

# **Vibration Measurement**

## **Vibration**

### ***Definition***

Basically, vibration is oscillating motion of a particle or body about a fixed reference point. Such motion may be simple harmonic (sinusoidal) or complex (non-sinusoidal). It can also occur in various modes - such as bending or translational modes - and, since the vibration can occur in more than one mode simultaneously, its analysis can be difficult.

### ***Units of vibration***

The units of vibration depend on the vibrational parameter, as follows:

- a) acceleration, measured in  $g$  or  $[m/s^2]$  ;
- b) velocity, measured in  $[m/s]$  ;
- c) displacement, measured in  $[m]$ .

### ***Some effects of vibration***

Vibration can cause damage to structures and machine sub-assemblies, resulting in mis-operation, excessive wear, or even fatigue failure.

Vibration may have adverse effects on human beings. The primary effects are task-performance interference; motion sickness; breathing and speech disturbance; and a hand-tool disease known as “white finger”, where the nerves in the fingers are permanently damaged, resulting in loss of touch sensitivity.

### ***Characteristics of vibration***

Vibration may be characterised by :

- a) the frequency in Hz;
- b) the amplitude of the measured parameter, which may be displacement, velocity, or acceleration.  
This is normally referred to as the vibration amplitude when expressed in units, but vibration level when expressed in decibels.

### ***Decibel notation applied to vibration measurement***

Because of the wide range of vibration amplitudes found in engineering, it is convenient to express the measured amplitude in decibels with reference to a fixed value. Reference values which are internationally accepted are as follows:

- a) for velocity, the reference is  $10^{-3}$  m/s ;
- b) for acceleration, the reference is  $10^{-5}$  m/s<sup>2</sup>.

Thus if the measured amplitude is  $A_i$  and the reference amplitude is  $A_0$ , the vibration level expressed in decibels is :

$$\text{vibration level} = 20 \log_{10} \frac{A_1}{A_0} \text{ dB}$$

**Example 11.1** If the measured vibrational acceleration amplitude of a vibrational body is 2g, express this in dB ref.  $10^{-5} \text{m/s}^2$ .

We have  $g = 9.81 \text{m/s}^2$ ,

$$\begin{aligned} \text{acceleration vibration level} &= 20 \log_{10} \frac{A_1}{A_0} \\ &= 20 \log_{10} \frac{2 \times 9.81 \text{ m/s}^2}{10^{-5} \text{ m/s}^2} \\ &= 125.8 \text{ dB} \end{aligned}$$

In practice, this may be rounded off to 126dB.

**Example 11.2**

If a mechanism has a vibrational velocity amplitude of 3.123m/s, express this in dB ref.  $10^{-3} \text{m/s}$ .

$$\begin{aligned} \text{velocity vibration level} &= 20 \log_{10} \frac{A_1}{A_0} \\ &= 20 \log_{10} \frac{3.123 \text{ m/s}}{10^{-3} \text{ m/s}} \\ &= 69.89 \text{ dB} \end{aligned}$$

In practice this would be rounded off to 70dB.

**Relationship between the vibration parameters**

Assuming that the vibration is simple harmonic motion, then

$$\begin{aligned} \text{displacement} & \quad x = A \sin \omega t \\ \text{velocity} & \quad v = A\omega \cos \omega t \\ \text{acceleration} & \quad a = -A\omega^2 \sin \omega t \end{aligned}$$

where

$$\begin{aligned} \omega &= 2\pi f \text{ rad/s} \\ f &= \text{frequency of vibration in Hz} \end{aligned}$$

Note that the frequencies are the same in each case, although there is a phase shift. The amplitudes of the above parameters are thus

$$\begin{aligned} \text{displacement amplitude} &= A \\ \text{velocity amplitude} &= A\omega \end{aligned}$$

$$\text{acceleration amplitude} = A\omega^2$$

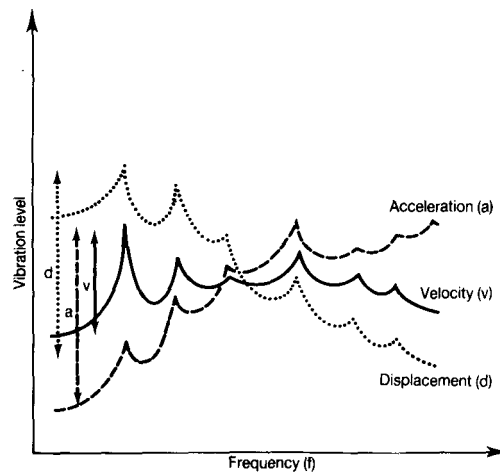
### **Which parameter?**

The choice of the best parameter to be measured depends on a number factors, including

- a) the type and size of the transducer available,
- b) the mass of the vibrating structure, and
- c) the frequency and amplitude characteristics of the vibration.

If the velocity, acceleration, and displacement amplitudes measured at various frequencies, the resulting graphs of amplitude vs. frequency are referred to as the vibration spectra, and the shape of graphs are referred to as the spectral shapes.

With instrumentation based on accelerometer transducers and integrator amplifiers, the user is free to choose between acceleration, velocity and displacement as the measurement parameter. The typical vibration spectra shown in Figure 1 are displays of the three parameters of a machine's vibration. Although they each have different average slopes their peaks occur at the same frequencies. In the example shown the amplitude range required to display the velocity spectrum is the smallest and thus occupies the least dynamic range. In addition it means that all the frequency components on this curve need a smaller relative change before they begin to influence the overall vibration level. The low frequency acceleration and high frequency displacement components of the spectra shown in Figure 1 need to exhibit much larger changes before they influence the overall vibration level. In general it is therefore advisable to display in turn each of the three parameters and choose the one which has the flattest spectrum. This will enable one to detect machine faults, which produce an increase in vibration level, at an early stage. In practice the velocity-frequency spectra of many industrial machines are shaped this way, i.e. they are quite flat over a wide range of frequencies. Since it is also a measure of vibrational energy present, the velocity parameter is the one in most common use and its use is supported by American and European standardisation organisations.



**Figure 1 : Vibration Spectra**

### **Effect of the transducer on the vibrating structure**

In general, the larger the mass of the vibration transducer, the greater its sensitivity. Unfortunately, the addition of the transducer's mass ( $m_1$ ) to the mass ( $m_0$ ) of the vibrating structure changes the resonant frequency of the vibrating system as follows:

$$\frac{f_1}{f_0} = \sqrt{\frac{m_0}{m_0 + m_1}}$$

where  $f_1$  = resonant frequency of the structure with the mass added

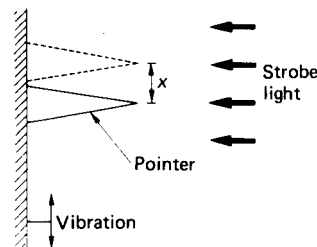
and  $f_0$  = resonant frequency of the structure before the transducer is added.

## Vibration-measuring devices

### Vibration Transducers

#### The Stroboscope Method

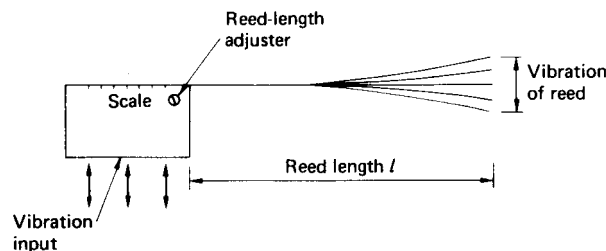
The fixed pointer or stud, shown in Figure 2, is attached to the vibrating surface and is used to give an indication of the displacement only. By using the light of a stroboscope to “freeze” or “slowly move” the stud, quite high-frequency small-amplitude vibrations may be measured. The typical upper range of frequency is quoted at 500 Hz for direct measurement.



**Figure 2 : The Stroboscope Method**

#### The Reed Vibrometer

The variable-length reed vibrometer shown in Figure 3 is used to measure the main frequency component of the vibration. In practice the length  $l$  is adjusted until the maximum reed vibration occurs, when its resonant frequency is the same as the frequency of the vibrating mechanism or structure. The length  $l$  is calibrated directly in Hz. A small mass may be added to the cantilever if the vibrometer is to be used for very-low frequency investigation, but the scale readings would then need to be corrected for the additional mass. The range of measurement is quoted as 5 Hz to 10kHz.



**Figure 3 : The Reed Vibrometer**

#### The Seismic-Mass Transducer

In instrumentation, seismic pickups are used to measure the motion of the surfaces to which they are fixed. They are sensitive to motion along one axis only, so if the motion is three dimensional, three

seismic pickups are needed to determine the components of the motion along three mutually perpendicular axes. The principal features of a seismic pickup are shown diagrammatically in Figure 4. The essential component is the seismic mass. This is a body of metal, suspended from a resilient support. This is a support whose deflection is proportional to the force applied to it. The inertia of the seismic mass causes it to lag behind the motion of the casing when the casing is accelerated, causing a deflection in the support. This deflection forms the input to a transducer, which produces a proportional output signal. In Figure 4 the transducer is represented by a potentiometer, but any suitable type of transducer may be used.

The damping shown in Figure 4 may consist only of the hysteresis of the support material, or it may be increased by filling the casing with a silicone fluid of suitable viscosity for example.

By choosing suitable values for the mass, the stiffness of the support and the damping, and by using an appropriate transducer, the same basic arrangement of seismic pickup can be designed as a displacement pickup, a velocity pickup or an acceleration pickup (accelerometer). The seismic pickup is essentially a damped spring-mass system, and will have a natural frequency of vibration given by:

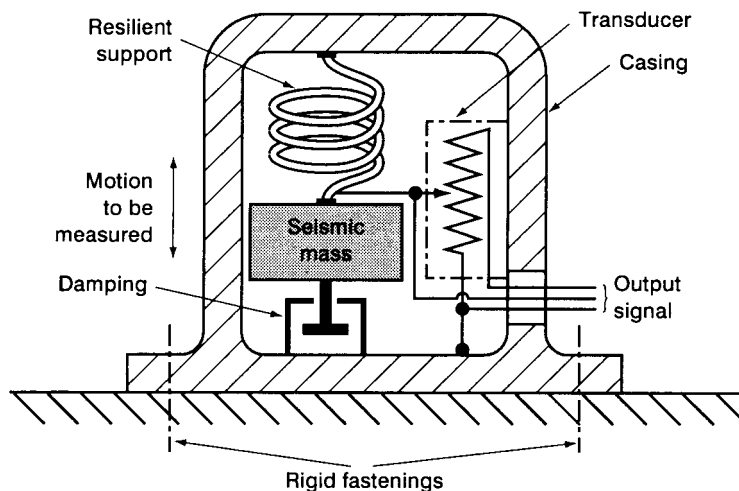
$$\omega_n = \sqrt{\frac{\lambda}{m}}$$

where

$\omega_n$  is the natural angular frequency (rad/s)

$\lambda$  is the spring stiffness (N/m)

$m$  is the mass (kg).



**Figure 4 : Seismic Mass Transducer**

### Displacement Pickups

This type of pickup is used to measure the displacement of a vibrating body when there is no fixed reference point available, for example in determining the movement of the chassis of a vehicle. We therefore want the seismic mass to behave (as far as possible) as though it was fixed in space. This can be arranged by using a relatively large seismic mass and a relatively 'floppy' resilient support. This gives a low value of  $\omega_n$  to the spring-mass system. Figure 5 shows the frequency response of such a pickup with various values of damping ratio  $\zeta$ .

$$\zeta = \frac{\text{Actual Damping}}{\text{Critical Damping}}$$

Critical damping is the value of damping which, if the mass is displaced from its equilibrium position and released, allows it to return in the shortest possible time without overshooting. If the actual damping is greater than critical ( $\zeta > 1$ ) the mass returns more slowly, again without overshooting. If the actual damping is less than critical ( $\zeta < 1$ ) the mass returns more quickly, but overshoots and oscillates about the equilibrium position with a decaying oscillation.

Figure 5(a) shows that for frequencies of vibration well above  $\omega_n$  the displacement of the seismic mass relative to the casing is practically equal to the displacement applied to the casing, while Figure 5(b) shows that those displacements will be nearly  $180^\circ$  out of phase with each other. This means that as the casing moves in one direction, the seismic mass moves in the opposite direction relative to it - it virtually stands still. Heavier damping reduces the phase lag somewhat from  $180^\circ$  but this is not usually important since we are usually more interested in the amplitude of a displacement than its phase angle. It can be shown mathematically that  $\zeta = 0.707$  gives the least variation of displacement ratio for values of  $(\omega/\omega_n) > 1$  (see Figure 5(a)). At this value of  $\zeta$  we can bring  $(\omega/\omega_n)$  down to about 1.75 before the error in displacement measurement exceeds 5%, so displacement pickups are often designed to have a damping ratio of about 0.7.

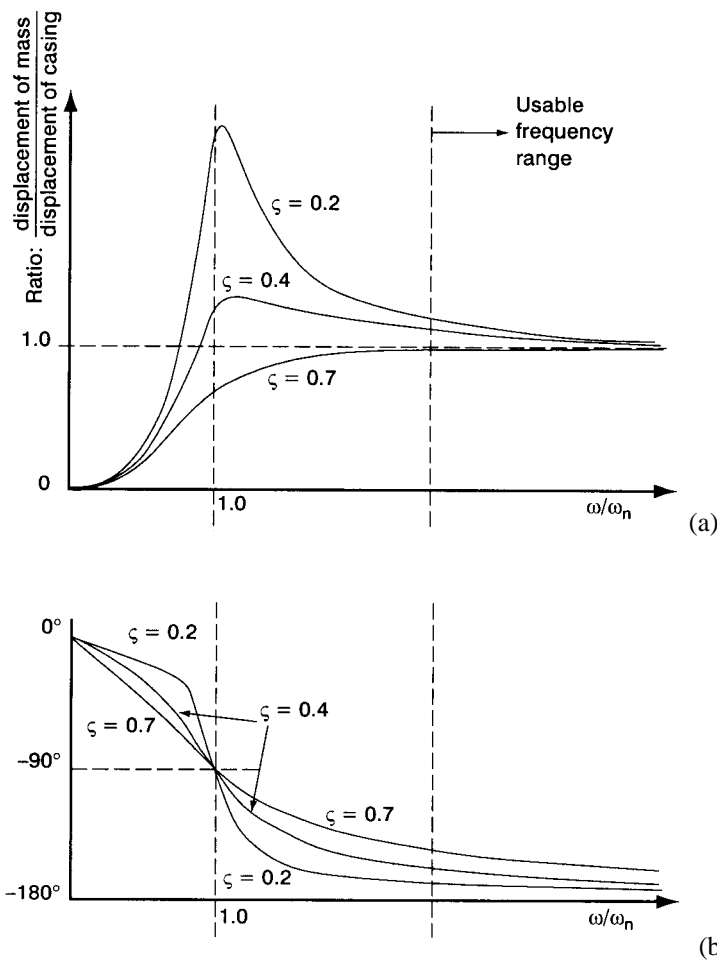


Figure 5 : Frequency response of a seismic displacement pickup (a) amplitude (b) phase

### Velocity Pickups

A signal proportional to velocity may be obtained from a vibration by:

- 1 differentiating the signal from a displacement pickup by passing it through a differentiating circuit
- 2 integrating the signal from an accelerometer by passing it through an integrating circuit
- 3 using a seismic velocity pickup. This is similar in principle to Figure 4 but with a velocity transducer in place of the displacement transducer.

Integrating from an accelerometer gives much more accurate results than differentiating from a displacement pickup, because differentiation amplifies any errors in the signal, whereas integration diminishes them. However, a velocity pickup gives a velocity signal directly, and this can be passed through an integrating circuit to give a displacement signal as well if required.

A velocity pickup is designed like a displacement pickup, to have a low value of  $\omega_n$  and to operate at angular frequencies well above  $\omega_n$ , so that the motion of the seismic mass is virtually the same as that of the casing but (almost) opposite in phase. The transducer is usually a coil of wire carried by the seismic mass. The coil is suspended in a radial magnetic field so that a voltage proportional to velocity is generated in the coil when it is vibrated axially. Figure 6 shows the construction of a typical velocity pickup. The seismic mass consists mainly of a central rod with its associated nuts, washers and coil former. The rod connects together two flexible diaphragms, whose stiffness' add to form the 'spring'. The coil former suspends the coil in a narrow annular slot in a cylindrical magnet, the magnetic field acting radially across the slot. The coil former may be made of metal, so that eddy current damping is provided.

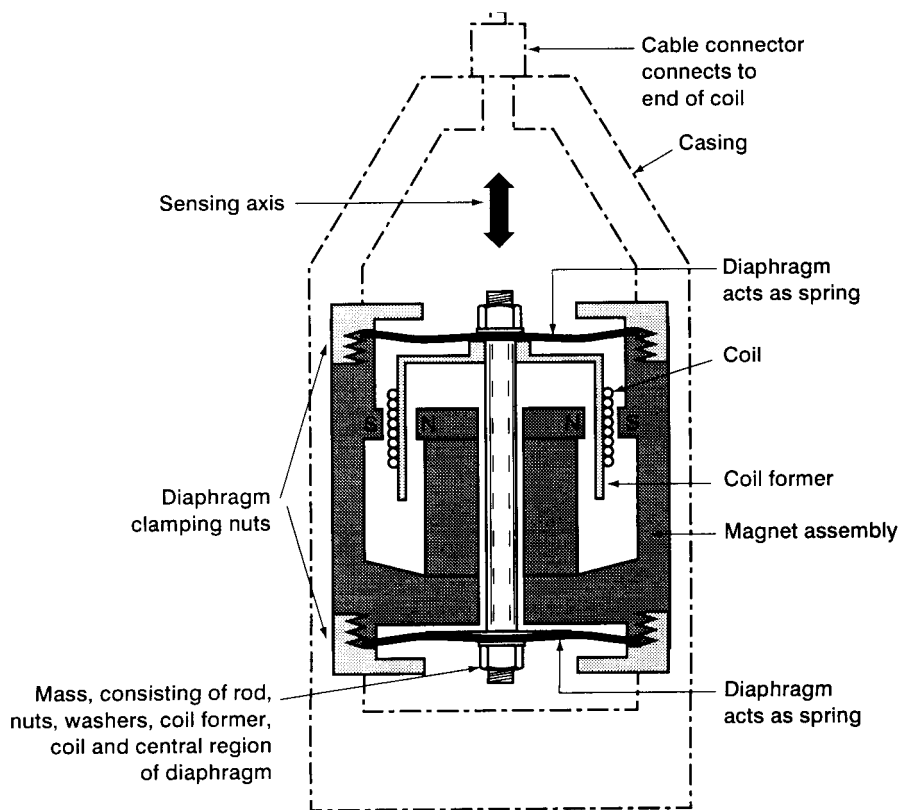


Figure 6 : Seismic Velocity Pickup

## Acceleration Pickups (accelerometers)

We have seen how, by designing the pickup system of Figure 4 to have a low value of  $\omega_n$  we can use it as a displacement pickup or a velocity pickup for angular frequencies well above  $\omega_n$ . To design it as an acceleration pickup we must go to the opposite extreme.

Figure 5 (a) shows that at angular frequencies well *below*  $\omega_n$  the displacement of the seismic mass relative to the casing tends to zero. Therefore at these much lower frequencies the seismic mass must be accelerating with the same acceleration as the casing. To give it these accelerations, corresponding forces must be applied by the spring because:

$$\text{force} = \text{mass} \times \text{acceleration.}$$

Therefore we can use the spring as a transducer, to tell us the force applied to the mass, its acceleration, and hence the acceleration of the casing. The graphs of Figure 7 show the ratio (acceleration of seismic mass)/(acceleration of casing) plotted against  $\omega/\omega_n$  on logarithmic scales, for various values of damping ratio  $\zeta$ . Because the horizontal scale is logarithmic, the left-hand end of the curves may be extrapolated to an infinitely small value of  $\omega/\omega_n$  the acceleration ratio remaining constant at 1.0.

The curves indicate that provided the damping ratio does not exceed 1.0, a seismic accelerometer will give accurate readings of acceleration for frequencies of vibration from zero up to about 0.2 of its undamped natural frequency. For heavier damping than this, the upper frequency limit will be somewhat less. Most accelerometers, however, use a piezoelectric crystal as a combined 'spring' and transducer, and the damping ratio of a crystal is almost zero - in fact the frequency response of a piezoelectric accelerometer can be assumed to be that shown by the curve for  $\zeta = 0.01$ . The ideal damping ratio would be  $\zeta = 0.7$  as this would allow accurate measurement up to about 0.5 of the undamped natural frequency. Clearly, to obtain the widest range of usable frequency response we want an accelerometer with the highest possible value of undamped natural frequency, and referring to the formula

$$\omega_n = \sqrt{\frac{\lambda}{m}}$$

we can see that this is given by a spring-mass system with a high value of spring stiffness,  $\lambda$ , and a low value of mass,  $m$ . For this reason a piezoelectric crystal is usually employed as the connection between the seismic mass and the casing, because it has a very high modulus of elasticity and so a very high spring stiffness. However it has the disadvantage that very low frequencies of vibration give time for the charge on the crystal to start to leak away, so there is a low frequency limit (usually about 5 Hz) below which the output of a piezoelectric accelerometer is unreliable. For even lower frequencies, and for the measurement of slowly varying or steady accelerations, some other form of spring and transducer must be used.



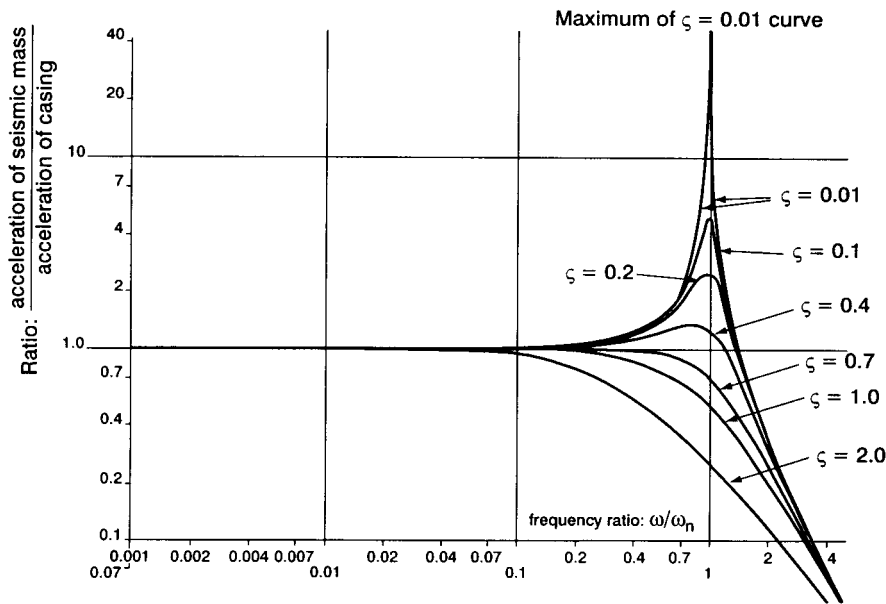


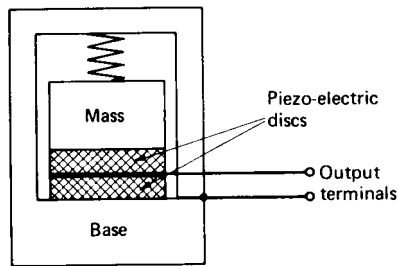
Figure 7 : Frequency Response of a Seismic Accelerometer

## Accelerometers

### Piezoelectric accelerometers

Tension cannot be applied to a crystal without using some kind of adhesive to make the tensile connection, and such a connection would be unreliable, so in the simplest form of piezoelectric accelerometer the crystal is kept permanently compressed by the seismic mass. Thus the effect of accelerations in alternate directions is an alternating increase and decrease in the compressive force on the crystal. The compressive pre-load is applied by screwing the seismic mass down on to the crystal to a given torque. The electrical connections to the crystal are made by metallising (depositing a thin film of metal on) the end faces. This type of construction gives an accelerometer which is rugged, but because the casing is part of the 'spring' in the spring-mass system, it may be subject to spurious inputs. These include temperature change (causing expansion or contraction of the casing), acoustic noise, base bending (distorting the casing), cross-axis motion, and magnetic fields.

One of the difficulties of measuring accelerations on a light, flexible structure is that the mass of the accelerometer may alter the frequency and amplitude of the vibration we are trying to measure. For such applications we need an accelerometer of the smallest and lightest possible type - a piezoelectric shear type. In this type of accelerometer a ring of piezoelectric material is bonded to a central pillar. The seismic mass is a metal ring bonded to the outside of the piezoelectric material. Acceleration in the direction of the sensing axis causes shear stresses in the piezoelectric material, which is arranged so that corresponding values of electrical charge appear between the central pillar and the seismic ring. Because the spring-mass system in this type of accelerometer consists only of the seismic ring and the piezoelectric material it is not sensitive to stresses caused by distortion of the casing or the base. Units can therefore be made very much lighter - one design has an outside diameter of 4 mm and a total mass of 0.14 g. Decreasing the seismic mass in this way has the advantage of increasing the undamped natural frequency, and hence of increasing the frequency range of the accelerometer, but it has the disadvantage of seriously reducing sensitivity, because the electrical output comes from the work done by the seismic mass. However, a reduced sensitivity may be acceptable if the accelerometer is only being used to find the natural frequency of vibration of a structure by finding the frequency of excitation at which the output of the accelerometer is a maximum.



**Figure 8 : Piezo-Electric Accelerometer**

### *Types of low-frequency accelerometer*

Strain gauges may be used as the transducer system; the strain gauges can be of either the unbonded type or the bonded type. Bonded strain gauges are usually applied to a thin flexible beam or cantilever which acts as the spring supporting the seismic mass. They are connected into a bridge circuit and positioned so as to have maximum sensitivity to accelerations along the sensing axis, minimum sensitivity to transverse accelerations, and to cancel out temperature effects. Damping is usually by means of silicone oil, to give a damping ratio of 0.6 to 0.8.

Other forms of transducer which may be used in accelerometers are the potentiometer, the linear variable differential transformer (LVDT), and the differential capacitor.

### *Servo accelerometers*

Servo accelerometers (another name for them is null-balance accelerometers) are used in preference to other types of accelerometer where greater accuracy is required. In the usual form of servo accelerometer the seismic mass is attached to the casing by material which has been thinned down, by machining, to make it flexible enough to act as a hinge. The seismic mass is maintained in an almost constant position relative to the casing by an automatic control system. This controls the position of the seismic mass by adjusting the current through an electromagnet which consists of a pair of coils attached to the seismic mass, and annular permanent magnets fixed to the casing. The current through the coils is also passed through a sensing resistance, R, so that an output voltage proportional to the acceleration is obtained from the voltage drop across R. The position of the seismic mass relative to the casing is sensed by an inductive or capacitive displacement transducer, the output of which is amplified and applied to the electromagnet to provide the restoring force.

Servo accelerometers have the following advantages over other types of accelerometer:

1. Because the mechanical spring is replaced by an electrical 'spring', linearity is improved and hysteresis eliminated.
2. Damping can be built into the characteristics of the electrical circuit and can therefore be made less sensitive to temperature change.
3. By introducing an offset current from an external source through the electromagnet coils, the servo accelerometer can be used as an acceleration controller.
4. Similarly, by means of offset currents, the static and dynamic performance of the accelerometer can be checked out before the start of expensive tests on a vehicle.

### *The calibration of accelerometers*

Accelerometers for the measurement of steady or slowly varying accelerations may be calibrated up to an acceleration of +1 g (the standard value of g is  $9.80665 \text{ m/s}^2$ ) by using the earth's gravitational

attraction. The accelerometer is mounted on a tilting table from which the angle  $\theta$  between the sensing axis and the vertical can be measured. At  $\theta = 0$  the force of gravity on the seismic mass is the same as the inertia force due to an acceleration of  $9.8 \text{ m/s}^2$ . At any other angle of  $\theta$  the corresponding acceleration is  $9.8 \cos \theta \text{ m/s}^2$ . For accurate calibration the true value of  $g$  at the location where the calibration is taking place should be used. The standard value, given above, is approximately correct for temperate latitudes, but  $g$  varies from  $9.832 \text{ m/s}^2$  at the poles to  $9.780 \text{ m/s}^2$  at the equator.

Some steady-state accelerometers have provision for applying known forces to the seismic mass along the sensing axis, by means of weights, so that if the value of the seismic mass is known, the accelerometer can be calibrated for accelerations greater than  $g$  by applying the equivalent of the inertia force. If the construction of the accelerometer does not permit this it may be mounted on a turntable so that its sensing axis is radial; the turntable is then run at known angular velocities of  $\omega \text{ rad/s}$ , so that known centripetal accelerations of  $\omega^2 r \text{ m/s}^2$  are applied, where  $r$  is the radius in meters to the center of the seismic mass.

Piezoelectric accelerometers cannot usually be calibrated by means of static loadings because their charge leaks away, although if the piezoelectric material is quartz the time constant of the leakage may be several days due to its high electrical insulation. It is usual, however, to calibrate piezoelectric accelerometers by shaking them with simple harmonic motion along the sensing axis, by means of an electro-mechanical exciter. For a primary calibration the amplitude of the motion is measured by means of an interferometer, using a laser as the light source and a phototransistor to convert the interference fringes into electrical pulses. By this means both the amplitude,  $x$ , and the angular frequency,  $\omega$ , of the motion may be accurately measured; the amplitude of the acceleration is then  $\omega^2 x$ .

For a secondary calibration, the accelerometer to be calibrated is mounted 'back-to-back' with one which has already been calibrated to act as a transfer standard, and the same simple harmonic motion is applied by the exciter to both. The acceleration applied to the accelerometer to be calibrated is then read from the one which has been previously calibrated.

### **Comparison of Vibration-Measuring Systems**

Table 1 compares some features of complete vibration-measuring systems and reveals that the accelerometer system, although the most expensive, covers the widest range of frequencies and vibration levels.

<b>Transducer</b>	<b>Parameter</b>	<b>Signal conditioner</b>	<b>Frequency range</b>	<b>Remarks</b>
Capacitive Inductive	Displacement	Amplitude modulation with bridge circuits	$0-0.1 f_c$ $f_c = \text{carrier frequency}$	Usually relative displacement only
Electro-magnetic	Velocity	May need an amplifier	15 to 1000Hz	Poor low frequency response
Piezo-electric	Acceleration	Charge amplifier	$0-0.3 f_n$ ( $f_n$ , the natural frequency, is typically 22 kHz)	Wide range of measurement. Typical $\pm 10000g$

**Table 1 : Comparison of vibration-measuring systems**

## Shock Measurement

Shock describes a type of motion where a moving body is brought suddenly to rest, often because of a collision. This is very common in industrial situations and usually involves a body being dropped and hitting the floor.

Shocks characteristically involve large-magnitude decelerations (e.g. 500g) which last for a very short time (e.g. 5ms). An instrument having a very high-frequency response is required for shock measurement, and for this reason, piezo-electric crystal-based accelerometers are commonly used. Again, other elements for analysing and recording the signal are required and described in the previous section. A storage oscilloscope is a suitable instrument for recording the output signal, as this allows the time duration as well as the acceleration levels in the shock to be measured. Alternatively, if a permanent record is required, the screen of a standard oscilloscope can be photographed. A further option is to record the output on magnetic tape, which facilitates computerised signal analysis.

### *Example*

A body is dropped from a height of 10 m and suffers a shock when it hits the ground. If the duration of the shock is 5 ms, calculate the magnitude of the shock in terms of g.

The equation of motion for a body falling under gravity gives the following expression for the terminal velocity,  $v$ :

$$v = (2gx)^{1/2}$$

where  $x$  is the height through which the body falls. Having calculated  $v$ , the average deceleration during the collision can be calculated as:

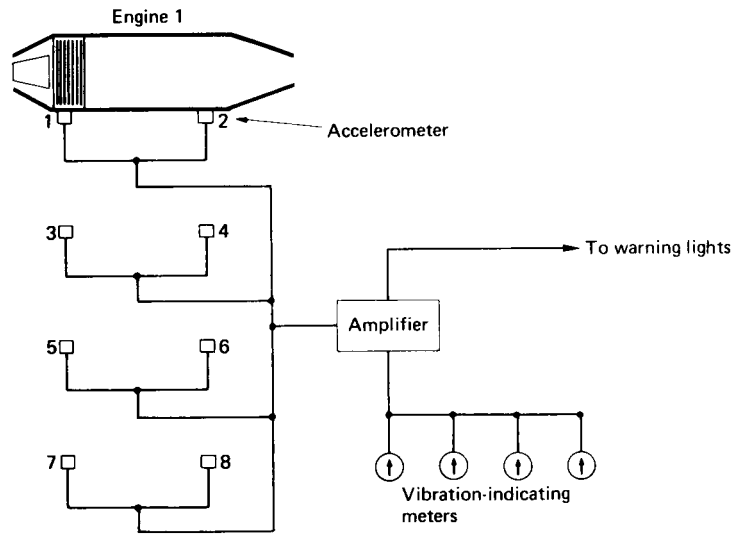
$$a = v/t$$

where  $t$  is the time duration of the shock. Substituting the appropriate numerical values into these expressions:

$$\begin{aligned}v &= (2 \times 9.81 \times 10)^{1/2} = 14.0 \text{ m/s} \\a &= 14.0/0.005 = 2801 \text{ m/s} = 286 \text{ g}\end{aligned}$$

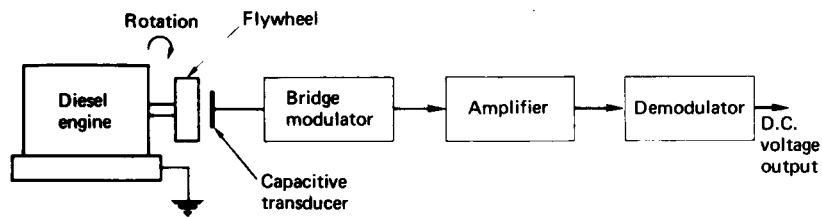
## Complete Vibration-Measuring Systems

An aircraft-engine vibration-measuring system is shown in Figure 9. The system uses four indicators and a switching device which permits the output signal of each transducer to be read individually. Four signal lamps indicate if the vibration levels exceed a pre-set alarm level.



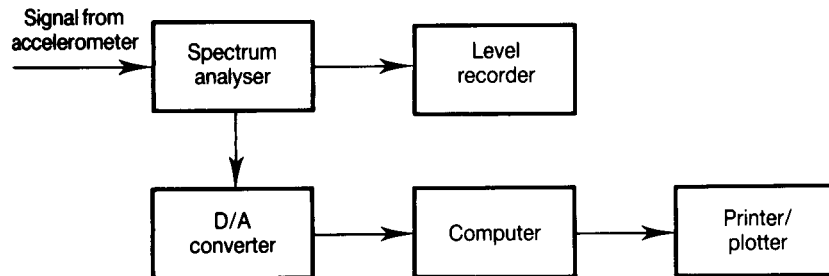
**Figure 9 : Aircraft Engine Vibration Measurement**

Figure 10 shows how capacitive (or inductive) transducers may be used to measure the axial vibration of a diesel engine. The system uses an amplitude-modulated system made up of an a.c.-excited capacitive bridge circuit, an amplifier, and a demodulator unit.



**Figure 10 : Measurement of Axial Vibration in a Diesel Engine**

A comprehensive vibration-measuring and analysing system is shown in Figure 11. The signal from the accelerometer and charge amplifier is frequency-analysed by a spectrum analyser. This unit gives two analogue outputs. One output provides the level recorder with an input signal for subsequent analysis, and the other output is converted into digital information for use with a computer.



**Figure 11 : Schematic of Complete Vibration Measuring System**

## *Exercises on Vibration Measurement*

- 1 Show by means of a simple labeled diagram the essential features of a seismic pickup. List the three main types of seismic pickup.
- 2 For a damped spring-mass system, sketch curves showing how the ratio (displacement of mass)/(displacement of support) varies with the exciting frequency, for various values of damping ratio. Hence show the usable frequency range of a seismic displacement pickup in relation to the undamped natural frequency of the spring-mass combination.
- 3 State three ways in which a signal proportional to the velocity of a vibration may be obtained, listing them in order of preference (assume that the necessary transducers are available) and giving your reasons for placing them in that order.
- 4 Draw a labeled diagram showing a section through a velocity pickup and explain how the signal is obtained. Is its usable frequency range above or below the undamped natural frequency of the spring-mass combination?
- 5
  - a) Describe how the seismic mass and crystal are arranged in:
    - i) an accelerometer in which piezoelectric material is in compression,
    - ii) an accelerometer in which piezoelectric material is in shear.
  - b) State the main advantage and disadvantage of each type relative to the other.
  - c) State an approximate value of the damping ratio of a piezoelectric accelerometer.
  - d) Can a piezoelectric accelerometer be used to measure accelerations which only change very slowly - if not, why not?
- 6 For a damped spring-mass system sketch the shape of graphs with logarithmic scales along both axes, which show how the ratio (acceleration of mass)/(acceleration of support) varies with the exciting frequency for various values of damping ratio between  $\zeta = 0.01$  and  $\zeta = 1.0$ . Indicate the undamped natural frequency, and the usable frequency range of an accelerometer.
- 7 Describe two alternative ways in which damping may be applied to the seismic mass of an accelerometer. State the ideal value for the damping ratio of such an instrument.
- 8
  - a) Explain the principle of a servo accelerometer.
  - b) State three advantages which a servo accelerometer has compared with other types of accelerometer.
- 9 How would the following calibrations usually be carried out?
  - a) The calibration of a strain-gauged accelerometer over the range + 1 g to -1 g only.
  - b) A primary calibration of a piezoelectric accelerometer (brief description only).
  - c) A secondary calibration of a piezoelectric accelerometer.

- 10 Which of the accelerometer types i) to iv), listed below, would you use for which of the following applications a) to d)? Give reasons for your answers. Assume that constraints such as cost, availability and size are not applicable.
- i) piezoelectric
  - ii) potentiometric
  - iii) strain-gauged
  - iv) servo.
- a) To measure accelerations with frequencies not greater than 1 Hz. Weight must be kept to a minimum.
- b) To measure accelerations with frequencies not greater than 1 Hz, where considerable electrical interference is present.
- c) To measure accelerations with the maximum possible accuracy.
- d) To measure accelerations with frequencies not less than 100 Hz. Weight must be kept to a minimum.
- 11 Express the following in decibels:  
20g acceleration, ref.  $10^{-5} \text{ m/s}^2$ ; 10m/s velocity, ref.  $10^{-3} \text{ m/s}$ .  
[145.85dB; 80dB]
- 12 If a vibration level is measured as 8g at 5Hz, express this in dB ref.  $10^{-5} \text{ m/s}^2$  and calculate the corresponding velocity in dB ref.  $10^{-3} \text{ m/s}$ .  
[138dB; 67.95dB]
- 13 A transducer having a mass of 0.07kg is attached to a vibrating mechanism. If mounting the transducer changes the resonant frequency of the mechanism (with transducer) by 1% , determine the mass of the mechanism alone.  
[3.5 kg]
- 14 An accelerometer and a charge amplifier are used to measure vibration levels. If the transducer's sensitivity is  $7 \text{ pC/g}$  and the charge amplifier's sensitivity is  $100 \text{ mV/pC}$ , determine the output voltage of system for an input acceleration of 3g.  
[2.1V]