, it displays velocity and displacement both.
- Because of its small size and usefulness for determining velocity and displacement besides acceleration, it is very widely used as a vibration measuring device and is termed as high frequency transducer.



- The voltage signals obtained from an accelerometer are usually very small which can be pre-amplified to see them bigger in size on oscilloscope.
- For getting velocity and displacement double integration device may be used and the results are obtained on screen.

Again considering equation

$$\frac{Z}{B} = \frac{(\omega/\omega_n)^2}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\varepsilon\,\omega/\omega_n]^2}} \qquad \frac{Z}{B} = \frac{r^2}{\sqrt{[1-r^2]^2 + [2\varepsilon r]^2}}$$

assuming $\omega/\omega_n =<< 1$

$$\frac{Z}{B} = \left(\frac{\omega}{\omega_n}\right)^2 . f \text{ or } Z = \frac{\omega^2 B}{\omega_n^2} f$$

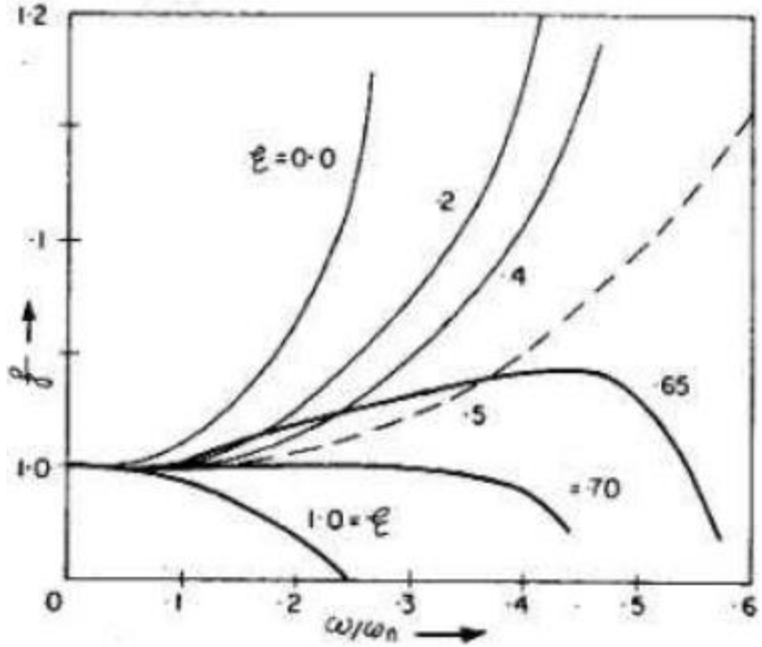
where *f* is a factor which remains constant for the useful range of accelerometer.

Where

$$f = \frac{1}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$$

In this equation $\omega^2 B$, the acceleration of the vibrating body. It is clearly seen that the acceleration is multiplied by a factor $(1/\ \omega_n^2)$.
To keep the value of factor f equal to 1 for *very* high range of ω/ω_n, ratio, ε should be high in value.
The amplitude *Z* becomes proportional to the acceleration provided the natural frequency remains constant.
Thus *Z* is treated proportional to the amplitude of acceleration to be measured

- With the help of equation $f = \frac{1}{\sqrt{(1-r^2)^2 + (2\varepsilon r)^2}}$ figure is drawn to show the linear response of the accelerometer.



- It is seen that for ε =0.7, there is complete linearity for accelerometer for $\omega/\omega_n \leq 0.25$. Thus the instrument with 100 Hz natural frequency will have a useful frequency range from 0 to 26 Hz at ε =0.7 and will provide very accurate results. For this purpose electromagnetic type accelerometers are widely used nowadays.

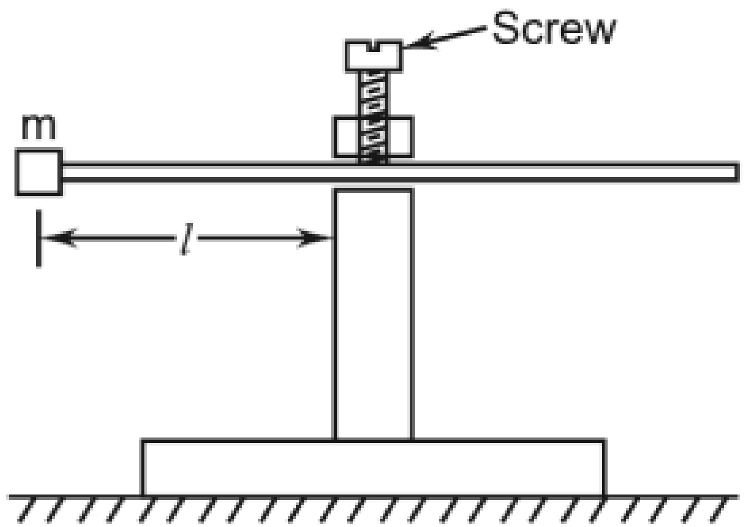
Frequency measuring devices

- The working of frequency measuring instruments is based on the principle of resonance.
- At resonance the amplitude of vibration is found to be maximum and then the excitation frequency is equal to the natural frequency of the instrument.
- Two types of instruments arc discussed here
    - Fullarton Tachometer
    - Fruhm Tachometer

Fullarton Tachometer:

- This instrument is known as single reed instrument.
- It consists of a thin strip carrying small mass attached at one of its free ends.
- The strip is treated *as a* cantilever the length of which is changed by means of a screw mechanism as shown in figure 4.18.

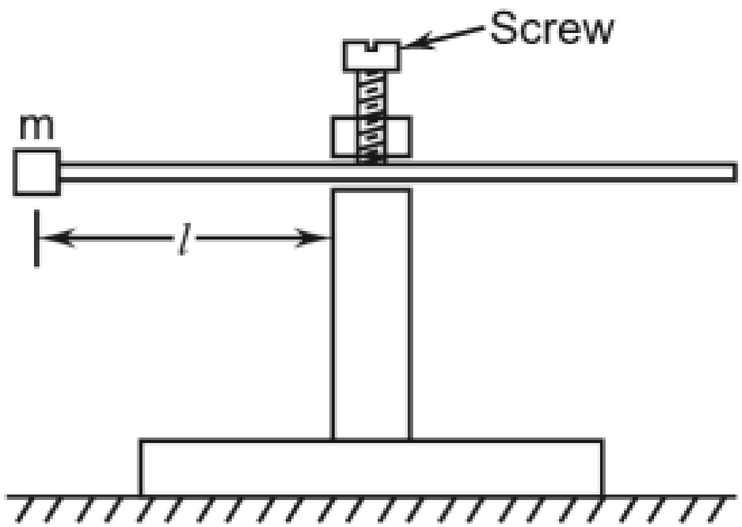
The strip of the instrument is pressed over the vibrating body to find its natural frequency.
We go on changing the length of the strip till amplitude of vibration is maximum.



Fullarton Tachometer (contd..)

- At the instant, the excitation frequency equals the natural frequency of cantilever strip which can be directly seen from the strip itself.
- The strip has different frequencies for its different lengths.
- The natural frequency can be determined with the help of this formula

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{ml^3}}$$



Fruhm Tachometer

- This is also known as multi reed instrument.
- It consists of several reed of known different natural frequencies.
- There may be a known series of frequencies for the reeds.
- Small difference in the frequencies of successive reeds will show more accurate results.
- The instrument is brought in contact with the vibrating body whose frequency is to be measured and one of the reeds will be having maximum amplitude and hence that reed will be showing the frequency of the vibrating body.
- The mathematical analysis involved in the calculation of the natural frequency of the vibrating body with the help of a Frahm's Reed Tachometer is discussed below :

Let m be the mass attached to the end of each reed of length *l* and E be the modulus of elasticity of the reed material

The static deflection of the reed considering it to be a cantilever fixed at one end is given by

$$x_{st} = \frac{mgl^3}{3EI}$$

Where $I = \frac{bd^3}{12}$ = moment of inertia of the reed about the base

We know that, $k\,x_{st} = m\,g$

where k = stiffness of the reed

So natural frequency of the reed = $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{m \cdot g}{x_{st} \cdot m}} = \frac{1}{2\pi}\sqrt{\frac{g}{x_{st}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g \cdot 3EI}{mgl^3}} = \frac{1}{2\pi}\sqrt{\frac{3EI}{ml^3}}$$

Fruhm Tachometer (contd..)

- Thus by having different values of mass *'m'* or length *l* of the reed, we can have a series of reeds with definite known frequencies.
- The one which has a frequency equal to the natural frequency of the vibrating body, vibrates with a large amplitude. Thus the frequency of the vibrating body can be determined easily by knowing the reed with maximum amplitude.
- The accuracy of the instrument depends upon the difference between the value of the natural frequency of the successive reeds.
- The instrument will be more accurate if the difference in the value of frequency is smaller. Refer figure.