Introduction

- Measurement of vibrations can be performed using different kind of vibration measuring instruments such as it is the important area of study in which different parameters of vibratory system or body can be studied and analyze the system.
- The vibrations are desirable as well as undesirable.
- In most of cases vibrations are measured in the field at which the machines or structures are installed and the test to be performed on it known as field tests.

- For the case in which vibrations are desirable, it is important that to know what amount of vibrations are needed to perform the specific task successfully.
- For example, vibration of mobile phone, vibratory shakers, feeders, quartz etc.
- For the case in which the vibrations are undesirable, it is also important that to protect the vibrating system or other nearer machines or structures from its harmful effect, which may cause failure of vibratory system or nearly machine or structure.

Reasons for the measurement of vibrations

- Some machines are running at high speeds which may cause resonant condition and they may get fail.
- In some situations the excessive vibrations may transfer to the nearby machines or structures.
- To check the health of the machines.
- To understand the dynamic behavior it is necessary to measure the vibrations.
- It helps to identify important parameters of system such as mass, stiffness, damping.

Classification of vibration measuring instruments

1. Classification base on contact
   1. Contact type
   2. Non-contact type
2. Classification base on display method
   1. Indicating type
   2. Recording type
3. Classification base on time base measurement
   1. Real time based
   2. Non-real time based
4. Classification base on power source
   1. Active system
   2. Passive system

1. Classification based on contact

I. Contact type

These types of vibration measuring instruments are in direct contact with the vibration machines. These instruments are compact in size.

e.g. Accelerometer

II. Non-contact type

These types of vibration measuring instruments are used when it is very difficult to use the contact type vibration measuring instruments. These types of instruments also small in size.

2. Classification based on display method

I. Indicating type

In these instruments, the measured data are displayed on the display unit of the instruments.

II. Recording type

These instruments are used to display and also to record the data for future analysis.

e.g. FFT analyzer

3. Classification based on time base measurement

I. Real time based

The real time based data can be measured using these instruments. These instruments are working based on microprocessor.

II. Non-real time based

These instruments are not real time based. The measure data can only display on the display unit of the instruments.

4. Classification based on power source

I. Active system

In these instruments, source of power is required to operate the instruments for vibration measurement.

e.g. FFT analyzer

II. Passive system

These instruments do not require any outside source of power to operate the instruments. They are compact, handy and battery operated.

e.g. Frahm’s tachometer

Vibration Measuring Instruments

- Displacement measuring instrument (Vibrometer)
- Velocity measuring instrument (Velometer)
- Acceleration measuring instrument (Accelerometer)

Vibration Analysis of a Mechanical System

- It is well known that the dynamic forces in a vibratory system depend on the displacement, velocity and acceleration components of a system:

  Spring force ∞ displacement

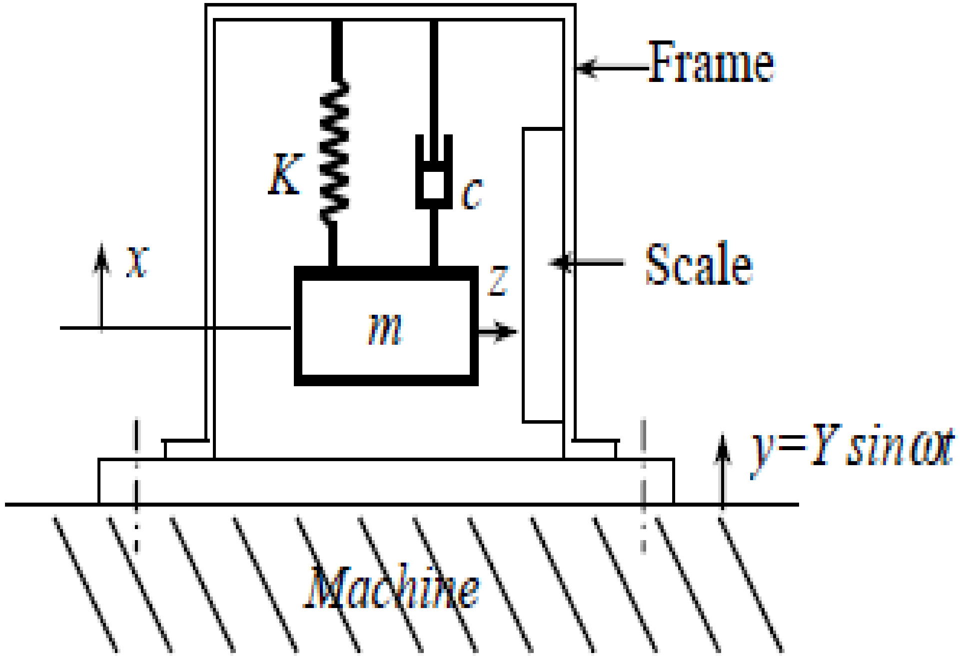
  Damping force ∞ velocity

  Inertia force ∞ acceleration

Vibration Analysis of a Mechanical System

- In vibration analysis of a mechanical system, it is required to measure the displacement, velocity and acceleration components of a system.
- An instrument, which is used to measure these parameters, is referred as vibration measuring instrument or seismic instrument.
- A simple model of seismic instrument is shown in Fig.
- The major requirement of a seismic instrument is to indicate an output, which represents an input such as the displacement amplitude, velocity or acceleration of a vibrating system as close as possible.

Seismic Instrument



Seismic Instrument

- *m-seismic mass*
- *c-damping coefficient of seismic unit*
- *K-stiffness of spring used in seismic unit*
- *x-absolute displacement of seismic mass*
- *y-base excitation (assume SHM)*
- *z=(x-y) displacement of seismic mass relative to frame*
- Seismic instruments are used to measure the displacement, velocity and acceleration components of a vibratory system. Basic theory of Seismic instruments is based on forced vibration considering the vibratory system under base excitation. A single Seismic instrument can be sued as vibrometer, velometrer and accelerometer using suitable calibration.

Derivation

To study the response of the system shown in Fig.5.1, we shall obtain the equation of motion of seismic mass:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + K(x - y) = 0 \qquad (1)$$

$$m\ddot{z} + c\dot{z} + Kz = -m\ddot{y} \qquad (2)$$

Considering base excitation to be SHM:

$$y(t) = Y\, sin\, \omega t \qquad (3)$$

$$m\ddot{z} + c\dot{z} + Kz = m\omega^2 Y \sin \omega t \qquad (4)$$

The above equation represents a equation of motion of a forced vibration with

$$m\omega^2 Y = F$$

Solution of governing differential equation is:

$$z(t) = z_c(t) + z_p(t)$$

z_c is the complimentary solution, which nullifies after some time. The total solution is thus, only steady state solution z_p

Let, the steady state solution of Eqn.(4) is:

$$z(t) = Z \sin(\omega t - \phi) \qquad (5)$$

Derivation

Eqn.(5) has to satisfy Eqn.(4). Substitute Eqn.(5) in (4) and draw force polygon as already studied in forced vibration. The amplitude of steady state vibration is:

$$Z = \frac{m\omega^2 Y}{\sqrt{(K - m\omega^2)^2 + (c\omega)^2}} \tag{6}$$

devide above equation by K

$$Z = \frac{r^2 Y}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \tag{7}$$

substitute eqn.(5.7) in eqn.(5.5)

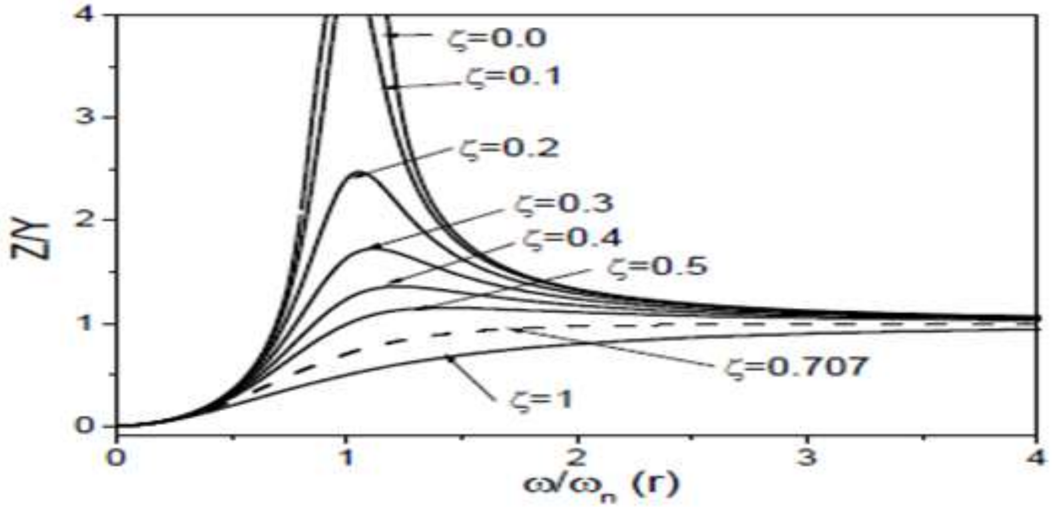
$$z(t) = \frac{r^2 Y}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \sin(\omega t - \phi) \tag{8}$$

the phase angle is:

$$\phi = \tan^{-1}\left(\frac{c\omega}{K - m\omega^2}\right) \tag{9}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right) \tag{10}$$

Vibrometer



Displacement measuring instrument (Vibrometer)

It is an instrument used to measure the displacement of a vibrating system.

In Eqn.(8) if,

$$\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \cong 1 \quad (11)$$

then,

$$z(t) = Y.\sin(\omega t - \phi) \quad (12)$$

Eqn.(11) is the condition for vibrometer.

Vibrometer

- Vibrometer or Seismometer is an instrument which measures the displacement of a vibrating machine or structure.
- It can be classified into following types:

1. Stylus Recording Instrument
2. Seismic Instrument or Seismometer or Vibration Pickup
3. Optical Recording Instrument
4. Simple Potentiometer
5. Capacitance Pickup
6. Mutual Inductance Pickup

Accelerometer

Acceleration measuring instrument (Accelerometer)

It is an instrument used to measure the acceleration of a vibrating system. The response of the seismic mass is given by Eqn.(8). Double differentiating the Eqn.(8), we get.

$$-z(t)\omega^2 = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}(-Y\omega^2\sin(\omega t-\phi)) \quad (13)$$

$$-z(t)\omega_n^2 = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}(-Y\omega^2\sin(\omega t-\phi)) \quad (14)$$

In above equation if

$$\frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \cong 1 \quad (15)$$

Then,

$$-z(t)\omega_n^2 = -Y\omega^2\sin(\omega t-\phi) \quad (16)$$

we have acceleration component of base excitation:

Eqn.(15) is the condition for accelerometer.

Accelerometer

- Acceleration pick-up or accelerometer is an instrument that measures the acceleration of a vibrating body.
- Seismometer can be used as an acceleration measuring instrument, if it satisfies the following conditions:

1. It's natural frequency should be very high.
2. It should generate output signal proportional to the relative acceleration of the vibrating body.

Numerical Problem

- A seismic instrument is mounted on a machine running at 1000 rpm. The natural frequency of the seismic instrument is 20 rad/sec. the instrument records relative amplitude of 0.5 mm. Compute the displacement, velocity and acceleration of the machine. Neglect the damping in seismic instrument.

Numerical Problem

Given data

ω_n=20 rad/s, ζ=0

Speed of the machine (N) = 1000 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1000)}{60}$$

=104.72 rad/s

Frequency ratio

$$r = \frac{\omega}{\omega_n} = \frac{104.72}{20} = 5.23$$

For seismic instrument

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

Numerical Problem

For the given system damping is neglected

$$Y = \frac{Z}{1.042} = \frac{0.5}{1.042} = 0.48mm$$

$$\frac{Z}{Y} = \frac{5.23^2}{1 - 5.23^2} = 1.042$$

Displacement of the machine:

$$Y = \frac{Z}{1.042} = \frac{0.5}{1.042} = 0.48mm$$

Velocity of the machine:

$$\omega.Y = (104.72)\ 0.48 = 50.26 \text{ mm/s}$$

Acceleration of the machine:

$$\omega^2.Y = (104.72)^2\ 0.48 = 5263.81 \text{ mm/s}^2$$

Numerical Problem

- A seismic instrument has natural frequency of 6 Hz. What is the lowest frequency beyond which the amplitude can be measured within 2% error. Neglect damping

Numerical Problem

Given data

$\omega_n = 6$ Hz, $\xi = 0$ and error $= 2\%$

Damping is neglected for given system

$$\frac{Z}{Y} = \frac{r^2}{1 - r^2}$$

$$\text{Error} = \frac{Z - Y}{Y} = 0.02$$

$$Z = Y + 0.02\ Y = 1.02\ Y$$

$$\frac{Z}{Y} = 1.02 = \frac{r^2}{1 - r^2}$$

$$1.02 - 1.02r^2 = r^2$$

$$r = 0.7034$$

The lowest frequency beyond which the amplitude can be measured within 2% error is:

$$\omega = r.\ \omega_n$$

$$\omega = (0.7034)\ 6$$

$$\omega = 4.22\ \text{Hz}$$

Conclusion

- Seismic instruments are used to measure the displacement, velocity and acceleration components of a vibratory system. Basic theory of Seismic instruments is based on forced vibration considering the vibratory system under base excitation. A single Seismic instrument can be sued as vibrometer, velometrer and accelerometer using suitable calibration.