2.1 : Concept of relative velocity

Q.1.State the relation between linear and angular terms like displacement,velocity and acceleration.

Ans:

1. **Displacement :** The linear displacement(s) is the distance travelled by a particle, it is measured in meters. The angular displacement θ is the angle turned by a link and it is measured in radians.
   Following is the relation between linear displacement and angular displacement.
   $$s=r\times\theta$$
   linear displacement = radius × angular displacement
2. **Velocity :** The linear velocity(v) is the distance travelled per unit time, and its unit is m/sec. Whereas the angular displacement (ω) is the angle turned per unit time and its unit is rad/sec. Following is the relation between linear velocity and angular velocity.
   $$v=r\times\omega$$
   Linear velocity= radius × angular velocity
3. **Acceleration** :The linear acceleration (a) is the rate of change of linear velocity and expressed in m/s^2 . Whereas the angular acceleration (α) is the rate of change of angular velocity and is expressed in $rad/sec^{2.}$. $a=r\times\alpha$
   Linear acceleration = radius × angular acceleration

Q.2. Define centripetal and tangential acceleration.

Centripetal acceleration:
The centripetal acceleration is the rate of change of tangential velocity. When an object is moving with uniform acceleration in circular direction, it is said to be experiencing the centripetal acceleration.

Tangential acceleration:
Tangential acceleration is a measure of how the tangential velocity of a point at a certain radius changes with time. Tangential acceleration is just like linear acceleration, but it's particular to the tangential direction, which is relevant to circular motion

Q.3.Define linear velocity, angular velocity, absolute velocity and state the relation between linear velocity and angular velocity

Term	**Definition**	**Mathematical/representation (optional)**
Linear velocity	Rate of change of ***linear displacement*** per unit time	$V = \frac{d_x}{d_t}$ m/sec
Angular velocity	Rate of change of ***angular displacement*** per unit time	$\omega = \frac{d_\theta}{d_t}$ rad/sec
Absolute velocity	Velocity of any point with respect any point ***fixed point***	V_{ao} ; velocity of point a w.r.t. o

Relation between linear and angular velocity: V = ω.r

Q.4.Find the velocity of point B and midpoint C of link AB shown in Figure (1).

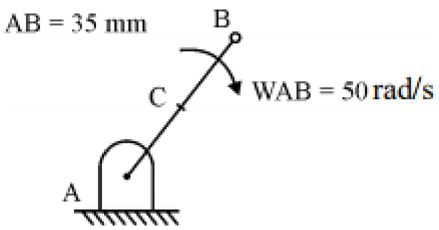


Fig. 1

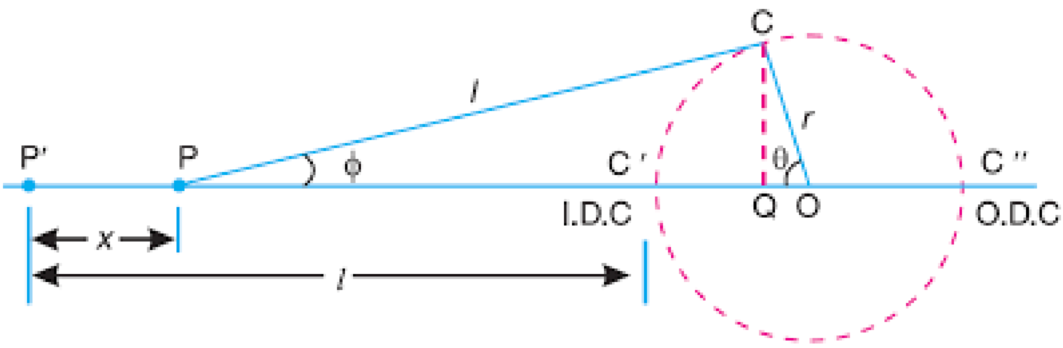
Solution :

Velocity of point B & C :

Vb = AB x wAB = 0.35 x 50 = 17.5 m/s

Vc = AC x wAB = 0.175 x 50 = 8.75 m/s

2.2 : Analytical method of finding velocity & acceleration



r= Radius of crank in meters

stroke =2 x radius of crank =2 r

l= length of connecting rod

(θ)= *Angle turned by crank from inner dead center*

Ratio $n = \frac{connecting\, rod\, length}{crank\, radius} = \frac{l}{r}$

Stroke = $2 \times crank\ radius = 2 \times r$

Part	Velocity	Acceleration
Crank	$\omega = \frac{2\pi N}{60}\ rad/sec$	0 {Because crank has constant angular velocity}
Piston /slider	$V_p = \omega . r[sin\theta + \frac{sin2\theta}{2n}]$......$m/sec$	$a_p = \omega^2 r[cos\theta + \frac{cos2\theta}{n}]..m/s^2$
Connecting rod	$\omega_c = \frac{\omega \cos\theta}{n}$ ....rad/sec	$\alpha_{cr} = -\omega^2[\frac{sin\theta}{n}]$ ....rad/sec^2

Numerical Problems :

1. The crank of a petrol engine is 4 cm and length of connecting rod is 16 cm.Find the velocity and acceleration of the piston at the instant when the crank is at $30^0$from IDC and rotating at 2500 rpm. Also, find the inertia force due to reciprocating parts having a mass of 0.3 kg.

{vel=6.37 m/s,acc=2716.965 m/s^2,force=815.09 N }

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2. The stroke of a steam engine is 15 cm and the connecting rod is 30 cm in length .Determine the velocity and acceleration of the piston when the crank has made 45^0measured from IDC position and rotates at 600 rpm. Also determine angular velocity and angular acceleration of the connecting rod.

{Vel piston=3.9 m/s,acc piston=209.36 m/s,ang vel of CR=11.107 rad/sec, ang acc of CR=-697.89 rad/sec^2}

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3. A reciprocating engine mechanism has its crank 10 cm long and the length of the connecting rod is 40 cm. Find the velocity and acceleration of the piston by analytical method when the crank rotates uniformly at angular speed of 10 rad/sec for the following positions 1) at idc 2) at $\theta = 30^0$3) at $\theta = 90^0$

{Ans (1) vel=0,acc=12.5m/s^2(2) vel=0.608 m/s,acc=9.91 m/s^2(3) vel=1.00 m/s.acc=-2.5 m/s^2}

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4. The crank and connecting rod of a steam engine are 0.3m and 1.5m in length. the crank rotates at 180 rpm clockwise. Determine the velocity and acceleration of the piston when the crank is at 40 degrees from the inner dead centre.

{Ans (1) vel=4.19 m/s,acc=85.26 }

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5. In a slider crank mechanism the length of the crank and connecting rod are 150mm and 600mm respectively. The crank position is $60^0$from inner dead centre. the crank shaft speed is 450 r.p.m. Using analytical method fine

- *Velocity and acceleration of the slider*
- *Velocity and acceleration of the connecting rod.*

{Vel piston=6.88m/s,acc piston=124.89 m/s^2,ang vel of CR=6.03 rad/sec, ang acc of CR=-480.70 rad/sec^2}

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6. In a slider crank mechanism the length of connecting rod is 80 cm, ratio of length of connecting rod to length of crank is 4. Determine (i) Velocity of slider (ii) Velocity of connecting rod, if crank makes an angle of 120° with IDC.

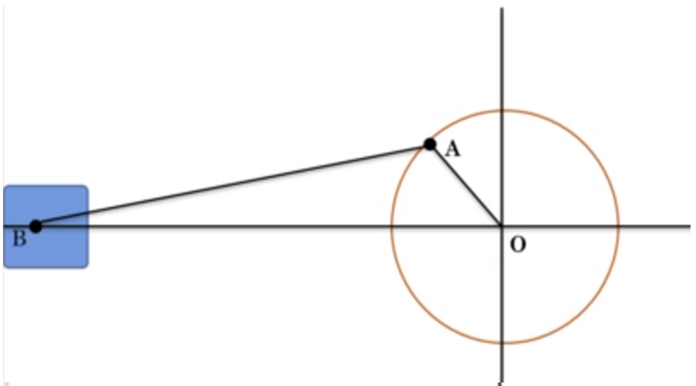
Klein's Construction Method

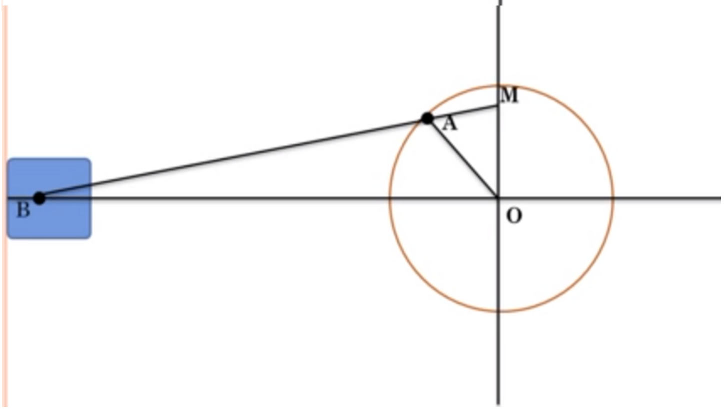
Q.1.Explain the Klein's construction method of finding the velocity and acceleration of various links/points in slider-crank Mechanism.

Ans: The Klein's construction is graphical method to obtain the velocity, acceleration of links or important points on the links. This construction is drawn directly on the configuration diagram. hence it does not involve two or three separate figures. However, it is applicable only for the slider-crank mechanism.

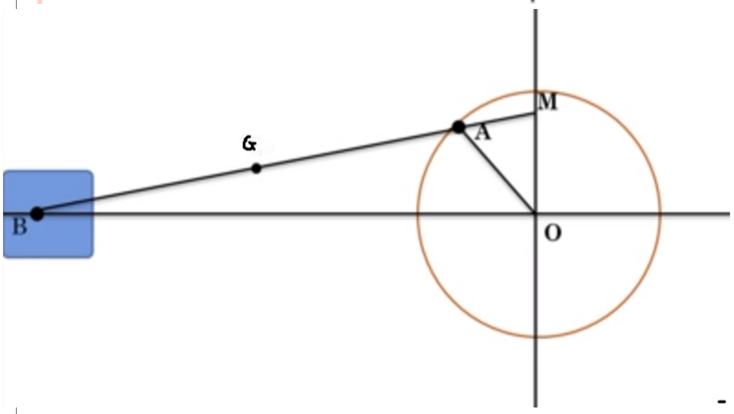
Steps to find the velocity and acceleration of various links

step 1) Draw the basic configuraion diagram by measuring the angle made by crank and also other dimension of crank and connecting rod.

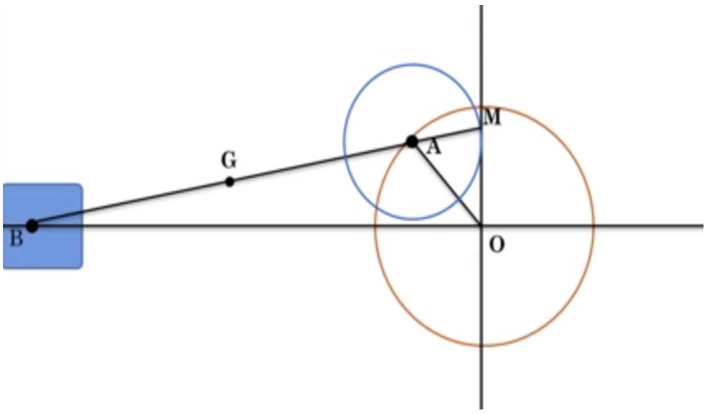




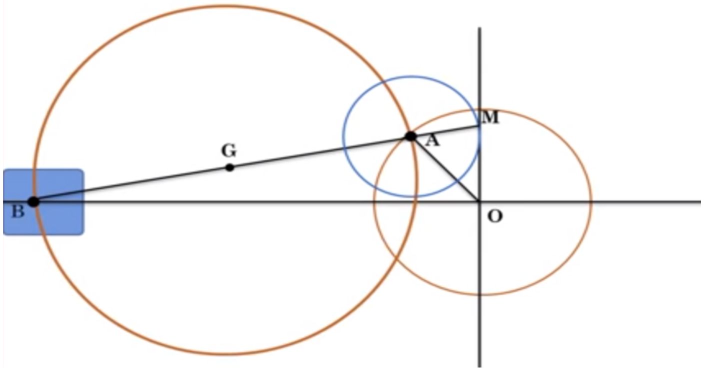
Step 2) Extend the connecting rod upto the vertical centre line of the crank circle and mark point M , the triangle created ΔOAM is the velocity triangle.



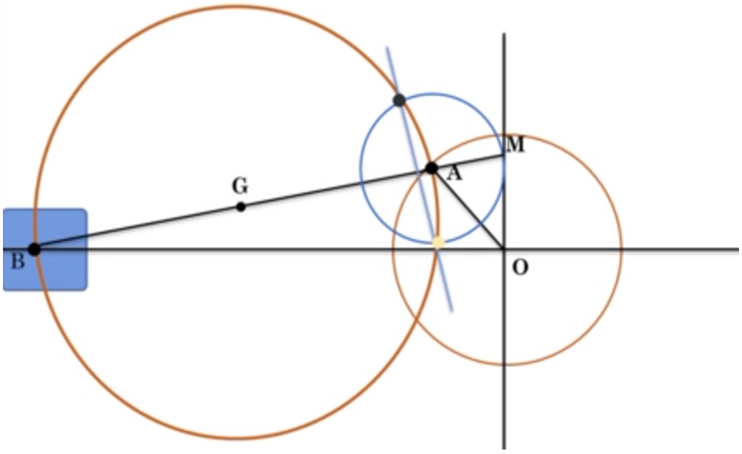
Step 3) Locate the midpoint of the connecting rod as point G.



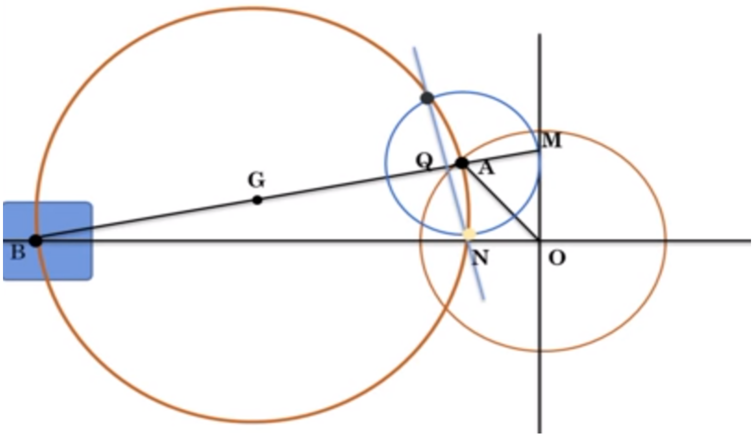
Step 4: With Centre as "A" and radius equal to AM draw the circle.



Step 5: With Centre as “G” and radius equal to GA or GB draw the circle.



Step 6: Both circles will intersect each other at two points,join these two points.



Step 7: This line will intersect the connecting rod at point “Q” and line of stroke at point “N”. Name these two points.

Step 8: Now OAM is the velocity triangle and the OAQN is the acceleration diagram. Which can be used to find the required velocity of acceleration of the links of various points on the links.

Relative Velocity Method (Graphical) 8 Marks

Important points to remember:

1. Relative velocity of a link is always perpendicular to link.
2. Angular velocity of crank is found by formula
   $\omega = \frac{2\pi N}{60}\, rad/sec.$

   This angular velocity is converted to linear velocity by formula
   $\nu_{AB} = l(AB) \times \omega\ m/s$

3) All fixed points in the mechanism are represented by a single point on velocity diagram.

4) For finding linear or angular velocity of any link use formula
   Linear velocity = Angular velocity ×Length of Link

- **Problems on Four Bar chain**

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1) In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40mm long and rotates at 120 r.p.m, while the link CD=80 mm oscillates about D. BC and AD are \equal length. FInd the angular velocity of link CD when angle isBAD=60^0

{Ans $\nu_{cd} = 0.385\, m/s\ and\ \omega_{cd} = 4.8\, rad/sec$ }

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2) PQRS is a four bar chain with PS fixed length of links are PQ = 62.5 mm, QR =175 mm, RS = 112.5 mm, PS = 200 mm. The crank PQ rotate at 10 rad/sec. in clockwise direction. Determine the angular velocity of point R, graphically by using relative velocity method.

{$\omega_{rs} = 3.82\, rad/sec$ }

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3) In a four bar chain mechanism, the lengths of links AB,BC,CD and DA are 60 mm,160mm,100mm and 200mm respectively. The link AB makes an angle of 60^0 with the fixed link AD and rotates at an angular velocity of 30 rad/sec. Find the angular velocity of link BC.

{Ans :$\omega_{BC} = 10.5\, rad/sec$ }

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4) In a four-link mechanism, the crank AB rotates at 36 rad/sec. The lengths of a link are AB = 200 mm, BC = 400 mm, CD = 450 mm and AD = 600 mm. AD is the fixed link. At the instant when AB is at right angles to AD determine the velocity of : (i) The midpoint of link BC (ii) A point on the link CD, 100 mm from the pin connecting the link CD & AD.

{vel of midpoint = 2.2 m/s , velocity of another point on link CD=1.4 m/sec}

Problems on Slider crank chain.

5) The crank and connecting rod of a steam engine are 0.5m and 2m long respectively. The crank makes 180 r.p.m in the clockwise direction. When it has turned 45^0 from inner dead centre position. Determine (1) Velocity of piston (2) Angular velocity of connecting rod (3) Velocity of point E on the connecting rod 1.5 m from the gudgeon pin.

{ $\nu_p = 8.15\, m/sec, \omega_{pb} = 3.4\, rad/sec, \nu_E = 8.5\, m/s$ }

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6) In a slider crank mechanism, the length of crank OB and connecting rod AB are 125 mm and 500 mm respectively. The centre of gravity G of the connecting rod is 275 mm from the slider. The crank speed is 600 rpm clockwise. When the crank has turned 45º from the inner dead centre position, determine (i) Velocity of slider ‘A’ (ii) Velocity of the point ‘G’ graphically. Draw the configuration diagram also.

{Velocity of slider Vector oa = Voa = 6.79 m/s, Velocity of conn. Rod Vector ab = VAB = 5.66 m/s Velocity of point ‘G’ Vector og = Vg = 7.2 m/s }

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7) In a single slider crank mechanism OAB, the crank OA(30 cm long) revolves at 200 rpm. The connecting rod AB is 75 cm long. Determine the velocity of Slider B and the velocity of midpoint of the connecting rod, when crank makes an angle of 35 degrees with the IDC.

{Velocity of slider =5.68 m/sec, velocity of midpoint 5.38 m/sec}

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8) In a single slider crank mechanism the crank OA is 7.5 cm long and rotates with uniform speed 60 rpm. The connecting rod AB is 30 cm long. Determine the linear velocity of the piston B, When crank makes an angle of 45 degrees with idc.

{ Velocity of piston :0.39 m/sec}

SUB TOPIC: 2.5 : Acceleration in Mechanisms (Graphical)

Important points regarding acceleration diagram:

- If any link carries only rectilinear motion with a constant velocity, then all points within the link shall have the same constant rectilinear velocity. Then such link will have no acceleration. Only in the case of variable rectilinear velocity, the link can have a rectilinear acceleration. The siders, pistons accordingly have only rectilinear acceleration and always along the straight line motion.
- Any link, if revolves around a fixed point. essentially carries a centripetal acceleration. This component of acceleration shall be written as $a^c \, or \, f^c$ and measured in m/sec^2.Further, if such link revolves at any constant ω, (for example : the crank of a slider - crank mechanism), then the endpoint of the link shall have a constant tangential velocity, hence no tangential acceleration. But if such link revolves with a variable angular velocity, then some tangential acceleration shall exist. The tangential acceleration, as per the first principles, shall be equal to $\frac{dv}{dx}$ and written as $a^t \, or \, f^t$, measured in m/sec^2. Normally, the input links of the mechanism are subjected to these options.
- Any link, having a mix motion; (like the connecting rod of a slider-crank mechanism) shall have some centripetal accelerations at its ends. The ends may have a tangential acceleration. Normally, the ends connected towards the output link, shall necessarily have some f^t. Accordingly, the ends of such link shall have a resultant acceleration, written as ‘a’ or ‘f’ $= \sqrt{((a^c)^2 + (a^t)^2} \, or \, \sqrt{(f^c)^2 + (f^t)^2}$.
- Any centripetal acceleration shall be always along the link. Any tangential acceleration will be always perpendicular to the centripetal acceleration. If it follows the direction of the angular velocity of the point, then it will be a real acceleration. If the it is seen opposite to the velocity, then it shall be taken as a negative acceleration or retardation.

Problems

(1) Crank OA of a mechanism is hinged at ‘O’ and rotates at an angular velocity of 20 rad/sec. and angular acceleration of 25 rad/sec2 . If crank OA is 50 mm long determine linear velocity, centripetal acceleration and tangential acceleration of a point A.

(2) Crank OA of a mechanism is hinged at ‘O’ and rotates at an angular velocity of 30 rad/sec. and angular acceleration of 40 rad/sec2 . If crank OA is 50 mm long determine linear velocity, centripetal acceleration and tangential acceleration of a point A.

(3) The crank and slider crank mechanism rotates clockwise at a constant speed of 300 rpm. the crank is 150 mm and the connecting rod is 600 mm long. Determine,

- linear velocity and acceleration of the midpoint of the connecting rod and
- angular velocity and angular acceleration of the connecting rod,
  When crank angle is 45 degrees from inner dead centre position.