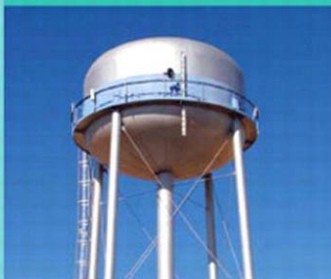


Fundamentals of Hydraulic Engineering Systems

Fourth Edition



Robert J. Houghtalen
A. Osman Akan
Ned H. C. Hwang

COMMON CONSTANTS AND CONVERSIONS

Physical Properties of Water

Units System*	Specific Weight (γ)	Density (ρ)	Viscosity (μ)	Kinematic Viscosity (ν)	Surface Tension (σ)	Vapor Pressure
<i>@ Normal conditions [20.2°C(68.4°F) and 760 mm Hg (14.7 lb/in²)]</i>						
SI	9,790 N/m ³	998 kg/m ³	1.00 × 10 ⁻³ N · sec/m ²	1.00 × 10 ⁻⁶ m ² /sec	7.13 × 10 ⁻² N/m	2.37 × 10 ³ N/m ²
BG	62.3 lb/ft ³	1.94 slug/ft ³	2.09 × 10 ⁻⁵ lb · sec/ft ²	1.08 × 10 ⁻⁵ ft ² /sec	4.89 × 10 ⁻³ lb/ft	3.44 × 10 ⁻¹ lb/in ²
<i>@ Standard conditions [4°C(39.2°F) and 760 mm Hg (14.7 lb/in²)]</i>						
SI	9,810 N/m ³	1,000 kg/m ³	1.57 × 10 ⁻³ N · sec/m ²	1.57 × 10 ⁻⁶ m ² /sec	7.36 × 10 ⁻² N/m	8.21 × 10 ² N/m ²
BG	62.4 lb/ft ³	1.94 slug/ft ³	3.28 × 10 ⁻⁵ lb · sec/ft ²	1.69 × 10 ⁻⁵ ft ² /sec	5.04 × 10 ⁻³ lb/ft	1.19 × 10 ⁻¹ lb/in ²

* Le Système International d'Unitès (SI) or the British gravitational system of units (BG)

Bulk Modulus of Elasticity, Specific Heat, Heat of Fusion/Vaporization

Bulk Modulus of Elasticity (water)* = 2.2×10^9 N/m² (3.2×10^5 lb/in.², or psi)

Specific Heat of Water** = 1 cal/g · °C (1.00 BTU/lbm · °F)

Specific Heat of Ice** = 0.465 cal/g · °C (0.465 BTU/lbm · °F)

Specific Heat of Water Vapor = 0.432 cal/g · °C (at constant pressure)

Specific Heat of Water Vapor = 0.322 cal/g · °C (at constant volume)

Heat of Fusion (Latent Heat) = 79.7 cal/g (144 BTU/lbm)

Heat of Vaporization = 597 cal/g (1.08×10^4 BTU/lbm)

* For typical pressure and temperature ranges.

** Under standard atmospheric pressure.

Common Constants

Design Constants	SI	BG
Standard Atmospheric Pressure	1.014×10^5 N/m ² (Pascals)	14.7 lb/in ²
	760 mm Hg	29.9 in. Hg
	10.3 m H ₂ O	33.8 ft H ₂ O
Gravitational Constant	9.81 m/sec ²	32.2 ft/sec ²

Useful Conversions

1 N ($\text{kg} \cdot \text{m}/\text{sec}^2$) = 100,000 dynes ($\text{g} \cdot \text{cm}/\text{sec}^2$)	1 hectare = 10,000 m^2 (100m by 100m)
1 acre = 43,560 ft^2	1 mi^2 = 640 acres
1 ft^3 = 7.48 gallons	1 hp = 550 $\text{ft} \cdot \text{lb}/\text{sec}$
1 ft^3/sec = 449 gallons/min (gpm)	

Conversion Factors: SI Units to BG Units

Unit Measure	To convert from	to	Multiply by
Area	m^2	ft^2	1.076×10^1
	cm^2	in^2	1.550×10^{-1}
	hectares	acres	2.471
Density	kg/m^3	slugs/ ft^3	1.940×10^{-3}
Force	N	lb	2.248×10^{-1}
Length	m	ft	3.281
	cm	in.	3.937×10^{-1}
	km	mi	6.214×10^{-1}
Mass	kg	slug	6.852×10^{-2}
Power	W	$\text{ft} \cdot \text{lb}/\text{sec}$	7.376×10^{-1}
	kW	hp	1.341×10^1
Energy	$\text{N} \cdot \text{m}$ (Joule)	$\text{ft} \cdot \text{lb}$	7.376×10^{-1}
Pressure	N/m^2 (Pascal)	lb/ft^2 (psf)	2.089×10^{-2}
	N/m^2 (Pascal)	lb/in^2 (psi)	1.450×10^{-4}
Specific Weight	N/m^3	lb/ft^3	6.366×10^{-3}
Temperature	$^{\circ}\text{C}$	$^{\circ}\text{F}$	$T_f = 1.8 T_c + 32^{\circ}$
Velocity	m/sec	ft/sec	3.281
Viscosity	$\text{N} \cdot \text{sec}/\text{m}^2$	$\text{lb} \cdot \text{sec}/\text{ft}^2$	2.089×10^{-2}
Viscosity (kinematic)	m^2/sec	ft^2/sec	1.076×10^1
Volume	m^3	ft^3	3.531×10^1
	liter	gal	2.642×10^{-1}
Volume flow rate (discharge)	m^3/sec	ft^3/sec (cfs)	3.531×10^1
	m^3/sec	gal/min	1.585×10^4

Fourth Edition

Fundamentals of Hydraulic Engineering Systems

Robert J. Houghtalen

Rose-Hulman Institute of Technology

A. Osman Akan

Old Dominion University

Ned H. C. Hwang

University of Miami, Emeritus

Prentice
Hall

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris
Montréal Toronto Delhi Mexico City Sao Paulo Sydney
Hong Kong Seoul Singapore Taipei Tokyo

Library of Congress Cataloging-in-Publication Data

Houghtalen, Robert J.

Fundamentals of hydraulic engineering systems / Robert J. Houghtalen, A. Osman Akan, Ned H. C. Hwang. – 4th ed.
p. cm.

Rev. ed. of: Fundamentals of hydraulic engineering systems / Ned H. C. Hwang, Robert J. Houghtalen. 1996.

Includes bibliographical references and index.

ISBN-13: 978-0-13-601638-0

ISBN-10: 0-13-601638-3

1. Hydraulics. 2. Hydraulic engineering. I. Akan, A. Osman. II. Hwang, Ned H. C. III. Hwang, Ned H. C.
Fundamentals of hydraulic engineering systems. IV. Title.

TC160.H86 2010

627–dc22

2009008672

Vice President and Editorial Director, ECS:

Marcia J. Horton

Acquisition Editor: *Holly Stark*

Editorial Assistant: *William Opatuch*

Director of Team-Based Project Management:

Vince O'Brien

Marketing Manager: *Tim Galligan*

Marketing Assistant: *Mack Patterson*

Senior Managing Editor: *Scott Disanno*

Production Project Manager: *Clare Romeo*

Operations Specialist: *Lisa McDowell*

Cover Designer: *Bruce Kenselaar*

Cover Illustration/Photo(s): *David Honan*

Manager, Rights and Permissions: *Zina Arabia*

Manager, Visual Research: *Beth Brenzel*

Image Permission Coordinator: *Karen Sanatar*

Composition: *Macmillan Publishing Solutions*

Full-Service Project Management: *Martha Wetherill*

Printer/Binder: *Hamilton Printing Co.*

Typeface: *10/12 Times*

Copyright © 2010, 1996 by Pearson Higher Education, Inc., Upper Saddle River, NJ 07458. All rights reserved. Manufactured in the United States of America. This publication is protected by Copyright and permissions should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. To obtain permission(s) to use materials from this work, please submit a written request to Pearson Higher Education, Permissions Department, One Lake Street, Upper Saddle River, NJ 07458.

Many of the designations by manufacturers and seller to distinguish their products are claimed as trademarks. Where those designations appear in this book, and the publisher was aware of a trademark claim, the designations have been printed in initial caps or all caps.

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages with, or arising out of, the furnishing, performance, or use of these programs.

Pearson Education Ltd., *London*

Pearson Education Singapore, Pte. Ltd

Pearson Education Canada, Inc.

Pearson Education—Japan

Pearson Education Australia PTY, Limited

Pearson Education North Asia, Ltd., *Hong Kong*

Pearson Educación de Mexico, S.A. de C.V.

Pearson Education Malaysia, Pte. Ltd.

Pearson Education, Inc., *Upper Saddle River, New Jersey*



10 9 8 7 6 5 4 3 2 1

ISBN-13: 978-0-13-601638-0

ISBN-10: 0-13-601638-3

Contents

PREFACE	<i>ix</i>
ACKNOWLEDGMENTS	<i>xiii</i>
INTRODUCTION	<i>xix</i>
1 FUNDAMENTAL PROPERTIES OF WATER	<i>1</i>
1.1 The Earth's Atmosphere and Atmospheric Pressure	2
1.2 The Three Phases of Water	2
1.3 Mass (Density) and Weight (Specific Weight)	3
1.4 Viscosity of Water	5
1.5 Surface Tension and Capillarity	7
1.6 Elasticity of Water	8
1.7 Forces in a Fluid Field	9
Problems	10

2	<i>WATER PRESSURE AND PRESSURE FORCES</i>	14
2.1	The Free Surface of Water	14
2.2	Absolute and Gauge Pressures	14
2.3	Surfaces of Equal Pressure	17
2.4	Manometers	18
2.5	Hydrostatic Forces on Flat Surfaces	22
2.6	Hydrostatic Forces on Curved Surfaces	27
2.7	Buoyancy	30
2.8	Flotation Stability	31
	Problems	35
3	<i>WATER FLOW IN PIPES</i>	51
3.1	Description of Pipe Flow	51
3.2	The Reynolds Number	52
3.3	Forces in Pipe Flow	54
3.4	Energy in Pipe Flow	56
3.5	Loss of Head from Pipe Friction	60
3.5.1	Friction Factor for Laminar Flow	60
3.5.2	Friction Factor for Turbulent Flow	62
3.6	Empirical Equations for Friction Head Loss	67
3.7	Friction Head Loss-Discharge Relationships	70
3.8	Loss of Head in Pipe Contractions	71
3.9	Loss of Head in Pipe Expansions	73
3.10	Loss of Head in Pipe Bends	75
3.11	Loss of Head in Pipe Valves	76
3.12	Method of Equivalent Pipes	80
3.12.1	Pipes in Series	80
3.12.2	Pipes in Parallel	81
	Problems	83
4	<i>PIPELINES AND PIPE NETWORKS</i>	88
4.1	Pipelines Connecting Two Reservoirs	88
4.2	Negative Pressure Scenarios (Pipelines and Pumps)	92
4.3	Branching Pipe Systems	97
4.4	Pipe Networks	104
4.4.1	The Hardy-Cross Method	105
4.4.2	The Newton Method	116
4.5	Water Hammer Phenomenon in Pipelines	119

- 4.6 Surge Tanks 127
- Problems 130

5 WATER PUMPS **143**

- 5.1 Centrifugal (Radial Flow) Pumps 143
- 5.2 Propeller (Axial Flow) Pumps 149
- 5.3 Jet (Mixed-Flow) Pumps 152
- 5.4 Centrifugal Pump Characteristic Curves 153
- 5.5 Single Pump and Pipeline Analysis 154
- 5.6 Pumps in Parallel or in Series 157
- 5.7 Pumps and Branching Pipes 161
- 5.8 Pumps and Pipe Networks 164
- 5.9 Cavitation in Water Pumps 165
- 5.10 Specific Speed and Pump Similarity 169
- 5.11 Selection of a Pump 171
- Problems 175

6 WATER FLOW IN OPEN CHANNELS **184**

- 6.1 Open-Channel Flow Classifications 186
- 6.2 Uniform Flow in Open Channels 188
- 6.3 Hydraulic Efficiency of Open-Channel Sections 194
- 6.4 Energy Principles in Open-Channel Flow 197
- 6.5 Hydraulic Jumps 203
- 6.6 Gradually Varied Flow 206
- 6.7 Classifications of Gradually Varied Flow 208
- 6.8 Computation of Water Surface Profiles 211
 - 6.8.1 Standard Step Method 212
 - 6.8.2 Direct Step Method 214
- 6.9 Hydraulic Design of Open Channels 221
 - 6.9.1 Unlined Channels 223
 - 6.9.2 Rigid Boundary Channels 225
- Problems 226

7 GROUNDWATER HYDRAULICS **232**

- 7.1 Movement of Groundwater 234
- 7.2 Steady Radial Flow to a Well 237
 - 7.2.1 Steady Radial Flow in Confined Aquifers 238
 - 7.2.2 Steady Radial Flow in Unconfined Aquifers 240

7.3	Unsteady Radial Flow to a Well	242
7.3.1	Unsteady Radial Flow in Confined Aquifers	242
7.3.2	Unsteady Radial Flow in Unconfined Aquifers	245
7.4	Field Determination of Aquifer Characteristics	248
7.4.1	Equilibrium Test in Confined Aquifers	248
7.4.2	Equilibrium Test in Unconfined Aquifers	250
7.4.3	Nonequilibrium Test	252
7.5	Aquifer Boundaries	256
7.6	Surface Investigations of Groundwater	261
7.6.1	The Electrical Resistivity Method	261
7.6.2	Seismic Wave Propagation Methods	262
7.7	Seawater Intrusion in Coastal Areas	263
7.8	Seepage Through Dam Foundations	267
7.9	Seepage Through Earth Dams	270
	Problems	271

8 HYDRAULIC STRUCTURES

281

8.1	Functions of Hydraulic Structures	281
8.2	Dams: Functions and Classifications	282
8.3	Stability of Gravity and Arch Dams	284
8.3.1	Gravity Dams	284
8.3.2	Arch Dams	288
8.4	Small Earth Dams	289
8.5	Weirs	291
8.6	Overflow Spillways	296
8.7	Side-Channel Spillways	299
8.8	Siphon Spillways	302
8.9	Culverts	305
8.10	Stilling Basins	310
	Problems	314

9 WATER PRESSURE, VELOCITY, AND DISCHARGE MEASUREMENTS

321

9.1	Pressure Measurements	321
9.2	Velocity Measurements	323
9.3	Discharge Measurements in Pipes	326

9.4	Discharge Measurements in Open Channels	331
9.4.1	Sharp-Crested Weirs	331
9.4.2	Broad-Crested Weirs	334
9.4.3	Venturi Flumes	335
	Problems	340
10	HYDRAULIC SIMILITUDE AND MODEL STUDIES	345
10.1	Dimensional Homogeneity	346
10.2	Principles of Hydraulic Similitude	347
10.3	Phenomena Governed by Viscous Forces: Reynolds Number Law	352
10.4	Phenomena Governed by Gravity Forces: Froude Number Law	355
10.5	Phenomena Governed by Surface Tension: Weber Number Law	357
10.6	Phenomena Governed by Both Gravity and Viscous Forces	358
10.7	Models for Floating and Submerged Bodies	358
10.8	Open-Channel Models	360
10.9	The Pi Theorem	362
	Problems	366
11	HYDROLOGY FOR HYDRAULIC DESIGN	371
11.1	The Hydrologic Cycle	372
11.2	Precipitation	376
11.3	Design Storm	380
11.4	Surface Runoff and Stream Flow	385
11.5	Rainfall-Runoff Relationships: The Unit Hydrograph	387
11.6	Rainfall-Runoff Relationships: SCS Procedures	393
11.6.1	Losses from Rainfall and Rainfall Excess	393
11.6.2	Time of Concentration	396
11.6.3	SCS Synthetic Unit Hydrograph	398
11.6.4	SCS Design Hydrograph	401
11.7	Storage Routing	402
11.8	Hydraulic Design: The Rational Method	410
11.8.1	Design of Stormwater-Collection Systems	412
11.8.2	Design of Stormwater Pipes	415
	Problems	418
12	STATISTICAL METHODS IN HYDROLOGY	430
12.1	Concepts of Probability	431
12.2	Statistical Parameters	431

12.3	Probability Distributions	435
12.3.1	Normal Distribution	435
12.3.2	Log-Normal Distribution	436
12.3.3	Gumbel Distribution	436
12.3.4	Log-Pearson Type III Distribution	437
12.4	Return Period and Hydrologic Risk	439
12.5	Frequency Analysis	440
12.5.1	Frequency Factors	440
12.5.2	Testing Goodness of Fit	443
12.5.3	Confidence Limits	445
12.6	Frequency Analysis Using Probability Graphs	448
12.6.1	Probability Graphs	448
12.6.2	Plotting Positions	448
12.6.3	Data Plotting and Theoretical Distributions	450
12.6.4	Estimating Future Magnitudes	451
12.7	Rainfall Intensity-Duration-Frequency Relationships	452
12.8	Applicability of Statistical Methods	455
	Problems	455

SYMBOLS **461**

ANSWERS TO SELECTED PROBLEMS **463**

INDEX **469**

Preface

This book provides a fundamental treatment of engineering hydraulics. It is primarily intended to serve as a textbook for undergraduate engineering students. However, it will serve as a very useful reference for practicing engineers who want to review basic principles and their applications in hydraulic engineering systems.

Engineering hydraulics is an extension of fluid mechanics in which many empirical relationships are applied and simplifying assumptions made to achieve practical engineering solutions. Experience has shown that many engineering students, possibly well versed in basic fluid mechanics, have trouble solving practical problems in hydraulics. This book is intended to bridge the gap between basic principles and the techniques applied to the design and analysis of hydraulic engineering systems. As such, the reader is exposed to many problems commonly encountered in practice and various solution scenarios including effective design procedures, equations, graphs/tables, and suggested computer software that can be used to advantage.

This book contains twelve chapters. The first five chapters cover the fundamentals of fluid statics, fluid dynamics, and pipe flow. The first chapter discusses fundamental properties of water as a fluid. In this chapter, the basic differences between the SI system (Le Système International d'Unités) and British units are discussed. The second chapter presents the concepts of water pressure and pressure forces on surfaces in contact with water. Chapter 3 introduces the basic principles of water flow in pipes. These principles are applied to the practical problems of pipelines and pipe networks in Chapter 4 with an emphasis on hydraulic systems. Chapter 5

discusses the theory, analysis, and design aspects of water pumps. The systems approach is again emphasized with detailed instruction on pump analysis within pipelines, branching pipe systems, and pipe networks, as well as pump selection and design considerations.

The next three chapters cover open channel flow, groundwater, and the design of various hydraulic structures. Water flow in open channels is presented in Chapter 6. Detailed discussions of uniform flow (normal depth), rapidly varied flow (hydraulic jumps), and gradually varied flow (classifications and water surface profiles) are included, along with open channel design. The hydraulics of wells and seepage problems are two key topics in Chapter 7 on groundwater. Well hydraulics includes coverage of equilibrium and non-equilibrium conditions in confined and unconfined aquifers. Chapter 8 introduces some of the most common hydraulic structures such as dams, weirs, spillways, culverts, and stilling basins. Functionality, hydraulic principles, practical considerations, and design procedures are provided.

The book ends with four ancillary topics: measurements, model studies, hydrology for hydraulic design, and statistical methods in hydrology. Chapter 9 discusses measurement of water pressure, velocity, and discharge in pipes and open channels. The proper use of scaled models is an essential part of hydraulic engineering. As such, the use of hydraulic models and the laws of engineering similitude are covered in Chapter 10. Flow rates are required for the design of all hydraulic structures; many are obtained using the principles of hydrology. The last two chapters introduce common techniques used for obtaining hydrologic design flows. Deterministic procedures are covered in Chapter 11 and statistical methods are covered in Chapter 12. In addition, the design of stormwater collection, transport, and storage systems (routing) is introduced in Chapter 11.

New to This Edition

Substantial revisions have been made to this new edition based on feedback from practitioners, college professors, and book reviewers. These revisions include:

- More than half of the homework problems provided in the book are new or revised from the previous edition.
- Chapter 3 includes two new sections: friction head loss-discharge relationships and the method of equivalent pipes.
- Chapter 4 now includes Hardy-Cross analysis procedures for pipe networks with multiple inflows from storage reservoirs (or water towers). In addition, matrix analysis of pipe networks is introduced using the Newton method.
- Chapter 5 on pumps now includes analysis procedures for branching pipe systems and pipe networks that contain pumps.
- Hydraulic design (sizing) of open channels is now contained in Chapter 6.
- Chapter 7 includes more thorough coverage of the well equations (equilibrium and non-equilibrium) for both confined and unconfined aquifers. In addition, field determination of aquifer characteristics is examined in greater detail.
- Chapter 8 contains a short section on safety (hydraulic currents) at low-head dams. Greater clarity is provided on the hydraulic principles guiding the design of culverts.
- Two chapters on hydrology are now available to accommodate professors who teach hydrology in combination with hydraulics. Common hydrologic methods for obtaining design flows, a primary requirement for hydraulic design, are presented in Chapter 11. Chapter 12, which is entirely new, contains common statistical methods in hydrology.

Faculty Information and Resources

This book is primarily used for a one-semester (16 weeks), three-hour course in the undergraduate curriculum. This edition continues to provide example problems and chapter problems in both SI and British units, leaning a little more heavily to SI. A prerequisite in fluid mechanics is not necessary, but is highly recommended. One of the authors uses the text for a hydraulic engineering course (fluid mechanics as a prerequisite) and begins in Chapter 4 after a quick review of the first three chapters. It is not possible to cover the book completely in a semester. However, many of the later chapters (groundwater, hydraulic structures, model studies, and hydrology) can be covered or deleted from the syllabus according to instructor preferences without a loss of continuity. Some adopters have used the book for a two semester sequence of hydraulics followed by hydrology.

There are 115 example problems and 560 homework problems, providing coverage for every major topic in the book. In general, the homework problems are sequenced according to Bloom's taxonomy; the earlier problems in a section measure comprehension and application and the later problems measure analysis and some synthesis. A **solutions manual** is available for college professors who adopt the book for their class. Three significant figures were used almost exclusively in all problem solutions. In addition, a **test manual** is available to those same professors to assist them in quickly creating tests for student assessment or assigning additional out-of-book problems. The test manual includes short-answer questions and two or three problems in each major section of the book (178 problems in all). The test manual could be used to assign supplemental homework problems. The solutions manual and test manual as well as PowerPoint figures of all images and tables from this book can be downloaded electronically off of our Instructor's Resource Center located at www.pearsonhighered.com <<http://www.pearsonhighered.com/>>. The material available through the Instructor's Resource Center is provided solely for the use of instructors in teaching their courses and assessing student learning. All requests for instructor access are verified against our customer database and/or through contacting the requestor's institution. Contact your local sales representative for additional assistance or support.

The authors have included many topics which would benefit greatly from the use of computer software. Some of these topics include: energy balance in pipelines (Sections 3.4–3.12); pipelines, branching pipe systems, and pipe networks (Sections 4.1–4.4); pump/pipeline system analysis (Sections 5.4–5.8); pump selection (Section 5.11); normal and critical depth in open channels (Sections 6.2–6.4); water surface profiles in open channels (Section 6.8); culvert analysis (Section 8.9); unit hydrographs, SCS procedures, and storage routing applications (Sections 11.5–11.7); and statistical methods in hydrology (Chapter 12). In engineering practice, an abundance of computer software is available in these topic areas to accelerate and simplify the design and analysis process. Some companies make use of in-house software written by their computer scientists and/or engineers. Other companies use software that is readily available from private software vendors and governmental agencies. The authors have written spreadsheets in some topic areas (evident in the solutions manual and available to course instructors who adopt the textbook) and occasionally used available software to check desk-top methods used in the solutions manual.

The authors encourage course instructors to have their students use off-the-shelf software and, in some cases, program their own solutions in spreadsheets or computer algebra packages (e.g., MathCAD, Maple, or Mathematica). Recommendations for task-specific, nonproprietary packages include "EPANET" (Environmental Protection Agency) for pipe networks, "HEC-RAS" (U.S. Army Corps of Engineers) for normal and critical depths and for water surface

profiles, and “HEC-HMS” (U.S. Army Corps of Engineers) for unit hydrographs and reservoir routing. Other software is freely available on the Internet for pressure pipe flow, open channel flow, and pump analysis/design. Internet postings include freely available MathCAD spreadsheets to solve many hydraulic engineering problems (e.g., <http://www2.latech.edu/~dmg/>; permission granted from D. M. Griffin, Jr., Ph.D., P.E., D.WRE). In addition, there is an abundance of proprietary packages that solve specific hydraulic problems that can be found quickly on the Internet. Almost all of the solution techniques are amenable to spreadsheet and/or computer algebra package programming and would make great student projects. (Please contact the authors, robert.houghtalen@gmail.com or oakan@odu.edu, if you need assistance.)

A number of homework problems in the book encourage the use of this computer software. In addition, “classroom computer exercises” are included to introduce appropriate software and its capabilities. These exercises are intended to be accomplished during class or lab time. They are meant to promote a cooperative learning environment where student teams are actively engaged in engineering analysis/design, and hopefully will promote some rich classroom discussion. The primary objective of the “classroom computer exercises” is the development of a deeper understanding of the subject matter, but they do require a computer classroom (or student laptops) loaded with the appropriate software. The senior author uses teams of two or three students to stimulate dialogue and enhance the learning process. Alternatively, students can proceed through the exercises for homework and bring printouts of their results to class to discuss.

Despite the gentle nudge to acquaint students with hydrologic and hydraulic software, the book does not become a slave to it. As previously mentioned, a number of topics encourage the use of software. The first few problems at the end of each of these sections (topics) require hand calculations. After the students are aware of the solution algorithms, more complex problems are introduced which can be solved with software. Consequently, the students will be in a position to anticipate the data the software needs to solve the problem and understand what the software is doing computationally. In addition, the instructor can ask many “what if” questions of the students in connection with these homework problems. This will greatly enhance their understanding of the subject matter without burdening them with tedious calculations.

Acknowledgments

I was honored to be asked by Ned Hwang to participate in writing the third edition of this textbook over a decade ago. He is now content to have me, and our new coauthor, Osman Akan, continue the project with this fourth edition. Ned did a terrific job in writing the first edition in the early 1980s. Writing a textbook is a massive undertaking, and it is to his credit that the book has been used by many colleges and universities for almost three decades. At one time, the third edition was being used by over 40 American universities with many international sales in Southeast Asia, Canada, and England.

Our new coauthor is a great addition to our team. Dr. Akan has published extensively in the hydrology and hydraulics literature. He is an excellent teacher and writer, having published other textbooks in these two fields. I was fortunate to have coauthored another book with him in the area of urban stormwater management. He has added significant content to the book, particularly in the areas of pipelines, pipe networks, pumps, open-channel flow, and groundwater flow. I am thankful to the Lord for the opportunity to work with Osman and Ned over the years; they have been good friends and mentors to me as I develop as a person and a professional.

I think I forgot how much time and effort it takes to revise a textbook since we put out the third edition in 1996. I am thankful to my wife, Judy, and my three sons for supporting me in this two-year effort. Most of the work was done while I was in the Sudan on sabbatical working for a humanitarian organization on bio-sand water filters. My oldest son Jesse spent many hours

reviewing the problems and solutions manual. He had just graduated with a B.S. degree in civil engineering and knows how important it is to students for the solutions to be correct and well explained. I continue to be grateful to Dr. Rooney Malcom, my master's advisor at North Carolina State University, for my early professional development. I am also grateful to the late Jerome Normann, my friend and mentor for many years who shared with me the fine art of professional practice in hydraulics and hydrology. In 1995–96, I spent a year's sabbatical working with Wright Water Engineers in Denver, Colorado honing my skills in hydraulic and hydrologic engineering. I continue to work with them on special projects and would like to thank master practitioners Jon Jones, Wayne Lorenz, and Ken Wright for teaching me so much about our honorable and valuable profession. Lastly, I want to thank Dave Honan for his photographs that grace the cover of the book and the opening page of each chapter. As a former student, he is likely to blend a career in civil engineering with professional photography.

I would also like to thank the reviewers: Forrest M. Holly Jr., University of Iowa; Keith W. Bedford, The Ohio State University; D. M. Griffin, Louisiana Tech; and Robert M. Sorensen, Lehigh University.

R. J. Houghtalen
Rose-Hulman Institute of Technology

It was a great honor for me to be asked by Robert and Ned to join them in writing this new edition. While I felt very privileged, finding ways to contribute to an already excellent book was somewhat challenging. I hope that the readers will find my contributions useful and meaningful.

Ned deserves the most credit for the existence of this book. He wrote the earlier editions with Robert's participation in the third edition. Many professors, including myself, and students used these editions as a textbook or a reference. Robert led the effort for the fourth edition and did an excellent job. I had the opportunity to coauthor another book with him in the recent past. He is a true scholar, a dedicated educator, and a fine individual. I feel very fortunate for having him as a colleague and friend for decades.

I am indebted to all the professors I had at Middle East Technical University, Ankara, Turkey, as an undergraduate student and at the University of Illinois as a graduate student. However, the late Ben C. Yen, my thesis and dissertation supervisor, always has a special place in my heart. I learned so much from him. Dr. Yen, a scholar and gentleman, remained my teacher, mentor, and friend until he passed away in 2001.

I am most indebted to my wife, Güzin, and my son, Doruk, both engineers, for their love and support during this project and always.

A. Osman Akan
Old Dominion University

The first attempt to arrange my lecture notes into a textbook was made in 1974. My colleagues at the University of Houston, particularly Professors Fred W. Rankin Jr. and Jerry Rogers, had provided valuable suggestions and assistance, which formed the foundation of this book. Dr. Rogers also carefully reviewed the first edition text after its completion.

I am also deeply indebted to my students, Drs. Travis T. Stripling, John T. Cox, James C. Chang, and Po-Ching Lu. All provided assistance during the various stages of preparation. Dr. Ahmed M. Sallam, who used a first draft of this book in a hydraulics course, provided many suggestions. Dr. Carlos E. Hita, who was coauthor of the second edition, offered valuable suggestions and also provided many of the sample problems used in the text.

My dear friend, Dr. David R. Gross, an ever-inquisitive physiologist (as was Jean Louise Poiseuille, 1799–1869) with a great interest in hydraulic matters, reviewed the first edition and offered many irrefutable criticisms.

During the preparation of the first edition, I was ill for some time. The continuous encouragement, loyalty, and love of Maria, Leon, and Leroy kept me on track during the dark hours that have since passed. To them also, I dedicate this book.

Ned H. C. Hwang
University of Miami, Emeritus

This page intentionally left blank

DEDICATION

To our beloved families

Judy, Jesse, Jamin, and Jared Houghtalen

and

Güzin and Doruk Akan

and

Maria, Leon, and Leroy Hwang

This page intentionally left blank

Introduction

Hydraulic systems are designed to transport, store, or regulate water. All hydraulic systems require the application of fundamental principles of fluid mechanics. However, many also require an understanding of hydrology, soil mechanics, structural analysis, engineering economics, geographic information systems, and environmental engineering for proper planning, design, construction, and operation.

Unlike some branches of engineering, each hydraulic project encounters a unique set of physical conditions to which it must conform. There are no standard solutions or simple hand-book answers. Hydraulic engineering relies on fundamental knowledge that must be applied to meet the special conditions of each project.

The shape and dimensions of hydraulic systems may vary from a small flow meter a few centimeters in size to a levee several hundred kilometers long. Generally hydraulic structures are relatively massive as compared with the products of other engineering disciplines. For this reason, the design of large hydraulic systems is site specific. It is not always possible to select the most desirable location or material for a particular system. Commonly, a hydraulic system is designed to suit the local conditions, which include topography, geology, ecology, social concerns, and the availability of native materials.

Hydraulic engineering is as old as civilization itself, testifying to the importance of water to human life. There is much evidence that hydraulic systems of considerable magnitude existed several thousand years ago. For example, a large-scale drainage and irrigation system built in

Egypt can be dated back to 3200 BC. Rather complex water supply systems, including several hundred kilometers of aqueducts, were constructed to bring water to ancient Rome. Dujonyen, a massive irrigation system in Siechuan, China, built nearly 2,500 years ago, is still in effective use today. The abundant knowledge resulting from these and other more recent practical applications of hydraulic engineering is indispensable.

In addition to the analytical approach, some modern hydraulic system design and operation depends on empirical formulas that produce excellent results in water works. No better replacement for these formulas is expected in the near future. Unfortunately, most of these empirical formulas cannot be analyzed or proven theoretically. In general, they are not dimensionally homogeneous. For this reason, the conversion of units from the British system to SI units and vice versa is more than just a matter of convenience. Sometimes the rigorous form (e.g., the Parshall equations for water flow measurements) must be maintained with its original units. In these cases, all quantities should be converted to the original units specified by the equation for computations.

The book leans toward the use of SI units. It is the most widely used unit system in the world and is particularly dominant in commerce and science. As the world has “flattened,” using the language of Thomas Friedman, the move to SI units has accelerated. The European Union will begin to ban non-SI markings on imported goods (e.g., pipes, pumps, etc.) in 2010. Roughly one-third of the problems in the book are written in British units to accommodate the mixture of units (BG and SI) used in the U.S. However, the chapters on hydrology use BG units almost exclusively. The transition to SI units in the field of hydrology in the U.S. does not seem to be progressing very quickly. A detailed table of conversions is provided on the inside front cover to assist the reader.



Fundamental Properties of Water

The word *hydraulic* comes from two Greek words: “hydor” (meaning water) and “aulos” (meaning pipe). Through the years, the definition of hydraulics has broadened beyond merely pipe flow. Hydraulic systems are designed to accommodate water at rest and in motion. The fundamentals of hydraulic engineering systems, therefore, involve the application of engineering principles and methods to the planning, control, transportation, conservation, and utilization of water.

It is important that the reader understand the physical properties of water to solve properly the various problems in hydraulic engineering systems. For example, the density, the surface tension, and the viscosity of water all vary, in one way or another, with water temperature. Density is a fundamental property that directly relates to the operation of a large reservoir. For example, change of density with temperature causes water to stratify in the summer, with warmer water on top of colder water. During the late fall, the surface water temperature drops rapidly, and water begins to sink toward the reservoir bottom. The warmer water near the bottom rises to the surface, resulting in “fall overturn” in northern climates. In the winter, the surface water freezes while warmer water remains insulated below the ice. The winter stratification is followed by a “spring

overtun.” The ice melts, and the surface water warms to 4° Celsius (C) (highest water density) and sinks to the bottom as the warmer water below rises. Similarly, variation of surface tension directly affects the evaporation loss from a large water body in storage; the variation of water viscosity with temperature is important to all problems involving water in motion.

This chapter discusses the fundamental physical properties of water that are important to problems in hydraulic engineering systems.

1.1 The Earth’s Atmosphere and Atmospheric Pressure

The earth’s atmosphere is a thick layer (approximately 1,500 km) of mixed gases. Nitrogen makes up approximately 78 percent of the atmosphere, oxygen makes up approximately 21 percent, and the remaining 1 percent consists mainly of water vapor, argon, and trace amounts of other gases. Each gas possesses a certain mass and consequently has a weight. The total weight of the atmospheric column exerts a pressure on every surface it contacts. At sea level and under normal conditions, the *atmospheric pressure* is approximately equal to $1.014 \times 10^5 \text{ N/m}^2$, or approximately 1 bar.* The pressure unit 1 N/m^2 is also known as 1 *pascal*, named after French mathematician Blaise Pascal (1623 to 1662).

Water surfaces that come in contact with the atmosphere are subjected to atmospheric pressure. In the atmosphere, each gas exerts a partial pressure independently of the other gases. The partial pressure exerted by the water vapor in the atmosphere is called the *vapor pressure*.

1.2 The Three Phases of Water

The water molecule is a stable chemical bond of oxygen and hydrogen atoms. The amount of energy holding the molecules together depends on the temperature and pressure present. Depending on its energy content, water may appear in solid, liquid, or gaseous form. Snow and ice are the solid forms of water; liquid is its most commonly recognized form; and moisture, water vapor in air, is water in its gaseous form. The three different forms of water are called its three *phases*.

To change water from one phase to another phase, energy must either be added or taken away from the water. The amount of energy required to change water from one phase to another is known as a *latent energy*. This amount of energy may be in the form of heat or pressure. One of the most common units of heat energy is the calorie (cal). One calorie is the energy required to increase the temperature of 1 gram (g) of water, in liquid phase, by 1°C. The amount of energy required to raise the temperature of a substance by 1°C is known as the *specific heat* of that substance. The latent heat and specific heat of all three phases of water are discussed next.

Under standard atmospheric pressure, the specific heat of water and ice are, respectively, 1.0 and 0.465 cal/g · °C. For water vapor, the specific heat under constant pressure is 0.432 cal/g · °C, and at constant volume it is 0.322 cal/g · °C. These values may vary slightly depending on the purity of the water. To melt 1 g of ice, changing water from its solid to liquid phase, requires a latent heat (*heat of fusion*) of 79.7 cal. To freeze water, the same amount of heat energy must be taken out of each gram of water, thus the process is reversed. *Evaporation*, the changing of liquid-phase water into its gaseous phase, requires a latent heat (*heat of vaporization*) of 597 cal/g.

Evaporation is a rather complex process. Under standard atmospheric pressure, water boils at 100°C. At higher elevations, where the atmospheric pressure is less, water boils at

* 1 atmospheric pressure = $1.014 \times 10^5 \text{ N/m}^2 = 1.014 \times 10^5 \text{ pascals}$
 = 1.014 bars = 14.7 lbs/in.²
 = 760 mm Hg = 10.33 m H₂O

TABLE 1.1 Vapor Pressure of Water

Temperature (°C)	Vapor Pressure		Temperature (°C)	Vapor Pressure	
	Atm	N/m ²		Atm	N/m ²
-5	0.004162	421	55	0.15531	15,745
0	0.006027	611	60	0.19656	19,924
5	0.008600	873	65	0.24679	25,015
10	0.012102	1,266	70	0.30752	31,166
15	0.016804	1,707	75	0.38043	38,563
20	0.023042	2,335	80	0.46740	47,372
25	0.031222	3,169	85	0.57047	57,820
30	0.041831	4,238	90	0.69192	70,132
35	0.055446	5,621	95	0.83421	84,552
40	0.072747	7,377	100	1.00000	101,357
45	0.094526	9,584	105	1.19220	120,839
50	0.121700	12,331	110	1.41390	143,314

temperatures lower than 100°C. This phenomenon may be explained best from a molecular-exchange viewpoint.

At the gas–liquid interface, there is a continual interchange of molecules leaving the liquid to the gas and molecules entering the liquid from the gas. Net evaporation occurs when more molecules are leaving than are entering the liquid; net *condensation* occurs when more molecules are entering than are leaving the liquid. Equilibrium exists when the molecular exchange at the gas–liquid interface are equal over a time interval.

Vapor molecules in the air exert a partial pressure on any contact surface that is known as the *vapor pressure*. This partial pressure combined with the partial pressures created by other gases in the atmosphere makes up the total atmospheric pressure.

If the temperature of a liquid is increased, the molecular energy is raised, causing a large number of molecules to leave the liquid. This, in turn, increases the vapor pressure. When the temperature reaches a point at which the vapor pressure is equal to the ambient atmospheric pressure, evaporation increases significantly, and boiling of the liquid takes place. The temperature at which a liquid boils is commonly known as the liquid’s *boiling point*. For water at sea level, the boiling point is 100°C. The vapor pressure of water is shown in Table 1.1.

In a closed system (e.g., pipelines or pumps), water vaporizes rapidly in regions where the pressure drops below the vapor pressure. This phenomenon is known as *cavitation*. The vapor bubbles formed in cavitation usually collapse in a violent manner when they move into higher pressure regions. This may cause considerable damage to a system. Cavitation in a closed hydraulic system can be avoided by maintaining the pressure above the vapor pressure everywhere in the system.

1.3 Mass (Density) and Weight (Specific Weight)

In the International System of Units (SI),* the unit of measurement for mass is either gram or kilogram (kg). The *density* of a substance is defined as the mass per unit volume. It is a property inherent in the molecular structure of the substance. This means that density depends not only on the size and weight of the molecules but also on the mechanisms by which the molecules are bonded together. The latter usually varies as a function of temperature and pressure. Because of

* From the French *Le Système International d’Unités*.

TABLE 1.2 Density and Specific Weight of Water

Temperature (°C)	Density (ρ , kg/m ³)	Specific Weight (γ , N/m ³)
0° (ice)	917	8,996
0° (water)	999	9,800
4°	1,000	9,810
10°	999	9,800
20°	998	9,790
30°	996	9,771
40°	992	9,732
50°	988	9,692
60°	983	9,643
70°	978	9,594
80°	972	9,535
90°	965	9,467
100°	958	9,398

its peculiar molecular structure, water is one of the few substances that expands when it freezes. The expansion of freezing water when contained in a closed container causes stresses on the container walls. These stresses are responsible for the bursting of frozen water pipes, the creation of cracks and holes in road pavement, and the weathering of rocks in nature.

Water reaches a maximum density at 4°C. It becomes less dense when further chilled or heated. The density of water is shown as a function of temperature in Table 1.2. Note that the density of ice is different from that of liquid water at the same temperature. We observe this phenomenon when we see ice float on water.

Seawater (or ocean water) contains dissolved salt. The molecules that make up the salt have more mass than the molecules they displace. Therefore, the density of seawater is about 4 percent more than that of freshwater. Thus, when freshwater meets seawater without sufficient mixing, as in Chesapeake Bay, salinity will increase with depth.

In the SI system, the weight of an object is defined by the product of its mass (m , in grams, kilograms, etc.) and the gravitational acceleration ($g = 9.81 \text{ m/sec}^2$ on Earth). The relationship* may be written as

$$W = mg \quad (1.1)$$

Weight in the SI system is usually expressed in the force units of newtons (N). One newton is defined as the force required to accelerate 1 kg of mass at a rate of 1 m/sec². The *specific weight* (weight per unit volume) of water (γ) can be determined by the product of the density (ρ) and the gravitational acceleration (g). The ratio of the specific weight of any liquid at a given temperature to that of water at 4°C is called the *specific gravity* of that liquid. Note that the specific weight of water is shown as a function of temperature in Table 1.2.

The unit of mass in the British system is the slug. One slug is defined as the mass of an object that requires 1 lb of force to reach an acceleration of 1 ft/sec².

* In the British (Imperial) system, the mass of an object is defined by its weight (ounce or pound) and the gravitational acceleration ($g = 32.2 \text{ ft/sec}^2$ on Earth). The relationship is written as

$$m = W/g \quad (1.1a)$$

Example 1.1

An aquarium holds 0.5 m^3 of water. The weight of the aquarium is $5,090 \text{ N}$ when full and 200 N when empty. Determine the temperature of the water.

Solution

The weight of the water in the aquarium is:

$$W(\text{water}) = 5,090 \text{ N} - 200 \text{ N} = 4,890 \text{ N}$$

The specific weight of the water is:

$$\gamma = 4,890\text{N}/(0.5 \text{ m}^3) = 9,780 \text{ N/m}^3$$

The temperature of the water can be obtained from Table 1.2:

$$T \approx 25^\circ\text{C}$$

1.4 Viscosity of Water

Water responds to shear stress by continuously yielding in angular deformation in the direction of the shear as shown in Figure 1.1. This leads to the concept of *viscosity*. The schematic diagram in Figure 1.1 represents the physical basis of viscosity. Consider that water fills the space between two parallel plates (lightweight plastic) that are a distance y apart. A horizontal force T is applied to the upper plate and moves it to the right at velocity v while the lower plate remains stationary. The shear force T is applied to overcome the water resistance R , and it must be equal to R because there is no acceleration involved in the process. The resistance per unit area of the upper plate (shear stress, $\tau = R/A = T/A$) has been found to be proportional to the rate of angular deformation in the fluid, $d\theta > dt$. The relationship may be expressed as

$$\tau \propto \frac{d\theta}{dt} = \frac{dx/dy}{dt} = \frac{dx/dt}{dy} = \frac{dv}{dy}$$

where $v = dx/dt$ is the velocity of the fluid element. Alternatively,

$$\tau = \mu \left(\frac{dv}{dy} \right) \tag{1.2}$$

The proportionally constant, μ , is the *absolute viscosity* of the fluid. Equation 1.2 is commonly known as *Newton's law of viscosity*. Most liquids abide by this relationship and are called *Newtonian fluids*. Liquids that do not abide by this linear relationship are known as *non-Newtonian fluids*. These include most house paints and human blood.

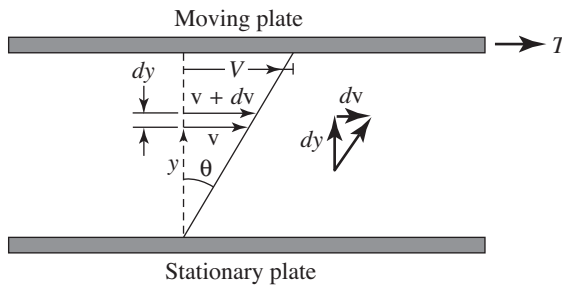


Figure 1.1 Shearing stresses in fluids

TABLE 1.3 Viscosities of Water and Air

Temperature (°C)	Water		Air	
	Viscosity (μ) N·sec/m ²	Kinematic Viscosity (ν) m ² /sec	Viscosity (μ) N·sec/m ²	Kinematic Viscosity (ν) m ² /sec
0	1.781×10^{-3}	1.785×10^{-6}	1.717×10^{-5}	1.329×10^{-5}
5	1.518×10^{-3}	1.519×10^{-6}	1.741×10^{-5}	1.371×10^{-5}
10	1.307×10^{-3}	1.306×10^{-6}	1.767×10^{-5}	1.417×10^{-5}
15	1.139×10^{-3}	1.139×10^{-6}	1.793×10^{-5}	1.463×10^{-5}
20	1.002×10^{-3}	1.003×10^{-6}	1.817×10^{-5}	1.509×10^{-5}
25	0.890×10^{-3}	0.893×10^{-6}	1.840×10^{-5}	1.555×10^{-5}
30	0.798×10^{-3}	0.800×10^{-6}	1.864×10^{-5}	1.601×10^{-5}
40	0.653×10^{-3}	0.658×10^{-6}	1.910×10^{-5}	1.695×10^{-5}
50	0.547×10^{-3}	0.553×10^{-6}	1.954×10^{-5}	1.794×10^{-5}
60	0.466×10^{-3}	0.474×10^{-6}	2.001×10^{-5}	1.886×10^{-5}
70	0.404×10^{-3}	0.413×10^{-6}	2.044×10^{-5}	1.986×10^{-5}
80	0.354×10^{-3}	0.364×10^{-6}	2.088×10^{-5}	2.087×10^{-5}
90	0.315×10^{-3}	0.326×10^{-6}	2.131×10^{-5}	2.193×10^{-5}
100	0.282×10^{-3}	0.294×10^{-6}	2.174×10^{-5}	2.302×10^{-5}

Absolute viscosity has the dimension of force per unit area (stress) multiplied by the time interval considered. It is usually measured in the unit of *poise* (named after French engineer-physiologist J. L. M. Poiseuille). The absolute viscosity of water at room temperature (20.2°C) is equal to 1 centipoise (cP), which is one one-hundredth (1/100) of a poise:

$$1 \text{ poise} = 0.1 \text{ N} \cdot \text{sec}/\text{m}^2 = 100 \text{ cP} \quad \text{or} \quad (1 \text{ N} \cdot \text{sec}/\text{m}^2 = 1,000 \text{ cP})$$

The absolute viscosity of air is approximately 0.018 cP (roughly 2 percent of water).

In engineering practice, it is often convenient to introduce the term *kinematic viscosity*, ν , which is obtained by dividing the absolute viscosity by the mass density of the fluid at the same temperature: $\nu = \mu/\rho$. The kinematic viscosity carries the unit of cm²/sec (with the unit of *stokes*, named after British mathematician G. G. Stoke). The absolute viscosities and the kinematic viscosities of pure water and air are shown as functions of temperature in Table 1.3.

Example 1.2

A flat plate of 50 cm² is being pulled over a fixed flat surface at a constant velocity of 45 cm/sec (Figure 1.1). An oil film of unknown viscosity separates the plate and the fixed surface by a distance of 0.1 cm. The force (T) required to pull the plate is measured to be 31.7 N, and the viscosity of the fluid is constant. Determine the viscosity (absolute).

Solution

The oil film is assumed to be Newtonian, therefore the equation $\tau = \mu(dv/dy)$ applies:

$$\mu = \tau/(dv/dy) = (T/A)/(\Delta v/\Delta y)$$

because $\tau = T/A$ and the velocity–distance relationship is assumed to be linear. Therefore,

$$\mu = [(31.7 \text{ N})/(50 \text{ cm}^2)] / [(45 \text{ cm}/\text{sec})/0.1 \text{ cm}]$$

$$\mu = 1.41 \times 10^{-3} \text{ N} \cdot \text{sec}/\text{cm}^2 [(100 \text{ cm})^2/(1 \text{ m})^2] = 14.1 \text{ N} \cdot \text{sec}/\text{m}^2$$

1.5 Surface Tension and Capillarity

Even at a small distance below the surface of a liquid body, liquid molecules are attracted to each other by equal forces in all directions. The molecules on the surface, however, are not able to bond in all directions and therefore form stronger bonds with adjacent liquid molecules. This causes the liquid surface to seek a minimum possible area by exerting *surface tension* tangent to the surface over the entire surface area. A steel needle floating on water, the spherical shape of dewdrops, and the rise and fall of liquid in capillary tubes are the results of surface tension.

Most liquids adhere to solid surfaces. The adhesive force varies, depending on the nature of the liquid and of the solid surface. If the adhesive force between the liquid and the solid surface is greater than the cohesion in the liquid molecules, then the liquid tends to spread over and wet the surface as shown in Figure 1.2 (a). If the cohesion is greater, then a small drop forms as in Figure 1.1. Water wets the surface of glass, but mercury does not. If we place a small-bore vertical glass tube into the free surface of water, the water surface in the tube rises. The same experiment performed with mercury will show that the mercury falls. These two typical cases are schematically presented in Figures 1.3 (a) and 1.3 (b). The phenomenon is commonly known as *capillary action*. The magnitude of the capillary rise (or depression), h , is determined by the balance of adhesive force between the liquid and solid surface and the weight of the liquid column above (or below) the liquid-free surface.

The angle θ at which the liquid film meets the glass depends on the nature of the liquid and the solid surface. The upward (or downward) motion in the tube will cease when the vertical component of the surface-tension force around the edge of the film equals the weight of the

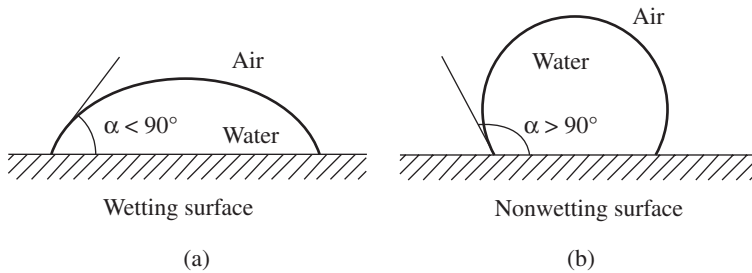


Figure 1.2 Wetting and nonwetting surfaces

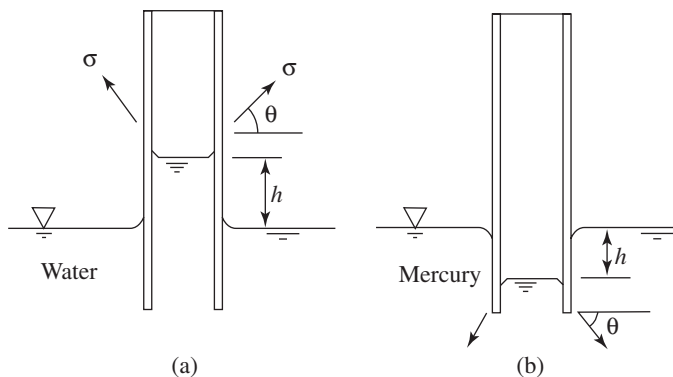


Figure 1.3 Capillary actions

TABLE 1.4 Surface Tension of Water

Surface Tension	Temperature (°C)									
	0	10	20	30	40	50	60	70	80	90
σ ($\times 10^{-2}$ N/m)	7.416	7.279	7.132	6.975	6.818	6.786	6.611	6.436	6.260	6.071
σ (dyne/cm)	74.16	72.79	71.32	69.75	68.18	67.86	66.11	64.36	62.60	60.71

raised (or lowered) liquid column. When the very small volume of liquid above (or below) the base of the curved meniscus is neglected, the relationship may be expressed as

$$(\sigma\pi D) \sin \theta = \frac{\pi D^2}{4}(\gamma h)$$

Hence,

$$h = \frac{4\sigma \sin \theta}{\gamma D} \quad (1.3)$$

where σ and γ are the surface tension and the unit weight of the liquid, respectively, and D is the inside diameter of the vertical tube.

The surface tension of a liquid is usually expressed in the units of force per unit length. Its value depends on the temperature and the electrolytic content of the liquid. Small amounts of salt dissolved in water tend to increase the electrolytic content and, hence, the surface tension. Organic matter (such as soap) decreases the surface tension in water and permits the formation of bubbles. The surface tension of pure water as a function of temperature is listed in Table 1.4.

1.6 Elasticity of Water

Water is commonly assumed to be incompressible under ordinary conditions. In reality, it is about 100 times more compressible than steel. It is necessary to consider the compressibility of water when water hammer issues are possible (see Chapter 4). The *compressibility* of water is inversely proportional to its *volume modulus of elasticity*, E_b , also known as the *bulk modulus of elasticity*. The pressure–volume relationship may be expressed as

$$\Delta P = -E_b \left(\frac{\Delta Vol}{Vol} \right) \quad (1.4)$$

where Vol is the initial volume, and ΔP and ΔVol are the corresponding changes in pressure and volume, respectively. The negative sign means that a positive change in pressure (i.e., pressure increase) will cause the volume to decrease (i.e., negative change). The bulk modulus of elasticity of water varies with both temperature and pressure. In the range of practical applications in typical hydraulic systems, a value of 2.2×10^9 N/m², or in BG* units, 3.2×10^5 lb/in² (psi) can be used.

* British gravitational system of units

Example 1.3

The density of seawater is $1,026 \text{ kg/m}^3$ at sea level. Determine the density of seawater on the ocean floor 2,000 m deep, where the pressure is approximately $2.02 \times 10^7 \text{ N/m}^2$.

Solution

The change of pressure at a depth of 2,000 m from the pressure at the water surface is

$$\Delta P = P - P_{\text{atm}} = 2.01 \times 10^7 \text{ N/m}^2$$

From Equation 1.4 we have

$$\Delta P = -E_b \left(\frac{\Delta Vol}{Vol_o} \right)$$

so that

$$\left(\frac{\Delta Vol}{Vol_o} \right) = \left(\frac{-\Delta P}{E_b} \right) = \frac{-2.01 \times 10^7}{2.20 \times 10^9} = -0.00914$$

Because

$$\rho = \left(\frac{m}{Vol} \right) \quad \therefore Vol = \left(\frac{m}{\rho} \right)$$

then

$$\Delta Vol = \left(\frac{m}{\rho} \right) - \left(\frac{m}{\rho_o} \right) \quad \therefore \frac{\Delta Vol}{Vol_o} = \left(\frac{\rho_o}{\rho} \right) - 1$$

so that

$$\rho = \left(\frac{\rho_o}{1 + \frac{\Delta Vol}{Vol_o}} \right) = \left(\frac{1,026 \text{ kg/m}^3}{1 - 0.00914} \right) = 1,040 \text{ kg/m}^3$$

1.7 Forces in a Fluid Field

Various types of forces may be exerted on a body of water at rest or in motion. In hydraulic practice, these forces usually include the effects of gravity, inertia, elasticity, friction, pressure, and surface tension.

These forces may be classified into three basic categories according to their physical characteristics:

1. body forces,
2. surface forces, and
3. line forces (or forces over a solid–liquid contact distance).

Body forces are forces that act on all particles in a body of water as a result of some external body or effect but not because of direct contact. An example of this is the gravitational force. It acts on all particles in any body of water as a result of the earth’s gravitational field, which may not be in direct contact with the particular water body of interest. Other body forces that are common in hydraulic practice include inertial forces and forces resulting from elastic effects. Body forces are usually expressed as force per unit mass (N/kg) or force per unit volume (N/m³).

Surface forces act on the surface of the water body by direct contact. These forces may be either external or internal. Pressure forces and friction forces are examples of external surface forces. The viscous force inside a fluid body may be viewed as an internal surface force. Surface forces are expressed as force per unit area (N/m^2).

Line forces act at the liquid surface normal to a line drawn in the surface. They often act along a linear solid–liquid interface. An example of a line force is surface tension. Line forces are expressed as force per unit length (N/m).

PROBLEMS (SECTION 1.2)

- 1.2.1. How much energy must be added to ice at -20°C to produce 250 liters of water at $+20^\circ\text{C}$?
- 1.2.2. Compute the heat energy (in calories) required to evaporate 1,200 g of water at 45°C under an ambient pressure of 0.9 bars.
- 1.2.3. At 0°C and at an absolute pressure of 911 N/m^2 , 100 g of water, 100 g of vapor, and 100 g of ice are in equilibrium in a sealed thermal container. Determine how much energy should be removed to freeze all the water and vapor.
- 1.2.4. What pressure must be maintained in a closed vessel to turn 100 liters of water at 10°C entirely into water vapor by using only 6.8×10^7 calories of energy?
- 1.2.5. A large kitchen pot initially contains 5 kg of water at 25°C . A burner is turned on and heat is added to the water at a rate of 500 cal/sec. Determine how many minutes it will take for one-half of the mass to evaporate at standard atmospheric pressure.
- 1.2.6. Determine the final temperature of a water–ice bath that is allowed to establish equilibrium in a thermal container. The water–ice bath is produced when 5 slugs of ice (specific heat = $0.46 \text{ BTU/lbm} \cdot ^\circ\text{F}$) at 20°F is mixed with 10 slugs of water (specific heat = $1.0 \text{ BTU/lbm} \cdot ^\circ\text{F}$) at 120°F . (Note: 1 slug = 32.2 lbm, and the heat of fusion is 144 BTU/lbm .)

(SECTION 1.3)

- 1.3.1. A container weighs 863 N when filled with water and 49 N when empty. How much water (at 20°C) does the container hold in cubic meters?
- 1.3.2. Derive the relationship between specific weight and density starting with Newton's second law ($F = ma$).
- 1.3.3. Mercury has a density of $13,600 \text{ kg/m}^3$. What are its specific weight and specific gravity?
- 1.3.4. A cylindrical water tank (Figure P1.3.4) is suspended vertically by its sides. The tank has a 10-ft diameter and is filled with 20°C water to a 3-ft depth. Determine the force exerted on the tank bottom.
- 1.3.5. A rocket carrying a tank of water weighing 7.85 kN on Earth lands on the moon where the gravitational acceleration is one-sixth that of Earth's. Find the mass and the moon weight of the water.
- 1.3.6. Determine the weight and specific gravity of a gallon of liquid if it has a mass of 0.258 slugs.
- 1.3.7. Determine how much 100 m^3 of water changes in volume when it is heated from 4°C (when water is the most dense) to 100°C (when it is the least dense).
- 1.3.8. The unit of force in the SI system is a newton. Convert one unit of force in this system to the British system in pounds.
- 1.3.9. The unit of energy in the SI system is a newton-meter (or joule). Convert one unit of energy in this system to the BG system (foot-pound).

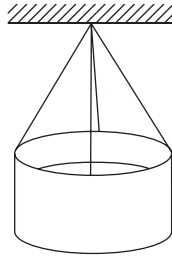


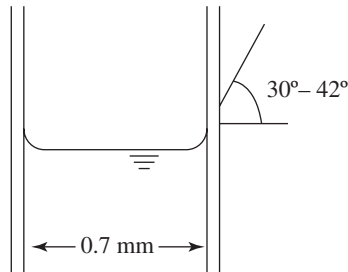
Figure P1.3.4

(SECTION 1.4)

- 1.4.1.** Compare the ratios of the absolute and kinematic viscosities of air and water at (a) 20°C and (b) 80°C. Discuss the differences.
- 1.4.2.** Convert the absolute and kinematic viscosity of water (Table 1.3) at 68°F (20°C) to the British system equivalent. (Check your result by looking in the front jacket of the book.)
- 1.4.3.** Establish the equivalence between
- absolute viscosity units of poise and $\text{lb} \cdot \text{sec}/\text{ft}^2$
 - kinematic viscosity units of stoke and the British system unit.
- 1.4.4.** Determine the force required to drag a small barge (10 ft by 30 ft) in a shallow canal (3 inches deep) in order to maintain a velocity of 5 ft/sec. Assume that the fluid is behaving in a Newtonian fashion and the water temperature is 68°F.
- 1.4.5.** A liquid flows with velocity distribution $v = y^2 - 2y$, where v is given in ft/sec and y in inches. Calculate the shear stresses at $y = 0, 1, 2, 3, 4$ if the viscosity is 375 centipoise (cP).
- 1.4.6.** A flat plate weighing 220 N slides down a 15° incline with a velocity of 2.5 cm/sec. A thin film of oil with a viscosity of $1.29 \text{ N} \cdot \text{sec}/\text{m}^2$ separates the plate from the ramp. If the plate is 50 cm by 75 cm, calculate the film thickness in cm.
- 1.4.7.** A piston slides downward with a constant velocity through a vertical pipe. The piston is 5.48 inches in diameter and 9.50 inches in length. An oil film between the piston and pipe wall resists the downward motion. If the film thickness is 0.002 in. and the cylinder weighs 0.5 pounds, estimate the velocity. (Assume the oil viscosity is $0.016 \text{ lb} \cdot \text{sec}/\text{ft}^2$ and the velocity distribution in the gap is linear.)
- 1.4.8.** SAE lubricating oil ($\mu = 0.0065 \text{ lb} \cdot \text{sec}/\text{ft}^2$) is used to fill a 1-in. space between two large stationary plates. How much force is required to pull a very thin plate (with a surface area of 2.00 ft^2) between the stationary plates at a speed of 1 ft/sec?
- 1.4.9.** Fluid viscosity can be measured by a rotating-cylinder viscometer, which consists of two concentric cylinders with a uniform gap between them. The liquid to be measured is poured into the gap between the two cylinders. For a particular liquid, the inner cylinder rotates at 2,000 rpm, and the outer cylinder remains stationary and measures a torque of $1.50 \text{ N} \cdot \text{m}$. The inner cylinder diameter is 5.0 cm, the gap width is 0.02 cm, and the liquid fills up to a height of 4.0 cm in the cylindrical gap. Determine the absolute viscosity of the liquid in $\text{N} \cdot \text{sec}/\text{m}^2$.
- 1.4.10.** A flat circular disk of radius 1.0 m is rotated at an angular velocity of 0.65 rad/sec over a fixed flat surface. An oil film separates the disk and the surface. If the viscosity of the oil is 16 times that of water (20°C) and the space between the disk and the fixed surface is 0.5 mm, what is the torque required to rotate the disk?

(SECTION 1.5)

- 1.5.1.** A capillary rise experiment is proposed for a high school physics class. The students are told that for water in clean glass tubes, the contact angle between liquid and glass (θ) is 90° . The students are asked to measure capillary rise in a series of tube diameters ($D = 0.05$ cm, 0.10 cm, 0.15 cm, 0.20 cm, etc.). They are then asked to graph the results and determine the approximate tube diameters that would produce capillary rises of 3 cm, 2 cm, and 1 cm. Predict the results if the water used in the experiment is 20°C .
- 1.5.2.** Surface tension is usually referred to as a line force (i.e., units of force per unit length) rather than a surface force (like pressure with units of force per unit area) or a body force (like specific weight with units of force per unit volume). Examine the development of Equation 1.3 and explain why the concept of a line force is a logical one.
- 1.5.3.** A liquid is observed to rise to a height of 0.6 in. in a 0.02 -in. glass tube. The contacting angle is determined to be 54° . Determine the surface tension of the liquid in lb/ft if its density is 1.94 slug/ft³.
- 1.5.4.** Laboratory experiments are being conducted using many opaque, saltwater tanks. Straight glass tubes (with a 0.25 cm inside diameter mounted on the tanks) are proposed to track the depth of the saltwater during the experiment. Determine the measurement error (in cm) resulting from the use of the tubes if the saltwater has a surface tension that is 20 percent greater than freshwater and the contact angle is 30° . (Assume the saltwater has a specific gravity of 1.03 and a temperature of 35°C .)
- 1.5.5.** A small amount of solvent is added to the ground water to change its electrolytic content. As a result, the contacting angle, θ , representing the adhesion between water and soil material, is increased from 30° to 42° while the surface tension decreases by 10 percent (Figure P1.5.5). The soil has a uniform pore size of 0.7 mm. Determine the change of capillary rise in the soil.

**Figure P1.5.5**

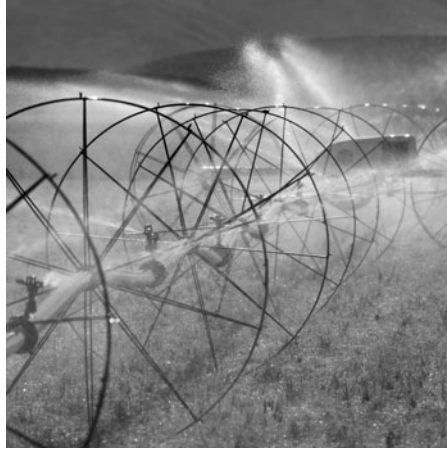
- 1.5.6.** The pressure inside a droplet of water is greater than the pressure outside. Split a droplet in half and identify forces. The bursting force is the pressure difference times the area that is balanced by the surface tension acting on the circumference. Derive an expression for the pressure difference.

(SECTION 1.6)

- 1.6.1.** Determine the bulk modulus of elasticity (in N/m^2) of a liquid that has a 1.10 percent volume decrease because of an increase in pressure from $1,000$ N/cm^2 to $11,000$ N/cm^2 .

- 1.6.2.** Determine the percentage change in density of water at 20°C when pressure is suddenly changed from 25 bars to 450,000 N/m².
- 1.6.3.** A steel tank holds 120 ft³ of water at atmospheric pressure (14.7 lbs/in.² and 68.4°F). The water is subjected to a 100-fold increase in pressure. Determine the initial weight and final density of the water if the volume decreases by 0.545 ft³.
- 1.6.4.** The pressure in a 150-cm diameter pipe 2,000 m long is 30 N/cm². Determine the amount of water that will enter the pipe if the pressure increases to 30 bars. Assume the pipe is rigid and does not increase its volume.

2



Water Pressure and Pressure Forces

2.1 The Free Surface of Water

When water fills a containing vessel, it automatically seeks a horizontal surface on which the pressure is constant everywhere. In practice, a free water surface is one that is not in contact with an overlying vessel cover. A free water surface may be subjected to atmospheric pressure (open vessel) or any other pressure that is exerted within the vessel (closed vessel).

2.2 Absolute and Gauge Pressures

A water surface in contact with the earth's atmosphere is subjected to atmospheric pressure, which is approximately equal to a 10.33-m-high column of water at sea level. In still water, any object located below the water surface is subjected to a pressure greater than atmospheric pressure. This additional pressure is often referred to as *hydrostatic pressure*. More precisely, it is the force per unit area acting in a normal direction on the surface of a body immersed in the fluid (in this case water).

To determine the variation of hydrostatic pressure between any two points in water (with a specific weight of γ), we may consider two arbitrary points A and B along an arbitrary x -axis, as shown in Figure 2.1. Consider that these points lie in the ends of a small prism of water having a cross-sectional area dA and a length L . P_A and P_B are the pressures at each

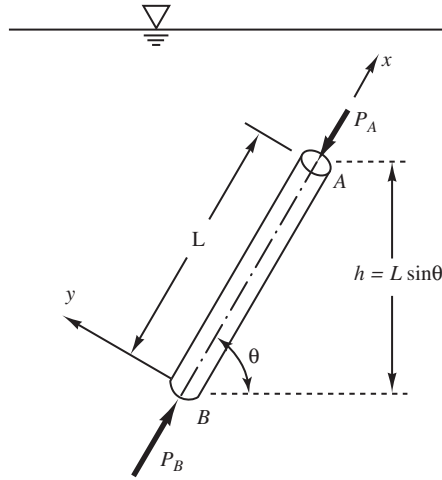


Figure 2.1 Hydrostatic pressure on a prism

end, where the cross-sectional areas are normal to the x -axis. Because the prism is at rest, all forces acting on it must be in equilibrium in all directions. For the force components in the x -direction, we may write

$$\Sigma F_x = P_A dA - P_B dA + \gamma L dA \sin \theta = 0$$

Note that $L \cdot \sin \theta = h$ is the vertical elevation difference between the two points. The above equation reduces to

$$P_B - P_A = \gamma h \tag{2.1}$$

Therefore, *the difference in pressure between any two points in still water is always equal to the product of the specific weight of water and the difference in elevation between the two points.*

If the two points are on the same elevation, $h = 0$ and $P_A = P_B$. In other words, *for water at rest, the pressure at all points in a horizontal plane is the same.* If the water body has a free surface that is exposed to *atmospheric pressure, P_{atm}* , we may position point A on the free surface and write

$$(P_B)_{abs} = \gamma h + P_A = \gamma h + P_{atm} \tag{2.2}$$

This pressure, $(P_B)_{abs}$, is commonly referred to as the *absolute pressure*.

Pressure gauges are usually designed to measure pressures above or below the atmospheric pressure. Pressure so measured, using atmospheric pressure as a base, is called *gauge pressure, P* . Absolute pressure is always equal to gauge pressure plus atmospheric pressure:

$$P = P_{abs} - P_{atm} \tag{2.3}$$

Figure 2.2 diagrammatically shows the relationship between the absolute and gauge pressure and two typical pressure-gauge dials. Comparing Equations 2.2 and 2.3, we have

$$P = \gamma h \tag{2.4}$$

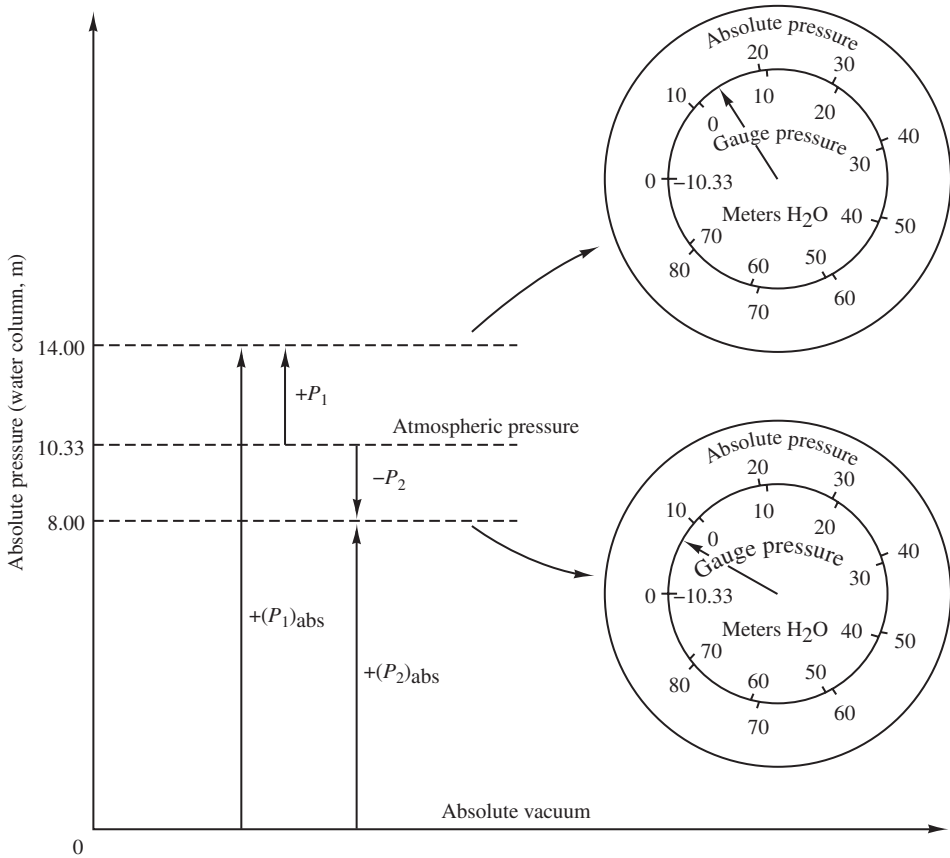


Figure 2.2 Absolute and gauge pressure

or

$$h = \frac{P}{\gamma} \tag{2.5}$$

Here the pressure is expressed in terms of the height of a water column h . In hydraulics it is known as the *pressure head*.

Equation 2.1 may thus be rewritten in a more general form as

$$\frac{P_B}{\gamma} - \frac{P_A}{\gamma} = \Delta h \tag{2.6}$$

meaning that the difference in pressure heads at two points in water at rest is always equal to the difference in elevation between the two points. From this relationship we can also see that any change in pressure at point B would cause an equal change at point A , because the difference in pressure head between the two points must remain the same value Δh . In other words, *a pressure applied at any point in a liquid at rest is transmitted equally and undiminished in all directions to every other point in the liquid*. This principle, also known as *Pascal's law*, has been made use of in the hydraulic jacks that lift heavy weights by applying relatively small forces.

Example 2.1

The diameters of cylindrical pistons *A* and *B* are 3 cm and 20 cm, respectively. The faces of the pistons are at the same elevation, and the intervening passages are filled with an incompressible hydraulic oil. A force *P* of 100 N is applied at the end of the lever, as shown in Figure 2.3. What weight *W* can the hydraulic jack support?

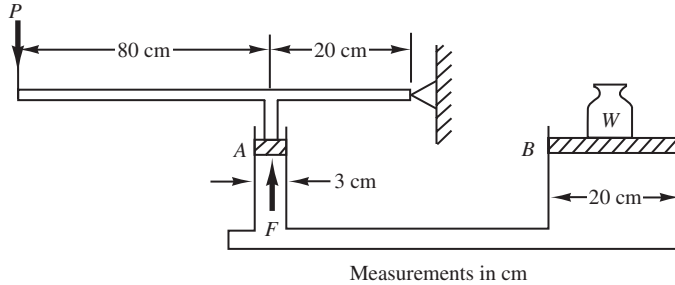


Figure 2.3 Hydraulic jack

Solution

Balancing the moments produced by *P* and *F* around the pin connection yields

$$(100 \text{ N})(100 \text{ cm}) = F(20 \text{ cm})$$

Thus,

$$F = 500 \text{ N}$$

From Pascal’s law, the pressure P_A applied at *A* is the same as that of P_B applied at *B*. Therefore,

$$P_A = \frac{F}{[(\pi \cdot 3^2)/4] \text{ cm}^2} \qquad P_B = \frac{W}{[(\pi \cdot 20^2)/4] \text{ cm}^2}$$

$$\frac{500 \text{ N}}{7.07 \text{ cm}^2} = \frac{W}{314 \text{ cm}^2}$$

$$\therefore W = 500 \text{ N} \left(\frac{314 \text{ cm}^2}{7.07 \text{ cm}^2} \right) = 2.22 \times 10^4 \text{ N}$$

2.3 Surfaces of Equal Pressure

The hydrostatic pressure in a body of water varies with the vertical distance measured from the free water surface. In general, all points on a horizontal surface in a static body of water are subjected to the same hydrostatic pressure, according to Equation 2.4. For example, in Figure 2.4 (a), points 1, 2, 3, and 4 have equal pressure, and the horizontal surface that contains these four points is a *surface of equal pressure*. However, in Figure 2.4 (b), points 5 and 6 are on the same horizontal plane but the pressures are not equal. This is because the water in the two tanks is not connected and the overlying depths to the free surfaces are different. Applying Equation 2.4 would produce different pressures. Figure 2.4 (c) displays tanks filled with two immiscible liquids of different densities. (Note: Immiscible liquids do not readily mix under normal conditions.) The horizontal surface (7, 8) that passes through the interface of the two liquids is an

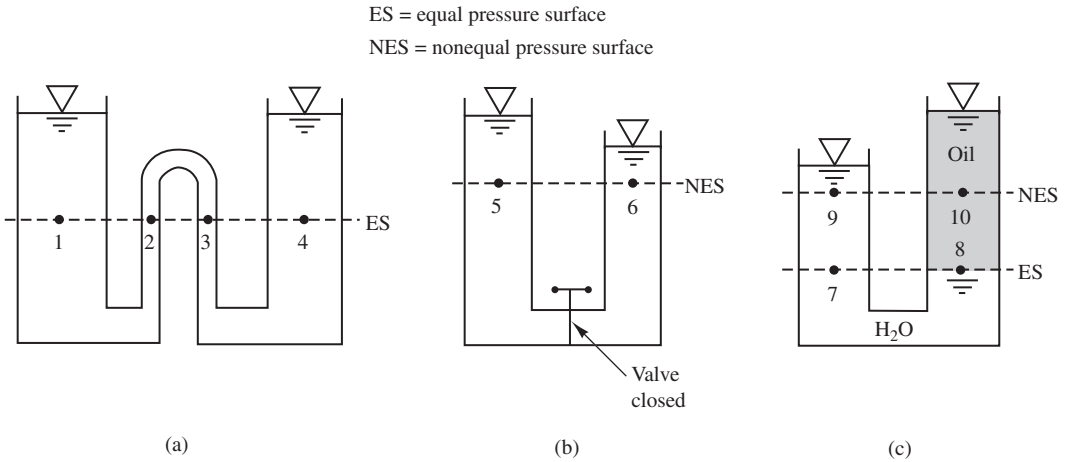


Figure 2.4 Hydraulic pressure in vessels

equal pressure surface. Applying Equation 2.4 at both points leads to the same pressure; we have the same fluid (water) at both locations (just below the interface at point 8), and both points are the same distance beneath the free water surface. However, points 9 and 10 are *not* on an equal pressure surface because they reside in different liquids. Verification would come from the application of Equation 2.4 using the different depths from the free surface to points 9 and 10 and the different specific weights of the fluids.

In summary, a surface of equal pressure requires that (1) the points on the surface be in the same liquid, (2) the points be at the same elevation (i.e., reside on a horizontal surface), and (3) the liquid containing the points be connected. The concept of equal pressure surface is a useful method in analyzing the strength or intensity of the hydrostatic pressure at various points in a container, as demonstrated in the following section.

2.4 Manometers

A *manometer* is a pressure-measurement device. It usually is a tube bent in the form of a “U” that contains a fluid of known specific gravity. The difference in elevations of the liquid surfaces under pressure indicates the difference in pressure at the two ends. Basically, there are two types of manometers:

1. An *open manometer* has one end open to atmospheric pressure and is capable of measuring the gauge pressure in a vessel.
2. A *differential manometer* has each end connected to a different pressure tap and is capable of measuring the pressure difference between the two taps.

The liquid used in a manometer is usually heavier than the fluids to be measured. It must form a distinct interface—that is, it must not mix with the adjacent liquids (i.e., immiscible liquids). The most frequently used manometer liquids are mercury (sp. gr. = 13.6), water (sp. gr. = 1.00), alcohol (sp. gr. = 0.9), and other commercial manometer oils of various specific gravities (e.g., from Meriam* Red Oil, sp. gr. = 0.827 to Meriam No. 3 Fluid, sp. gr. = 2.95).

* Meriam Process Technologies, Cleveland, Ohio 44102

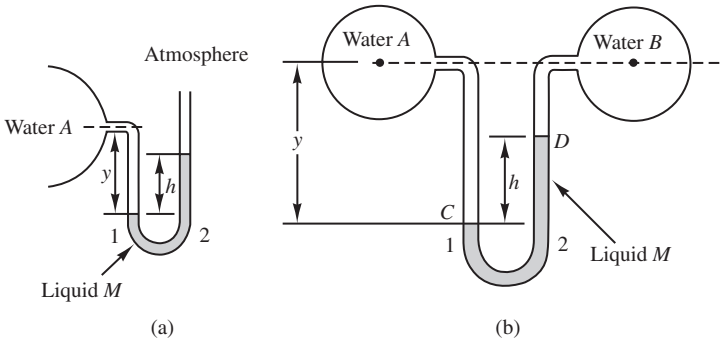


Figure 2.5 Types of manometers: (a) open manometer and (b) differential manometer

Figure 2.5 (a) shows a schematic of a typical open manometer; Figure 2.5 (b) shows a schematic of a typical differential manometer. It is obvious that the higher the pressure in vessel A, the larger the difference, h , in the surface elevations in the two legs of the manometer. A mathematical calculation of pressure in A, however, involves the densities of the fluids and the geometry involved in the entire measuring system.

A simple step-by-step procedure is suggested for pressure computation.

- Step 1. Make a sketch of the manometer system, similar to that in Figure 2.5, and approximately to scale.
- Step 2. Draw a horizontal line through the lower surface of the manometer liquid (point 1). The pressure at points 1 and 2 must be the same since the system is in static equilibrium.
- Step 3. (a) For open manometers, the pressure on 2 is exerted by the weight of the liquid M column above 2; and the pressure on 1 is exerted by the weight of the column of water above 1 plus the pressure in vessel A. The pressures must be equal in value. This relation may be written as follows:

$$\gamma_M h = \gamma y + P_A \quad \text{or} \quad P_A = \gamma_M h - (\gamma y)$$

- (b) For differential manometers, the pressure on 2 is exerted by the weight of the liquid M column above 2, the weight of the water column above D , and the pressure in vessel B, whereas the pressure on 1 is exerted by the weight of the water column above 1 plus the pressure in vessel A. This relationship may be expressed as:

$$\gamma_M h + \gamma (y - h) + P_B = \gamma y + P_A$$

or

$$\Delta P = P_A - P_B = h(\gamma_M - \gamma)$$

Either one of these equations can be used to solve for P_A . Of course, in the case of the differential manometer, P_B must be known. The same procedure can be applied to any complex geometry, as demonstrated in the following example.

Example 2.2

A mercury manometer (sp. gr. = 13.6) is used to measure the pressure difference in vessels A and B, as shown in Figure 2.6. Determine the pressure difference in pascals (N/m^2).

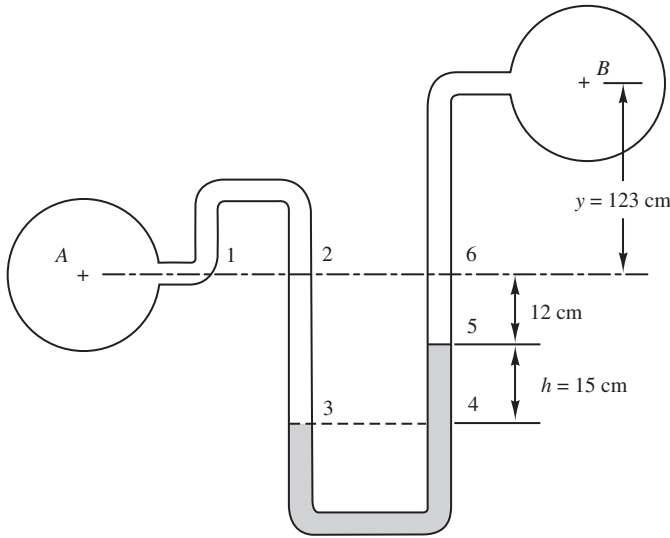


Figure 2.6

Solution

The sketch of the manometer system (step 1) is shown in Figure 2.6. Points 3 and 4 (P_3, P_4) are on a surface of equal pressure (step 2) and so are the vessel A and points 1 and 2 (P_1, P_2):

$$P_3 = P_4$$

$$P_A = P_1 = P_2$$

The pressures at points 3 and 4 are, respectively (step 3),

$$P_3 = P_2 + \gamma (27 \text{ cm}) = P_A + \gamma (27 \text{ cm})$$

$$P_4 = P_B + \gamma (135 \text{ cm}) + \gamma_M (15 \text{ cm})$$

Now

$$P_3 = P_A + \gamma (27 \text{ cm}) = P_4 = P_B + \gamma (135 \text{ cm}) + \gamma_M (15 \text{ cm})$$

and noting that $\gamma_M = \gamma$ (sp. gr.)

$$\Delta P = P_A - P_B = \gamma (135 \text{ cm} - 27 \text{ cm}) + \gamma_M (15 \text{ cm})$$

$$\Delta P = \gamma [108 + (13.6)(15)] \text{ cm} = (9790 \text{ N/m}^3)(3.12 \text{ m})$$

$$\Delta P = 30,500 \text{ N/m}^2 \text{ (pascals) or } 30.6 \text{ kilo-pascals}$$

The open manometer, or U-tube, requires readings of liquid levels at two points. In other words, any change in pressure in the vessel causes a drop of liquid surface at one end and a rise in the other. A *single-reading manometer* can be made by introducing a reservoir with a larger cross-sectional area than that of the tube into one leg of the manometer. A typical single-reading manometer is shown in Figure 2.7.

Because of the large area ratio between the reservoir and the tube, a small drop of surface elevation in the reservoir will cause an appreciable rise in the liquid column of the other leg. If there is an increase in pressure, ΔP_A will cause the liquid surface in the reservoir to drop by a small amount Δy . Then

$$A\Delta y = ah \tag{2.7}$$

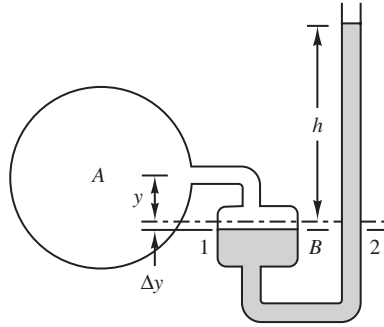


Figure 2.7 Single-reading manometer

where A and a are cross-sectional areas of the reservoir and the tube, respectively.

Applying step 2 to points 1 and 2, we may generally write

$$\gamma_A(y + \Delta y) + P_A = \gamma_B(h + \Delta y) \tag{2.8}$$

Simultaneous solution of Equations 2.7 and 2.8 give the value of P_A , the pressure in the vessel, in terms of h . All other quantities in Equations 2.7 and 2.8— A , a , y , γ_A , and γ_B —are quantities predetermined in the manometer design. A single reading of h will thus determine the pressure.

Because Δy can be made negligible by introducing a very large A/a ratio, the above relationship may be further simplified to

$$\gamma_A y + P_A = \gamma_B h \tag{2.9}$$

Thus, the height reading h is a measure of the pressure in the vessel.

The solution of practical hydraulic problems frequently requires the difference in pressure between two points in a pipe or a pipe system. For this purpose, differential manometers are frequently used. A typical differential manometer is shown in Figure 2.8.

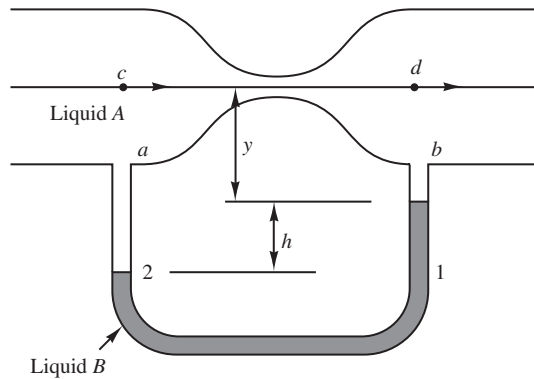


Figure 2.8 A differential manometer installed in a flow-measurement system

The same computation steps (steps 1, 2, and 3) suggested previously can be readily applied here, too. When the system is in static equilibrium, the pressure at the same elevation points, 1 and 2, must be equal. We may thus write

$$\gamma_A(y + h) + P_c = \gamma_B h + \gamma_A y + P_d$$

Hence, the pressure difference, ΔP , is expressed as

$$\Delta P = P_c - P_d = (\gamma_B - \gamma_A)h \tag{2.10}$$

2.5 Hydrostatic Forces on Flat Surfaces

Determining the total (or resultant) hydrostatic force on structures produced by hydrostatic pressure is often critical in engineering design and analysis. To determine the magnitude of this force, let's examine an arbitrary area AB (Figure 2.9) on the back face of a dam that inclines at an angle θ . Next, place the x -axis on the line where the surface of the water intersects with the dam surface (i.e., into the page) with the y -axis running downward along the surface or face of the dam. Figure 2.9 (a) shows a plan (front) view of the area and Figure 2.9 (b) shows the projection of AB on the dam surface.

We may assume that the plane surface AB is made up of an infinite number of horizontal strips, each having a width of dy and an area of dA . The hydrostatic pressure on each strip may be considered constant because the width of each strip is very small. For a strip at depth h below the free surface, the pressure is

$$P = \gamma h = \gamma y \sin \theta$$

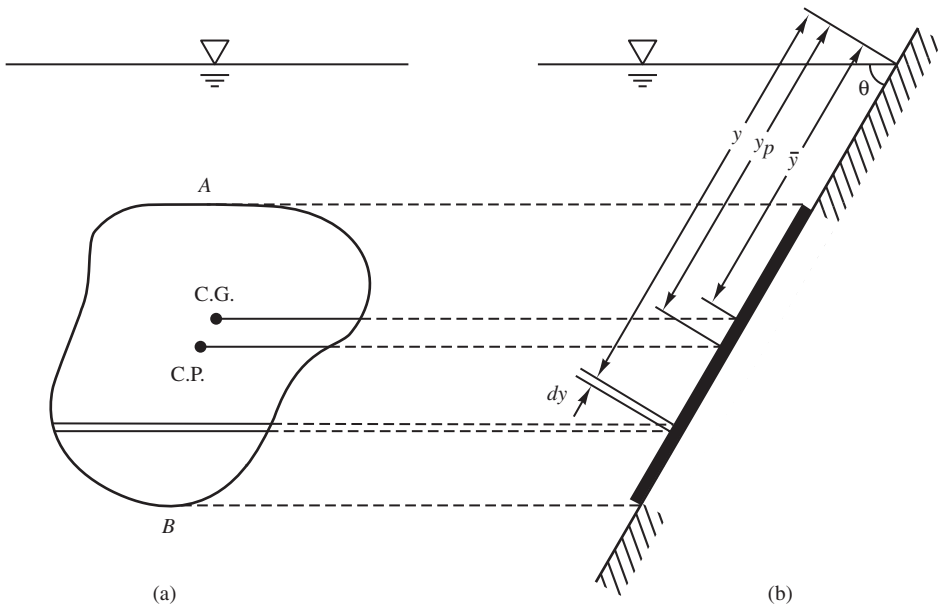


Figure 2.9 Hydrostatic pressure on a plane surface

The total pressure force on the strip is the pressure times the area

$$dF = \gamma y \sin \theta dA$$

The total pressure force (resultant force) over the entire AB plane surface is the sum of pressure on all the strips

$$\begin{aligned} F &= \int_A dF = \int_A \gamma y \sin \theta dA = \gamma \sin \theta \int_A y dA \\ &= \gamma \sin \theta A \bar{y} \end{aligned} \tag{2.11}$$

where $\bar{y} = \int_A y dA / A$ is the distance measured from the x -axis to the centroid (or the center of gravity, C.G.) of the AB plane (Figure 2.9).

Substituting \bar{h} , the vertical distance of the centroid below the water surface, for $\bar{y} \sin \theta$, we have

$$F = \gamma \bar{h} A \tag{2.12}$$

This equation states that *the total hydrostatic pressure force on any submerged plane surface is equal to the product of the surface area and the pressure acting at the centroid of the plane surface.*

Pressure forces acting on a plane surface are distributed over every part of the surface. They are parallel and act in a direction normal to the surface. These parallel forces can be analytically replaced by a single *resultant force* F of the magnitude shown in Equation 2.12. The resultant force also acts normal to the surface. The point on the plane surface at which this resultant force acts is known as the *center of pressure* (C.P., Figure 2.9). Considering the plane surface as a free body, we see that the distributed forces can be replaced by the single resultant force at the pressure center without altering any reactions or moments in the system. Designating y_p as the distance measured from the x -axis to the center of pressure, we may thus write

$$F y_p = \int_A y dF$$

Hence,

$$y_p = \frac{\int_A y dF}{F} \tag{2.13}$$

Substituting the relationships $dF = \gamma y \sin \theta dA$ and $F = \gamma \sin \theta A \bar{y}$, we may write Equation 2.13 as

$$y_p = \frac{\int_A y^2 dA}{A \bar{y}} \tag{2.14}$$

in which $\int_A y^2 dA = I_x$ and $A \bar{y} = M_x$ are, respectively, the moment of inertia and the static moment of the plane surface AB with respect to the x -axis. Therefore,

$$y_p = \frac{I_x}{M_x} \tag{2.15}$$

With respect to the centroid of the plane, this may be written as

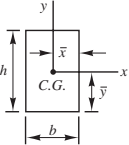
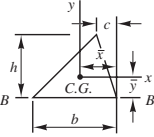
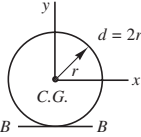
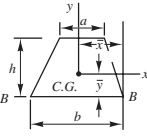
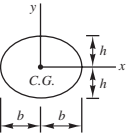
$$y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y} \tag{2.16}$$

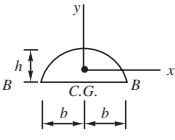
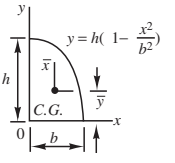
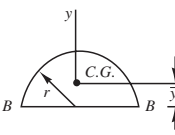
where I_0 is the moment of inertia of the plane with respect to its own centroid, A is the plane surface area, and \bar{y} is the distance between the centroid and the x -axis.

The center of pressure of any submerged plane surface is always below the centroid of the surface area (i.e., $y_p > \bar{y}$). This must be true because all three variables in the first term on the right-hand side of Equation 2.16 are positive, making the term positive. That term is added to the centroidal distance (\bar{y}).

The centroid, area, and moment of inertia with respect to the centroid of certain common geometrical plane surfaces are given in Table 2.1.

TABLE 2.1 Surface Area, Centroid, and Moment of Inertia of Certain Simple Geometrical Plates

Shape	Area	Centroid	Moment of Inertia About the Neutral x -Axis
Rectangle 	bh	$\bar{x} = \frac{1}{2}b$ $\bar{y} = \frac{1}{2}h$	$I_0 = \frac{1}{12}bh^3$
Triangle 	$\frac{1}{2}bh$	$\bar{x} = \frac{b + c}{3}$ $\bar{y} = \frac{h}{3}$	$I_0 = \frac{1}{36}bh^3$
Circle 	$\frac{1}{4}\pi d^2$	$\bar{x} = \frac{1}{2}d$ $\bar{y} = \frac{1}{2}d$	$I_0 = \frac{1}{64}\pi d^4$
Trapezoid 	$\frac{h(a + b)}{2}$	$\bar{y} = \frac{h(2a + b)}{3(a + b)}$	$I_0 = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)}$
Ellipse 	πbh	$\bar{x} = b$ $\bar{y} = h$	$I_0 = \frac{\pi}{4}bh^3$

Shape	Area	Centroid	Moment of Inertia About the Neutral x -Axis
Semi-ellipse 	$\frac{\pi}{2}bh$	$\bar{x} = b$ $\bar{y} = \frac{4h}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)}{72\pi}bh^3$
Parabolic section 	$\frac{2}{3}bh$	$\bar{y} = \frac{2}{5}h$ $\bar{x} = \frac{3}{8}b$	$I_0 = \frac{8}{175}bh^3$
Semicircle 	$\frac{1}{2}\pi r^2$	$\bar{y} = \frac{4r}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)r^4}{72\pi}$

Example 2.3

A vertical trapezoidal gate with its upper edge located 5 m below the free surface of water is shown in Figure 2.10. Determine the total pressure force and the center of pressure on the gate.

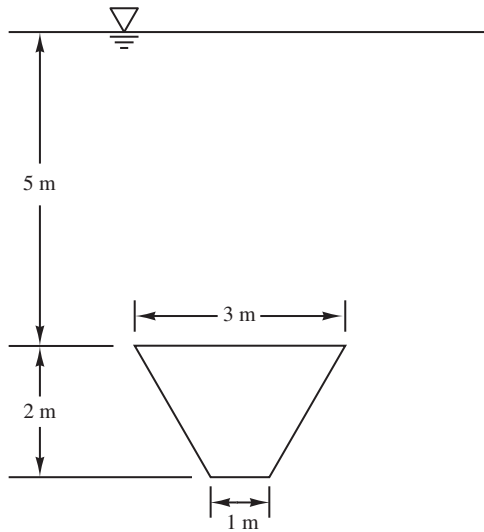


Figure 2.10

Solution

The total pressure force is determined using Equation 2.12 and Table 2.1.

$$\begin{aligned}
 F &= \gamma \bar{h} A \\
 &= 9,790 \left[5 + \frac{2[(2)(1) + 3]}{3(1 + 3)} \right] \left[\frac{2(3 + 1)}{2} \right] \\
 &= 2.28 \times 10^5 \text{ N} = 228 \text{ kN}
 \end{aligned}$$

The location of the center of pressure is

$$y_p = \frac{I_0}{A\bar{y}} + \bar{y}$$

where (from Table 2.1)

$$\begin{aligned}
 I_0 &= \frac{2^3[1^2 + 4(1)(3) + 3^2]}{36(1 + 3)} = 1.22 \text{ m}^4 \\
 \bar{y} &= 5.83 \text{ m} \\
 A &= 4.00 \text{ m}^2
 \end{aligned}$$

Thus,

$$y_p = \frac{1.22}{4(5.83)} + 5.83 = 5.88 \text{ m}$$

below the water surface.

Example 2.4

An inverted semicircular gate (Figure 2.11) is installed at 45° with respect to the free water surface. The top of the gate is 5 ft below the water surface in the vertical direction. Determine the hydrostatic force and the center of pressure on the gate.

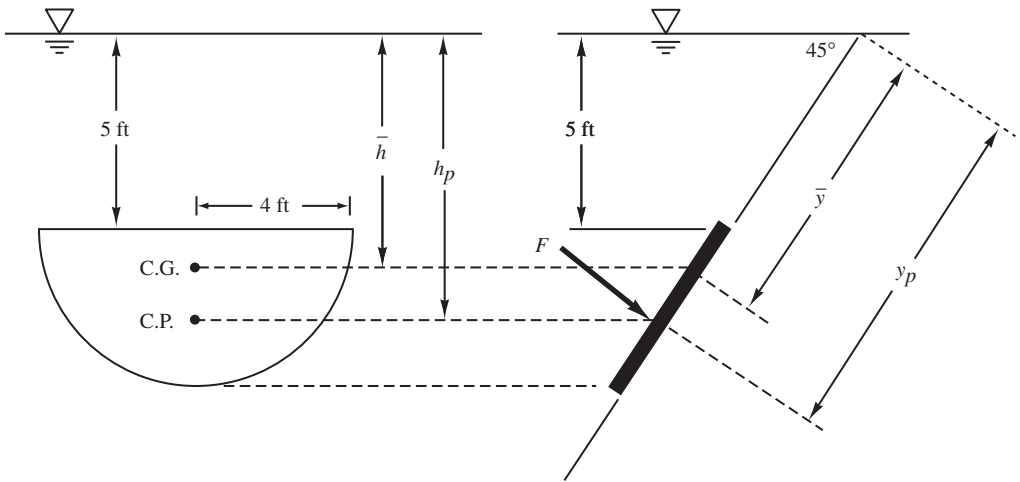


Figure 2.11

Solution

The total pressure force is

$$F = \gamma \bar{y} \sin \theta A$$

where

$$A = \frac{1}{2}[\pi (4)^2] = 25.1 \text{ ft}^2$$

and

$$\bar{y} = 5 \sec 45^\circ + \frac{44}{3\pi} = 8.77 \text{ ft}$$

Therefore,

$$F = 62.3(\sin 45^\circ)(8.77)(25.1) = 9,700 \text{ lbs}$$

This is the total hydrostatic force acting on the gate. The location of the center of pressure is

$$y_p = \frac{I_0}{A\bar{y}} + \bar{y}$$

where (from Table 2.1)

$$I_0 = \frac{(9\pi^2 - 64)}{72\pi} r^4 = 28.1 \text{ ft}^4$$

Therefore,

$$y_p = \frac{28.1}{25.1(8.77)} + 8.77 = 8.90 \text{ ft}$$

This is the inclined distance measured from the water surface to the center of pressure.

2.6 Hydrostatic Forces on Curved Surfaces

The hydrostatic force on a curved surface can be analyzed best by resolving the total pressure force on the surface into its horizontal and vertical components. (Remember that hydrostatic pressure acts normal to a submerged surface.) Figure 2.12 shows the curved wall of a container gate that has a unit width normal to the plane of the page.

Because the water body in the container is stationary, every part of the water body must be in equilibrium or each of the force components must satisfy the equilibrium conditions—that is, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

In the free body diagram of the water contained in ABA' , equilibrium requires the horizontal pressure exerted on plane surface $A'B$ (the vertical projection of AB) to be equal and opposite the horizontal pressure component F_H (the force that the gate wall exerts on the fluid). Likewise, the vertical component, F_V , must equal the total weight of the water body above gate AB . Hence, the horizontal and vertical pressure force on the gate may be expressed as

$$\begin{aligned} \Sigma F_x &= F_{A'B} - F_H = 0 \\ \therefore F_H &= F_{A'B} \\ \Sigma F_y &= F_V - (W_{AA'} + W_{ABA'}) = 0 \\ \therefore F_V &= W_{AA'} + W_{ABA'} \end{aligned}$$

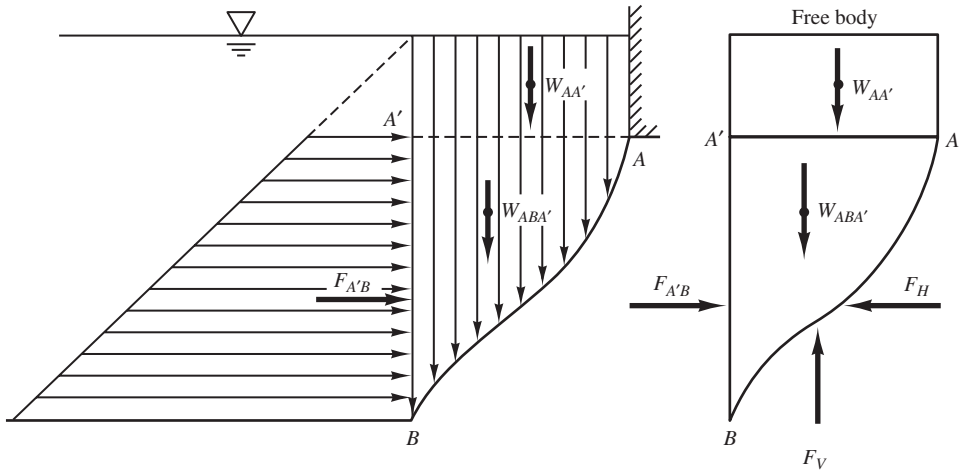


Figure 2.12 Hydrostatic pressure on a curved surface

Therefore, we may make the following statements.

1. *The horizontal component of the total hydrostatic pressure force on any surface is always equal to the total pressure on the vertical projection of the surface. The resultant force of the horizontal component can be located through the center of pressure of this projection.*
2. *The vertical component of the total hydrostatic pressure force on any surface is always equal to the weight of the entire water column above the surface extending vertically to the free surface. The resultant force of the vertical component can be located through the centroid of this column.*

Example 2.5

Determine the total hydrostatic pressure and the center of pressure on the 5-m-long, 2-m-high quadrant gate in Figure 2.13.

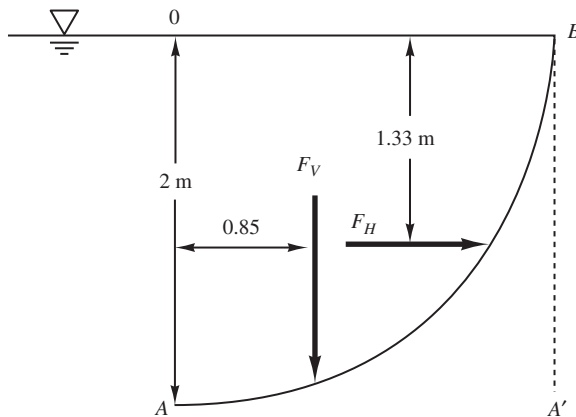


Figure 2.13

Solution

The horizontal component is equal to the hydrostatic pressure force on the projection plane $A'B$.

$$F_H = \gamma \bar{h} A = (9,790 \text{ N/m}^3) \left(\frac{1}{2} (2 \text{ m}) \right) [(2 \text{ m})(5 \text{ m})] = 97,900 \text{ N}$$

The location of the horizontal component is $y_p = I_0/A\bar{y} + \bar{y}$, where $A = 10 \text{ m}^2$ (projected area) and $I_0 = [(5 \text{ m})(2 \text{ m})^3]/12 = 3.33 \text{ m}^4$, $y_p = (3.33 \text{ m}^4)/[(10 \text{ m}^2)(1 \text{ m})] + 1 = 1.33 \text{ m}$ below the free surface. The vertical component is equal to the weight of the water in the volume AOB . The direction of this pressure component is downward.

$$F_V = \gamma (Vol) = (9,790 \text{ N/m}^3) \left(\frac{1}{4} \pi (2 \text{ m})^2 \right) (5 \text{ m}) = 154,000 \text{ N}$$

The pressure center is located at $4(2)/3\pi = 0.85 \text{ m}$ (Table 2.1), and the resultant force is

$$F = \sqrt{(97,900)^2 + (154,000)^2} = 182,000 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{F_V}{F_H} \right) = \tan^{-1} \frac{154,000}{97,900} = 57.6^\circ$$

Example 2.6

Determine the total hydrostatic pressure and the center of pressure on the semicylindrical gate shown in Figure 2.14.

Solution

The horizontal component of the hydrostatic pressure force on the projection plane $A'B'$ per unit width can be expressed as

$$F_H = \gamma \bar{h} A = \gamma \left(\frac{H}{2} \right) (H) = \frac{1}{2} \gamma H^2$$

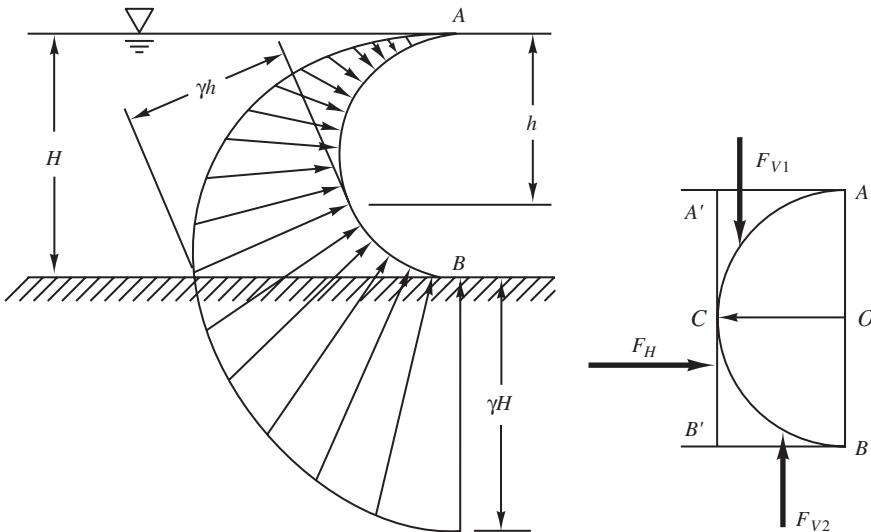


Figure 2.14

The pressure center of this component is located at a distance of $H/3$ from the bottom.

The vertical component can be determined as follows. The volume $AA'C$ over the upper half of the gate, AC , produces a downward vertical pressure force component:

$$F_{V_1} = -\gamma \left(\frac{H^2}{4} - \frac{\pi H^2}{16} \right)$$

The vertical pressure force component exerted by the water on the lower half of the gate, CB , is upward and equivalent to the weight of water replaced by the volume $AA'CB$:

$$F_{V_2} = \gamma \left(\frac{H^2}{4} + \frac{\pi H^2}{16} \right)$$

By combining these two components, one can see that the direction of the resultant vertical force is upward and equal to the weight of the water replaced by the volume ACB .

$$F_V = F_{V_1} + F_{V_2} = \gamma \left[-\left(\frac{H^2}{4} - \frac{\pi H^2}{16} \right) + \left(\frac{H^2}{4} + \frac{\pi H^2}{16} \right) \right] = \gamma \frac{\pi}{8} H^2$$

The resultant force is then

$$F = \gamma H^2 \sqrt{\frac{1}{4} + \frac{\pi^2}{64}}$$

$$\theta = \tan^{-1} \frac{F_V}{F_H} = \tan^{-1} \left(\frac{\pi}{4} \right) = 38.1^\circ$$

Because all pressure forces are concurrent at the center of the gate, point O , the resultant force must also act through point O .

2.7 Buoyancy

Archimedes discovered (~250 B.C.) that *the weight of a submerged body is reduced by an amount equal to the weight of the liquid displaced by the body*. *Archimedes' principle*, as we now call it, can be easily proven by using Equation 2.12.

Assume that a solid body of arbitrary shape, AB , is submerged in water as shown in Figure 2.15. A vertical plane MN may then be drawn through the body in the direction normal to the page. One observes that the horizontal pressure force components in the direction of the paper, F_H and F'_H , must be equal because they both are calculated using the same vertical projection area MN . The horizontal pressure force components in the direction normal to the page must also be equal for the same reason; they share the same projection in the plane of the page.

The vertical pressure-force component can be analyzed by taking a small vertical prism ab with a cross-sectional area dA . The vertical pressure force on top of the prism ($\gamma h_1 dA$) acts downward. The vertical force on the bottom of the prism ($\gamma h_2 dA$) acts upward. The difference gives the resultant vertical force component on the prism (*buoyancy force*)

$$F_V = \gamma h_2 dA - \gamma h_1 dA = \gamma (h_2 - h_1) dA \uparrow$$

which is exactly equal to the weight of the water column ab replaced by the prism. In other words, the weight of the submerged prism is reduced by an amount equal to the weight of the liquid replaced by the prism. A summation of the vertical forces on all the prisms that make up the entire submerged body AB gives the proof of Archimedes' principle.

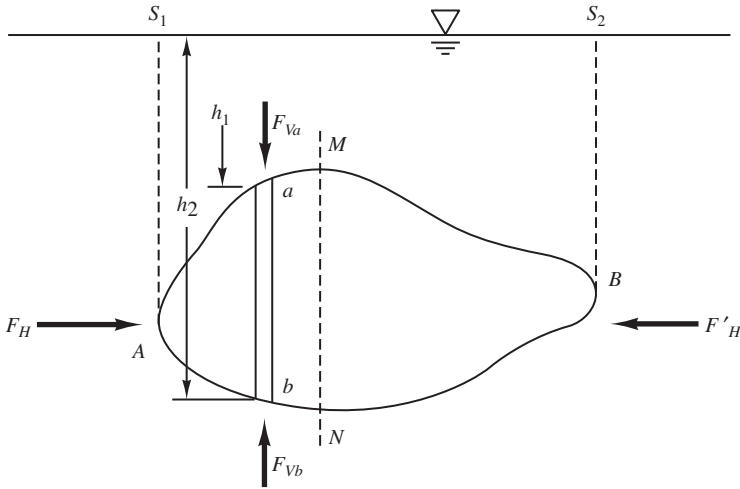


Figure 2.15 Buoyancy of a submerged body

Archimedes' principle may also be viewed as the difference of vertical pressure forces on the two surfaces ANB and AMB . The vertical pressure force on surface ANB is equal to the weight of the hypothetical water column (volume of S_1ANBS_2) acting upward; and the vertical pressure force on surface AMB is equal to the weight of the water column S_1AMBS_2 acting downward. Because the volume S_1ANBS_2 is larger than the volume S_1AMBS_2 by an amount exactly equal to the volume of the submerged body $AMBN$, the net difference is a force equal to the weight of the water that would be contained in the volume $AMBN$ acting upward. This is the buoyancy force acting on the body.

A floating body is a body partially submerged resulting from a balance of the body weight and buoyancy force.

2.8 Flotation Stability

The stability of a floating body is determined by the relative positions of the *center of gravity* of body G and the *center of buoyancy* B , which is the center of gravity of the liquid volume replaced by the body, as show in Figure 2.16.

The body is in equilibrium if its center of gravity and its center of buoyancy lie on the same vertical line, as in Figure 2.16 (a). This equilibrium may be disturbed by a variety of causes (e.g., wind or wave action), and the floating body is made to heel or list through an angle θ as shown in Figure 2.16 (b). When the floating body is in the heeled position, the center of gravity of the body remains unchanged, but the center of buoyancy, which is now the center of gravity of area $a'cb'$, has been changed from B to B' . The buoyant force $\gamma \cdot Vol$, acting upward through B' , and the weight of the body W , acting downward through G , constitute a couple, $W \cdot X$, which resists further overturning and tends to restore the body to its original equilibrium position.

By extending the line of action of the buoyant force through the center of buoyancy B' , we see that the vertical line intersects the original axis of symmetry $c-t$ at a point M . The point M is known as the *metacenter* of the floating body, and the distance between the center of gravity and the metacenter is known as the *metacentric height*. The metacentric height is a measure of the flotation stability of the body. When the angle of inclination is small, the position of M does not

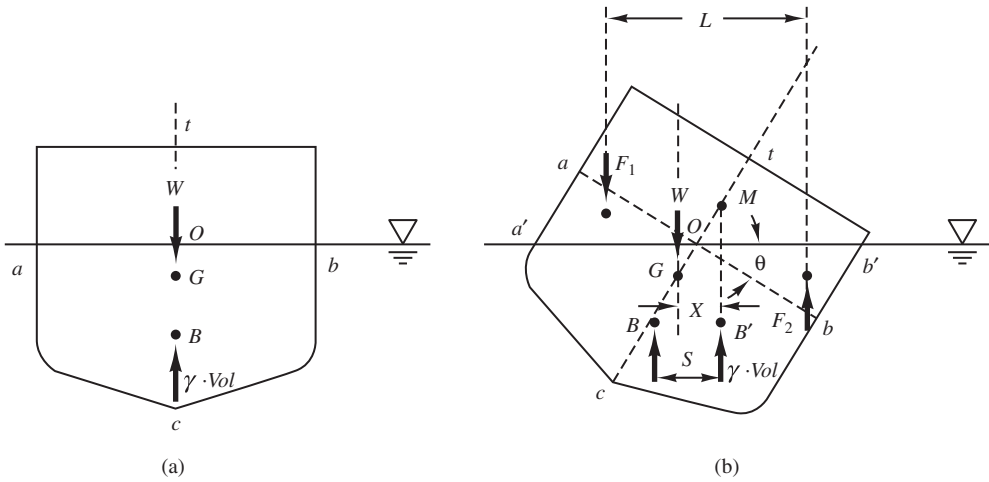


Figure 2.16 Center of buoyancy and metacenter of a floating body

change materially with the tilting position. The metacentric height and the righting moment can be determined in the following way.

Because tilting a floating body does not change the total body weight, the total displacement volume is not changed. The roll through an angle θ only changes the shape of the displaced volume by adding the immersion wedge bob' and subtracting the emersion wedge aoa' . In this new position, the total buoyancy force ($\gamma \cdot Vol$) is shifted through a horizontal distance S to B' . This shift creates a couple F_1 and F_2 because of the new immersion and emersion wedges. The moment of the resultant force $(\gamma \cdot Vol)_{B'}$ about point B must equal the sum of the moments of the component forces:

$$\begin{aligned}
 (\gamma Vol)_{B'}(S) &= (\gamma Vol)_B(\text{zero}) + \text{moment of the force couple} \\
 &= 0 + \gamma Vol_{\text{wedge}}L
 \end{aligned}$$

or

$$\begin{aligned}
 (\gamma Vol)_{B'}S &= \gamma Vol_{\text{wedge}}L \\
 S &= \frac{Vol_{\text{wedge}}}{Vol}L \quad (a)
 \end{aligned}$$

where Vol is the total volume submerged, Vol_{wedge} is the volume of wedge bob' (or aoa'), and L is the horizontal distance between the centers of gravity of the two wedges.

But, according to the geometric relation, we have

$$S = \overline{MB} \sin \theta \quad \text{or} \quad \overline{MB} = \frac{S}{\sin \theta} \quad (b)$$

Combining Equations (a) and (b), we get

$$\overline{MB} = \frac{Vol_{\text{wedge}}L}{Vol \sin \theta}$$

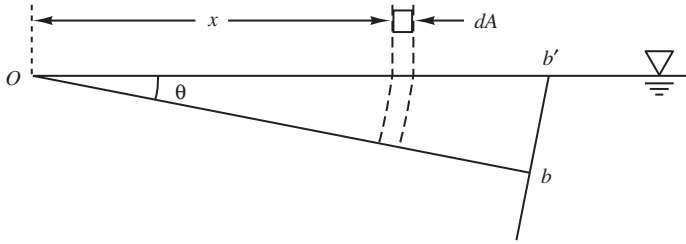


Figure 2.17

For a small angle, $\sin \theta \approx \theta$, the previous relationship may be simplified to

$$\overline{MB} = \frac{Vol_{\text{wedge}}L}{Vol \theta}$$

The buoyancy force produced by wedge bob' , as depicted in Figure 2.17, can be estimated by considering a small prism of the wedge. Assume that the prism has a horizontal area, dA , and is located at a distance x from axis of rotation O . The height of the prism is $x(\tan \theta)$. For a small angle θ , it may be approximated by $x\theta$. Thus, the buoyancy force produced by this small prism is $\gamma x\theta dA$. The moment of this force about the axis of rotation O is $\gamma x^2\theta dA$. The sum of the moments produced by each of the prisms in the wedge gives the moment of the immersed wedge. The moment produced by the force couple is, therefore,

$$\gamma Vol_{\text{wedge}}L = FL = \int_A \gamma x^2 \theta dA = \gamma \theta \int_A x^2 dA$$

But $\int_A x^2 dA$ is the moment of inertia of the waterline cross-sectional area of the floating body about the axis of rotation O .

$$I_0 = \int_A x^2 dA$$

Hence, we have

$$Vol_{\text{wedge}}L = I_0\theta$$

For small angles of tilt, the moment of inertia for upright cross section aob , and the inclined cross section $a'ob'$ may be approximated by a constant value. Therefore,

$$\overline{MB} = \frac{I_0}{Vol} \tag{2.17}$$

The metacentric height, defined as the distance between the metacenter M and the center of gravity G , can be estimated:

$$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{Vol} \pm \overline{GB} \tag{2.18}$$

The distance between the center of gravity and the center of buoyancy \overline{GB} in the upright position, shown in Figure 2.16, can be determined by the sectional geometry or the design data of the vessel.

The \pm sign indicates the relative position of the center of gravity with respect to the center of buoyancy. For greater flotation stability, it is advantageous to make the center of gravity as low as possible. If G is lower than B , then \overline{GB} would be added to the distance to \overline{MB} and produce a larger value of \overline{GM} .

The righting moment, when tilted as depicted in Figure 2.16 (b), is

$$M = W\overline{GM}\sin\theta \tag{2.19}$$

The stability of buoyant bodies under various conditions may be summarized as follows.

1. A floating body is stable if the center of gravity is below the metacenter. Otherwise, it is unstable.
2. A submerged body is stable if the center of gravity is below the center of buoyancy.

Example 2.7.

A 3 m \times 4 m rectangular box caisson is 2 m deep (Figure 2.18). It has a draft of 1.2 m when it floats in an upright position. Compute (a) the metacentric height and (b) the righting moment in seawater (sp. gr. = 1.03) when the angle of heel (list) is 8°.

Solution

From Equation 2.18

$$\overline{GM} = \overline{MB} - \overline{GB}$$

where

$$\overline{MB} = \frac{I_0}{Vol}$$

and I_0 is the waterline area moment of inertia of the box about its longitudinal axis through O . Therefore,

$$\begin{aligned} \overline{GM} &= \frac{\frac{1}{12}Lw^3}{Lw(1.2)} - \left(\frac{h}{2} - \frac{1.2}{2}\right) \\ &= 0.225 \text{ m} \end{aligned}$$

(Note: $L = 4 \text{ m}$, $w = 3 \text{ m}$, $h = 2 \text{ m}$.)

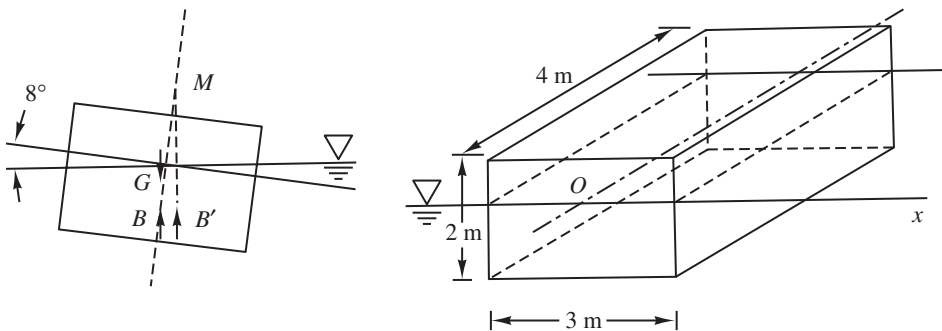


Figure 2.18

The specific gravity of seawater is 1.03; from Equation 2.19, the righting moment is

$$\begin{aligned}
 M &= W\overline{GM} \sin \theta \\
 &= [(9,790 \text{ N/m}^3)(1.03)\{(4 \text{ m})(3 \text{ m})(1.2 \text{ m})\}](0.225 \text{ m})(\sin 8^\circ) \\
 &= 4,550 \text{ N}\cdot\text{m}
 \end{aligned}$$

PROBLEMS (SECTION 2.2)

- 2.2.1.** Collapse depth (or crush depth) is the submerged depth that a submarine cannot exceed without collapsing because of the surrounding water pressure. The collapse depth of modern submarines is not quite a kilometer (730 m). Assuming seawater to be incompressible (sp. gr. = 1.03), what is the crush depth pressure in N/m^2 and psi (lb/in.^2)? Is the pressure you computed absolute or gauge pressure?
- 2.2.2.** A cylindrical water tank (Figure P2.2.2) is suspended vertically by its sides. The tank has a 10-ft diameter and is filled with 20°C water to a 3-ft depth. Determine the force exerted on the tank bottom using two separate calculations (a) based on the weight of the water, and (b) based on the hydrostatic pressure on the bottom of the tank.

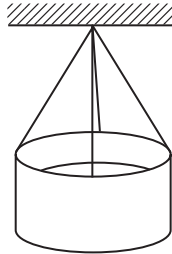


Figure P2.2.2

- 2.2.3.** The simple barometer in Figure P2.2.3 uses water at 30°C as the liquid indicator. The liquid column rises to a height of 9.8 m from an original height of 8.7 m in the vertical tube. Compute the new atmospheric pressure, neglecting surface tension effects. What is the percentage error if the direct reading is used and vapor pressure is ignored?

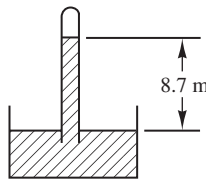


Figure P2.2.3

- 2.2.4.** Mercury is often used in barometers as depicted in Figure P2.2.3. This is because the vapor pressure of mercury is low enough to be ignored, and because it is so dense (sp. gr. = 13.6) the tube can be shortened considerably. With the atmospheric pressure found in Problem 2.2.3 (99.9 kN/m^2 at 30°C), determine the column height in meters (and feet) if mercury is used.
- 2.2.5.** A storage tank ($6 \text{ m} \times 6 \text{ m} \times 6 \text{ m}$) is filled with water. Determine the force on the bottom and on each side.

- 2.2.6.** A 30-ft-high, 1-ft-diameter pipe is welded to the top of a cubic container (3 ft \times 3 ft \times 3 ft). The container and pipe are filled with water at 20°C. Determine the weight of the water and the pressure forces on the bottom and sides of the container.
- 2.2.7.** A closed tank contains a liquid (sp. gr. = 0.80) under pressure. The pressure gauge depicted in Figure P2.2.7 registers a pressure of 4.50×10^4 N/m² (pascals). Determine the pressure at the bottom of the tank and the height of the liquid column that will rise in the vertical tube.

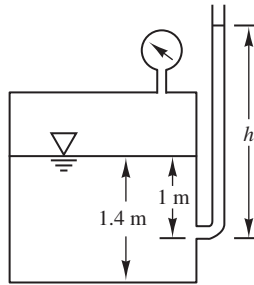


Figure P2.2.7

- 2.2.8.** An underwater storage tank was constructed to store natural gas offshore. Determine the gas pressure in the tank (in pascals and psi; lb/in.²) when the water elevation in the tank is 6 m below sea level (Figure P2.2.8). The specific gravity of seawater is 1.03.

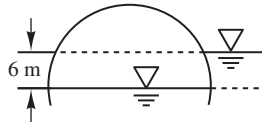


Figure P2.2.8

- 2.2.9.** A closed tank contains oil with a specific gravity 0.85. If the gauge pressure at a point 10 feet below the oil surface is 23.7 psi (lb/in.²), determine the absolute pressure and gauge pressure (in psi) in the air space at the top of the oil surface.
- 2.2.10.** A multiple-piston hydraulic jack has two output pistons, each with an area of 250 cm². The input piston, whose area is 25 cm², is connected to a lever that has a mechanical advantage of 9:1. If a 50-N force is exerted on the lever, how much pressure (kN/m²) is developed in the system? How much force (kN) will be exerted by each output piston?

(SECTION 2.4)

- 2.4.1.** Referring to Figure 2.4 (c), if the height of water (at 4°C) above point 7 is 52.3 cm, what is the height of the oil (sp. gr. = 0.85) above point 8? (Note: Point 9 is 42.5 cm above point 7.)
- 2.4.2.** A significant amount of mercury is poured into a U-tube with both ends open to the atmosphere. If water is poured into one leg of the U-tube until the water column is 3 feet above the mercury–water meniscus, what is the elevation difference between the mercury surfaces in the two legs?
- 2.4.3.** An open tank in a petroleum company lab contains a layer of oil on top of a layer of water. The water height is 4 times the oil height h . The oil has a specific gravity of 0.82. If the gauge pressure at the bottom of the tank indicates 26.3 cm of mercury, what is the oil height h ?

- 2.4.4. A mercury (sp. gr. = 13.6) manometer is used to measure water pressure in a pipe. Referring to Figure 2.5 (a), the value of y is 3.40 cm and the value of h is 2.60 cm. Determine the pressure in the pipe.
- 2.4.5. A manometer is mounted on a city water-supply pipe to monitor water pressure, as shown in Figure P2.4.5. However, the manometer reading of 3 ft (Hg) may be incorrect. If the pressure in the pipe is measured independently and found to be 16.8 lb/in.² (psi), determine the correct value of the reading h .

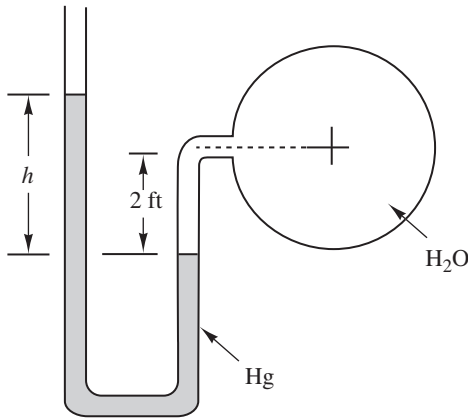


Figure P2.4.5

- 2.4.6. An open manometer, shown in Figure P2.4.6, is installed to measure pressure in a pipe carrying an oil (sp. gr. = 0.82). If the manometer liquid is carbon tetrachloride (sp. gr. = 1.60), determine the pipe pressure (in meters of water-column height).

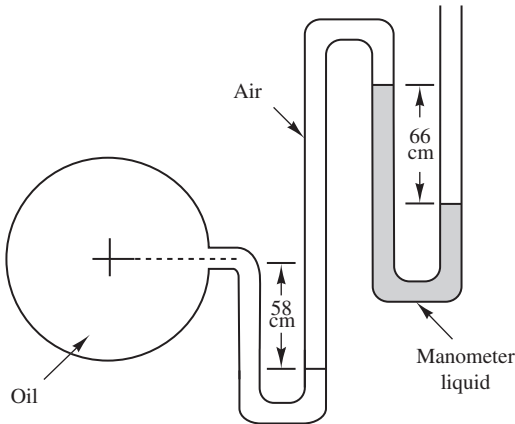


Figure P2.4.6

- 2.4.7. In Figure P2.4.7, a single-reading mercury manometer is used to measure water pressure in the pipe. What is the pressure (in psi) if $h_1 = 6.9$ in. and $h_2 = 24.0$ in.?

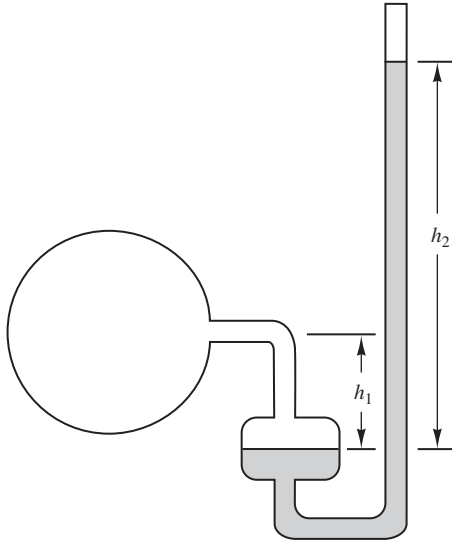


Figure P2.4.7

- 2.4.8. In Figure P2.4.7, determine the water pressure [in kilo-pascals (kPa)] in the pipe if $h_1 = 20.0$ cm and $h_2 = 67.0$ cm. Also determine the change in liquid height h_1 for a 10 cm change in h_2 if the diameter of the manometer tube is 0.5 cm and the diameter of the manometer fluid reservoir is 5 cm.
- 2.4.9. In Figure P2.4.9, water is flowing in pipe A and oil (sp. gr. = 0.82) is flowing in pipe B. If mercury is used as the manometer liquid, determine the pressure difference between A and B in psi.

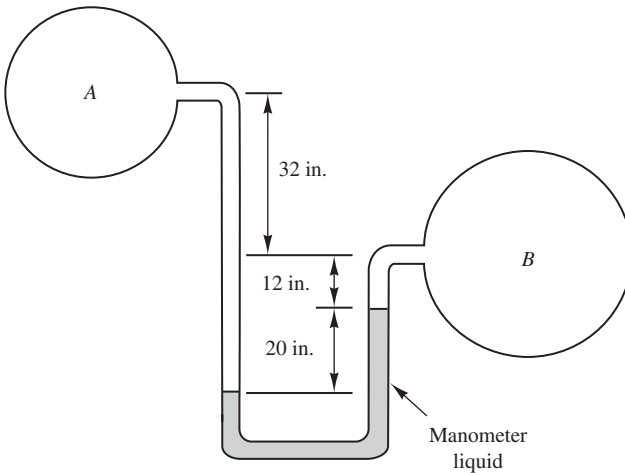


Figure P2.4.9

- 2.4.10. A micromanometer consists of two reservoirs and a U-tube as shown in Figure P2.4.10. Given that the densities of the two liquids are ρ_1 and ρ_2 , determine an expression for the pressure difference ($P_1 - P_2$) in terms of ρ_1 , ρ_2 , h , d_1 , and d_2 .

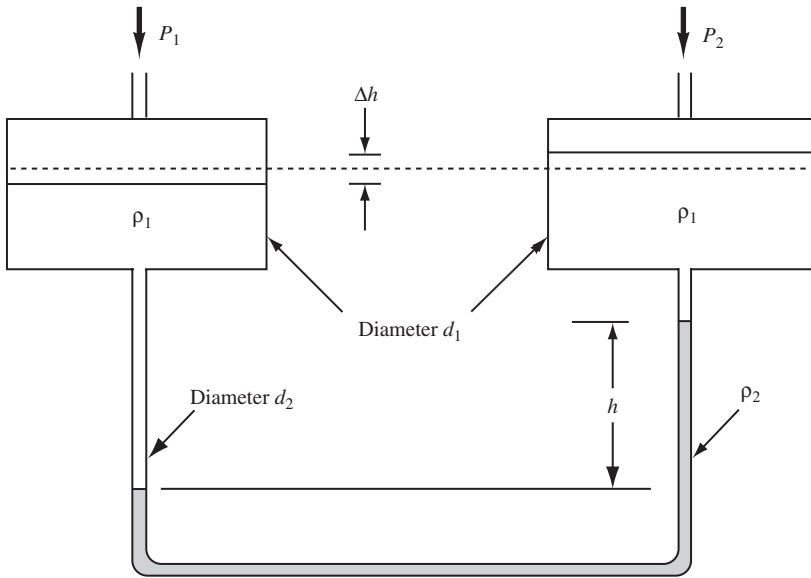


Figure P2.4.10

2.4.11. For the system of manometers shown in Figure P2.4.11, determine the differential reading h . Two different manometry fluids are being used with different specific gravities.

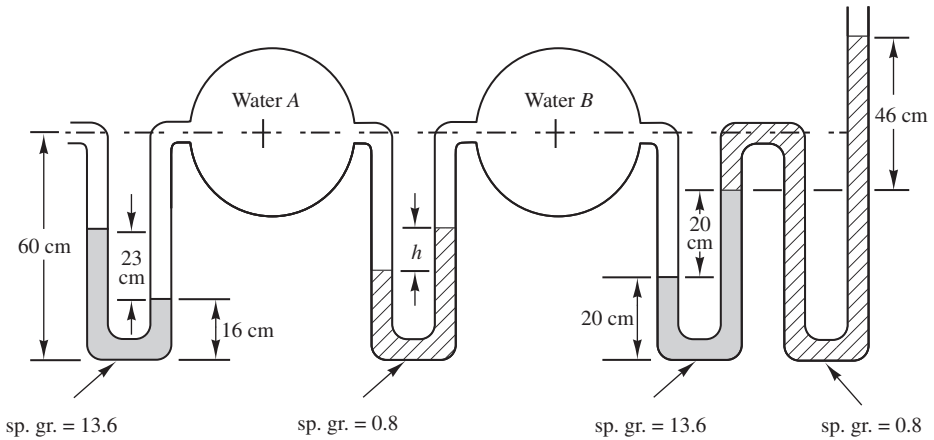


Figure P2.4.11

2.4.12. Determine the air pressure (kPa and cm of Hg) in the sealed left tank of Figure P2.4.12 if $E_A = 32.5$ m.

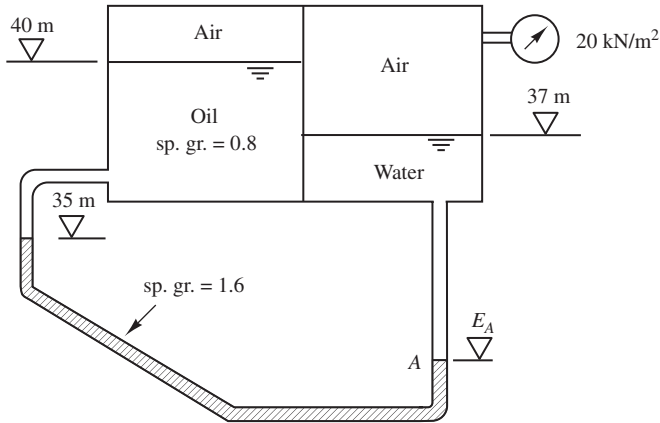


Figure P2.4.12

(SECTION 2.5)

- 2.5.1. A vertical gate keeps water from flowing in a triangular irrigation channel. The channel has a 4-m top width and a 3-m depth. If the channel is full, what is the magnitude of the hydrostatic force on the triangular gate and its location?
- 2.5.2. A concrete dam with a triangular cross section (Figure P2.5.2) is built to hold 30 ft of water. Determine the hydrostatic force on a unit length of the dam and its location. Also, if the specific gravity of concrete is 2.67, what is the moment generated with respect to the toe of the dam, A ? Is the dam safe?

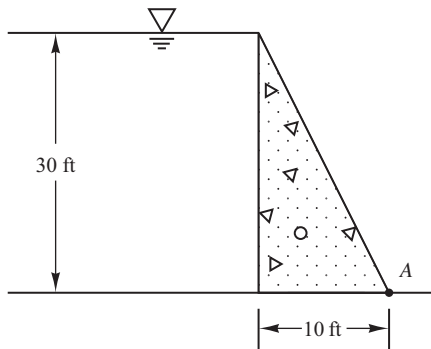


Figure P2.5.2

- 2.5.3. A 1-m-diameter circular (plane) gate is mounted into an inclined wall (45°). The center of the gate is located 1 m (vertically) below the water surface. Determine the magnitude of the hydrostatic force and its location with respect to the surface of the water along the incline.
- 2.5.4. A vertical plate, composed of a square and a triangle, is submerged so that its upper edge coincides with the water surface (Figure P2.5.4). What is the height to length ratio such that the pressure force on the square is equal to the pressure force on the triangle?

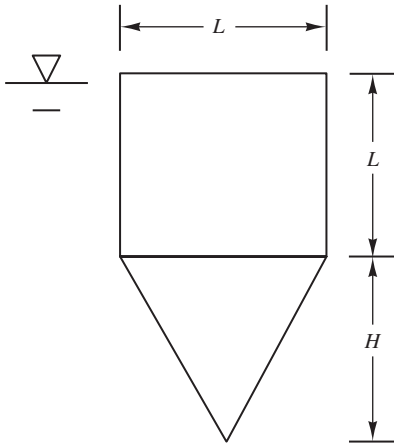


Figure P2.5.4 Front view of vertically submerged plate.

2.5.5. The rectangular gate in Figure P2.5.5 is hinged at *A* and separates water in the reservoir from the tail water tunnel. If the uniformly thick gate has a dimension of 2 m × 3 m and weighs 20 kN, what is the maximum height *h* for which the gate will stay closed? (Hint: Assume the water level *h* does not rise above the hinge.)

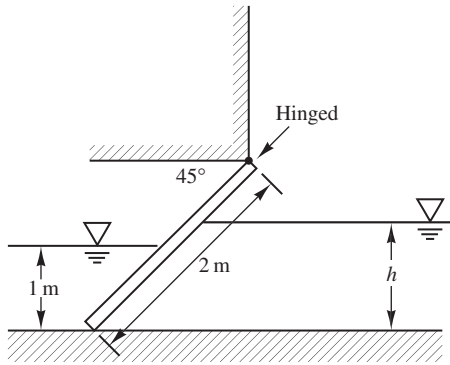


Figure P2.5.5

2.5.6. A circular gate is installed on a vertical wall as shown in Figure P2.5.6. Determine the horizontal force *P* necessary to hold the gate closed if the gate diameter is 6 ft and *h* = 7 ft. Neglect friction at the pivot.

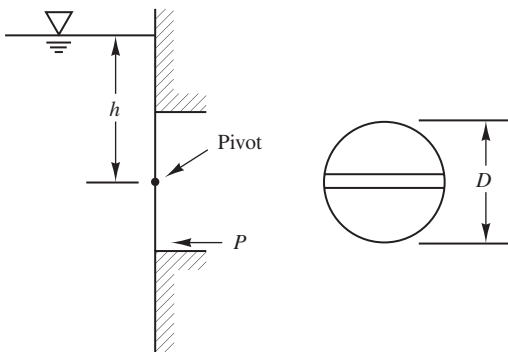


Figure P2.5.6

2.5.7. Figure P2.5.7 shows a 10-ft high (H), vertical, rectangular gate. The gate opens automatically when h increases to 4 ft. Determine the location of the horizontal axis of rotation $O-O'$.

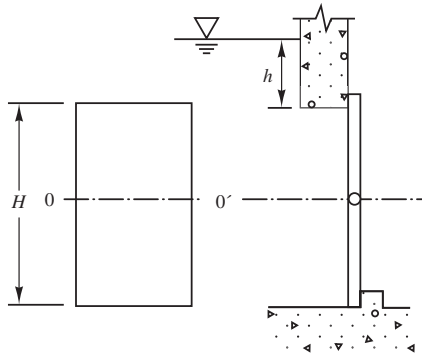


Figure P2.5.7

2.5.8. Calculate the magnitude and the location of the resultant pressure force on the annular gate shown in Figure P2.5.8.

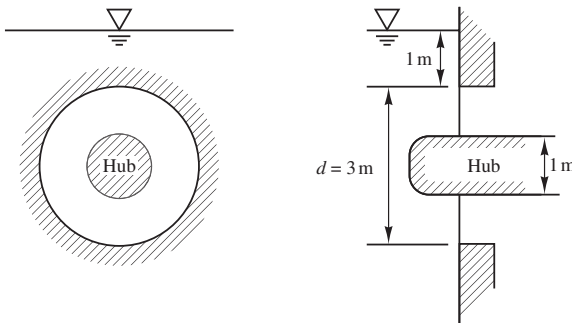


Figure P2.5.8

2.5.9. Calculate the magnitude and the location of the resultant pressure force on the annular gate shown in Figure P2.5.8 if the round central hub is replaced by a square hub (1 m by 1 m).

2.5.10. In Figure P2.5.10, the wicket dam is 5 m high and 3 m wide and is pivoted at its center. Determine the reaction force in the supporting member AB .

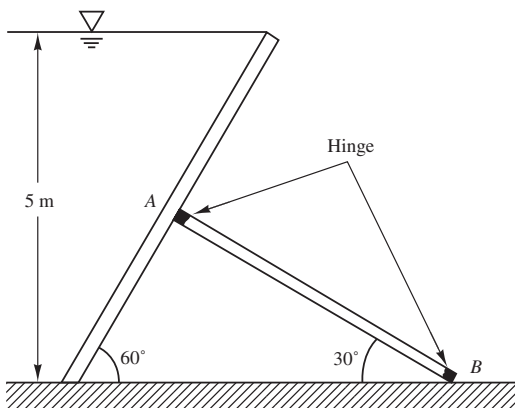


Figure P2.5.10

2.5.11. Determine the depth of the water (d) in Figure P2.5.11 that will cause the gate to open (lay down). The gate is rectangular and is 8 ft wide. Neglect the weight of the gate in your computations. At what depth will it close?

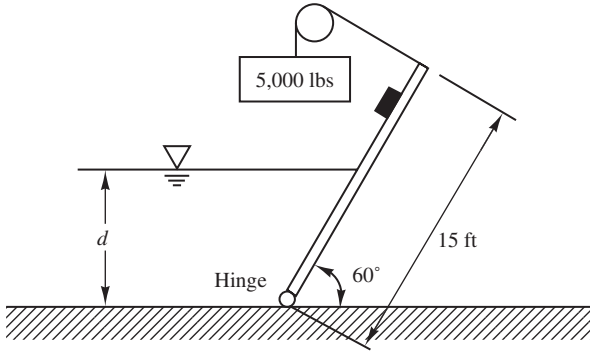


Figure P2.5.11

2.5.12. Neglecting the weight of the hinged gate, determine the depth h at which the gate will open in Figure P2.5.12.

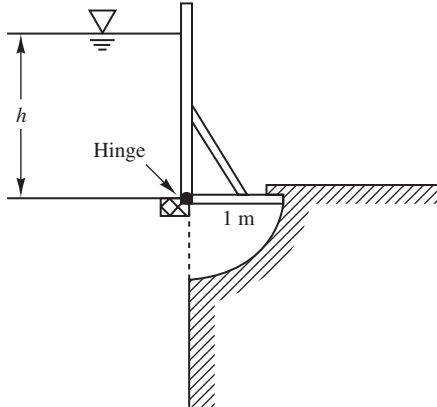


Figure P2.5.12

2.5.13. The circular gate shown in Figure P2.5.13 is hinged at the horizontal diameter. If it is in equilibrium, what is the relationship between h_A and h_B as a function of γ_A , γ_B , and d ?

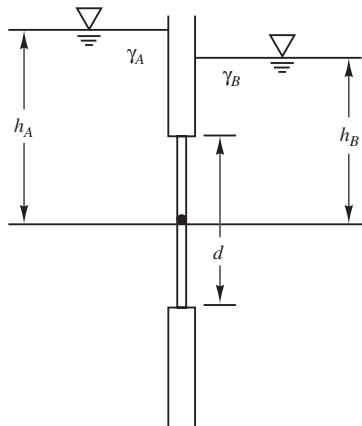


Figure P2.5.13

- 2.5.14. A sliding gate 10 ft wide and 6 ft high is installed in a vertical plane and has a coefficient of friction against the guides of 0.2. The gate weighs 3 tons, and its upper edge is 27 ft below the water surface. Calculate the vertical force required to lift the gate.

(SECTION 2.6)

- 2.6.1. A 10-m-long curved gate depicted in Figure P2.6.1 is retaining a 6-m depth of water in a storage tank. Determine the magnitude and direction of the total hydrostatic force on the gate. Does the force pass through point A? Explain.

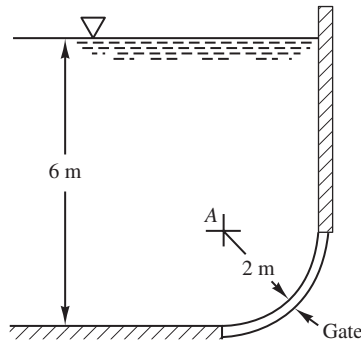


Figure P2.6.1

- 2.6.2. A hemispherical viewing port in a marine museum (Figure P2.6.2) has a 1-m radius, and the top of the port is 3 meters below the surface of the water (h). Determine the magnitude, direction, and location of the total hydrostatic force on the viewing port. (Assume the saltwater has a sp. gr. = 1.03.)

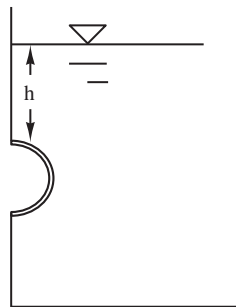


Figure P2.6.2

- 2.6.3. An inverted hemispherical shell of diameter d as shown in Figure P2.6.3 is used to cover a tank filled with water at 20°C. Determine the minimum weight the shell needs to be to hold itself in place if the diameter is 6 feet.

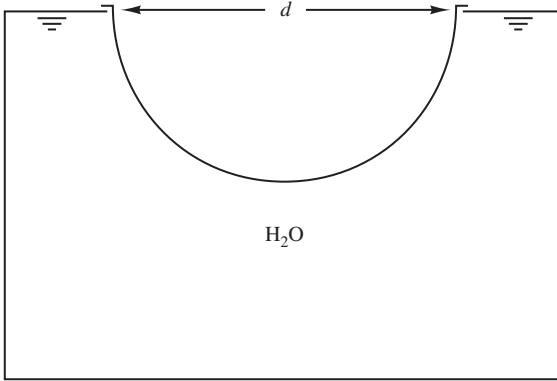


Figure P2.6.3

2.6.4. The corner plate of a barge hull is curved, with a radius of 1.75 m. The barge is leaking, and the depth of submergence (draft) is 4.75 m as depicted in Figure P2.6.4. The water on the inside is up to level A, producing hydrostatic pressure on the inside as well as the outside. Determine the resultant horizontal hydrostatic pressure force and the resultant vertical hydrostatic force on plate AB per unit length of hull.

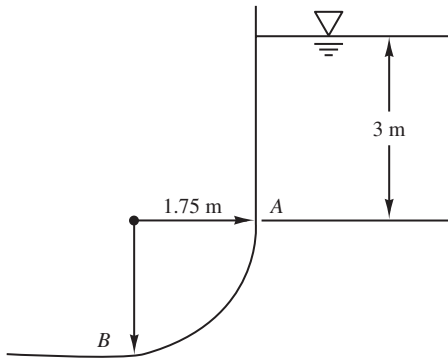


Figure P2.6.4

2.6.5. The tainter gate section shown in Figure P2.6.5 has a cylindrical surface with a 12-m radius; it is supported by a structural frame hinged at O. The gate is 10 m long (in the direction perpendicular to the page). Determine the magnitude, direction, and location of the total hydrostatic force on the gate (Hint: Determine the horizontal and vertical force components.)

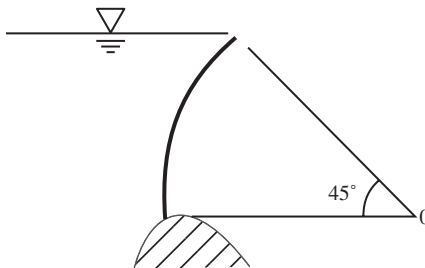


Figure P2.6.5

2.6.6. Calculate the magnitude, direction, and location of the total hydrostatic pressure force (per unit length) on the gate shown in Figure P2.6.6.

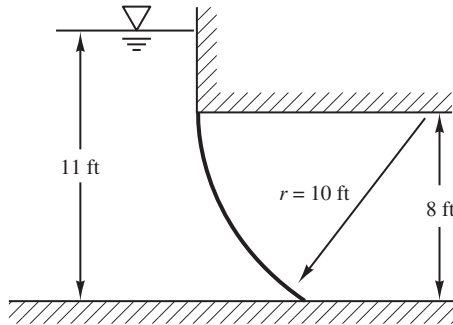


Figure P2.6.6

2.6.7. A 4-ft-diameter cylindrical tank lays on its side with its central axis horizontal. A 1.5-ft diameter pipe extends vertically upward from the middle of the tank. Oil (sp. gr. = 0.9) fills the tank and pipe to a level of 8 ft above the top of the tank. What is the hydrostatic force on one end of the tank? What is the total hydrostatic pressure force on one side (semicircle) of the tank if it is 10 ft long?

2.6.8. Calculate the horizontal and vertical forces acting on the curved surface ABC in Figure P2.6.8.

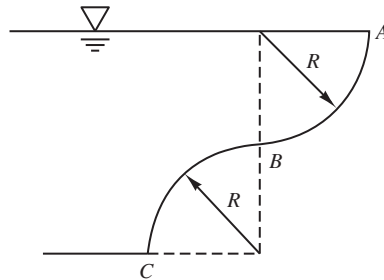


Figure P2.6.8

2.6.9. Calculate the magnitude and location of the vertical and horizontal components of the hydrostatic force on the surface shown in Figure P2.6.9 (quadrant on top of the triangle, both with a unit width). The liquid is water, and the radius $R = 4$ ft.

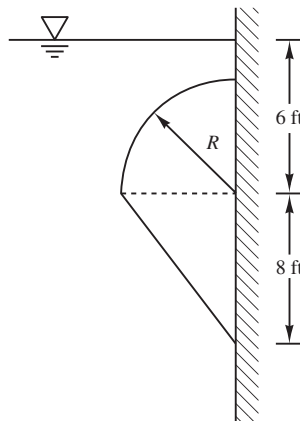


Figure P2.6.9

2.6.10. In Figure P2.6.10 a homogeneous cone plugs the 0.1-m-diameter orifice between reservoir A that contains water and reservoir B that contains oil (sp. gr. = 0.8). Determine the specific weight of the cone if it unplugs when h_0 reaches 1.5 m.

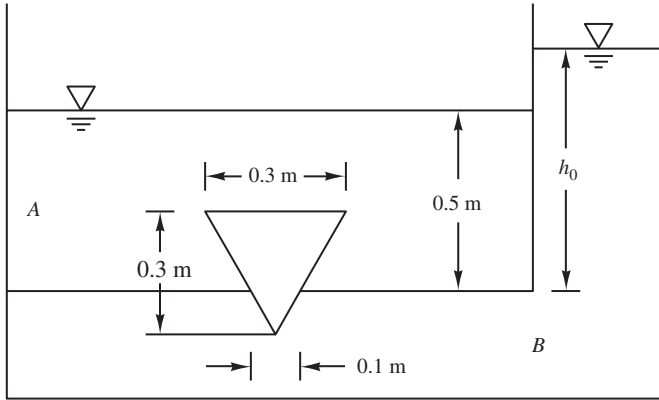


Figure P2.6.10

2.6.11. What would be the specific weight of the cone if reservoir B in Figure P2.6.10 contains air at a pressure of $8,500 \text{ N/m}^2$ instead of oil?

2.6.12. The homogeneous cylinder (sp. gr. = 2.0) in Figure P2.6.12 is 1 m long and $\sqrt{2}$ m in diameter and blocks a 1-m² opening between reservoirs A and B (sp. gr._A = 0.8, sp. gr._B = 1.5). Determine the magnitude of the horizontal and vertical components of the hydrostatic force on the cylinder.

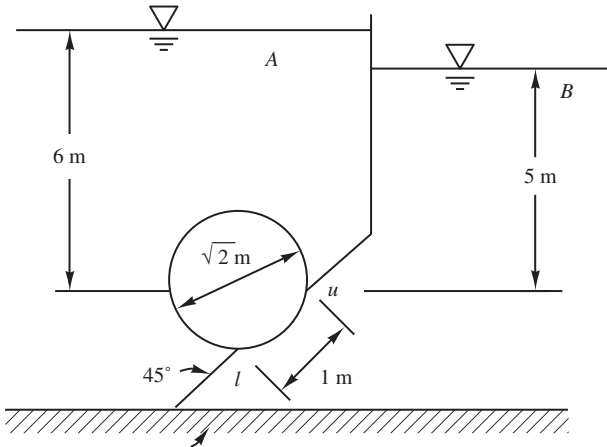


Figure P2.6.12

(SECTION 2.8)

2.8.1. A piece of irregularly shaped metal weighs 301 N. When the metal is completely submerged in water, it weighs 253 N. Determine the specific weight and the specific gravity of the metal.

2.8.2. The solid floating prism show in Figure P2.8.2 has two components. Determine γ_A and γ_B in terms of γ if $\gamma_B = 1.5 \cdot \gamma_A$.

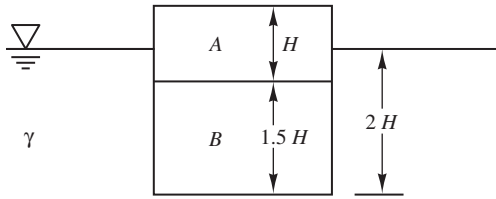


Figure P2.8.2

2.8.3. A solid brass sphere of 30-cm diameter is used to hold a cylindrical buoy in place (Figure P2.8.3) in seawater (sp. gr. = 1.03). The buoy (sp. gr. = 0.45) has a height of 2 m and is tied to the sphere at one end. What rise in tide, h , will be required to lift the sphere off the bottom?

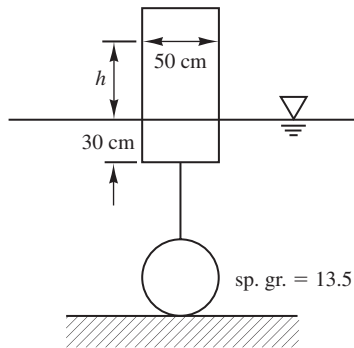


Figure P2.8.3

- 2.8.4. Three people are in a boat with an anchor. If the anchor is thrown overboard, will the lake level rise, fall, or stay the same theoretically? Explain.
- 2.8.5. A freshwater cylindrical anchor ($h = 1.2$ ft and $D = 1.5$ ft) is made of concrete (sp. gr. = 2.7). What is the maximum tension in the anchor line before the anchor is lifted from the lake bottom if the anchor line is at an angle of 60° with respect to the bottom?
- 2.8.6. In Figure P2.8.6, the spherical buoy of radius R opens the square gate AB when water rises to the half-buoy height. Determine R if the weight of the buoy and the gate are negligible.

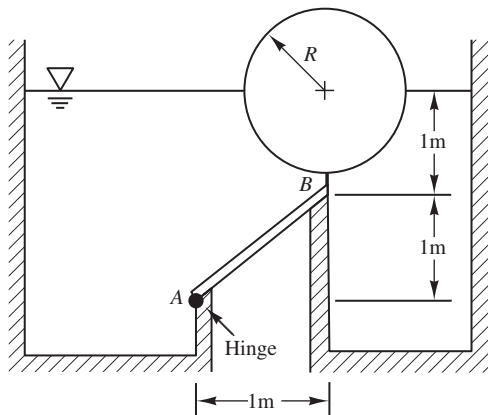


Figure P2.8.6

2.8.7. The floating rod shown in Figure P2.8.7 weighs 150 lbs, and the water's surface is 7 ft above the hinge. Calculate the angle θ assuming a uniform weight and buoyancy distribution.

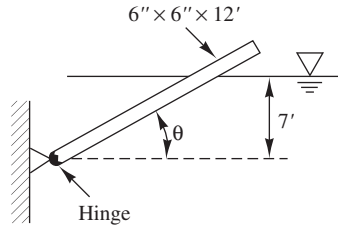


Figure P2.8.7

2.8.8. A rectangular barge is 14 m long, 6 m wide, and 2 m deep. The center of gravity is 1.0 m from the bottom, and the barge draws 1.5 m of seawater (sp. gr. = 1.03). Find the metacentric height and the righting moments for the following angles of heel (or list): 4° , 8° , and 12° .

2.8.9. Figure P2.8.9 shows a buoy that consists of a wooden pole 25 cm in diameter and 2 m long with a spherical weight at the bottom. The specific gravity of the wood is 0.62, and the specific gravity of the bottom weight is 1.40. Determine (a) how much of the wooden pole is submerged in the water, (b) the distance to the center of buoyancy from the water level, (c) the distance to the center of gravity from the water level, and (d) the metacentric height.

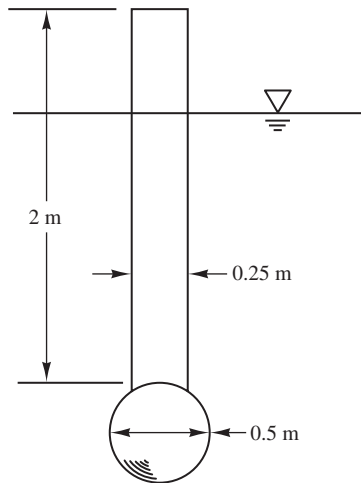


Figure P2.8.9

2.8.10. A wooden block is 2 m long, 1 m wide, and 1 m deep. Is the floating block stable if the metacenter is at the same point as the center of gravity? Explain.

2.8.11. A subway tunnel is being constructed across the bottom of a harbor. The process involves tugboats that pull floating cylindrical sections (or *tubes* as they are often called) across the harbor and sink them in place, where they are welded to the adjacent section already on the harbor bottom. The cylindrical tubes are 50 feet long with a diameter of 36 feet. When in place for the tugboats, the tubes are submerged vertically to a depth of 42 feet, and 8 feet of the tube is above the water (sp. gr. = 1.02). To accomplish this, the tubes are flooded with 34 feet of water on the inside. Determine the metacentric height and estimate the righting moment when the tubes are tipped

through a heel (list) angle of 4° by the tugboats. (Hint: Assume the location of the center of gravity can be determined based on the water contained inside the tubes and the container weight is not that significant.)

- 2.8.12.** A 12-m-long, 4.8-m-wide, and 4.2-m-deep rectangular pontoon has a draft of 2.8 m in seawater (sp. gr. = 1.03). Assuming the load is uniformly distributed on the bottom of the pontoon to a depth of 3.4 m, and the maximum design angle of list is 15° , determine the distance that the center of gravity can be moved from the center line toward the edge of the pontoon.

3



Water Flow in Pipes

3.1 Description of Pipe Flow

In hydraulics, the term *pressure pipe flow* refers to full water flow in closed conduits of circular cross sections under a certain pressure gradient. For a given discharge (Q), pipe flow at any location can be described by the pipe cross section, the pipe elevation, the pressure, and the flow velocity in the pipe.

The *elevation* (h) of a particular section in the pipe is usually measured with respect to a horizontal reference datum such as *mean sea level* (MSL). The *pressure* in a pipe generally varies from one point to another, but a mean value is normally used at a given cross section. In other words, the regional pressure variation in a given cross section is commonly neglected unless otherwise specified.

In most engineering computations, the section *mean velocity* (V) is defined as the discharge (Q) divided by the cross-sectional area (A):

$$V = \frac{Q}{A} \quad (3.1)$$

The velocity distribution within a cross section in a pipe, however, has special meaning in hydraulics. Its significance and importance in hydraulics are discussed next.

3.2 The Reynolds Number

Near the end of the nineteenth century, British engineer Osborne Reynolds performed a very carefully prepared pipe flow experiment. Figure 3.1 shows the schematics of a typical setup for the Reynolds experiment. A long, straight, glass pipe of small bore was installed in a large tank with glass sides. A control valve (C) was installed at the outlet end of the glass pipe to regulate the outflow. A small bottle (B) filled with colored water and a regulating valve at the bottle's neck were used to introduce a fine stream of colored water into the entrance of the glass pipe when the flow was initiated. Water in the large tank was allowed to settle very quietly in a room for several hours so that water in every part of the tank became totally stationary. Valve C was then partially opened to allow a very slow flow in the pipe. At this time, the colored water appeared as a straight line extending to the downstream end, indicating *laminar flow* in the pipe. The valve was opened up slowly to allow the pipe flow rate to increase gradually until a certain velocity was reached; at that time, the thread of color suddenly broke up and mixed with the surrounding water, which showed that the pipe flow became *turbulent* at this point.

Reynolds found that the transition from laminar to turbulent flow in a pipe actually depends not only on the velocity but also on the pipe diameter and the viscosity of the fluid. Furthermore, he postulated that the onset of turbulence was related to a particular index number. This dimensionless ratio is commonly known as the *Reynolds number* (N_R) (see also Chapter 10) and it can be expressed as

$$N_R = \frac{DV}{\nu} \quad (3.2)$$

When expressing Reynolds number for pipe flow, D is the pipe diameter, V is the mean velocity, and ν is the kinematic viscosity of the fluid, defined by the ratio of absolute viscosity (μ) and the fluid density (ρ).

$$\nu = \frac{\mu}{\rho} \quad (3.3)$$

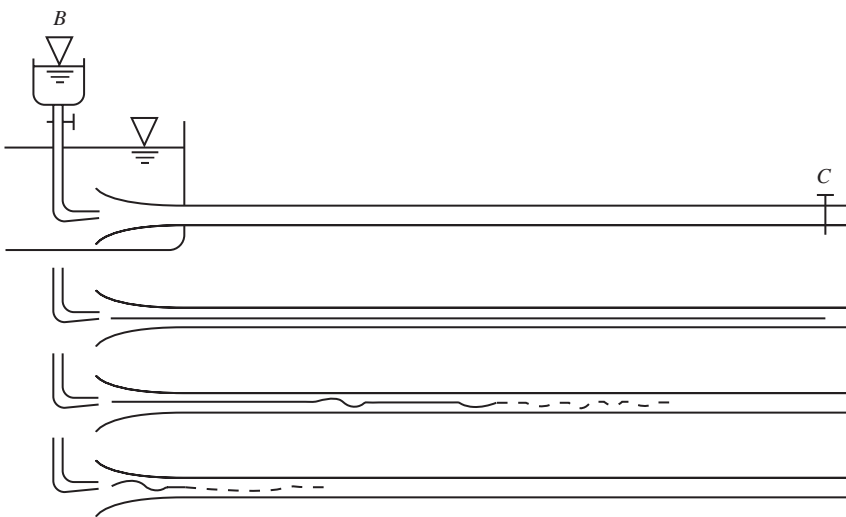


Figure 3.1 Reynolds' apparatus

It has been found and verified by many carefully prepared experiments that for flows in circular pipes the *critical Reynolds number* is approximately 2,000. At this point, the laminar pipe flow changes to a turbulent one. The transition from laminar to turbulent flow does not happen at exactly $N_R = 2,000$ but varies from approximately 2,000 to 4,000 based on differences in experimental conditions. This range of Reynolds number between laminar and turbulent flow is commonly known as the *critical zone*, which will be discussed more fully later.

Laminar flow occurs in a circular pipe when fluid flows in orderly laminae; this is analogous to the telescoping of a large number of thin-walled concentric tubes. The outer tube adheres to the pipe wall while the tube next to it moves with a very slight velocity. The velocity of each successive tube increases gradually and reaches a maximum velocity near the center of the pipe. In this case, the velocity distribution takes the form of a paraboloid of revolution with the mean velocity V equal to one-half of the maximum center line velocity, as shown in Figure 3.2.

In turbulent flow, the turbulent motion causes the slower water particles adjacent to the pipe wall to mix continuously with the high-speed particles in the midstream. As a result, the low-speed particles near the pipe wall are accelerated because of momentum transfer. For this reason, the velocity distribution in turbulent flow is more uniform than laminar flow. The velocity profiles in turbulent pipe flows have been shown to take the general form of a logarithmic curve in revolution. Turbulent mixing activities increase with the Reynolds number; hence, the velocity distribution becomes flatter as the Reynolds number increases.

Under ordinary circumstances, water loses energy as it flows through a pipe. A major portion of the energy loss is caused by

1. friction against the pipe walls and
2. viscous dissipation occurring throughout the flow.

Wall friction on a moving column of water depends on the roughness of the wall material (e) and the velocity gradient $[(dV/dr)|_{r=D/2}]$ at the wall (see Equation 1.2). For the same flow rate, it is evident in Figure 3.2 that turbulent flow has a higher wall velocity gradient than that of laminar flow; hence, a higher friction loss may be expected as the Reynolds number increases. At the same time, momentum transfer of water molecules between layers is intensified as the flow becomes more turbulent, which indicates an increasing rate of viscous dissipation in the flows. As a consequence, the rate of energy loss in pipe flow varies as a function of the Reynolds

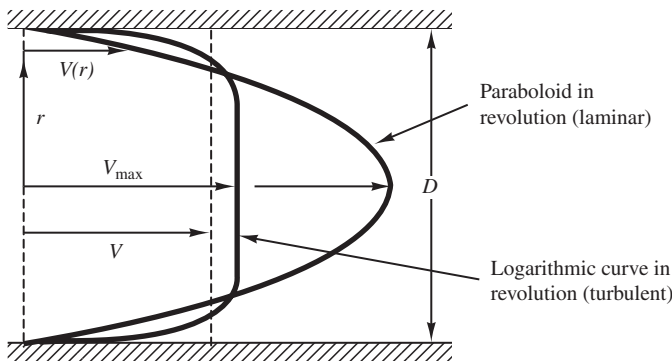


Figure 3.2 Velocity profiles of laminar and turbulent flows in circular pipes

number and the roughness of the pipe wall. The ramifications of this in engineering applications will be discussed later in the chapter.

Example 3.1

A 40-mm-diameter circular pipe carries water at 20°C. Calculate the largest flow rate for which laminar flow can be expected.

Solution

The kinematic viscosity of water at 20°C is $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{sec}$ (Table 1.3 or book jacket). Taking $N_R = 2,000$ as the conservative upper limit for laminar flow, the velocity can be determined as

$$N_R = \frac{DV}{\nu} = \frac{(0.04 \text{ m})V}{1.00 \times 10^{-6} \text{ m}^2/\text{sec}} = 2,000$$

$$V = 2000(1.00 \times 10^{-6}/0.04) = 0.05 \text{ m/sec}$$

The flow rate is

$$Q = AV = \frac{\pi}{4}(0.04)^2(0.05) = 6.28 \times 10^{-5} \text{ m}^3/\text{sec}$$

3.3 Forces in Pipe Flow

Figure 3.3 shows a section of water flowing in a circular pipe. For a general description of the flow, the cross-sectional area and the elevation of the pipe are allowed to vary along the axial direction of the flow.

A *control volume* is considered between sections 1–1 and 2–2. After a short time interval dt , the mass originally occupying the control volume, has advanced to a new position, between sections 1'–1' and 2'–2'.

For incompressible and steady flows, the mass flux (fluid mass) that enters the control volume, $\rho dVol_{1-1'}$, should equal the mass flux that leaves the control volume, $\rho dVol_{2-2'}$. This is the principle of *conservation of mass*:

$$\rho \frac{dVol_{1-1'}}{dt} = \rho A_1 \frac{dS_1}{dt} = \rho A_1 V_1 = \rho A_2 V_2 = \rho Q$$

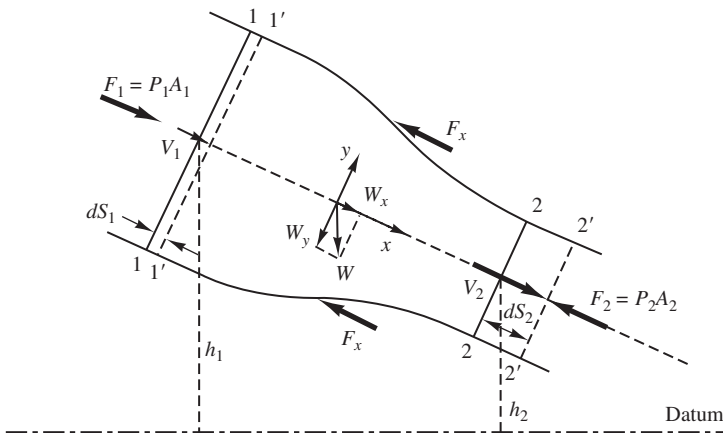


Figure 3.3 General description of flow in pipes

or

$$A_1V_1 = A_2V_2 = Q \tag{3.4}$$

Equation 3.4 is known to hydraulic engineers as the *continuity equation* for incompressible, steady flow.

Applying Newton’s second law to the moving mass in the control volume yields

$$\Sigma \vec{F} = m\vec{a} = m \frac{d\vec{V}}{dt} = \frac{m\vec{V}_2 - m\vec{V}_1}{\Delta t} \tag{3.5}$$

In this equation, both the forces and velocities are vector quantities. They must be balanced in every direction considered. Along the axial direction of the flow, the external forces exerted on the control volume may be expressed as

$$\Sigma F_x = P_1A_1 - P_2A_2 - F_x + W_x \tag{3.6}$$

where $V_1, V_2, P_1,$ and P_2 are the velocities and pressures at sections 1–1 and 2–2, respectively. F_x is the axial direction force exerted on the control volume by the wall of the pipe. W_x is the axial component of the weight of the liquid in the control volume.

By recognizing $m/\Delta t$ in Equation 3.5 as mass flow rate (ρQ), the principle of *conservation of momentum* (or the impulse-momentum equation) may be expressed, for the axial direction, as

$$\Sigma F_x = \rho Q(V_{x_2} - V_{x_1}) \tag{3.7a}$$

Similarly, for the other directions,

$$\Sigma F_y = \rho Q(V_{y_2} - V_{y_1}) \tag{3.7b}$$

$$\Sigma F_z = \rho Q(V_{z_2} - V_{z_1}) \tag{3.7c}$$

In general, we may write in vector quantities

$$\Sigma \vec{F} = \rho Q(\vec{V}_2 - \vec{V}_1) \tag{3.7}$$

Example 3.2

A horizontal nozzle (Figure 3.4) discharges 0.01 m³/sec of water at 4°C into the air. The supply pipe’s diameter ($d_A = 40$ mm) is twice as large as the nozzle diameter ($d_B = 20$ mm). The nozzle is held in place by a hinge mechanism. Determine the magnitude and direction of the reaction force at the hinge, if the gauge pressure at A is 500,000 N/m². (Assume the weight supported by the hinge is negligible.)

Solution

The hinge force resists the pressure and momentum change in the system. This force can be computed from the conservation of momentum equation. The hydrostatic forces are

$$\begin{aligned} F_{x,A} &= PA_A = (500,000 \text{ N/m}^2)[(\pi/4)(0.04 \text{ m})^2] = 628 \text{ N} \\ F_{y,A} &= 0 \text{ (all flow in the } x\text{-direction); } F_{x,B} = F_{y,B} = 0 \text{ (atmospheric pressure)} \\ V_A &= Q/A_A = (0.01 \text{ m}^3/\text{sec}) / [(\pi/4)(0.04 \text{ m})^2] = 7.96 \text{ m/sec} = V_{x,A}, V_{y,A} = 0 \\ V_B &= Q/A_B = (0.01 \text{ m}^3/\text{sec}) / [(\pi/4)(0.02 \text{ m})^2] = 31.8 \text{ m/sec} \\ V_{x,B} &= (31.8 \text{ m/sec})(\cos 60^\circ) = 15.9 \text{ m/sec} \\ V_{y,B} &= (31.8 \text{ m/sec})(\sin 60^\circ) = 27.5 \text{ m/sec} \end{aligned}$$

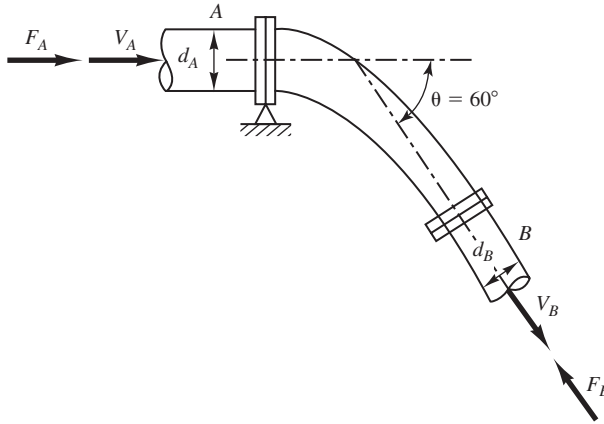


Figure 3.4 Flow through a horizontal nozzle

Now

$$\Sigma F_x = \rho Q(V_{x_B} - V_{x_A})$$

with the sign convention ($\rightarrow +$). Assuming F_x is negative and substituting yields

$$\begin{aligned} 628 \text{ N} - F_x &= (998 \text{ kg/m}^3)(0.01 \text{ m}^3/\text{sec})[(15.9 - 7.96) \text{ m/sec}]; \\ F_x &= 549 \text{ N} \leftarrow \end{aligned}$$

Both forces and velocities are vector quantities that must adhere to the sign convention. Because the sign of F_x ended up positive, our assumed direction was correct. Likewise,

$$\Sigma F_y = \rho Q(V_{y_B} - V_{y_A})$$

with the sign convention ($\uparrow +$). Assuming F_y is negative and substituting yields

$$\begin{aligned} -F_y &= (998 \text{ kg/m}^3)(0.01 \text{ m}^3/\text{sec})[(-27.5 - 0) \text{ m/sec}]; \\ F_y &= 274 \text{ N} \downarrow \end{aligned}$$

The resultant force is

$$F = [(549 \text{ N})^2 + (274 \text{ N})^2]^{1/2} = 614 \text{ N}$$

and its direction is

$$\theta = \tan^{-1}(F_y/F_x) = 26.5^\circ \swarrow$$

3.4 Energy in Pipe Flow

Water flowing in pipes may contain energy in various forms. The major portion of the energy is contained in three basic forms:

1. kinetic energy,
2. potential energy, and
3. pressure energy.

The three forms of energy may be demonstrated by examining the flow in a general section of pipe, as shown in Figure 3.3. This section of pipe flow can represent the concept of a stream tube that is a cylindrical passage with its surface everywhere parallel to the flow velocity; therefore, the flow cannot cross its surface.

Consider the control volume depicted in Figure 3.3. In time interval dt , the water particles at section 1–1 move to 1'–1' with the velocity V_1 . In the same time interval, the particles at section 2–2 move to 2'–2' with the velocity V_2 . To satisfy the continuity condition,

$$A_1V_1dt = A_2V_2dt$$

The work done by the pressure force acting on section 1–1 in the time dt is the product of the total pressure force and the distance through which it acts, or

$$P_1A_1dS_1 = P_1A_1V_1dt \tag{3.8}$$

Similarly, the work done by the pressure force on section 2–2 is

$$-P_2A_2dS_2 = -P_2A_2V_2dt \tag{3.9}$$

and is negative because P_2 is in the opposite direction to the distance dS_2 traveled.

The work done by gravity on the entire mass of water in moving from 1122 to 1'1'2'2' is the same as the work done if 111'1' were moved to the position 222'2' and the mass 1'1'22 was left undistributed. The gravity force acting on the mass 111'1' is equal to the volume A_1V_1dt times the specific weight $\gamma = \rho g$. If h_1 and h_2 represent the elevations of the center of mass of 111'1' and 222'2' above a certain horizontal datum line, respectively, the work done by gravity force to move the mass from h_1 to h_2 is

$$\rho g A_1 V_1 dt (h_1 - h_2) \tag{3.10}$$

The net gain in kinetic energy of the entire mass is

$$\frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 = \frac{1}{2} \rho A_1 V_1 dt (V_2^2 - V_1^2) \tag{3.11}$$

Because the total work done on a given mass by all the forces is equal to the change in kinetic energy, Equations 3.8 to 3.11 may be combined to give

$$P_1 Q dt - P_2 Q dt + \rho g Q dt (h_1 - h_2) = \frac{1}{2} \rho Q dt (V_2^2 - V_1^2)$$

Dividing both sides by $\rho g Q dt$ results in the *Bernoulli equation* expressed in terms of energy per unit weight of water, which is referred to as *head*. Thus,

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 \tag{3.12}$$

Therefore, the algebraic sum of the *velocity head*, the *pressure head*, and the *elevation head* accounts for nearly all the energy contained in a unit weight of water flowing through a particular section of pipe. In reality, however, a certain amount of energy loss occurs when the water mass flows from one section to another. Accounting for this loss in engineering applications is discussed next.

Figure 3.5 depicts schematically the heads at two locations along a pipeline. At section 1, the upstream section, the three heads are $V_1^2/2g$, P_1/γ , and h_1 . (Note that the energy per unit

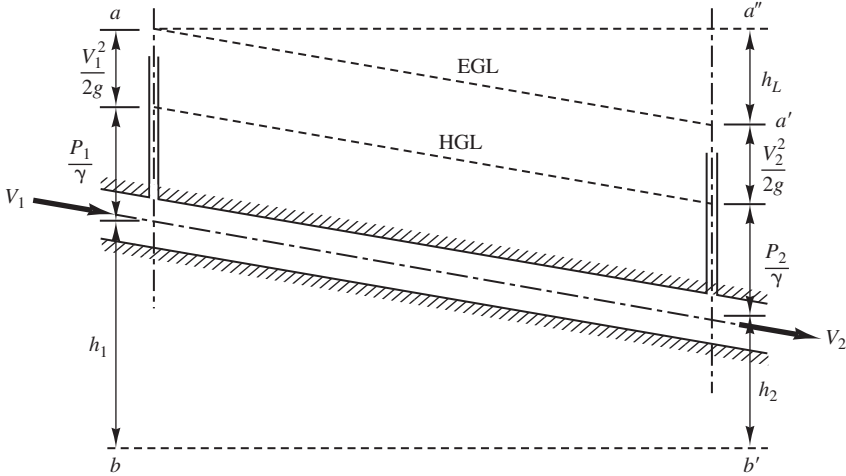


Figure 3.5 Total energy and head loss in pipe flow

weight of water results in a length or height dimensionally.) The algebraic sum of these three heads gives the point *a* above the energy datum. The distance measured between points *a* and *b* represents the total head, or the total energy contained in each unit weight of water that passes through section 1.

$$H_1 = \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 \tag{3.13}$$

During the journey between the upstream and downstream sections, a certain amount of hydraulic energy is lost because of friction (i.e., primarily converted to heat). The remaining energy in each unit weight of water at section 2 is represented by the distance between points *a'* and *b'* in Figure 3.5. Once again, this is the total head and is the sum of the velocity head, the pressure head, and the elevation head.

$$H_2 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 \tag{3.14}$$

The elevation difference between points *a'* and *a''* represents the *head loss* (h_L), between sections 1 and 2. The energy relationship between the two sections can be written in the following form:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L \tag{3.15}$$

This relationship is known as the *energy equation*, but occasionally it is mistakenly called *Bernoulli's equation* (which does not account for losses or assumes they are negligible). For a horizontal pipe of uniform size, it can be shown that the head loss results in a pressure drop in the pipe because the velocity heads and the elevation heads are equal.

$$\frac{P_1 - P_2}{\gamma} = h_L \tag{3.15a}$$

Figure 3.5 displays a few other noteworthy hydraulic engineering concepts. For example, a line may be drawn through all of the points that represent total energy along the pipe. This is called the *energy grade line* (EGL). The slope of the EGL represents the rate at which energy is being lost along the pipe. A distance $V^2/2g$ below the EGL is the *hydraulic grade line* (HGL). These concepts will be discussed in later sections.

Example 3.3

A 25-cm circular pipe carries 0.16 m³/sec of water under a pressure of 200 Pa. The pipe is laid at an elevation of 10.7 m above mean sea level in Freeport, Texas. What is the total head measured with respect to MSL?

Solution

The continuity condition (Equation 3.4), requires that

$$Q = AV$$

Hence,

$$V = \frac{Q}{A} = \frac{0.16 \text{ m}^3/\text{sec}}{(\pi/4)(0.25 \text{ m})^2} = 3.26 \text{ m/sec}$$

The total head measured with respect to mean sea level is

$$\frac{V^2}{2g} + \frac{P}{\gamma} + h = \frac{(3.26 \text{ m/sec})^2}{2(9.81 \text{ m/sec}^2)} + \frac{200 \text{ N/m}^2}{9,790 \text{ N/m}^3} + 10.7 \text{ m} = 11.3 \text{ m}$$

Example 3.4

The elevated water tank shown in Figure 3.6 is being drained to an underground storage location through a 12-in. diameter pipe. The flow rate is 3,200 gallons per minutes (gpm), and the total head loss is 11.5 ft. Determine the water surface elevation in the tank.

Solution

An energy relationship (Equation 3.15) can be established between section 1 at the reservoir surface and section 2 at the end of the pipe.

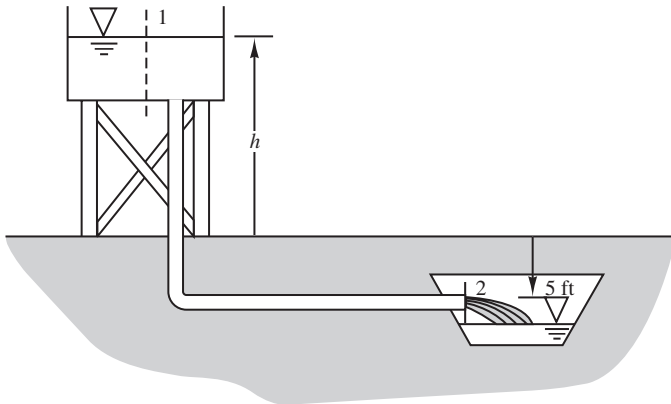


Figure 3.6 Flow from an elevated water tank

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$$

The water velocity in the reservoir, being small compared with that in the pipe, can be neglected. Furthermore, both sections are exposed to atmospheric pressure so that

$$P_1 = P_2 = 0$$

The mean velocity is

$$V = \frac{Q}{A} = \frac{3,200 \text{ gpm}}{\pi r^2} = \left[\frac{3,200 \text{ gpm}}{\pi (0.5 \text{ ft})^2} \right] \left[\frac{1 \text{ ft}^3/\text{sec}}{449 \text{ gpm}} \right] = 9.07 \text{ ft/sec}$$

Now solving the energy relationship by setting the datum at the ground elevation yields

$$h = h_1 = \frac{V^2}{2g} + h_2 + h_L = \frac{(9.07 \text{ ft/sec})^2}{2(32.2 \text{ ft/sec}^2)} - 5 \text{ ft} + 11.5 \text{ ft} = 7.78 \text{ ft}$$

3.5 Loss of Head from Pipe Friction

Energy loss resulting from friction in a pipeline is commonly termed the *friction head loss* (h_f). This is the loss of head caused by pipe wall friction and the viscous dissipation in flowing water. Friction loss is sometimes referred to as the *major loss* because of its magnitude, and all other losses are referred to as *minor losses*. Several studies have been performed during the past century on laws that govern the loss of head by pipe friction. It has been learned from these studies that resistance to flow in a pipe is

1. independent of the pressure under which the water flows,
2. linearly proportional to the pipe length (L),
3. inversely proportional to some power of the pipe diameter (D),
4. proportional to some power of the mean velocity (V), and
5. related to the roughness of the pipe, if the flow is turbulent.

Several experimental equations have been developed in the past. Some of these equations have been used faithfully in various hydraulic engineering practices.

The most popular pipe flow equation was derived by Henri Darcy (1803 to 1858), Julius Weisbach (1806 to 1871), and others about the middle of the nineteenth century. The equation takes the following form:

$$h_f = f \left(\frac{L}{D} \right) \frac{V^2}{2g} \quad (3.16)$$

This equation is commonly known as the *Darcy-Weisbach equation*. It is conveniently expressed in terms of the velocity head in the pipe. Moreover, it is dimensionally uniform since in engineering practice the *friction factor* (f) is treated as a dimensionless numerical factor; h_f and $V^2/2g$ are both in units of length.

3.5.1 Friction Factor for Laminar Flow

In laminar flow, f can be determined by balancing the viscous force and the pressure force at the two end sections of a horizontal pipe separated by a distance L . In a cylindrical pipe section of radius r (Figure 3.7), the difference in pressure force between the two ends of the cylinder is

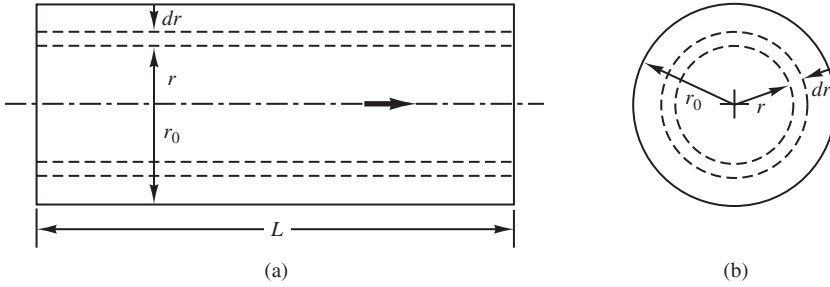


Figure 3.7 Geometry of a circular pipe

$(P_1 - P_2) \pi r^2$, and the viscous force on the cylinder is equal to $(2\pi r L)\tau$. The values of shear stress τ have been shown, in Equation 1.2, to be $\mu(dv/dr)$. Under equilibrium conditions, when the pressure force and the viscous force on the cylinder of water are balanced, the following expression results:

$$-2\pi r L \left(\mu \frac{dv}{dr} \right) = (P_1 - P_2) \pi r^2$$

The minus sign is used because the velocity decreases as the radial position (r) increases (i.e., dv/dr is always negative in pipe flow). This equation can be integrated to give the general expression of flow velocity in terms of r :

$$v = \left(\frac{P_1 - P_2}{4\mu L} \right) (r_0^2 - r^2) \tag{3.17}$$

where r_0 is the inner radius of the pipe, and the equation shows that the velocity distribution in laminar pipe flow is a parabolic function of radius r . The total discharge through the pipe can be obtained by integrating the discharge through the elemental area, $(2\pi r) dr$.

$$\begin{aligned} Q &= \int dQ = \int v dA = \int_{r=0}^{r=r_0} \frac{P_1 - P_2}{4\mu L} (r_0^2 - r^2) (2\pi r) dr \\ &= \frac{\pi r_0^4 (P_1 - P_2)}{8\mu L} = \frac{\pi D^4 (P_1 - P_2)}{128\mu L} \end{aligned} \tag{3.18}$$

This relationship is also known as the *Hagen-Poiseuille law** of laminar flow. The mean velocity is

$$\begin{aligned} V &= \frac{Q}{A} = \frac{\pi D^4 (P_1 - P_2)}{128\mu L} \left(\frac{1}{(\pi/4) D^2} \right) \\ V &= \frac{(P_1 - P_2) D^2}{32\mu L} \end{aligned} \tag{3.19}$$

* Experimentally derived by G. W. Hagen (1839) and later independently obtained by J. L. M. Poiseuille (1840).

For a horizontal uniform pipe, the energy equation (3.15) leads to

$$h_f = \frac{P_1 - P_2}{\gamma}$$

Thus, the Darcy-Weisbach equation may be written as

$$\frac{P_1 - P_2}{\gamma} = f \left(\frac{L}{D} \right) \frac{V^2}{2g} \quad (3.20)$$

Combining Equations 3.19 and 3.20, we have

$$f = \frac{64 \mu g}{\gamma V D} \quad (3.20a)$$

Because $\gamma = \rho g$,

$$f = \frac{64 \mu}{\rho V D} = \frac{64}{N_R} \quad (3.21)$$

which indicates a direct relationship between the friction factor (f) and the Reynolds number (N_R) for laminar pipe flow. It is independent of the surface roughness of the pipe.

3.5.2 Friction Factor for Turbulent Flow

When the Reynolds number approaches a higher value—say, $N_R \gg 2,000$ —the flow in the pipe becomes practically turbulent and the value of f then becomes less dependent on the Reynolds number but more dependent on the *relative roughness* (e/D) of the pipe. The quantity e is a measure of the average roughness height of the pipe wall irregularities, and D is the pipe diameter. The roughness height of commercial pipes is commonly described by providing a value of e for the pipe material. It means that the selected pipe has the same value of f at high Reynolds numbers as would be obtained if a smooth pipe were coated with sand grains of a uniform size e . The roughness height for certain common commercial pipe materials is provided in Table 3.1.

It has been determined that immediately next to the pipe wall there exists a very thin layer of flow commonly referred to as the *laminar sublayer* even if the pipe flow is turbulent. The thickness of the laminar sublayer δ' decreases with an increase in the pipe's Reynolds number. A pipe is said to be *hydraulically smooth* if the average roughness height is less than the thickness of the laminar sublayer. In hydraulically smooth pipe flow, the friction factor is not affected by the surface roughness of the pipe.

Based on laboratory experimental data, it has been found that if $\delta' > 1.7e$, then the effect of surface roughness is completely submerged by the laminar sublayer and the pipe flow is hydraulically smooth. In this case, Theodore Von Kármán* developed an equation for the friction factor:

*Theodore Von Kármán, "Mechanische Ähnlichkeit und Turbulenz" (Mechanical similitude and turbulence), *Proc. 3rd International Congress for Applied Mechanics, Stockholm*, Vol. I (1930).

TABLE 3.1 Roughness Heights, e , for Certain Common Pipe Materials

Pipe Material	e (mm)	e (ft)
Brass	0.0015	0.000005
Concrete		
Steel forms, smooth	0.18	0.0006
Good joints, average	0.36	0.0012
Rough, visible form marks	0.60	0.002
Copper	0.0015	0.000005
Corrugated metal (CMP)	45	0.15
Iron (common in older water lines, except ductile or DIP, which is widely used today)		
Asphalt lined	0.12	0.0004
Cast	0.26	0.00085
Ductile; DIP—cement mortar lined	0.12	0.0004
Galvanized	0.15	0.0005
Wrought	0.045	0.00015
Polyvinyl chloride (PVC)	0.0015	0.000005
Polyethylene, high density (HDPE)	0.0015	0.000005
Steel		
Enamel coated	0.0048	0.000016
Riveted	0.9 ~ 9.0	0.003–0.03
Seamless	0.004	0.000013
Commercial	0.045	0.00015

$$\frac{1}{\sqrt{f}} = 2 \log \left(\frac{N_R \sqrt{f}}{2.51} \right) \tag{3.22}$$

At high Reynolds numbers, δ' becomes very small. If $\delta' < 0.08e$, it has been found that f becomes independent of the Reynolds number and depends only on the relative roughness height. In this case, the pipe behaves as a *hydraulically rough pipe*, and von Kármán found that f can be expressed as

$$\frac{1}{\sqrt{f}} = 2 \log \left(3.7 \frac{D}{e} \right) \tag{3.23}$$

Between these two extreme cases, if $0.08e < \delta' < 1.7e$, the pipe behaves neither smoothly nor completely roughly. C. F. Colebrook* devised an approximate relationship for this intermediate range:

$$\frac{1}{\sqrt{f}} = -\log \left(\frac{e}{3.7D} + \frac{2.51}{N_R \sqrt{f}} \right) \tag{3.24}$$

* C. F. Colebrook, "Turbulent flow in pipes, with particular reference to the transition region between smooth and rough pipe laws," *Jour. Ist. Civil Engrs.*, London (Feb. 1939).

In the early 1940s, it was somewhat cumbersome to use any of these implicit equations in engineering practice. A convenient chart was prepared by Lewis F. Moody* (Figure 3.8); it is commonly called the *Moody diagram of friction factors for pipe flow*.

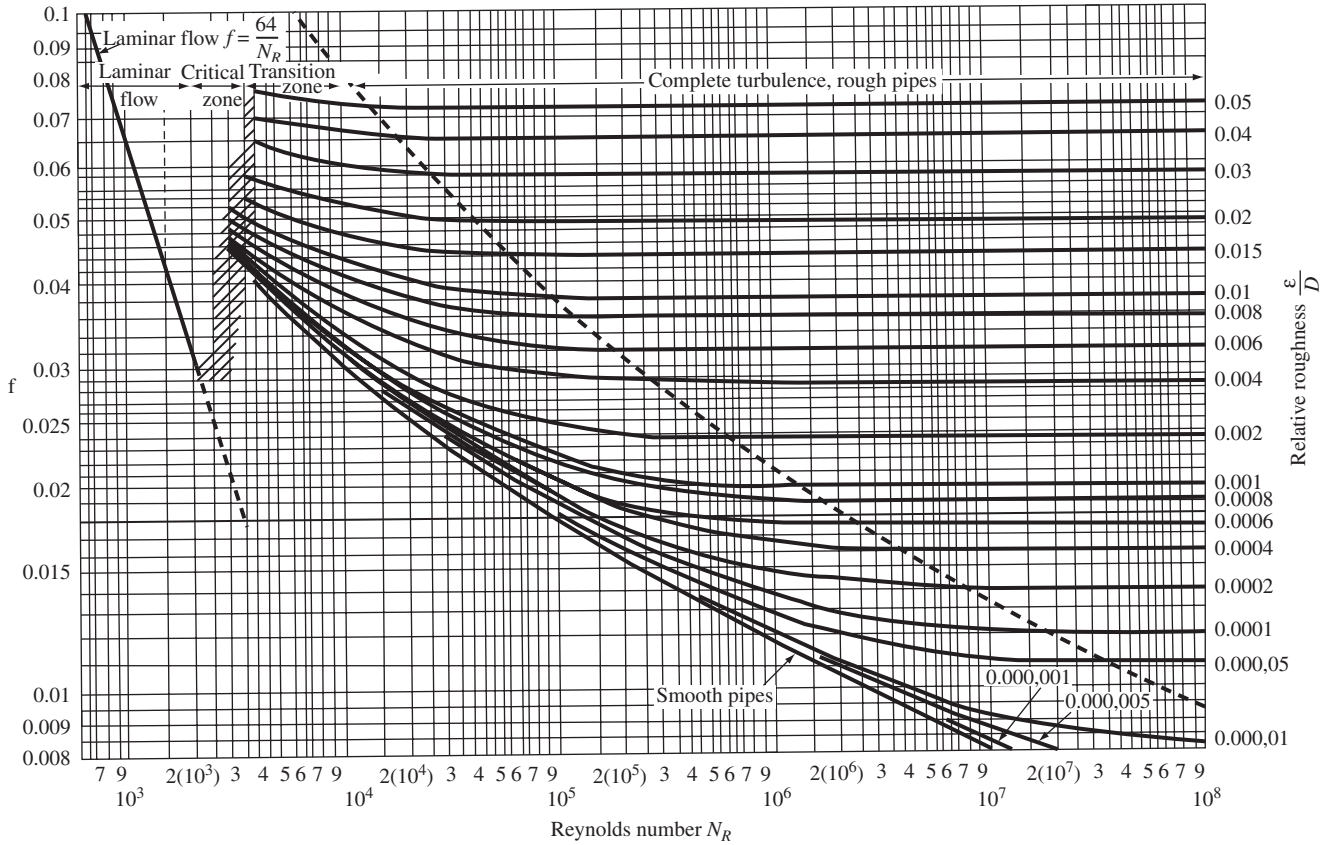


Figure 3.8 Friction factors for flow in pipes: the Moody diagram. *Source:* From L. F. Moody, “Friction factors for pipe flow,” *Trans. ASME*, vol. 66, 1944.

* L. F. Moody, “Friction factors for pipe flow,” *Trans. ASME*, 66 (1944).

The chart clearly shows the four zones of pipe flow:

1. a laminar flow zone where the friction factor is a simple linear function of the Reynolds number,
2. a critical zone where values are uncertain because the flow might be neither laminar nor truly turbulent,
3. a transitional zone where f is a function of both the Reynolds number and the relative roughness of the pipe, and
4. a zone of fully developed turbulence where the value of f depends solely on the relative roughness and is independent of the Reynolds number.

Figure 3.8 may be used together with Table 3.1 to obtain the friction factor f for circular pipes.

Subsequent to the development of the Moody diagram, the Swamee-Jain equation* was proposed to solve for the friction factor once N_R is known.

$$f = \frac{0.25}{\left[\log \left(\frac{e/D}{3.7} + \frac{5.74}{N_R^{0.9}} \right) \right]^2} \tag{3.24a}$$

This explicit expression is supposed to provide a very accurate estimate (within 1 percent) of the implicit Colebrook-White equation for $10^{-6} < e/D < 10^{-2}$ and $5,000 < N_R < 10^8$.

Example 3.5

Compute the discharge capacity of 3-m concrete (rough) pipe carrying water at 10°C. It is allowed to have a head loss of 2 m/km of pipe length.

Solution

From Equation 3.16, the friction head loss in the pipe is

$$h_f = f \left(\frac{L}{D} \right) \frac{V^2}{2g}$$

Hence,

$$\begin{aligned} 2 \text{ m} &= f \left(\frac{1000 \text{ m}}{3 \text{ m}} \right) \frac{V^2}{2(9.81 \text{ m/sec}^2)} \\ V^2 &= \frac{0.118 \text{ m}^2/\text{sec}^2}{f} \end{aligned} \tag{1}$$

From Table 3.1, taking $e = 0.60 \text{ mm}$, we obtain for the 3-m cast-iron pipe:

$$\frac{e}{D} = 2.00 \times 10^{-4} = 0.0002$$

At 10°C, the kinematic viscosity of water is $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{sec}$.

Therefore,

$$N_R = \frac{DV}{\nu} = \frac{3V}{1.31 \times 10^{-6}} = (2.29 \times 10^6) V \tag{2}$$

* P. K. Swamee and A. K. Jain, "Explicit equations for pipe-flow problems," *Journal of the Hydraulics Division*, ASCE 102(5): 657–664, 1976.

By using Figure 3.8, Equations (1) and (2) are solved by iteration until both conditions are satisfied. The iteration procedure is demonstrated as follows.

The Moody diagram (Figure 3.8) is used to find f . However, V is not available, so N_R cannot be solved. But the e/D value and the Moody diagram can be used to obtain a trial f value by assuming flow is in the complete turbulence regime. This is generally a good assumption for water-transmission systems because the viscosity of water is low and the velocities are high, yielding high N_R values. Thus, with $e/D = 0.0002$, an $f = 0.014$ is obtained from the Moody diagram assuming complete turbulence (i.e., reading directly across the Moody diagram from the relative roughness on the right to the associated f value on the left). Using this friction factor in Equation (1) we obtain $V = 2.90$ m/sec; using this value in Equation (2), we get $N_R = 6.64 \times 10^6$. This Reynolds number and the e/D value are then taken to the Moody diagram to obtain a new friction factor, $f = 0.014$, which is unchanged from the assumed friction factor. (If the friction factor was different, then additional iterations would be performed until the *trial* f and the *computed* f are essentially equal.) Now using the final flow velocity, the discharge is calculated as

$$Q = AV = [(\pi/4)(3 \text{ m})^2](2.90 \text{ m/sec}) = 20.5 \text{ m}^3/\text{sec}$$

Note: The Darcy-Weisbach equation for friction head loss (Equation 3.16), Reynolds number (Equation 3.2), and Colebrook's friction factor relationship (Equation 3.24) or the Swamee-Jain relationship (Equation 3.24a) can be solved simultaneously by a computer algebra software system (e.g., Mathcad, Maple, or Mathematica) and should yield the same result. However, the relationships are highly nonlinear, and a good initial estimate may be required to avoid numerical instability.

Example 3.6

Estimate the size of a uniform, horizontal welded-steel pipe installed to carry $14.0 \text{ ft}^3/\text{sec}$ of water at 70°F (approximately 20°C). The allowable pressure loss resulting from friction is 17 ft/mi of pipe length.

Solution

The energy equation can be applied to two pipe sections 1 mile apart:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$$

For a uniformly sized, horizontal pipe with no localized (minor) head losses,

$$V_1 = V_2; h_1 = h_2; h_L = h_f$$

and the energy equation reduces to

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = h_f = 17 \text{ ft}$$

From Equation 3.16,

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{Q^2}{2g(\pi D^2/4)^2} = \frac{8fLQ^2}{g\pi^2 D^5}$$

Therefore,

$$D^5 = \frac{8fLQ^2}{g\pi^2 h_f} = 1,530f \quad (\text{a})$$

where $L = 5,280 \text{ ft}$, and $h_f = 17 \text{ ft}$. At 20°C , $\nu = 1.08 \times 10^{-5} \text{ ft}^2/\text{sec}$. Assuming welded-steel roughness to be in the lower range of riveted steel, $e = 0.003 \text{ ft}$, the diameter can then be found using the Moody chart (Figure 3.8) by means of an iteration procedure as follows.

Let $D = 2.5$ ft, then

$$V = \frac{Q}{A} = \frac{14 \text{ ft}^3/\text{sec}}{\pi(1.25 \text{ ft})^2} = 2.85 \text{ ft/sec}$$

and

$$N_R = \frac{VD}{\nu} = \frac{(2.85 \text{ ft/sec})(2.5 \text{ ft})}{1.08 \times 10^{-5} \text{ ft}^2/\text{sec}} = 6.60 \times 10^5$$

$$e/D = \frac{0.003 \text{ ft}}{2.5 \text{ ft}} = 0.0012$$

Entering these values into the Moody chart, we get $f = 0.021$. A better estimate of D can now be obtained by substituting this friction factor into Equation (a), which gives

$$D = [(1,530)(0.021)]^{1/5} = 2.0 \text{ ft}$$

A second iteration yields $V = 4.46$ ft/sec, $N_R = 8.26 \times 10^5$, $e/D = 0.0015$, $f = 0.022$, and $D = 2.02$ ft ≈ 2.00 ft. More iterations will produce the same result. Once again, a computer algebra system (e.g., Mathcad, Maple, or Mathematica) should yield the same result.

3.6 Empirical Equations for Friction Head Loss

Throughout the history of civilization, hydraulic engineers have built systems to deliver water for people to use. The hydraulic design of these systems was significantly aided in the twentieth century by empirical equations, which are actually formulas in the strict sense. Generally speaking, these design equations were developed from experimental measurements of fluid flow under a certain range of conditions. Some do not have a sound analytical basis. For this reason, the empirical equations may not be dimensionally correct; if so, they can only be applicable to the conditions and ranges specified. The two equations discussed below contain empirical roughness coefficients that depend on the roughness of the tested pipes and not relative roughness, further limiting their usefulness.

One of the best examples is the *Hazen-Williams equation*, which was developed for water flow in larger pipes ($D \geq 5$ cm, approximately 2 in.) within a moderate range of water velocity ($V \leq 3$ m/sec, approximately 10 ft/sec). This equation has been used extensively for the designing of water-supply systems in the United States. The Hazen-Williams equation, originally developed for the British measurement system, has been written in the form

$$V = 1.318 C_{HW} R_h^{0.63} S^{0.54} \tag{3.25}$$

where S is the slope of the *energy grade line*, or the head loss per unit length of the pipe ($S = h_f/L$), and R_h is the *hydraulic radius*, defined as the water cross-sectional area (A) divided by the *wetted perimeter* (P). For a circular pipe, with $A = \pi D^2/4$ and $P = \pi D$, the hydraulic radius is

$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} \tag{3.26}$$

The Hazen-Williams coefficient, C_{HW} , is not a function of the flow conditions (i.e., Reynolds number). Its values range from 140 for very smooth, straight pipes down to 90 or even 80 for old, unlined tuberculated pipes. Generally, the value of 100 is taken for average conditions. The values of C_{HW} for commonly used water-carrying conduits are listed in Table 3.2.

TABLE 3.2 Hazen-Williams Coefficient, C_{HW} , for Different Types of Pipes

Pipe Materials	C_{HW}
Brass	130–140
Cast iron (common in older water lines)	
New, unlined	130
10-year-old	107–113
20-year-old	89–100
30-year-old	75–90
40-year-old	64–83
Concrete or concrete lined	
Smooth	140
Average	120
Rough	100
Copper	130–140
Ductile iron (cement mortar lined)	140
Glass	140
High-density polyethylene (HDPE)	150
Plastic	130–150
Polyvinyl chloride (PVC)	150
Steel	
Commercial	140–150
Riveted	90–110
Welded (seamless)	100
Vitrified clay	110

Note that the coefficient in the Hazen-Williams equation shown in Equation 3.25, 1.318, has units of $\text{ft}^{0.37}/\text{sec}$. Therefore, Equation 3.25 is applicable *only* for the British units in which the velocity is measured in feet per second and the hydraulic radius (R_h) is measured in feet. Because $1.318 \text{ ft}^{0.37}/\text{sec} = 0.849 \text{ m}^{0.37}/\text{sec}$, the Hazen-Williams equation in SI units may be written in the following form:

$$V = 0.849 C_{HW} R_h^{0.63} S^{0.54} \quad (3.27)$$

where the velocity is measured in meters per second and R_h is measured in meters.

Example 3.7

A 100-m-long pipe with $D = 20$ cm and $C_{HW} = 120$ carries a discharge of 30 L/sec. Determine the head loss in the pipe.

Solution

$$\text{Area: } A = \frac{\pi D^2}{4} = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2$$

$$\text{Wetted perimeter: } P = \pi D = 0.2\pi = 0.628 \text{ m}$$

$$\text{Hydraulic radius: } R_h = A/P = \frac{0.0314}{0.628} = 0.0500 \text{ m}$$

Applying Equation 3.27,

$$V = \frac{Q}{A} = 0.849 C_{HW} R_h^{0.63} S^{0.54}$$

$$\frac{0.03}{0.0314} = 0.849 (120) (0.05)^{0.63} \left(\frac{h_f}{100}\right)^{0.54}$$

$$h_f = 0.579 \text{ m}$$

Another popular empirical equation is the *Manning equation*, which was originally developed in metric units. The Manning equation has been used extensively for open-channel designs (discussed in detail in Chapter 6). It is also quite commonly used for pipe flows. The Manning equation may be expressed in the following form:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{3.28}$$

where the velocity is measured in meters per second and the hydraulic radius is measured in meters. The n is *Manning’s coefficient of roughness*, specifically known to hydraulic engineers as *Manning’s n*.

In British units, the Manning equation is written as

$$V = \frac{1.486}{n} R_h^{2/3} S^{1/2} \tag{3.29}$$

where R_h is measured in feet and the velocity is measured in units of feet per second. The coefficient in Equation 3.29 serves as a unit conversion factor because $1 \text{ m}^{1/3}/\text{sec} = 1.486 \text{ ft}^{1/3}/\text{sec}$. Table 3.3 contains typical values of n for water flow in common pipe materials.

TABLE 3.3 Manning’s Roughness Coefficient, n , for Pipe Flows

Type of Pipe	Manning’s n	
	Min.	Max.
Brass	0.009	0.013
Cast iron	0.011	0.015
Cement mortar surfaces	0.011	0.015
Cement rubble surfaces	0.017	0.030
Clay drainage tile	0.011	0.017
Concrete, precast	0.011	0.015
Copper	0.009	0.013
Corrugated metal (CMP)	0.020	0.024
Ductile iron (cement mortar lined)	0.011	0.013
Glass	0.009	0.013
High-density polyethylene (HDPE)	0.009	0.011
Polyvinyl chloride (PVC)	0.009	0.011
Steel, commercial	0.010	0.012
Steel, riveted	0.017	0.020
Vitrified sewer pipe	0.010	0.017
Wrought iron	0.012	0.017

Example 3.8

A horizontal pipe (old cast iron) with a 10-cm uniform diameter is 200 m long. If the measured pressure drop is 24.6 m of water, what is the discharge?

Solution

$$\text{Area: } A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$$

$$\text{Wetted perimeter: } P = \pi D = 0.1\pi = 0.314 \text{ m}$$

$$\text{Hydraulic radius: } R_h = A/P = \frac{0.00785}{0.314} = 0.0250 \text{ m}$$

$$\text{Energy slope: } S = h_f/L = \frac{24.6 \text{ m}}{200 \text{ m}} = 0.123$$

$$\text{Manning's roughness coefficient: } n = 0.015 \text{ (Table 3.3)}$$

Substituting the above quantities into the Manning equation, Equation 3.28, we have

$$V = \frac{Q}{A} = \frac{1}{n}R_h^{2/3}S^{1/2}$$

$$Q = \frac{1}{0.015}(0.00785)(0.025)^{2/3}(0.123)^{1/2} = 0.0157 \text{ m}^3/\text{sec}$$

3.7 Friction Head Loss–Discharge Relationships

Many engineering problems involve determination of the friction head loss in a pipe given the discharge. Therefore, expressions relating the friction head loss to discharge are convenient. Noting that $A = \pi D^2/4$ and $g = 32.2 \text{ ft/sec}^2$ for the British unit system, we can rearrange the Darcy-Weisbach equation, Equation 3.16, as

$$h_f = fL \frac{0.0252 Q^2}{D^5} \quad (3.30)$$

As discussed previously, the friction factor generally depends on the pipe size, roughness, and the Reynolds number. However, an inspection of the Moody diagram (Figure 3.8) reveals that the graphed lines become horizontal at high Reynolds numbers where the friction factor depends only on the e/D ratio for fully turbulent flow. In other words, for a given pipe size and material, f is constant for fully turbulent flow. This is generally the case for most water-transmission systems because the viscosity of water is low and the velocities are high, which yields high N_R values. For practical purposes, Equation 3.30 is written as

$$h_f = KQ^m \quad (3.31)$$

where $K = (0.0252 \cdot f \cdot L)/D^5$ and $m = 2.0$. Other friction head loss equations can also be expressed in the form of Equation 3.31 as summarized in Table 3.4. Note that in Equation 3.31 m is dimensionless, and the dimension of K depends on the friction equation and the unit system chosen.

There is an abundance of computer software available to solve the pipe flow equations that have been discussed (i.e., Darcy-Weisbach, Hazen-Williams, and Manning). Some of these computer programs are free and readily available on the Internet as pipe flow calculators. Others are proprietary [e.g., FlowMaster, PIPE FLO, and Pipe Flow Wizard (Great Britain)], tend to be more robust, and have better print options for technical reports. For most of these computer programs, any four of the five variables (L , D , Q , h_f , and the loss coefficient) are required to solve for the desired variable. Spreadsheet programs can easily be written to accomplish the same thing.

TABLE 3.4 Friction Equations Expressed in the Form of $h_f = KQ^m$

Equation	m	K (BG System)	K (SI System)
Darcy-Weisbach	2	$\frac{0.0252 fL}{D^5}$	$\frac{0.0826 fL}{D^5}$
Hazen-Williams	1.85	$\frac{4.73 L}{D^{4.87} C_{HW}^{1.85}}$	$\frac{10.7 L}{D^{4.87} C_{HW}^{1.85}}$
Manning	2	$\frac{4.64n^2L}{D^{5.33}}$	$\frac{10.3n^2L}{D^{5.33}}$

3.8 Loss of Head in Pipe Contractions

A sudden contraction in a pipe usually causes a marked drop in pressure in the pipe because of both the increase in velocity and the loss of energy to turbulence. The phenomenon of a sudden contraction is schematically represented in Figure 3.9.

The vertical distance measured between the energy grade line and the pipe centerline represents the total head at any particular location along the pipe. The vertical distance measured between the hydraulic grade line and the pipe centerline represents the pressure head (P/γ), and the distance between the EGL and HGL is the velocity head ($V^2/2g$) at that location. After point B , the HGL begins to drop as the stream picks up speed, and a region of stagnant water appears at the corner of contraction C . Immediately downstream from the contraction, the streamlines separate from the pipe wall and form a high-speed jet that reattaches to the wall at point E . The phenomenon that takes place between C and E is known to hydraulic engineers as the *vena contracta*, which will be discussed in detail in Chapter 9. Most of the energy loss in a pipe contraction takes place between C and D where the jet stream velocity is high and the pressure is low. A certain amount of pressure recovers between D and E as the jet stream gradually dissipates and normal pipe flow resumes. Downstream from point E , the EGL and HGL lines become parallel to each other again,

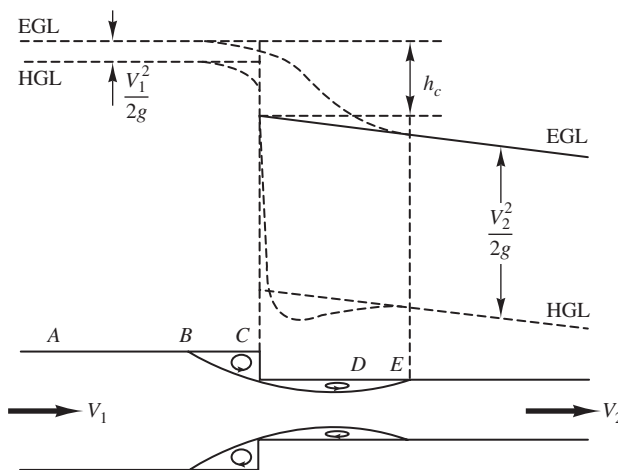


Figure 3.9 Head loss and pressure variation resulting from sudden contraction

TABLE 3.5 Values of the Coefficient K_c for Sudden Contraction

Velocity in Smaller Pipe (m/sec)	Sudden Contraction Coefficients, K_c (Ratio of Smaller to Larger Pipe Diameters, D_2/D_1)									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.49	0.49	0.48	0.45	0.42	0.38	0.28	0.18	0.07	0.03
2	0.48	0.48	0.47	0.44	0.41	0.37	0.28	0.18	0.09	0.04
3	0.47	0.46	0.45	0.43	0.40	0.36	0.28	0.18	0.10	0.04
6	0.44	0.43	0.42	0.40	0.37	0.33	0.27	0.19	0.11	0.05
12	0.38	0.36	0.35	0.33	0.31	0.29	0.25	0.20	0.13	0.06

but they take a steeper slope (than in the pipe upstream of the contraction) where a higher rate of energy dissipation (resulting from friction) in the smaller pipe is expected.

The loss of head in a sudden contraction may be represented in terms of velocity head in the smaller pipe as

$$h_c = K_c \left(\frac{V_2^2}{2g} \right) \tag{3.32}$$

Here K_c is the coefficient of contraction. Its value varies with the ratio of contraction, D_2/D_1 , and the pipe velocity as shown in Table 3.5.

Head loss from pipe contraction may be greatly reduced by introducing a gradual pipe transition known as a *confusor* as shown in Figure 3.10. The head loss in this case may be expressed as

$$h'_c = K'_c \left(\frac{V_2^2}{2g} \right) \tag{3.33}$$

The values of K'_c vary with the transition angle α and the area ratio A_2/A_1 , as shown in Figure 3.11.

The loss of head at the entrance of a pipe from a large reservoir is a special case of loss of head resulting from contraction. Because the water cross-sectional area in the reservoir is very large compared with that of the pipe, a ratio of contraction of zero may be taken. For a square-edged entrance, where the entrance of the pipe is flush with the reservoir wall as shown in Figure 3.12 (a), the K_c values shown for $D_2/D_1 = 0.0$ in Table 3.5 are used.

The general equation for an *entrance head loss* is also expressed in terms of the velocity head of the pipe:

$$h_e = K_e \left(\frac{V^2}{2g} \right) \tag{3.34}$$

The approximate values for the *entrance loss coefficient* (K_e) for different entrance conditions are shown in Figure 3.12(a-d).

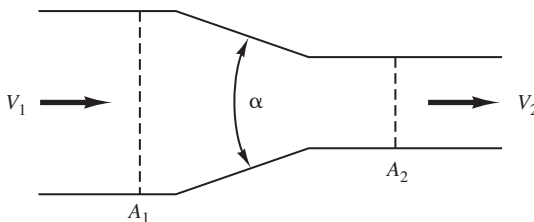


Figure 3.10 Pipe confusor

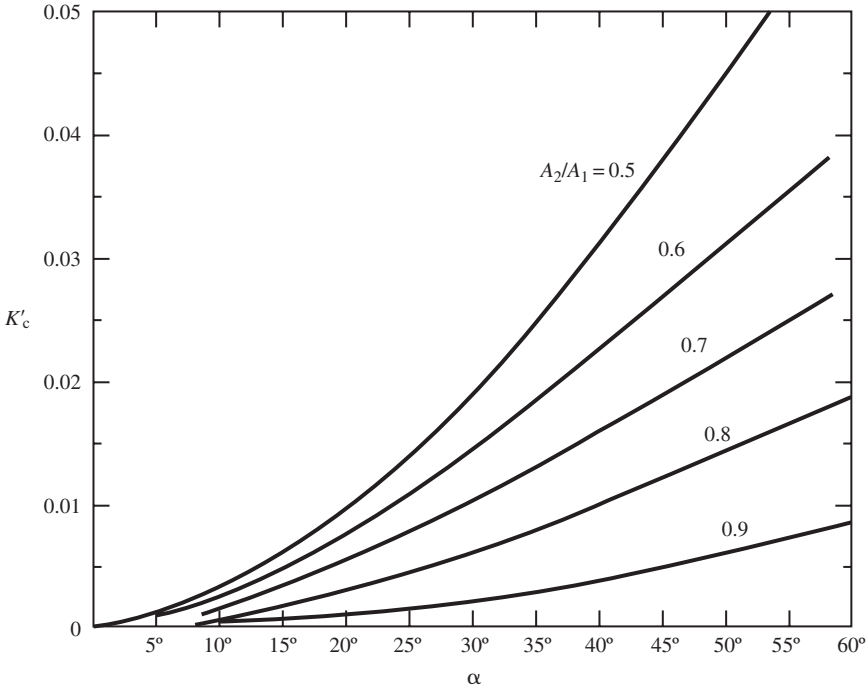


Figure 3.11 Coefficient K'_c for pipe conusors. *Source:* From Chigong Wu et al., *Hydraulics* (Chengdu, Sichuan, China: The Chengdu University of Science and Technology Press, 1979).

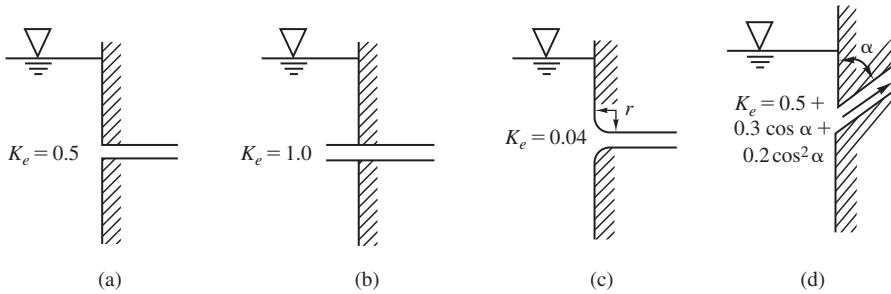


Figure 3.12 Coefficient K_e for pipe entrances

3.9 Loss of Head in Pipe Expansions

The behavior of the energy grade line and the hydraulic grade line in the vicinity of a sudden pipe expansion is schematically depicted in Figure 3.13. At the corner of the sudden expansion A, the stream line separates from the wall of the large pipe and leaves an area of relative stagnation between A and B, in which a large recirculating eddy is formed to fill the space. Most of the energy loss in a sudden expansion takes place between A and B where the stream lines reattach to the wall. A recovery of pressure may take place here as a result of the decrease of velocity in the pipe. The high-speed jet stream gradually slows down and reaches equilibrium at point C. From

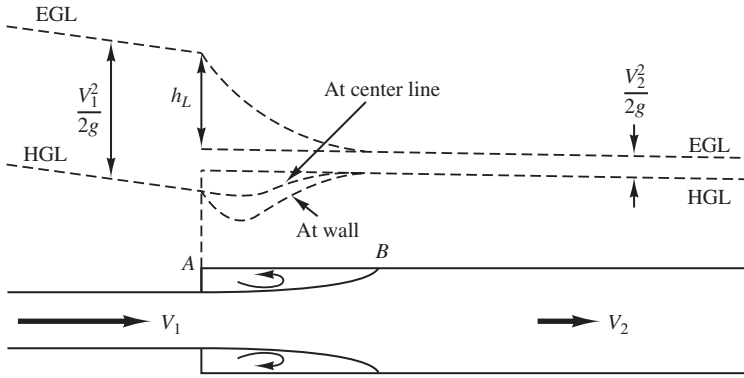


Figure 3.13 Head loss from sudden expansion

this point downstream, the normal pipe flow conditions resume and the energy grade line takes on a smaller slope than that of the approaching pipe, as expected.

The loss of head from a sudden expansion in a pipe can be derived from the momentum considerations (e.g., Daugherty and Franzini, 1976*). The magnitude of the head loss may be expressed as

$$h_E = \frac{(V_1 - V_2)^2}{2g} \tag{3.35}$$

Physically, this equation shows that the change in velocities expressed as a velocity head is the head loss in the sudden expansion.

The head loss resulting from pipe expansions may be greatly reduced by introducing a gradual pipe transition known as a *diffusor*, which is depicted in Figure 3.14. The head loss in this pipe transition case may be expressed as

$$h'_E = K'_E \frac{(V_1^2 - V_2^2)}{2g} \tag{3.36}$$

The values of K'_E vary with the diffusor angle (α):

α	10°	20°	30°	40°	50°	60°	75°
K'_E	0.08	0.31	0.49	0.60	0.67	0.72	0.72

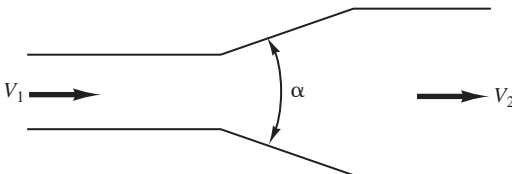


Figure 3.14 Pipe diffusor

* R. L. Daugherty and J. B. Franzini, *Fluid Mechanics with Engineering Applications* (New York: McGraw-Hill Book Company, 1977).

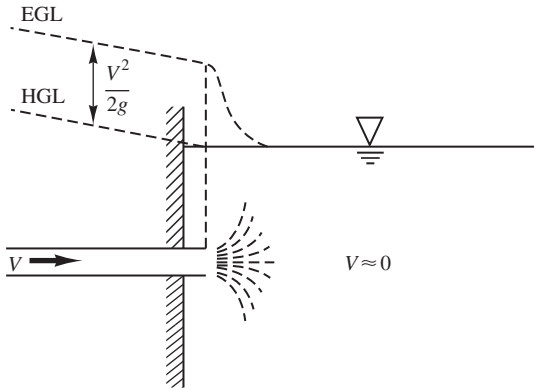


Figure 3.15 Exit (discharge) head loss

A submerged pipe discharging into a large reservoir is a special case of head loss from expansion. The flow velocity (V) in the pipe is discharged from the end of a pipe into a reservoir that is so large that the velocity within it is negligible. From Equation 3.35 we see that the entire velocity head of the pipe flow is dissipated and that the *exit (discharge) head loss* is

$$h_d = K_d \frac{V^2}{2g} \tag{3.37}$$

where the exit (discharge) loss coefficient $K_d = 1.0$. The phenomenon of exit loss is shown in Figure 3.15.

3.10 Loss of Head in Pipe Bends

Pipe flow around a bend experiences an increase of pressure along the outer wall and a decrease of pressure along the inner wall. A certain distance downstream from the bend, the velocity and pressure resume normal distributions. To achieve this, the inner wall pressure must rise back to the normal value. The velocity near the inner wall of the pipe is lower than that at the outer wall, and it must also increase to the normal value. The simultaneous demand of energy may cause separation

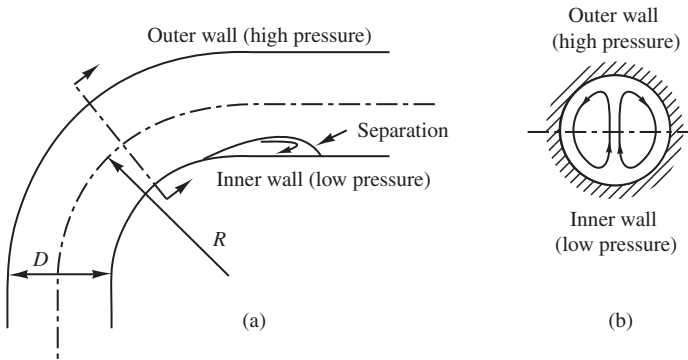


Figure 3.16 Head loss at a bend: (a) flow separation in a bend and (b) secondary flow at a bend

of the stream from the inner wall as shown in Figure 3.16 (a). In addition, the unbalanced pressure at the bend causes a secondary current as in Figure 3.16 (b). This transverse current and the axial velocity form a pair of spiral flows that persist as far as 100 diameters downstream from the bend. Thus, the head loss at the bend is combined with the distorted flow conditions downstream from the bend until the spiral flows are dissipated by viscous friction.

The head loss produced at a bend was found to be dependent on the ratio of the radius of curvature of the bend (R) to the diameter of the pipe (D) (Figure 3.16). Because the spiral flow produced by a bend extends some distance downstream from the bend, the head loss produced by different pipe bends placed close together cannot be treated by simply adding the losses of each one separately. The total loss of a series of bends placed close together depends not only on the spacing between the bends but also on the direction of the bends. Detailed analysis of head loss produced by a series of bends is a rather complex matter, and it can only be analyzed on an individual case-by-case basis.

In hydraulic design the loss of head due to a bend, in excess of that which would occur in a straight pipe of equal length, may be expressed in terms of the velocity head as

$$h_b = K_b \frac{V^2}{2g} \quad (3.38)$$

For a smooth pipe bend of 90° , the values of K_b for various values of R/D as determined by Beij* are listed in the following table. The bend loss has also been found nearly proportional to the angle of the bend (α) for pipe bends other than 90° in steel pipes and drawn tubings. Pipe manufacturers are more than willing to supply prospective buyers with loss coefficients for bends, contractions, confusers, expansions, and diffusers.

R/D	1	2	4	6	10	16	20
K_b	0.35	0.19	0.17	0.22	0.32	0.38	0.42

3.11 Loss of Head in Pipe Valves

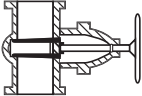
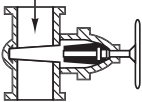
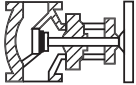


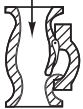
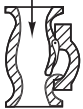

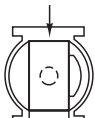
Valves are installed in pipelines to control flow by imposing high head losses. Depending on how the particular valve is designed, a certain amount of energy loss usually takes place even when the valve is fully open. As with other losses in pipes, the head loss through valves may also be expressed in terms of velocity head in the pipe:

$$h_v = K_v \frac{V^2}{2g} \quad (3.39)$$

The values of K_v vary with the type and design of the valves. When designing hydraulic systems, it is necessary to determine the head losses through any valves that are present. Valve manufacturers are more than willing to supply prospective buyers with loss coefficients. The values of K_v for common valves are listed in Table 3.6.

* K. H. Beij, "Pressure Losses for Fluid Flow in 90° Pipe Bends," *Jour. Research Natl. Bur. Standards*, 21 (1938).

TABLE 3.6 Values of K_v for Common Hydraulic Valves

A. Gate valves			
	Closed		
	Open		$K_v = 0.15$ (fully open)
B. Globe valves			
	Closed		
	Open		$K_v = 10.0$ (fully open)
C. Check valves			
	Closed	Hinge (swing type)	Swing type: $K_v = 2.5$ (fully open)
			Ball type: $K_v = 70.0$ (fully open)
	Open		Lift type: $K_v = 12.0$ (fully open)
D. Rotary valves			
	Closed		
	Open		$K_v = 10.0$ (fully open)

Example 3.9

Figure 3.17 shows two sections of cast-iron pipe connected in series that transport water from a reservoir and discharge it into air through a rotary valve at a location 100 m below the water surface elevation. If the water temperature is 10°C, and square-edged connections are used, what is the discharge?

Solution

The energy equation can be written for section 1 at the reservoir surface and section 3 at the discharge end as

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + h_3 + h_L$$

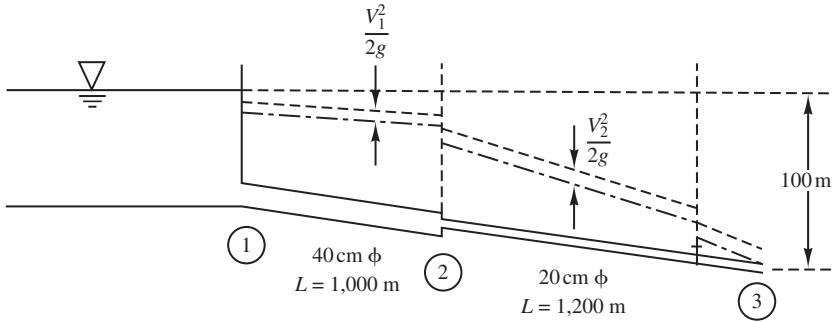


Figure 3.17 Flow through a pipeline

Selecting the reference datum at section 3 yields $h_3 = 0$. Because the reservoir and the discharge end are both exposed to atmospheric pressure and the velocity head at the reservoir can be neglected, we have

$$h_1 = 100 = \frac{V_3^2}{2g} + h_L$$

The total available energy, 100 m of water column, is equal to the velocity head at the discharge end plus all the head losses incurred in the pipeline system. This relationship, as shown in Figure 3.17, may be expressed as (noting that $V_3 = V_2$):

$$h_e + h_{f1} + h_c + h_{f2} + h_v + \frac{V_2^2}{2g} = 100$$

where h_e is the entrance head loss. For a square-edged entrance, Equation 3.34 and Figure 3.12 yield

$$h_e = (0.5) \frac{V_1^2}{2g}$$

The head loss from friction in pipe section 1–2 is h_{f1} . From Equation 3.16,

$$h_{f1} = f_1 \frac{1,000}{0.40} \frac{V_1^2}{2g}$$

The head loss from the sudden contraction at section 2 is h_c . From Table 3.5 and Equation 3.32 (assume $K_c = 0.33$ for the first trial),

$$h_c = K_c \frac{V_2^2}{2g} = 0.33 \frac{V_2^2}{2g}$$

The head loss from friction in pipe section 2–3 is h_{f2} :

$$h_{f2} = f_2 \frac{1,200}{0.20} \frac{V_2^2}{2g}$$

The head loss at the valve is h_v . From Table 3.6 and Equation 3.39,

$$h_v = K_v \frac{V_2^2}{2g} = (10) \frac{V_2^2}{2g}$$

Therefore,

$$100 = \left(1 + 10 + f_2 \frac{1,200}{0.20} + 0.33 \right) \frac{V_2^2}{2g} + \left(f_1 \frac{1,000}{0.40} + 0.5 \right) \frac{V_1^2}{2g}$$

From the continuity equation (Equation 3.4), we have

$$\begin{aligned} A_1 V_1 &= A_2 V_2 \\ \frac{\pi}{4}(0.4)^2 V_1 &= \frac{\pi}{4}(0.2)^2 V_2 \\ V_1 &= 0.25 V_2 \end{aligned}$$

Substituting V_1 in the above relationship gives

$$V_2^2 = \frac{1,960}{11.4 + 156 f_1 + 6,000 f_2}$$

To evaluate f_1 and f_2 we have:

$$\begin{aligned} N_{R_1} &= \frac{D_1 V_1}{\nu} = \frac{0.4}{1.31 \times 10^{-6}} V_1 = (3.05 \times 10^5) V_1 \\ N_{R_2} &= \frac{D_2 V_2}{\nu} = \frac{0.2}{1.31 \times 10^{-6}} V_2 = (1.53 \times 10^5) V_2 \end{aligned}$$

where $\nu = 1.31 \times 10^{-6}$ at 10°C . For the 40-cm pipe, $e/D = 0.00065$, which yields $f_1 \approx 0.018$ (assuming complete turbulence). For the 20-cm pipe, $e/D = 0.0013$, so $f_2 \approx 0.021$. Solving the above equation for V_2 yields the following:

$$\begin{aligned} V_2^2 &= \frac{1,960}{11.4 + 156(0.018) + 6,000(0.021)} \\ V_2 &= 3.74 \text{ m/sec} \quad \text{and} \quad V_1 = 0.25(3.74 \text{ m/sec}) = 0.935 \text{ m/sec} \end{aligned}$$

Hence,

$$\begin{aligned} N_{R_1} &= 3.05 \times 10^5 (3.74) = 1.14 \times 10^6; & f_1 &= 0.018 \\ N_{R_2} &= 1.53 \times 10^5 (0.935) = 1.43 \times 10^5; & f_2 &= 0.0225 \end{aligned}$$

These f values do not agree with the assumed values, so a second trial must be made. For the second trial, we assume that $K_c = 0.35$, $f_1 = 0.018$, and $f_2 = 0.0225$. Repeating the above calculation, we obtain $V_2 = 3.62 \text{ m/sec}$, $V_1 = 0.905 \text{ m/sec}$, $N_{R_1} = 1.10 \times 10^6$, $N_{R_2} = 1.38 \times 10^5$. Now from Figure 3.8, $f_1 = 0.018$; $f_2 = 0.0225$.

Therefore, the discharge is

$$Q = A_2 V_2 = \frac{\pi}{4}(0.2 \text{ m})^2(3.62 \text{ m/sec}) = 0.114 \text{ m}^3/\text{sec}$$

Note: The energy equation, the Reynolds number expressions, Colebrook's friction factor relationship, or the Swamee-Jain equation and the continuity equation can be solved simultaneously by a computer algebra software system (e.g., Mathcad, Maple, or Mathematica) and should yield the same result. However, the relationships are highly nonlinear, and a good initial estimate may be required to avoid numerical instability.

3.12 Method of Equivalent Pipes

The method of equivalent pipes is used to facilitate the analysis of pipe systems containing several pipes in series or in parallel. An equivalent pipe is a hypothetical pipe that produces the same head loss as two or more pipes in series or parallel for the same discharge. The expressions presented for equivalent pipes account for the losses from friction only.

3.12.1 Pipes in Series

The method of equivalent pipes saves very little computation time when applied to pipes in series. However, the method is included herein for the sake of completeness.

Consider pipes 1 and 2 shown in Figure 3.18. Suppose the pipes' diameters, lengths, and friction factors are known. We want to find a single pipe, E , that is hydraulically equivalent to 1 and 2 in series. For the two systems shown in Figure 3.18 to be equivalent, neglecting the head loss from pipe expansions and contractions, we must have

$$Q_1 = Q_2 = Q_E \quad (3.39)$$

and

$$h_{f_E} = h_{f_1} + h_{f_2} \quad (3.40)$$

Suppose we employ the Darcy-Weisbach equation to solve the series pipe problem. In terms of the discharge (Q), Equation 3.16 becomes

$$h_f = f \frac{8LQ^2}{g\pi^2 D^5} \quad (3.41)$$

Then writing Equation 3.41 for pipes 1, 2, and E , substituting into Equation 3.40, and simplifying with $Q_E = Q_1 = Q_2$, we obtain

$$f_E \frac{L_E}{D_E^5} = f_1 \frac{L_1}{D_1^5} + f_2 \frac{L_2}{D_2^5} \quad (3.42)$$

Any combination of f_E , L_E , and D_E that satisfies Equation 3.42 is acceptable. To find the characteristics of the hypothetical equivalent pipe, we pick two of the three unknowns— f_E , L_E , and D_E —arbitrarily and calculate the third from Equation 3.42. In other words, an infinite number of single hypothetical pipes are hydraulically equivalent to two pipes in series. For N pipes in series, an equivalent pipe can be found using

$$f_E \frac{L_E}{D_E^5} = \sum_{i=1}^N f_i \frac{L_i}{D_i^5} \quad (3.43)$$

We should note that Equations 3.42 and 3.43 are valid for the Darcy-Weisbach equation only. The equivalent pipe relationships for the Hazen-Williams and Manning equations are given in Table 3.7. These relationships are valid both for the British and the SI unit systems.

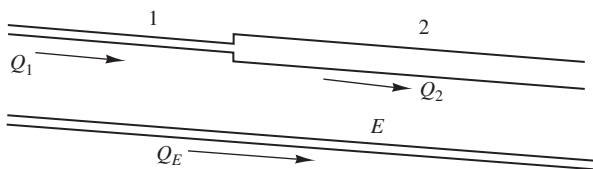


Figure 3.18 Pipes in series

TABLE 3.7 Equivalent Pipe Equations

Equation	Pipes in Series	Pipes in Parallel
Darcy-Weisbach	$f_E \frac{L_E}{D_E^5} = \sum_{i=1}^N f_i \frac{L_i}{D_i^5}$	$\sqrt{\frac{D_E^5}{f_E L_E}} = \sum_{i=1}^N \sqrt{\frac{D_i^5}{f_i L_i}}$
Manning	$\frac{L_E n_E^2}{D_E^{5.33}} = \sum_{i=1}^N \frac{L_i n_i^2}{D_i^{5.33}}$	$\sqrt{\frac{D_E^{5.33}}{n_E^2 L_E}} = \sum_{i=1}^N \sqrt{\frac{D_i^{5.33}}{n_i^2 L_i}}$
Hazen-Williams	$\frac{L_E}{C_{HWE}^{1.85} D_E^{4.87}} = \sum_{i=1}^N \frac{L_i}{C_{HWi}^{1.85} D_i^{4.87}}$	$\sqrt[1.85]{\frac{C_{HWE}^{1.85} D_E^{4.87}}{L_E}} = \sum_{i=1}^N \sqrt[1.85]{\frac{C_{HWi}^{1.85} D_i^{4.87}}{L_i}}$

3.12.2 Pipes in Parallel

The method of equivalent pipes is a very powerful tool for analyzing pipe systems containing pipes in parallel. Consider the system with parallel pipes 1 and 2 shown in Figure 3.19. Suppose we want to determine a single pipe that is equivalent to pipes 1 and 2 in parallel. The two systems will be equivalent if

$$h_{f_1} = h_{f_2} = h_{f_E} \tag{3.44}$$

and

$$Q_E = Q_1 + Q_2 \tag{3.45}$$

In considering the two requirements for parallel pipe flow (Equations 3.44 and 3.45), the flow equation is the most intuitive. However, the friction loss equality is the most critical to the solution process. Basically, the friction loss equality states that flow from one junction to another produces equal head losses regardless of the path taken. This concept is important in the solution of pipe network problems that will be covered in the next chapter.

To solve the parallel pipe problem, we can rearrange Equation 3.41 as

$$Q = \left\{ \frac{g \pi^2 D^5 h_f}{8 f L} \right\}^{1/2} \tag{3.46}$$

Writing Equation 3.46 for pipes 1, 2, and E, substituting into Equation 3.45, and simplifying with $h_{f_1} = h_{f_2} = h_{f_E}$, we obtain

$$\sqrt{\frac{D_E^5}{f_E L_E}} = \sqrt{\frac{D_1^5}{f_1 L_1}} + \sqrt{\frac{D_2^5}{f_2 L_2}} \tag{3.47}$$

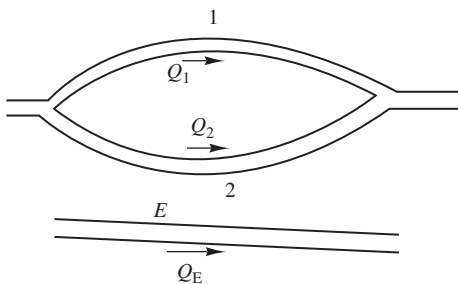


Figure 3.19 Pipes in parallel

For N pipes in parallel, Equation 3.47 can be generalized to obtain

$$\sqrt{\frac{D_E^5}{f_E L_E}} = \sum_{i=1}^N \sqrt{\frac{D_i^5}{f_i L_i}} \quad (3.48)$$

Again, two of the three unknowns— f_E , L_E , and D_E —can be chosen arbitrarily, and the third one is obtained from Equation 3.48. Also note that Equations 3.47 and 3.48 are applicable for the Darcy-Weisbach equation. Refer to Table 3.7 for the Hazen-Williams and Manning equations.

Example 3.10

Pipes AB and CF in Figure 3.20 have a diameter of 4 ft, possess a Darcy-Weisbach friction factor of 0.02, and carry a discharge of 120 cubic feet per second (cfs). The length of AB is 1,800 ft and that of CF is 1,500 ft. Branch 1 is 1,800 ft long and has a diameter of 3 ft and a friction factor of 0.018. Branch 2 has a length of 1,500 ft, a diameter of 2 ft, and a friction factor of 0.015. (a) Determine the total head loss resulting from friction between points A and F , and (b) determine the discharge in each of the two branches (1 and 2).

Solution

- (a) We will first determine a hypothetical pipe that is hydraulically equivalent to the two pipes in parallel, branches 1 and 2. Let us arbitrarily pick a diameter of 4 ft and a friction factor of 0.02 for the equivalent pipe. By using Equation 3.47,

$$\sqrt{\frac{4^5}{0.02 L_E}} = \sqrt{\frac{3^5}{(0.018)(1,800)}} + \sqrt{\frac{2^5}{(0.015)(1,500)}}$$

Solving for L_E , we obtain $L_E = 3,310$ ft. Then

$$h_{f_{AF}} = h_{f_{AB}} + h_{f_{BC}} + h_{f_{CF}}$$

and by using Equation 3.30,

$$\begin{aligned} h_{f_{AF}} &= (0.02)(1,800) \frac{0.0252 (120)^2}{4^5} + (0.02)(3,310) \frac{0.0252 (120)^2}{4^5} \\ &+ (0.02)(1,500) \frac{0.0252 (120)^2}{4^5} \\ h_{f_{AF}} &= 12.8 + 23.5 + 10.6 = 46.9 \text{ ft} \end{aligned}$$

- (b) For branch 1,

$$h_{f_{BC}} = 23.5 = (0.018)(1,800) \frac{0.0252 Q_1^2}{3^5}$$

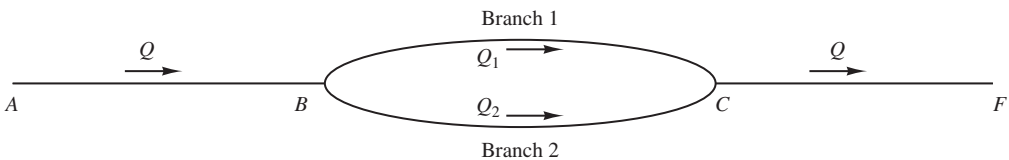


Figure 3.20 Flow through parallel pipes

Solving for Q_1 , we obtain $Q_1 = 83.6$ cfs. Likewise, for branch 2

$$h_{f_{BC}} = 23.5 = (0.015)(1,500) \frac{0.0252 Q_2^2}{2^5}$$

and we obtain, $Q_2 = 36.4$ cfs. Note that $Q_1 + Q_2 = 120$ cfs, which satisfies mass balance.

PROBLEMS (SECTION 3.3)

- 3.3.1.** A jet of water exits a nozzle heading in the positive x -direction and strikes a flat plate at a 90° angle. The water then sprays through a 360° arc (y - and z -directions) exiting the plate. If the nozzle has a 20-cm diameter and the flow has a 3.44 m/sec velocity, what is the force exerted on the plate by the water?
- 3.3.2.** At a firefighter's convention, a competition pits two contestants in mock combat. Each is armed with a fire hose and a shield. The object is to push the opponent backward a certain distance with the spray. A choice of shields is offered. One shield is a flat garbage can lid; the other is a hemispherical lid that directs the water back toward the opponent. Which shield would you choose? Why?
- 3.3.3.** A 1-in.-diameter jet flowing in the negative x -direction hits a fixed blade and deflects the water through a 180° angle. If the force on the blade is 233 lbs, what is the velocity of the jet? (Assume the blade is frictionless.)
- 3.3.4.** At the end of a 0.6-m-diameter pipe, the pressure is $270,000 \text{ N/m}^2$. The pipe is connected to a 0.3-m-diameter nozzle. If the flow rate is $1.1 \text{ m}^3/\text{sec}$ in the positive x -direction, what is the total force on the connection?
- 3.3.5.** Water flows through a horizontal, 0.5-m-diameter pipe with a flow rate of $0.9 \text{ m}^3/\text{sec}$. It is ejected from the pipe through a 0.25-m-diameter nozzle. If the force that holds the nozzle in place is 43.2 kN, determine the water pressure in the pipe just upstream of the nozzle.
- 3.3.6.** Water flowing in a positive x -direction passes through a 90° elbow in a 6-inch-diameter pipeline and heads in a positive y -direction with a flow rate of $3.05 \text{ ft}^3/\text{sec}$. Compute the magnitude and direction of the force exerted by the elbow. The pressure upstream of the elbow is 15.1 psi; just downstream it is 14.8 psi.
- 3.3.7.** Determine the magnitude and direction of the reaction force at a 90° pipe bend that is passing a mass flow rate of 985 kg/sec. The diameter of the bend is 60 cm, and the pressure head is 10 m just upstream of the bend and 9.8 m downstream of the bend. (Assume that the water is flowing in a positive x -direction entering the bend and a positive y -direction leaving the bend.)
- 3.3.8.** Water flows through a reducing pipe bend and is deflected 30° in the horizontal plane. The velocity is 4 m/sec entering the bend (15-cm diameter) with a pressure of 250 kPa. The pressure leaving the bend is 130 kPa (7.5-cm diameter). Determine the anchoring force required to hold the bend in place. (Assume that the water is flowing in a positive x -direction entering the bend and continues in a positive x -direction and a positive y -direction after the bend.)

(SECTION 3.5)

- 3.5.1.** A commercial steel pipe, 1.5 m in diameter, carries $3.5 \text{ m}^3/\text{sec}$ of water at 20°C . Determine the friction factor and the flow regime (i.e., laminar–critical zone; turbulent–transitional zone; turbulent–smooth pipe; or turbulent–rough pipe).
- 3.5.2.** Water at 68°F flows at the rate of 628 cfs (ft^3/sec) through 100 feet of horizontal corrugated metal pipe 10 ft in diameter. Determine the friction factor and flow regime (i.e., laminar, critical zone, turbulent; transitional zone, turbulent; smooth pipe; or turbulent, rough pipe).

- 3.5.3.** A homeowner is interested in supplying water (20°C) to her woodshop that is about 30 m from her house. However, she is concerned about the lack of water pressure. Calculate the pressure drop she can expect for a flow rate of 10 liters per minute through a 1.5-cm copper pipe. Assume minor losses are negligible.
- 3.5.4.** A 15-in. galvanized iron pipe is installed on a 1/50 slope (uphill) and carries water at 68°F (20°C). What is the pressure drop in the 65-ft-long pipe when the discharge is 18 cfs (ft³/sec)? Assume minor losses are negligible.
- 3.5.5.** The commercial steel (seamless) pipeline depicted in Figure P3.5.5 is 100 m long and has a diameter of 0.4 m. Determine the height of the water tower (h) if the velocity of flow is 7.95 m/sec. Assume minor losses are negligible and the water temperature is 4°C.

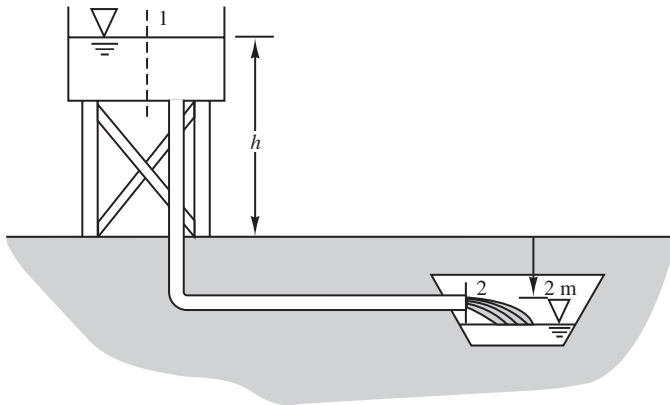


Figure P3.5.5

- 3.5.6.** A 30-cm circular cast-iron pipeline, 2 km long, carries water at 10°C. What is the maximum discharge if a 4.6-m head loss is allowed?
- 3.5.7.** Two sections, A and B, are 4.5 km apart along a 4-m-diameter riveted-steel pipe in its best condition. A is 100 m higher than B. If the water temperature is 20°C and the pressure heads measured at A and B are 8.3 m and 76.7 m, respectively, what is the flow rate? Assume minor losses are negligible.
- 3.5.8.** A smooth concrete pipe (1.5-ft diameter) carries water from a reservoir to an industrial treatment plant 1 mile away and discharges it into the air over a holding tank. The pipe leaving the reservoir is 3 ft below the water surface and runs downhill on a 1:100 slope. Determine the flow rate (in ft³/sec) if the water temperature is 40°F (4°C) and minor losses are negligible.
- 3.5.9.** Drawings to an old buried pipeline have been lost. At the entrance and exit of the pipe, two pressure gauges measure a pressure drop of 16.3 psi (lb/in.²). If the 6-in. galvanized iron pipe carries water at 68°F with a flow rate of 1.34 cfs (ft³/sec), what is the length of the horizontal underground line? Ignore minor losses.
- 3.5.10.** Water at 20°C is transported through a 200-m-long wrought iron pipe with a head loss of 9.8 m. Determine the diameter of the pipe required to convey 10 L/sec.
- 3.5.11.** A water main made of rough concrete loses 43 psi (lb/in.²) of pressure over a 1-mi horizontal length. Estimate the size of the pipe required to carry 16.5 cfs (ft³/sec) of water at 68°F (20°C). Assume minor losses are negligible.
- 3.5.12.** The city water company wants to transport 1,800 m³ of water per day to a plant from a reservoir 8 km away. The water surface elevation at the reservoir is 6 m above the entrance of the pipe, and the water surface in the holding tank is 1 m above the exit of the pipe. The pipe will be laid on a

1/500 slope. What is the minimum diameter of a concrete (good joints) pipe that may be used if the water temperature varies between 4°C and 20°C? Assume minor losses are negligible.

- 3.5.13.** Equation 3.19 defines the mean velocity for laminar flow using the Hagen-Poiseuille law. Equation 3.20 gives the Darcy-Weisbach equation applied to a horizontal uniform pipe. Derive Equation 3.20a, showing all steps in the process.
- 3.5.14.** Water flows through a 20-cm commercial steel pipe at 20°C. The flow rate is 80 liters per second, and pressure is constant throughout the length of the pipe. Determine the slope of the pipe. Assume minor losses are negligible.
- 3.5.15.** An old water-supply pipeline contains a long segment that is horizontal and 30 cm in diameter (cast iron). A leak along an inaccessible portion of the buried pipeline is highly likely. A pair of pressure gauges located upstream of the leak indicate a pressure drop of 23,000 N/m². Another pair of pressure gauges located downstream of the leak indicate a pressure drop of 20,900 N/m². The distance between the gauges in each pair is 100 m. Determine the magnitude of the leak. Assume minor losses are negligible and that the water temperature is 20°C.

(SECTION 3.7)

- 3.7.1.** A 6-km cast-iron (new) pipeline conveys 320 L/sec of water at 30°C. If the pipe diameter is 30 cm, compare the head loss calculated from (a) the Darcy-Weisbach equation, (b) the Hazen-Williams equation, and (c) the Manning equation. Use computer software to verify your results.
- 3.7.2.** A proposed 2-mile pipeline is required to carry 77.6 ft³/sec of water at 4°C (39°F) between two reservoirs. The receiving reservoir is located near Denver, Colorado, with a water surface elevation of 5,280 ft MSL. The supply reservoir is being constructed in the foothills with an estimated water surface elevation of 5,615 ft MSL. Based on flow demands, determine the required water surface elevation of the supply reservoir using the (a) Hazen-Williams equation and the (b) Manning equation. The 2.5-ft-diameter pipeline will be made from riveted steel (smooth). Ignore minor losses and verify your results with computer software.
- 3.7.3.** Two reservoirs 1,200 m apart are connected by a 50-cm smooth concrete pipe. If the two reservoirs have an elevation difference of 5 m, determine the discharge (20°C) in the pipe by (a) the Darcy-Weisbach equation, (b) the Hazen-Williams equation, and (c) the Manning equation. Assume minor losses are negligible and verify your results with computer software.
- 3.7.4.** Do some research to find two or three additional empirical equations involving head loss in pipe lines. List the author(s) and limitations of each equation.
- 3.7.5.** Use the Hazen-Williams equation and the Manning equation to calculate the flow rate for Problem 3.5.7, which was solved using the Darcy-Weisbach equation and yielded $Q = 78.8 \text{ m}^3/\text{sec}$. Compare the results and discuss the differences. Assume minor losses are negligible and verify your results with computer software.
- 3.7.6.** A buried, horizontal concrete pipe ($n = 0.012$) of unknown length needs to be replaced. The length is not known because the original plans for the pipeline route have been lost. The pressure head drop along the segment that needs to be replaced is 29.9 ft. If the flow rate is 30.0 cfs (ft³/sec), what length of new pipe is needed? How much would the answer change (in percent) if the n value was difficult to determine and could be assigned a value of 0.013 instead of 0.012? Assume minor losses are negligible and verify your results with computer software.
- 3.7.7.** The elevation difference between two reservoirs 2,000 m apart is 20 m. Compute the flow rate if (a) a 30-cm commercial steel ($C_{HW} = 140$) pipeline connects the reservoirs and (b) two 20-cm commercial steel pipelines are used instead. Ignore minor losses and verify your results with computer software.
- 3.7.8.** A concrete tunnel ($n = 0.013$) with a semicircular cross section (radius = 1.0 ft) flows full with a discharge of 15 cfs (ft³/sec). What is the head loss in 1,200 ft? Can you verify your results with computer software?

- 3.7.9.** A cast-iron, horizontal pipeline was installed 20 years ago with a Hazen-Williams coefficient of 130. The pipeline is 2,000 m long and has a diameter of 30 cm. Significant tuberculation has occurred since it was installed, and tests are run to determine the existing C_{HW} . A pressure drop of 366,000 pascals is measured over the pipeline length for a flow rate of $0.136 \text{ m}^3/\text{sec}$. Determine the existing Hazen-Williams coefficient. Assume minor losses are negligible and verify your results with computer software.

(SECTION 3.11)

- 3.11.1.** A 30-cm diameter pipe conveys water at the rate of 106 L/sec. Determine the contraction head loss if the diameter is suddenly reduced to 15 cm. Compare this with the head loss incurred when the 15-cm pipe suddenly expands to 30 cm.
- 3.11.2.** In Problem 3.11.1, the head loss from the abrupt contraction and expansion were calculated to be 0.606 m and 1.03 m, respectively. With the same flow rate and geometry, determine the corresponding head losses if a 15° confusor and a 15° diffuser are used to reduce the head losses.
- 3.11.3.** Valve manufacturers want to be able to supply the loss coefficients of their products to prospective buyers. They usually perform experiments to determine the coefficients. Determine the loss coefficient for a valve if water flows through the 8-cm valve at the rate of $0.04 \text{ m}^3/\text{s}$ and produces a pressure drop of 100 kPa.
- 3.11.4.** A check valve (swing) and a globe valve are installed in series in an 8-in. pipe. Determine the flow rate if the pressure drop across the valves is 5.19 psi ($\text{lbs}/\text{in.}^2$). Assume that the friction losses are negligible because of the short pipe length.
- 3.11.5.** Water flows through a sudden contraction in a horizontal plane, passing from a 60-cm pipe to a 30-cm pipe. The pressure on the upstream side of the contraction is 285 kPa and on the downstream side is 265 kPa. Determine the flow rate.
- 3.11.6.** Water flows through a 4-cm, wrought iron, horizontal pipeline from point A to point B. The pipeline is 50 meters long and contains a fully open gate valve and two elbows ($R/D = 4$). If the pressure at B (downstream) is 192 kPa and the flow rate is $0.006 \text{ m}^3/\text{s}$, what is the pressure at point A?
- 3.11.7.** Water flows from a storage tank through a horizontal, 6-inch pipe (cast iron) and discharges water (20°C) into the atmosphere at the end. The pipe is 500 feet long, contains two 1-ft-radius bends and a fully open gate valve, and the entrance is square edged. Calculate the flow rate if the depth of water in the storage tank is 60.2 feet above the inlet (and outlet) of the pipe.
- 3.11.8.** A 75-m long cast-iron pipe, 15 cm in diameter, connects two tanks (open to the atmosphere) that have a water (20°C) surface elevation difference of 5 m. The entrance of the pipe from the supply tank is square edged, and the pipe contains a 90° bend with a sharp, 15-cm radius. Determine the flow rate in the pipe.
- 3.11.9.** A cylindrical tank 5 m in diameter is filled with water to a 3-m depth. A very short horizontal pipe with a 20-cm diameter and a rotary valve is used to drain the tank from the bottom. How long does it take to drain 50 percent of the tank?
- 3.11.10.** A 34-m-high water tower supplies water to a residential area by means of a 20-cm-diameter, 800-m-long commercial steel pipe. To increase the pressure head at the delivery point, engineers are considering the replacement of 94 percent of the pipe length with a larger (30-cm-diameter) steel pipe connected by a 30° confusor back to the remaining smaller pipe. If the peak water (20°C) demand is $0.10 \text{ m}^3/\text{sec}$, what is the pressure head that would be gained by this strategy?
- 3.11.11.** A pipe goes through a sudden contraction to half its diameter and then a sudden expansion back to its original size. Which loss is greater—the expansion loss or the contraction loss? Prove your answer.

(SECTION 3.12)

- 3.12.1. Derive an expression for N pipes in parallel using the Manning equation.
- 3.12.2. Derive an expression for N pipes in parallel using the Hazen-Williams equation.
- 3.12.3. Redo Example 3.10 using the Manning equation, assuming that $n = 0.013$ for all the pipes.
- 3.12.4. Redo Example 3.10 using the Hazen-Williams equation and assuming that $C_{HW} = 100$ for all the pipes.
- 3.12.5. Pipes AB and CF in Figure P3.12.5 have a diameter of 3 m and a Darcy-Weisbach friction factor of 0.02. The length of AB is 1000 m and that of CF is 900 m. The discharge in pipe AB is $60 \text{ m}^3/\text{sec}$. Branch 1 is 1,000 m long, and it has a diameter of 2 m and a friction factor of 0.018. Branch 2 has a length of 800 m, diameter of 3 m, and a friction factor of 0.02. A discharge of $20 \text{ m}^3/\text{sec}$ is added to the flow at point B and $10 \text{ m}^3/\text{sec}$ is taken out at point C as shown in the figure. (a) Determine the total head loss due to friction between section A and F , and (b) determine the discharge in each of branches 1 and 2.

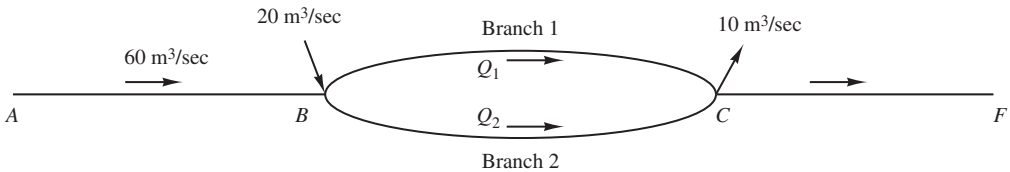
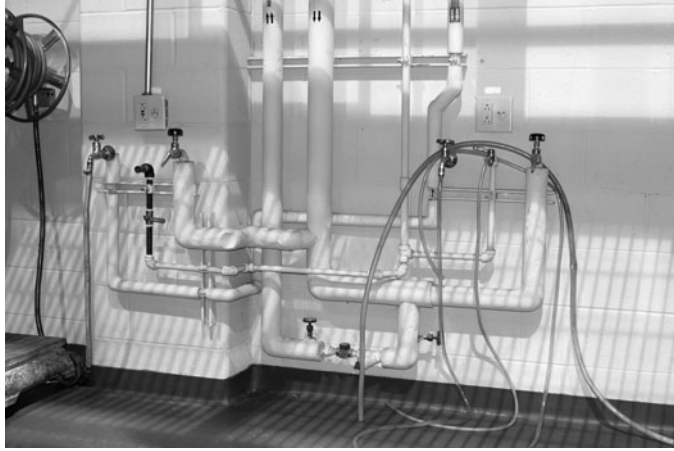


Figure P3.12.5

- 3.12.6. Can the method of equivalent pipes be used to find a single hypothetical pipe that is equivalent to the pipe system of Example 3.10? If your answer is yes, determine an equivalent pipe. If your answer is no, explain your answer.
- 3.12.7. Can the method of equivalent pipes be used to find a single hypothetical pipe that is equivalent to the pipe system of Problem 3.12.5? If your answer is yes, determine an equivalent pipe. If your answer is no, explain your answer.

4



Pipelines and Pipe Networks

In general, when a number of pipes are connected together to transport water for a given project, they perform as a system that may include series pipes, parallel pipes, branching pipes, elbows, valves, meters, and other appurtenances. The arrangement is known as a *pipeline* if all elements are connected in series. Otherwise, the arrangement is a *pipe network*.

Although the basic knowledge of pipe flow discussed in Chapter 3 is applicable to each individual pipe in the system, the design and analysis of a pipeline or pipe network does create certain complex problems unique to the system. This is particularly true if the system consists of a large number of pipes like those that frequently occur in the water-distribution networks of large metropolitan areas.

The physical phenomena and problems that are pertinent to pipelines and pipe networks, as well as the special techniques developed for the analysis and design of such systems, are discussed in the following sections.

4.1 Pipelines Connecting Two Reservoirs

A *pipeline* is a system of one or more pipes connected in series and designed to transport water from one location (often a reservoir) to another. There are three principal types of pipeline problems.

1. Given the flow rate and the pipe combinations, determine the total head loss.
2. Given the allowable total head loss and the pipe combinations, determine the flow rate.
3. Given the flow rate and the allowable total head loss, determine the pipe diameter.

The first type of problem can be solved by a direct approach, but the second and third types involve iterative procedures as shown in the following examples.

Example 4.1

Two cast-iron pipes in series connect two reservoirs (Figure 4.1). Both pipes are 300 m long and have diameters of 0.6 m and 0.4 m, respectively. The elevation of the water surface (WS) in reservoir A is 80 m. The discharge of 10°C water from reservoir A to reservoir B is 0.5 m³/sec. Find the elevation of the surface of reservoir B. Assume a sudden contraction at the junction and a square-edged entrance.

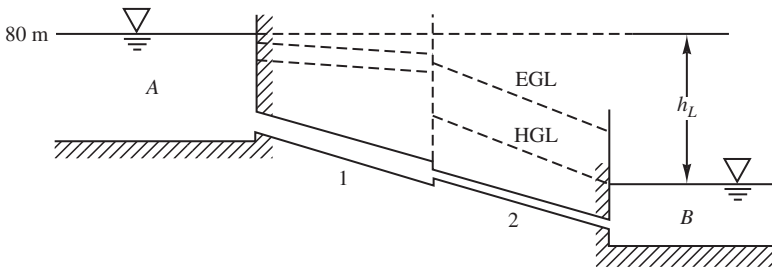


Figure 4.1

Solution

Applying the energy equation (Equation 3.15) between the reservoir surfaces A and B,

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$$

Because $P_A = P_B = 0$, and the velocity heads can be neglected in a reservoir,

$$h_B = h_A - h_L$$

Designating subscript 1 for the upstream pipe and subscript 2 for the downstream pipe to compute the head loss, we may write

$$V_1 = \frac{Q}{A_1} = \frac{0.5}{(\pi/4)(0.6)^2} = 1.77 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.5}{(\pi/4)(0.4)^2} = 3.98 \text{ m/sec}$$

$$N_{R1} = \frac{V_1 D_1}{\nu} = \frac{1.77(0.6)}{1.31 \times 10^{-6}} = 8.11 \times 10^5$$

$$N_{R2} = \frac{V_2 D_2}{\nu} = \frac{3.98(0.4)}{1.31 \times 10^{-6}} = 1.22 \times 10^6$$

From Table 3.1, we have

$$\frac{e}{D_1} = \frac{0.26}{600} = 0.00043$$

$$\frac{e}{D_2} = \frac{0.26}{400} = 0.00065$$

From the Moody chart (Figure 3.8), we have

$$f_1 = 0.017 \text{ and } f_2 = 0.018$$

For the total head loss,

$$h_L = h_e + h_{f_1} + h_c + h_{f_2} + h_d.$$

From Equations 3.16, 3.32, 3.34, and 3.37, we may write

$$h_L = \left(0.5 + f_1 \frac{L_1}{D_1}\right) \frac{V_1^2}{2g} + \left(0.21 + f_2 \frac{L_2}{D_2} + 1\right) \frac{V_2^2}{2g} = 13.3 \text{ m}$$

The elevation of the surface of reservoir *B* is

$$h_B = h_A - h_L = 80 - 13.3 = 66.7 \text{ m}$$

Example 4.2

Pipeline *AB* connects two reservoirs. The difference in elevation between the two reservoirs is 33 ft. The pipeline consists of an upstream section, $D_1 = 30$ in. and $L_1 = 5,000$ ft; and a downstream section, $D_2 = 21$ in. and $L_2 = 3,500$ ft. The pipes are concrete (smooth) and are connected end to end with a sudden reduction of area. Assume the water temperature is 68°F. Compute the discharge capacity.

Solution

The energy equation can be written as follows between the reservoir surfaces:

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$$

By eliminating zero and small terms, we can rewrite this as

$$h_L = h_A - h_B = 33 \text{ ft}$$

Because the discharge is not yet known, the velocity in each pipe can only be assumed to be V_1 and V_2 , respectively. The total energy equation, as above, will contain these two assumed quantities. It cannot be solved directly, so an iteration procedure is used. For water temperature at 68°F, $\nu = 1.08 \times 10^{-5}$ ft²/sec. The corresponding Reynolds numbers may be expressed as

$$N_{R_1} = \frac{V_1 D_1}{\nu} = \frac{V_1 (2.5)}{1.08 \times 10^{-5}} = (2.31 \times 10^5) V_1 \quad (1)$$

$$N_{R_2} = \frac{V_2 D_2}{\nu} = \frac{V_2 (1.75)}{1.08 \times 10^{-5}} = (1.62 \times 10^5) V_2 \quad (2)$$

From the continuity condition, $A_1 V_1 = A_2 V_2$, we have

$$\frac{\pi}{4} (2.5)^2 (V_1) = \frac{\pi}{4} (1.75)^2 (V_2)$$

$$V_2 = 2.04 V_1 \quad (3)$$

Substituting Equation (3) into Equation (2) we get

$$N_{R_2} = (1.62 \times 10^5) (2.04 V_1) = (3.30 \times 10^5) V_1$$

The energy equation may be written as

$$33 = \left[0.5 + f_1 \left(\frac{5,000}{2.5} \right) \right] \frac{V_1^2}{2g} + \left[0.18 + f_2 \left(\frac{3,500}{1.75} \right) + 1 \right] \frac{V_2^2}{2g}$$

$$33 = (0.084 + 31.1f_1 + 129f_2)V_1^2$$

From Table 3.1, $e/D_1 = 0.00024$ and $e/D_2 = 0.00034$. As a first trial, try $f_1 = 0.014$ and $f_2 = 0.015$ (assuming complete turbulence, generally a good assumption for water-transmission systems because the viscosity of water is low and the velocities or pipe diameters are large, which yields high N_R values). Thus

$$33 = [0.084 + 31.1(0.014) + 129(0.015)]V_1^2$$

$$V_1 = 3.67 \text{ ft/sec}$$

$$N_{R_1} = 2.31 \times 10^5(3.67) = 8.48 \times 10^5$$

$$N_{R_2} = 3.30 \times 10^5(3.67) = 1.21 \times 10^6$$

From Figure 3.8, $f_1 = 0.0155$ and $f_2 = 0.016$. These values do not agree with the values assumed previously. For the second trial, assume that $f_1 = 0.0155$ and $f_2 = 0.016$.

$$V_1 = 3.54 \text{ ft/sec} \quad N_{R_1} = 8.17 \times 10^5 \quad N_{R_2} = 1.17 \times 10^6$$

We obtain, from Figure 3.8, $f_1 = 0.0155$ and $f_2 = 0.016$. These values are the same as the assumed values, suggesting that $V_1 = 3.54 \text{ ft/sec}$ is the actual velocity in the upstream pipe. Therefore, the discharge is

$$Q = A_1V_1 = \frac{\pi}{4}(2.5)^2(3.54) = 17.4 \text{ cfs (ft}^3\text{/sec)}$$

Example 4.3

A concrete pipeline is installed to deliver $6 \text{ m}^3\text{/sec}$ of water (10°C) between two reservoirs 17 km apart. If the elevation difference between the two reservoirs is 12 m, what is the required pipe size?

Solution

As in the previous examples, the energy relationship between the two reservoirs is

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$$

Thus,

$$h_L = h_A - h_B = 12 \text{ m}$$

The mean velocity may be obtained by using the continuity condition, Equation 3.4:

$$V = \frac{Q}{A} = \frac{6}{(\pi/4)D^2} = \frac{7.64}{D^2}$$

and

$$N_R = \frac{DV}{\nu} = \frac{D \left(\frac{7.64}{D^2} \right)}{1.31 \times 10^{-6}} = \frac{5.83 \times 10^6}{D}$$

Neglect the minor losses (for long pipes of $L/D \gg 1,000$, minor losses may be neglected). Therefore, the energy loss contains only the friction loss term. Equation 3.16 gives

$$12 = f \left(\frac{L}{D} \right) \frac{V^2}{2g} = f \left(\frac{L}{D} \right) \frac{Q^2}{2gA^2} = f \left(\frac{17,000}{D} \right) \left(\frac{6^2}{2(9.81)(\pi/4)^2 D^4} \right)$$

Simplified,

$$0.000237 = \frac{f}{D^5} \quad (a)$$

For concrete pipes, $e = 0.36$ mm (average), and assume that $D = 2.5$ m for the first trial,

$$\frac{e}{D} = \frac{0.36}{2500} = 0.00014$$

and $N_R = 2.33 \times 10^6$. From Figure 3.8 we obtain $f = 0.0135$. Substituting these values into Equation (a), we see that

$$D = \left(\frac{0.0135}{0.000237} \right)^{1/5} = 2.24$$

Hence, a different pipe diameter must be used for the second iteration. Use $D = 2.24$ m. Hence, $e/D = 0.00016$ and $N_R = 2.6 \times 10^6$. From Figure 3.8, we obtain $f = 0.0136$. From Equation (a), we now have

$$D = \left(\frac{0.0136}{0.000237} \right)^{1/5} = 2.25 \text{ m}$$

The value on the right-hand side is considered close enough to that on the previous iteration, and the pipe diameter of 2.25 m is selected.

Note: For all of these pipeline problems, simultaneous equations can be established and solved using computer algebra software (e.g., Mathcad, Maple, or Mathematica). The equations that are required include the energy equation, the Darcy-Weisbach equation for friction head loss, minor loss equations, the continuity equation, Reynolds number, and Colebrook’s friction factor relationship or the Swamee-Jain equation. However, the relationships are highly nonlinear, and a good initial estimate may be required to avoid numerical instability.

4.2 Negative Pressure Scenarios (Pipelines and Pumps)

Pipelines used to transport water from one location to another over a long distance usually follow the natural contour of the land. Occasionally, a section of the pipeline may be raised to an elevation that is above the local hydraulic grade line (HGL), as shown in Figure 4.2. As discussed in Chapter 3, the vertical distance measured between the energy grade line (EGL) and the hydraulic grade line at any location along the pipeline is the velocity head $V^2/2g$ at that location. The vertical distance measured between the hydraulic grade line and the pipeline is the local pressure head (P/γ). In the vicinity of the pipeline summit (S in Figure 4.2), the pressure head may take on a negative value.

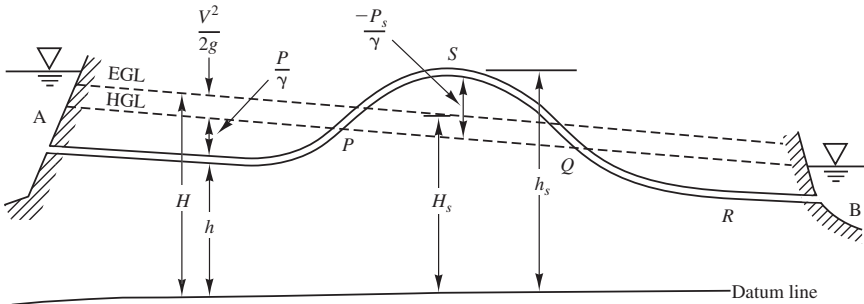


Figure 4.2 Elevated section in a pipeline

Negative pressure scenarios in pipelines are not difficult to understand in light of the principles of hydraulic head. Recall that the total head

$$H = \left(\frac{V^2}{2g} + h \right) + \frac{P}{\gamma}$$

must equal the vertical distance between the datum and the energy grade line at any location in the pipeline. At the summit, for example, the elevation head (h_s) is the vertical distance from the datum to the centerline of the pipe. The velocity head ($V^2/2g$) is also a fixed positive value. The sum $V^2/2g + h_s$ may become larger than H_s , the total head at the summit. If this occurs, the pressure head (P_s/γ) must take on a negative value. *Negative gauge pressure* (in reference to atmospheric pressure as zero, $P_{\text{atm}} = 0$) exists in a pipeline wherever the pipeline is raised above the hydraulic grade line (between P and Q in Figure 4.2). This negative pressure reaches a maximum value at the summit, $-(P_s/\gamma)$. Water flow from S to R must flow against the pressure gradient. In other words, it flows from a point of lower pressure toward a point of higher pressure. This is possible because water always flows toward lower-energy locations, and in the closed conduit the elevation head decrease more than compensates for the pressure head increase. For example, if a unit weight of water flowing from S to R experiences a pressure increase equal to 3 m of water column, then the elevation of S must be at least 3 m higher than the elevation of R . In fact, the difference in elevation between S and R must equal 3 m plus the loss of head between S and R . Or, more generally, the elevation difference between any two points 1 and 2 in a pipeline is

$$\Delta h_{1-2} = \left(\frac{P_2}{\gamma} - \frac{P_1}{\gamma} \right) + h_f \quad (4.1)$$

For design purposes, it is important to maintain pressure at all points in a pipeline above the vapor pressure of water. As we discussed in Chapter 1, the vapor pressure (gauge) of water is approximately equal to a negative water column height of 10 m at 20°C. When the pressure in a pipe drops below this value, water will be vaporized locally to form vapor pockets that separate the water in the pipe. These vapor pockets collapse in regions of higher pressure downstream. The action of vapor collapse is very violent, causing vibrations and sound that can greatly damage the pipeline. The entire process is often referred to as *cavitation*. It is important to note that the vaporization (boiling) of water in pipelines occurs even at normal atmospheric temperatures if the pressure drop is of sufficient magnitude.

Theoretically, a pipeline may be designed to allow the pressure to fall to the level of vapor pressure at certain sections in the pipeline. For example, the vapor pressure of water at 20°C is 2,335 N/m² (Table 1.1). This is an absolute pressure; gauge pressure is found by subtracting atmospheric pressure (1.014×10^5 N/m²) or, in terms of pressure head, $(P_{\text{vapor}} - P_{\text{atm}})/\gamma = (2,335 - 101,400)/9,790 = -10.1$ m. However, in practice it is generally not acceptable to allow the pressure to fall to vapor-pressure levels. Water usually contains dissolved gases that will vaporize well before the vapor-pressure point is reached. Such gases return to the liquid phase very slowly. They usually move with water in the form of large bubbles that reduce the effective flow area and thus disrupt the flow. For this reason, negative pressure should not be allowed to exceed approximately two-thirds of standard atmospheric-pressure head (10.3 m of H₂O) in any section of the pipeline (i.e., about -7.0 m of H₂O, gauge pressure).

Example 4.4

A PVC pipeline, 40 cm in diameter and 2,000 m long, carries water at 10°C between two reservoirs as shown in Figure 4.2. The two reservoirs have a water surface elevation difference of 30 m. At midlength, the pipeline must be raised to carry the water over a small hill. Determine the maximum height the pipeline may be raised (at the summit, S) above the lower reservoir's water surface elevation in order to prevent cavitation.

Solution

Pipeline problems are most often solved by using the energy equation. However, two points must be chosen to balance energy. In choosing the points, you generally identify where information is needed and where the most information is available in terms of energy. In this problem, the elevation of the summit is requested, and the most information is available at the reservoir water surfaces (i.e., where velocity head and pressure heads are negligible). Because the elevation of the summit is requested with respect to the lower reservoir (B), start by balancing energy between these two points. This yields

$$\frac{V^2}{2g} + \frac{P_s}{\gamma} + h_s = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$$

Setting the datum at the water surface of reservoir B , all terms on the right-hand side of the equation disappear except the head loss term. The head loss includes the friction loss (h_f) and the exit loss (h_d). Rewriting the equation and substituting known values results in

$$\frac{V^2}{2g} - 10.2 \text{ m} + h_s = \left(f \frac{1,000 \text{ m}}{0.4 \text{ m}} + 1.0 \right) \frac{V^2}{2g} \quad (1)$$

where an allowable vapor-pressure head of -10.2 m (gauge) is used. The vapor pressure of water at 10°C is $1,266 \text{ N/m}^2$ (Table 1.1) or, in terms of pressure head, $(P_{\text{vapor}} - P_{\text{atm}})/\gamma = (1,266 - 101,400)/9,800 = -10.2 \text{ m}$. It is obvious from Equation (1) that the flow rate in the pipeline, which allows us to determine the pipeline velocity, will have to be determined in order to find the allowable height of the summit, h_s .

In the previous section, pipeline flow rates between reservoirs were determined by balancing energy between the reservoir surfaces. In this case, the total head loss between the two reservoirs is 30 m, which includes the entrance loss (h_e), the friction loss in the pipeline (h_f), and the discharge loss (h_d). Therefore, we may write

$$h_A - h_B = 30 = \left(K_e + f \frac{L}{D} + K_d \right) \frac{V^2}{2g}$$

Assuming a square-edged entrance and complete turbulence, $e/D = 0.0015 \text{ mm}/400 \text{ mm} = 0.0000375$, basically a smooth pipe that requires N_R . Thus, try $f = 0.015$, which is in the middle of the Moody diagram (Figure 3.8) for smooth pipes. Substituting into the energy equation yields

$$30 = \left(0.5 + (0.015) \frac{2,000}{0.4} + 1 \right) \frac{V^2}{2(9.81)}$$

Hence, $V = 2.77 \text{ m/sec}$. We must check the assumed friction factor (f) using this velocity. The relative roughness remains the same, and the Reynolds number is

$$N_R = \frac{VD}{\nu} = \frac{(2.77)(0.4)}{1.31 \times 10^{-6}} = 8.46 \times 10^5$$

The Moody diagram now reads $f = 0.012$, which is different from the assumed value. The energy relationship is rewritten as follows:

$$30 = \left(0.5 + (0.012) \frac{2,000}{0.4} + 1 \right) \frac{V^2}{2(9.81)}$$

from which $V = 3.09$ m/sec. Now, $N_R = 9.44 \times 10^5$, and the Moody chart gives $f = 0.012$, which matches our previous value. Rearranging Equation (1) and substituting for V and f yields the maximum height (h_s) the pipeline may be raised (at the summit) above the surface elevation of the lower reservoir to prevent cavitation:

$$\begin{aligned} h_s &= 10.2 + \left(0.012 \frac{1,000}{0.4} + 1.0 \right) \frac{V^2}{2g} - \frac{V^2}{2g} \\ &= 10.2 + \left[(0.012) \frac{1,000}{0.4} \right] \frac{(3.09)^2}{2(9.81)} = 24.8 \text{ m} \end{aligned}$$

Note that other dissolved gases may vaporize if the gauge pressure is allowed to drop below -7.0 m, which is a more conservative design than the -10.2 m used here.

Pumps may be needed in a pipeline to lift water from a lower elevation or simply to boost the rate of flow. Pumps add energy to water in pipelines by increasing the pressure head. The details of pump design and selection will be discussed in Chapter 5, but the analysis of pressure head and energy (in the form of head) provided by a pump to a pipeline system are discussed here. The computations for pump installation in a pipeline are usually carried out by separating the pipeline system into two sequential parts: the *suction side* and the *discharge side*.

Figure 4.3 shows a typical pump installation in a pipeline and the associated EGL and HGL. The head provided by the pump to the system (H_p) is represented by the vertical distance between the low point (L) and the high point (M) on the energy grade line (at the inlet and outlet of the pump). The elevation of M represents the total head at the outlet of the pump that delivers the water to the receiving reservoir (R). An energy equation can be written between the supply reservoir (S) and the receiving reservoir as

$$H_S + H_P = H_R + h_L \tag{4.2}$$

where H_S and H_R are the position heads in the supply and receiving reservoirs, respectively (i.e., generally the water surface elevations). H_P is the head added by the pump, and h_L is the total head loss in the system.

Additional information is evident from the EGL and HGL in Figure 4.3. The suction side of the system from the supply reservoir (section 1–1) to the inlet of the pump (section 2–2) is subjected to negative pressure, whereas the discharge side from the outlet of the pump (section 3–3) to the receiving reservoir (section 4–4) is subjected to positive pressure. The change from negative pressure to positive pressure is the result of the pump adding energy to the water, mainly in the form of pressure head. Also note that the EGL drops significantly and quickly coming out of the supply reservoir. This is because of the friction loss in the vertical portion of the suction pipe, the entrance loss, the strainer loss, and the 90° bend. Also note that the 135° bends in the discharge line are assumed to have negligible losses as noted by the absence of discontinuities in the EGL. It is left to the reader to comment on whether the slope of the EGL in the discharge line should be uniform based on the fact that a portion of the pipe is not horizontal.

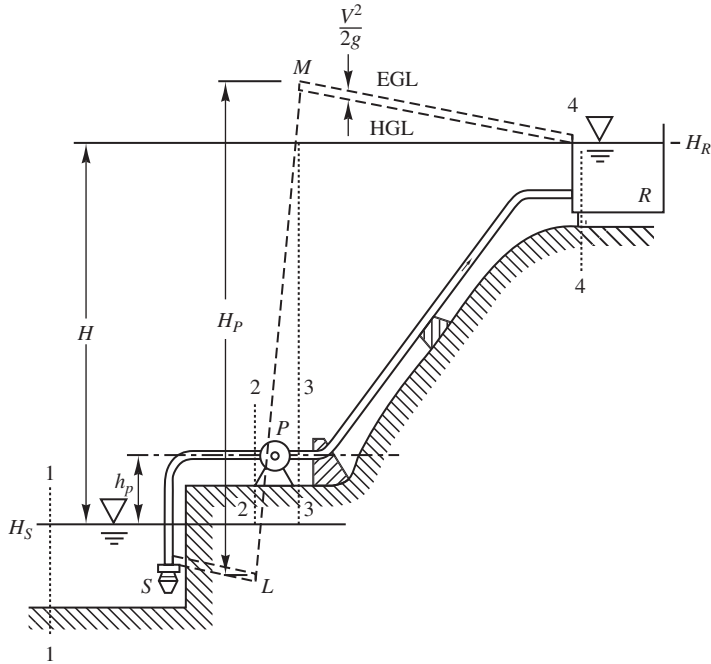


Figure 4.3 Energy grade line and hydraulic grade line of a pumping station

Example 4.5

A pump is necessary to lift water from the clear well (reservoir) at a water-treatment plant to a storage tower 50-ft high and some distance away. A flow rate of 15 cubic feet per second (cfs) is required (68°F). The 15-in. pipeline ($e/D = 0.00008$) between the two reservoirs is 1,500-ft long and contains minor losses that amount to 15 times the velocity head. Determine the pressure head required from the pump. Also determine the pressure head on the suction side of the pump if it is 10 ft above the water surface of the clear well and 100 ft down the pipeline.

Solution

From Equation 4.2,

$$H_p = H_R - H_S + h_L = 50 \text{ ft} + h_L$$

where

$$h_L = \left(f \frac{L}{D} + 15 \right) \frac{V^2}{2g}$$

For the flow rate required, the velocity and Reynolds number are

$$V = \frac{Q}{A} = \frac{4(15)}{(\pi/4)(1.25)^2} = 12.2 \text{ ft/sec}$$

$$N_R = \frac{VD}{\nu} = \frac{12.2(1.25)}{1.08 \times 10^{-5}} = 1.41 \times 10^6$$

And with $e/D = 0.00008$, the Moody chart gives $f = 0.013$. The energy loss in the pipeline is

$$h_L = \left((0.013) \frac{1,500}{1.25} + 15 \right) \frac{(12.2)^2}{2(32.2)} = 70.7 \text{ ft}$$

The minimum pressure head the pump must provide is

$$H_p = 70.7 + 50 = 120.7 \text{ ft}$$

(Note: A certain amount of pressure head must be added to compensate for the energy loss that occurs at the pump when it is in operation.)

Balancing energy between the clear well and the suction side of the pump (sections 1–1 and 2–2 in Figure 4.3) gives

$$H_s = h_2 + \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_L$$

where $H_s = 0$ (datum) and the energy loss is

$$h_L = \left(K_e + f \frac{L}{D} \right) \frac{V^2}{2g}$$

Assuming $K_e = 4.0$ (entrance loss with strainer), we have

$$h_L = \left(4.0 + 0.013 \frac{100}{1.25} \right) \frac{(12.2)^2}{2(32.2)} = 11.6 \text{ ft}$$

Therefore, the pressure head on the suction side of the pump is

$$\frac{P_2}{\gamma} = 0.0 - 10 - \frac{(12.2)^2}{2(32.2)} - 11.6 = -23.9 \text{ ft}$$

This is above the vapor pressure of water of 0.344 lb/in.² (at 68°F, found in front jacket of book and Table 1.1) which equates to a gauge pressure of -14.4 lb/in.² ($P_{\text{vapor}} - P_{\text{atm}} = 0.344 \text{ lb/in.}^2 - 14.7 \text{ lb/in.}^2$) and converts to a pressure head of -33.3 ft of water [$(P_{\text{vapor}} - P_{\text{atm}})/\gamma = (-14.4 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)/62.3 \text{ lb/ft}^3$]. Therefore, the water in the pipeline will not vaporize. However, it is on the threshold of what is allowed in practice (-7 m = -23.0 ft) based on other dissolved gases in the water that could vaporize.

4.3 Branching Pipe Systems

Branching pipe systems are the result of more than two pipelines converging at a junction. These systems must simultaneously satisfy two basic conditions. First, the total amount of water brought by pipes to a junction must always be equal to that carried away from the junction by the other pipes (conservation of mass). Second, all pipes that meet at the junction must share the same energy level at the junction (conservation of energy).

The hydraulics of branching pipe systems at a junction can be best demonstrated by the classical *three-reservoir problem* in which three reservoirs of different elevation are connected to a common junction J as shown in Figure 4.4. Given the lengths, diameters, and material of all pipes involved, as well as the water elevation in each of the three reservoirs, the discharges to or from each reservoir (Q_1 , Q_2 , and Q_3) can be determined. If an open-ended vertical tube (*piezometer*) is installed at the junction, the water elevation in the tube will rise to the elevation P . The vertical distance between P and J is a direct reading of the pressure head at the junction. The elevation of P is the total energy level (position head plus pressure head) if the velocity head is assumed to be negligible at the junction where all of the flows are coming together. Therefore, the difference in elevation between the water surface at reservoir A and elevation P represents the friction losses in transporting water from A to J , as indicated by h_{f_1} . (Minor losses are neglected; friction losses often dominate in large water-conveyance systems.) Likewise, the elevation

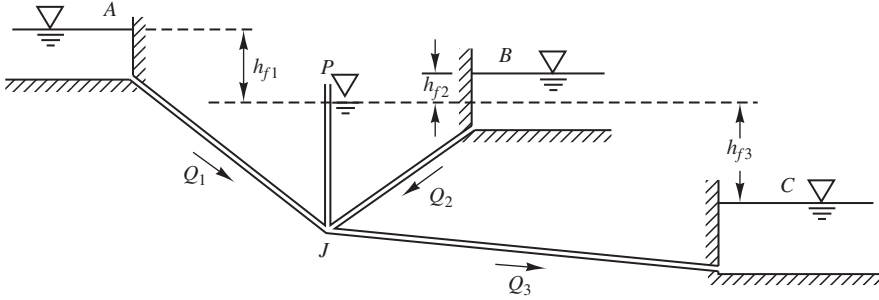


Figure 4.4 Branching pipes connecting three reservoirs

difference between reservoir B and P (h_{f_2}) represents the friction losses in transporting water from B to J ; h_{f_3} represents the friction losses in transporting water from J to C .

Because the mass of water brought to the junction must equal the mass of water taken from the junction, we may simply write

$$Q_3 = Q_1 + Q_2 \quad (4.3)$$

or

$$\Sigma Q = 0$$

at the junction (assuming that density remains constant).

These types of problems can be solved iteratively. Not knowing the discharge in each pipe, we may first assume a total energy elevation, P , at the junction. This assumed elevation establishes the friction head losses h_{f_1} , h_{f_2} , and h_{f_3} for each of the three pipes. From this set of head losses and the given pipe diameters, lengths, and materials, friction loss equations yield a set of values for the discharges Q_1 , Q_2 , and Q_3 . If the assumed total energy elevation P is correct, then the computed Q 's should satisfy the mass balance condition stated above—that is,

$$\Sigma Q = Q_1 + Q_2 - Q_3 = 0 \quad (4.4)$$

Otherwise, a new elevation P is assumed for the second iteration. The computation of another set of Q 's is performed until the above condition is satisfied. The correct values of discharge in each pipe are thus obtained.

Note that if the assumed water surface elevation in the piezometer tube is higher than the elevation in reservoir B , then Q_1 should be equal to $Q_2 + Q_3$; if it is lower, then $Q_1 + Q_2 = Q_3$. The error in the trial Q 's indicates the direction in which the assumed piezometric elevation (P) should be set for the next trial. Numerical techniques such as the bisection method can be programmed and applied to the error to quickly obtain the correct result.

To understand the concept, it may be helpful to plot the computed trial values of P against ΣQ . The resulting flow residual (ΣQ) may be positive or negative for each trial. However, with values obtained from three trials, a curve may be plotted as shown in Example 4.6. The correct discharge is indicated by the intersection of the curve with the vertical axis. The computation procedure is demonstrated in the following example.

Example 4.6

In Figure 4.5, the three reservoirs *A*, *B*, and *C* are connected by pipes to a common junction, *J*. Pipe *AJ* is 1,000 m long and 30 cm in diameter, pipe *BJ* is 4,000 m long and 50 cm in diameter, and pipe *CJ* is 2,000 m long and 40 cm in diameter. The pipes are made of concrete for which $e = 0.6$ mm may be assumed. Determine the discharge in each pipe if the water temperature is 20°C. (Assume that minor losses are negligible.)

Solution

Let subscripts 1, 2, and 3 represent, respectively, pipes *AJ*, *BJ*, and *CJ*. As a first trial, we assume the pressure head at *J*, represented by the elevation P , is 110 m. Q_1 , Q_2 , and Q_3 may be calculated as follows.

Balancing energy between reservoir *A* and the junction (*J*), and using the Darcy-Weisbach equation to compute the head loss resulting from friction, results in

$$h_{f_1} = 120 - 110 = f_1 \left(\frac{L_1}{D_1} \right) \frac{V_1^2}{2g} = f_1 \left(\frac{1,000}{0.3} \right) \frac{V_1^2}{2(9.81)}$$

Assuming fully developed turbulent flow,

$$\frac{e_1}{D_1} = \frac{0.6}{300} = 0.002$$

and from the Moody diagram, $f_1 = 0.024$, which yields $V_1 = 1.57$ m/sec from the energy equation. Hence, $N_R = V_1 D_1 / \nu = 4.71 \times 10^5$. Rechecking the Moody diagram, $f_1 = 0.024$ is correct, therefore

$$Q_1 = V_1 A_1 = (1.57) \left[\frac{\pi}{4} (0.3)^2 \right] = 0.111 \text{ m}^3/\text{sec}$$

Similarly, for reservoir *B*,

$$h_{f_2} = 110 - 100 = f_2 \left(\frac{L_2}{D_2} \right) \frac{V_2^2}{2g} = f_2 \left(\frac{4,000}{0.5} \right) \frac{V_2^2}{2(9.81)}$$

$$\frac{e_2}{D_2} = \frac{0.6}{500} = 0.0012$$

and from the Moody diagram, $f_2 = 0.0205$ which yields $V_2 = 1.09$ m/sec and $N_R = 5.45 \times 10^5$. From the Moody chart, a better estimation for f is obtained: $f = 0.021$. This yields $V_2 = 1.08$ m/sec and $N_R = 5.40 \times 10^5$. Because no further approximation is necessary, we may write

$$Q_2 = V_2 A_2 = (1.08) \left[\frac{\pi}{4} (0.5)^2 \right] = 0.212 \text{ m}^3/\text{sec}$$

For reservoir *C*,

$$h_{f_3} = 110 - 80 = f_3 \left(\frac{L_3}{D_3} \right) \frac{V_3^2}{2g} = f_3 \left(\frac{2,000}{0.4} \right) \frac{V_3^2}{2(9.81)}$$

$$\frac{e_3}{D_3} = \frac{0.6}{400} = 0.0015$$

and from the Moody diagram, $f_3 = 0.022$, which yields $V_3 = 2.31$ m/sec and $N_R = 9.24 \times 10^5$. Because $f_3 = 0.022$ is correct,

$$Q_3 = V_3 A_3 = (2.31) \left[\frac{\pi}{4} (0.4)^2 \right] = 0.290 \text{ m}^3/\text{sec}$$

Hence, the summation of flows into the junction (*J*) is

$$\Sigma Q = Q_1 - (Q_2 + Q_3) = -0.391 \text{ m}^3/\text{sec}$$

As the second trial, we assume $P = 100$ m. Similar computations are repeated to obtain

$$Q_1 = 0.157 \text{ m}^3/\text{sec} \quad Q_2 = 0.0 \text{ m}^3/\text{sec} \quad Q_3 = 0.237 \text{ m}^3/\text{sec}$$

Hence,

$$\Sigma Q = (Q_1 + Q_2) - Q_3 = -0.080 \text{ m}^3/\text{sec}$$

The computations are repeated again for $P = 90$ to obtain

$$Q_1 = 0.193 \text{ m}^3/\text{sec} \quad Q_2 = 0.212 \text{ m}^3/\text{sec} \quad Q_3 = 0.167 \text{ m}^3/\text{sec}$$

Hence,

$$\Sigma Q = (Q_1 + Q_2) - Q_3 = 0.238 \text{ m}^3/\text{sec}$$

With the values computed above, a graph may be constructed with P plotted against the corresponding values of ΣQ , as shown in Figure 4.5. The curve intersects the $\Sigma Q = 0$ line at $P = 99.0$ m, which is used to compute the final set of discharges. We obtain

$$Q_1 = 0.161 \text{ m}^3/\text{sec} \quad Q_2 = 0.065 \text{ m}^3/\text{sec} \quad Q_3 = 0.231 \text{ m}^3/\text{sec}$$

Hence, the condition $\Sigma Q = (Q_1 + Q_2) - Q_3 = 0$ is satisfied (within $-0.005 \text{ m}^3/\text{sec}$).

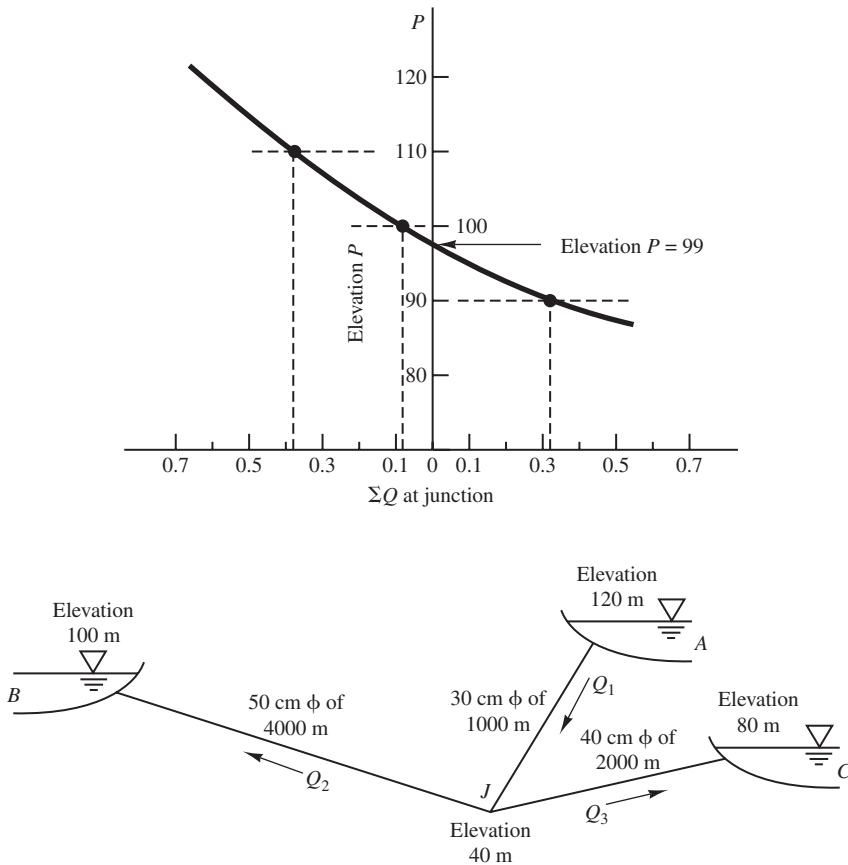


Figure 4.5 Three-reservoir problem

Note: The energy equation for each pipe, the Darcy-Weisbach equation for friction head loss (or, alternatively, the Hazen-Williams equation or the Manning equation), the Reynolds number expression, the Von Kármán equation for the friction factor assuming complete turbulence, Colebrook’s implicit friction factor equation when N_R is available (or, alternatively, the explicit Swamee-Jain equation), and mass balance at the junction can be solved simultaneously by a computer algebra software system (e.g., Mathcad, Maple, or Mathematica) and should yield the same result. Likewise, a simple spreadsheet program can be formulated to quickly perform the iterations. (See Problem 4.3.1.)

Example 4.7

A horizontal, galvanized iron pipe system consists of a 10-in-diameter, 12-ft-long main pipe between the two junctions 1 and 2, as depicted in Figure 4.6. A gate valve is installed at the downstream end immediately before junction 2. The branch pipe has a 6-in. diameter and is 20 ft long. It consists of two 90° elbows ($R/D = 2.0$) and a globe valve. The system carries a total discharge of 10 cfs of water at 40°F. Determine the discharge in each of the pipes when the valves are both fully opened.

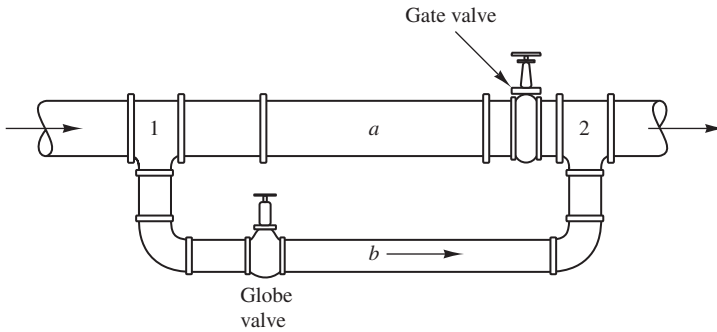


Figure 4.6

Solution

The cross-sectional area of pipes a and b are, respectively,

$$A_a = \frac{\pi}{4} \left(\frac{10}{12} \right)^2 = 0.545 \text{ ft}^2 \quad A_b = \frac{\pi}{4} \left(\frac{6}{12} \right)^2 = 0.196 \text{ ft}^2$$

Mass balance requires that

$$10 \text{ cfs} = A_a V_a + A_b V_b = 0.545 V_a + 0.196 V_b \quad (a)$$

where V_a and V_b are the velocities in pipes a and b , respectively. The head loss between junctions 1 and 2 along the main pipe is

$$h_a = f_a \left(\frac{L_a}{D_a} \right) \frac{V_a^2}{2g} + 0.15 \frac{V_a^2}{2g}$$

The second term accounts for the fully open gate valve (Table 3.6). The head loss between junctions 1 and 2 along the branch pipe is

$$h_b = f_b \left(\frac{L_b}{D_b} \right) \frac{V_b^2}{2g} + 2(0.19) \frac{V_b^2}{2g} + 10 \frac{V_b^2}{2g}$$

The second term accounts for the elbow losses; the third term accounts for the fully open globe valve (Table 3.6).

Since the head losses through both pipes must be the same, $h_a = h_b$, we have

$$\left[f_a \left(\frac{12}{0.833} \right) + 0.15 \right] \frac{V_a^2}{2g} = \left[f_b \left(\frac{20}{0.5} \right) + 0.38 + 10 \right] \frac{V_b^2}{2g}$$

or

$$(14.4f_a + 0.15)V_a^2 = (40f_b + 10.4)V_b^2 \quad (b)$$

Equations (a) and (b) can be solved simultaneously for V_a and V_b once friction factors have been established. For galvanized iron, Table 3.1 gives

$$\left(\frac{e}{D} \right)_a = \frac{0.0005}{0.833} = 0.00060$$

and

$$\left(\frac{e}{D} \right)_b = \frac{0.0005}{0.50} = 0.0010$$

Assuming complete turbulence, the Moody diagram yields the following f values:

$$f_a = 0.0175 \quad \text{and} \quad f_b = 0.020$$

as a first approximation.

Substituting the above values into Equation (b), we have

$$[14.4(0.0175) + 0.15]V_a^2 = [40(0.020) + 10.4]V_b^2$$

$$0.402V_a^2 = 11.2V_b^2$$

$$V_a = \sqrt{\frac{11.2}{0.402}}V_b = 5.28V_b$$

Substituting V_a into Equation (a), we find

$$10 = 0.545(5.28V_b) + 0.196V_b = 3.07V_b$$

$$V_b = \frac{10}{3.07} = 3.26 \text{ ft/sec}$$

Hence, $V_a = 5.28V_b = 17.2 \text{ ft/sec}$. The corresponding Reynolds numbers are calculated to verify the assumed friction factors. For pipe a ,

$$N_{R_a} = \frac{V_a D_a}{\nu} = \frac{17.2(0.833)}{1.69 \times 10^{-5}} = 8.48 \times 10^5$$

The Moody chart gives $f = 0.0175$, which matches the original assumption. For pipe b ,

$$N_{R_b} = \frac{V_b D_b}{\nu} = \frac{3.26(0.5)}{1.69 \times 10^{-5}} = 9.65 \times 10^4$$

The Moody chart gives $f_b = 0.0225 \neq 0.020$

Equations (a) and (b) are solved again using the new value of f_b :

$$[14.4(0.0175) + 0.15]V_a^2 = [40(0.0225) + 10.4]V_b^2$$

$$V_a = 5.30V_b$$

Substituting V_a into Equation (a),

$$10 = 0.545(5.30V_b) + 0.196V_b$$

$$V_b = 3.24 \text{ ft/sec} \quad \text{and} \quad V_a = 5.30(3.24) = 17.2 \text{ ft/sec}$$

Therefore, the discharges are

$$Q_a = A_a V_a = 0.545(17.2) = 9.37 \text{ cfs}$$

and

$$Q_b = A_b V_b = 0.196(3.24) = 0.635 \text{ cfs}$$

Branching pipes connecting more than three reservoirs to a junction (Figure 4.7) are not common in hydraulic engineering. However, problems of multiple (more than three) reservoirs can be solved by following the same principles.

Assume that the water elevation in the piezometer reaches level P at junction J . The differences in water surface elevations between reservoirs A and B , A and C , and A and D are, respectively, H_1 , H_2 , and H_3 . The head losses between reservoirs A , B , C , and D and the junction are, respectively, h_{f1} , h_{f2} , h_{f3} , and h_{f4} , as shown in Figure 4.7. A set of four independent equations may be written in the following general form for the four reservoirs:

$$H_1 = h_{f1} - h_{f2} \tag{4.5}$$

$$H_2 = h_{f1} + h_{f3} \tag{4.6}$$

$$H_3 = h_{f1} + h_{f4} \tag{4.7}$$

$$\Sigma Q_j = 0 \tag{4.8}$$

For each of the pipe branches, the head loss may be expressed in the form of the Darcy-Weisbach equation,* Equation 3.16, as follows:

$$h_{f1} = f_1 \left(\frac{L_1}{D_1} \right) \frac{V_1^2}{2g} = f_1 \left(\frac{L_1}{D_1} \right) \frac{Q_1^2}{2gA_1^2}$$

$$h_{f2} = f_2 \left(\frac{L_2}{D_2} \right) \frac{V_2^2}{2g} = f_2 \left(\frac{L_2}{D_2} \right) \frac{Q_2^2}{2gA_2^2}$$

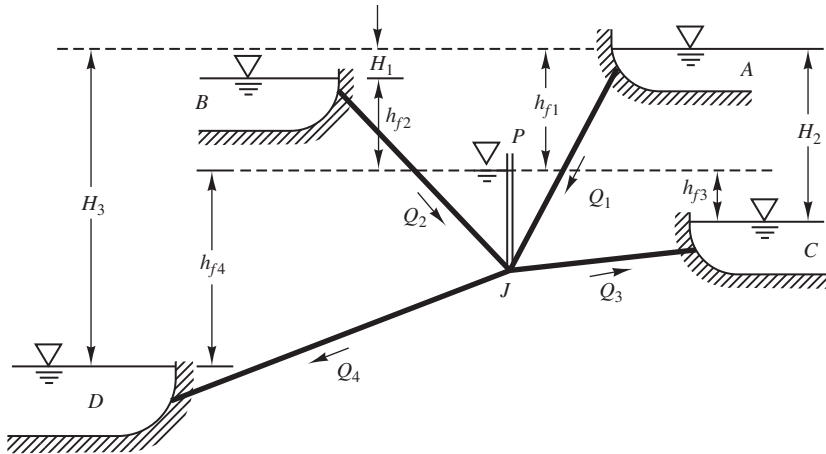


Figure 4.7 Multiple reservoirs connected at a junction

* Empirical equations for friction head loss, such as the Hazen-Williams equation and the Manning equation, may be applied using analogous expressions.

$$h_{f_3} = f_3 \left(\frac{L_3}{D_3} \right) \frac{V_3^2}{2g} = f_3 \left(\frac{L_3}{D_3} \right) \frac{Q_3^2}{2gA_3^2}$$

$$h_{f_4} = f_4 \left(\frac{L_4}{D_4} \right) \frac{V_4^2}{2g} = f_4 \left(\frac{L_4}{D_4} \right) \frac{Q_4^2}{2gA_4^2}$$

Substituting these relationships into Equations 4.5, 4.6, and 4.7 yields

$$H_1 = \frac{1}{2g} \left(f_1 \frac{L_1}{D_1} \frac{Q_1^2}{A_1^2} - f_2 \frac{L_2}{D_2} \frac{Q_2^2}{A_2^2} \right) \quad (4.9)$$

$$H_2 = \frac{1}{2g} \left(f_1 \frac{L_1}{D_1} \frac{Q_1^2}{A_1^2} + f_3 \frac{L_3}{D_3} \frac{Q_3^2}{A_3^2} \right) \quad (4.10)$$

$$H_3 = \frac{1}{2g} \left(f_1 \frac{L_1}{D_1} \frac{Q_1^2}{A_1^2} + f_4 \frac{L_4}{D_4} \frac{Q_4^2}{A_4^2} \right) \quad (4.11)$$

and

$$\Sigma Q_j = 0 \quad (4.8)$$

Equations 4.8 through 4.11 can then be solved simultaneously for the four unknowns: Q_1 , Q_2 , Q_3 , and Q_4 . These values are the flow rates for each of the pipe branches shown. This procedure could be applied to any number of reservoirs connected to a common junction.

4.4 Pipe Networks

Water-supply distribution systems in municipal districts are usually constructed of a large number of pipes interconnected to form loops and branches. Although the calculations of flow in a pipe network involve a large number of pipes and may become tedious, the solution scenario is based on the same principles that govern flow in pipelines and branching pipes previously discussed. In general, a series of simultaneous equations can be written for the network. These equations are written to satisfy the following conditions:

1. At any junction, $\Sigma Q = 0$ based on the conservation of mass (**junction equation**).
2. Between any two junctions, the total head loss is independent of the path taken based on the conservation of energy (**loop equation**).

Depending on the number of unknowns, it is usually possible to set up a sufficient number of independent equations to solve the problem. A typical problem would be to determine the flow distribution in each pipe of a network (Figure 4.8) when the inflows (Q_1 and Q_2) and outflows (Q_3 and Q_4) to the network are known. These equations may then be solved simultaneously.

For the simple network shown in Figure 4.8, a set of 12 independent equations (eight junction equations and four loop equations) is needed to solve for the flow distribution in the 12 pipes. As a general rule, a network with m loops and n junctions provides a total of $m + (n - 1)$ independent equations. For more complex networks, the number of equations increases proportionally. At a certain point it will be obvious that algebraic solution of the network equations

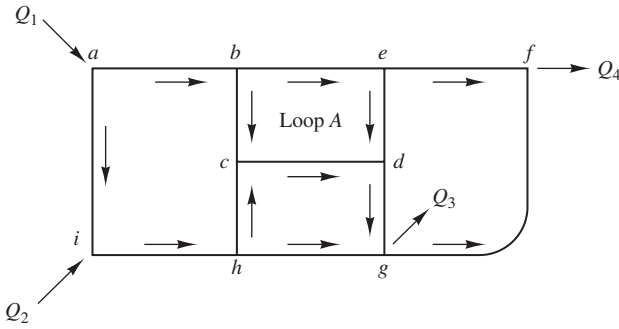


Figure 4.8 Schematic of a pipe network

becomes impractical. For most engineering applications, pipe-network solutions are obtained using computer software designed specifically for the task. Two algorithms commonly used for pipe network analysis are outlined below.

4.4.1 The Hardy-Cross Method

The Hardy-Cross method utilizes successive flow approximations based on the two conditions stated previously for each junction and loop in the pipe network. In loop A shown in Figure 4.8, the arrowheads indicate the presumed flow direction. This loop must satisfy the conditions of mass and energy balance.

1. At each junction *b, c, d, and e*, the total inflow must equal the total outflow.
2. The loss of head from flow in the counterclockwise direction along pipes *bc* and *cd* must equal the loss of head from flow in the clockwise direction along pipes *be* and *ed*.

To initiate the process, a distribution of flows in each pipe is estimated such that the total inflow equals the total outflow at each junction throughout the pipe network. For a network with *n* junctions, (*n* - 1) junction equations can be established to determine the flow rates in the system. Once flows are established for the first (*n* - 1) junctions, the flows to and from the last junction are fixed and thus dependent. The estimated flows, along with the pipe diameters, lengths, friction coefficients, and other network data (e.g., connectivity, junction elevations) are required for the Hardy-Cross method. The loss of head resulting from the estimated flow rates in all pipes is then computed for the network.

The possibility of the assumed flow distribution satisfying the *m* loop equations is small. Invariably, the estimated pipe flows need to be adjusted until head losses in the clockwise direction equal head losses in the counterclockwise direction within each loop. The successive computational procedure uses the loop equations, one at a time, to correct the assumed pipe flows and thus equalizes the head losses in the loop. Because the flow balance at each junction must be maintained, a given correction to flow in any pipe (e.g., pipe *be*) in the clockwise direction requires a corresponding flow correction of the same magnitude in the clockwise direction in the other pipes (pipes *bc, cd, and ed*). The successive flow corrections to equalize the head loss are discussed next.

Knowing the diameter, length, and roughness of a pipe, we see that the head loss in the pipe is a function of the flow rate, *Q*. Applying the Darcy-Weisbach equation (3.16), we may write

$$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g} = \left[f\left(\frac{L}{D}\right)\frac{1}{2gA^2}\right]Q^2 = KQ^2 \tag{4.12}$$

In any network loop, such as a loop A , the total head loss in the clockwise direction (hereafter designated with the subscript c) is the sum of the head losses in all pipes that carry flow in the clockwise direction around the loop:

$$\Sigma h_{fc} = \Sigma K_c Q_c^2 \quad (4.13)$$

Similarly, the loss of head in the counterclockwise direction (subscript cc) is

$$\Sigma h_{fcc} = \Sigma K_{cc} Q_{cc}^2 \quad (4.14)$$

Using the assumed flow rates, Q 's, it is not expected that these two values will be equal during the first trial, as mentioned previously. The difference,

$$\Sigma K_c Q_c^2 - \Sigma K_{cc} Q_{cc}^2$$

is the *closure error* of the first trial.

We need to determine a flow correction ΔQ that, when subtracted from Q_c and added to Q_{cc} , will equalize the two head losses. Thus, the correction ΔQ must satisfy the following equation:

$$\Sigma K_c (Q_c - \Delta Q)^2 = \Sigma K_{cc} (Q_{cc} + \Delta Q)^2$$

Expanding the terms in the parentheses on both sides of the equation, we have

$$\Sigma K_c (Q_c^2 - 2Q_c \Delta Q + \Delta Q^2) = \Sigma K_{cc} (Q_{cc}^2 + 2Q_{cc} \Delta Q + \Delta Q^2)$$

Assuming that the correction term is small compared with both Q_c and Q_{cc} , we may simplify the previous expression by dropping the last term on each side of the equation and write

$$\Sigma K_c (Q_c^2 - 2Q_c \Delta Q) = \Sigma K_{cc} (Q_{cc}^2 + 2Q_{cc} \Delta Q)$$

From this relationship, we may solve for ΔQ :

$$\Delta Q = \frac{\Sigma K_c Q_c^2 - \Sigma K_{cc} Q_{cc}^2}{2(\Sigma K_c Q_c + \Sigma K_{cc} Q_{cc})} \quad (4.15)$$

If we take Equation 4.12 and divide it by Q on both sides, we have

$$KQ = \frac{h_f}{Q} \quad (4.16)$$

Equations 4.13, 4.14, and 4.16 can be substituted into Equation 4.15 to obtain

$$\Delta Q = \frac{(\Sigma h_{fc} - \Sigma h_{fcc})}{2 \left(\Sigma \frac{h_{fc}}{Q_c} + \Sigma \frac{h_{fcc}}{Q_{cc}} \right)} \quad (4.17a)$$

Equation 4.17a is appropriate when the Manning equation (3.28) is used to determine friction head losses instead of the Darcy-Weisbach equation. However, when the Hazen-Williams equation (3.27) is used, the equation should be

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{1.85 \left(\Sigma \frac{h_{fc}}{Q_c} + \Sigma \frac{h_{fcc}}{Q_{cc}} \right)} \quad (4.17b)$$

Once the error magnitude is established, a second iteration uses this correction to determine a new flow distribution. The computation results from the second iteration are expected to give a closer match of the two head losses along the c and cc directions in loop A. Note that pipes bc , cd , and ed in loop A are each common to two loops and, therefore, need to be subjected to double corrections, one from each loop. The successive computational procedure is repeated until each loop in the entire network is balanced (mass and energy) and the corrections become negligibly small.

The Hardy-Cross method can best be described using an example problem. Even though commercial computer software is available to solve these laborious calculations, the student should go through the procedure a few times with small networks. By becoming familiar with the algorithms, a more judicious and appreciative use of the computer software can be expected.

Example 4.8

A water-supply distribution system for an industrial park is schematically shown in Figure 4.9 (a). The demands on the system are currently at junctions C, G, and F with flow rates given in liters per second. Water enters the system at junction A from a water storage tank on a hill. The water surface elevation in the tank is 50 m above the elevation of point A in the industrial park. All the junctions have the same elevation as point A. All pipes are aged ductile iron ($e = 0.26$ mm) with lengths and diameters provided in the table below. Calculate the flow rate in each pipe. Also determine if the pressure at junction F will be high enough to satisfy the customer there. The required pressure is 185 kPa.

A table of pipe and system geometry is a convenient way to organize the available information and make some preliminary calculations. The table below has been set up for that purpose. The first column identifies all of the pipes in the network. Column 2 contains flow rates for each pipe, which were estimated to initiate the Hardy-Cross algorithm. These estimated flow rates are shown in brackets on the system

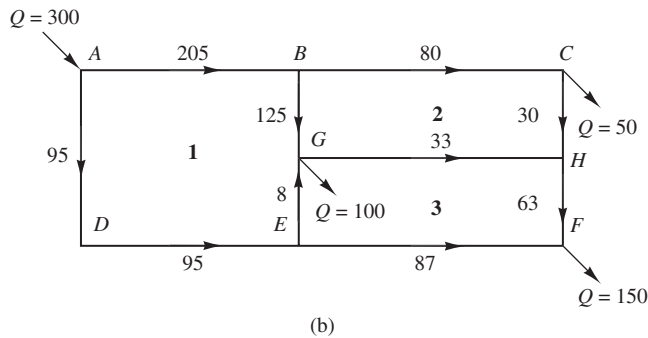
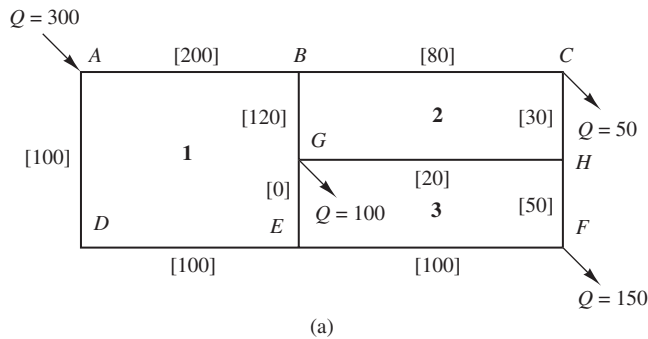


Figure 4.9

schematic in Figure 4.9 (a). Note that mass balance was maintained at each junction. Flow direction in the table is indicated using the junction letters that define the pipe. For example, flow in pipe *AB* is from junction *A* to junction *B*. Friction factors (column 6) are found assuming complete turbulence and read from the Moody diagram using e/D or, alternatively, from Equation 3.23. The “*K*” coefficient (column 7) is used later in the procedure to obtain the head loss in each pipe according to Equation 4.12.

Pipe	Flow (m ³ /sec)	Length (m)	Diameter (m)	e/D	f	K (sec ² /m ⁵)
<i>AB</i>	0.20	300	0.30	0.00087	0.019	194
<i>AD</i>	0.10	250	0.25	0.00104	0.020	423
<i>BC</i>	0.08	350	0.20	0.00130	0.021	1,900
<i>BG</i>	0.12	125	0.20	0.00130	0.021	678
<i>GH</i>	0.02	350	0.20	0.00130	0.021	1,900
<i>CH</i>	0.03	125	0.20	0.00130	0.021	678
<i>DE</i>	0.10	300	0.20	0.00130	0.021	1,630
<i>GE</i>	0.00	125	0.15	0.00173	0.022	2,990
<i>EF</i>	0.10	350	0.20	0.00130	0.021	1,900
<i>HF</i>	0.05	125	0.15	0.00173	0.022	2,990

Solution

The Hardy-Cross method utilizes a relaxation technique (method of successive approximations). Using the estimated flow rates, head losses are found in each pipe by using Equation 4.12, one loop at a time. Equation 4.17a is then used to determine a flow correction and thus improve the flow estimate. The same procedure is applied to all the remaining loops, and then the cycle repeats itself. The process is ended when the flow corrections become acceptably small. At this point, conservation of mass is satisfied for each junction, and the loss of head around each loop is the same for counterclockwise and clockwise flow (conservation of energy).

We will proceed through the calculations by using a series of tables; explanations will follow each table. Having said all that, let us commence with loop 1.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
1	<i>AB</i>	0.200	194	7.76	38.8	0.205
	<i>BG</i>	0.120	678	9.76	81.3	0.125
	<i>GE</i>	0.000	2,990	0.00	0.0	0.005
	<i>AD</i>	(0.100)	423	(4.23)	(42.3)	(0.095)
	<i>DE</i>	(0.100)	1,630	(16.3)	(163.0)	(0.095)

The flows listed in column 3 are the original estimates. Flows in loop 1 that are counterclockwise are placed in parentheses. Head losses are computed from:

$$h_f = KQ^2$$

The flow correction is found using Equation 4.17a.

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(7.76 + 9.76) - (4.23 + 16.3)}{2[(38.8 + 81.3) + (42.3 + 163.0)]} = -0.005 \text{ m}^3/\text{sec}$$

The negative sign on the flow adjustment indicates that counterclockwise head losses dominate ($\Sigma h_{fcc} > \Sigma h_{fc}$). Therefore, the flow correction of 0.005 m³/sec is applied in the clockwise direction [column 7 (“New Q ”).]. This will help to equalize the losses in the next iteration. Now we continue with loop 2.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
2	<i>BC</i>	0.080	1,900	12.2	152.5	0.078
	<i>CH</i>	0.030	678	0.61	20.3	0.028
	<i>BG</i>	(0.125)	678	(10.6)	(84.8)	(0.127)
	<i>GH</i>	(0.020)	1,900	(0.76)	(38.0)	(0.022)

Because pipe *BG* is shared by loops 1 and 2, the revised flow from the calculation in loop 1 is used here. Note that in loop 1 the flow in *BG* is clockwise; in loop 2, the flow is counterclockwise. The flow correction is found to be

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(12.2 + 0.61) - (10.6 + 0.76)}{2[(152.5 + 20.3) + (84.8 + 38.0)]} = +0.002 \text{ m}^3/\text{sec}$$

Clockwise flow dominates the loss so the correction of 0.002 m³/sec is added in the counterclockwise direction. We complete the first iteration by correcting flows in loop 3.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
3	<i>GH</i>	0.022	1,900	0.92	41.8	0.035
	<i>HF</i>	0.050	2,990	7.48	149.6	0.063
	<i>GE</i>	(0.005)	2,990	(0.07)	(14.0)	0.008
	<i>EF</i>	(0.100)	1,900	(19.0)	(190.0)	(0.087)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(0.92 + 7.48) - (0.07 + 19.0)}{2[(41.8 + 149.6) + (14.0 + 190.0)]} = -0.013 \text{ m}^3/\text{sec}$$

Counterclockwise head losses dominate so the flow correction is added in the clockwise direction. Note that this is a large enough correction to reverse the flow direction in *GE*; it will be labeled *EG* the next time. We now begin the second iteration with loop 1.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
1	<i>AB</i>	0.205	194	8.15	39.8	0.205
	<i>BG</i>	0.127	678	10.9	85.8	0.127
	<i>AD</i>	(0.095)	423	(3.82)	(40.2)	(0.095)
	<i>DE</i>	(0.095)	1,630	(14.7)	(154.7)	(0.095)
	<i>EG</i>	(0.008)	2,990	(0.19)	(23.8)	(0.008)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(8.15 + 10.9) - (3.82 + 14.7 + 0.19)}{2[(39.8 + 85.8) + (40.2 + 154.7 + 23.8)]} = +0.000 \text{ m}^3/\text{sec}$$

The correction is very small (< 0.0005 m³/sec). We continue on to loop 2.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
2	<i>BC</i>	0.078	1,900	11.6	148.7	0.080
	<i>CH</i>	0.028	678	0.53	18.9	0.030
	<i>BG</i>	(0.127)	678	(10.9)	(85.8)	(0.125)
	<i>GH</i>	(0.035)	1,900	(2.33)	(66.6)	(0.033)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(11.6 + 0.53) - (10.9 + 2.33)}{2[(148.7 + 18.9) + (85.8 + 66.6)]} = -0.002 \text{ m}^3/\text{sec}$$

Once again the correction appears to be acceptably small. Finally, we check loop 3.

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
3	<i>GH</i>	0.033	1,900	2.07	62.7	0.033
	<i>HF</i>	0.063	2,990	11.9	188.9	0.063
	<i>EG</i>	0.008	2,990	0.19	23.8	0.008
	<i>EF</i>	(0.087)	1,900	(14.4)	(165.5)	(0.087)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(2.07 + 11.9 + 0.19) - (14.4)}{2[(62.7 + 188.9 + 23.8) + (165.5)]} = -0.000 \text{ m}^3/\text{sec}$$

Because the correction is very small on all three loops, the flow rates are accepted and the process is ended. The final flows appear in Figure 4.9 (b).

The following table below summarizes information about the pipe system. Final head losses are determined by using final flows and Equation 4.12. The head loss is converted to a pressure drop in the last column ($\Delta P = \gamma h_f$).

Pipe	Q (L/sec)	Length (m)	Diameter (cm)	h_f (m)	ΔP (kPa)
<i>AB</i>	205	300	30	8.2	80.3
<i>AD</i>	95	250	25	3.8	37.2
<i>BC</i>	80	350	20	12.2	119.4
<i>BG</i>	125	125	20	10.6	103.8
<i>GH</i>	33	350	20	2.1	20.6
<i>CH</i>	30	125	20	0.6	5.9
<i>DE</i>	95	300	20	14.7	143.9
<i>EG</i>	8	125	15	0.2	2.0
<i>EF</i>	87	350	20	14.4	141.0
<i>HF</i>	63	125	15	11.9	116.5

Now we determine whether or not the pressure at junction *F* is large enough to satisfy the customer at that location. We will use an energy balance to do this. But first, because the water surface elevation in the tank is 50 m above junction *A*, the pressure there is

$$P = \gamma h = (9790 \text{ N/m}^3)(50 \text{ m}) = 489.5 \text{ kPa}$$

The pressure at junction *F* may now be determined by subtracting the pressure drops in pipes *AD*, *DE*, and *EF* or any alternative route from *A* to *F*. (In this case, all the junctions are at the same elevation and the variations in the velocity heads are negligible, so an energy balance only involves pressure heads and friction head losses.) Therefore,

$$P_F = P_A - \Delta P_{AD} - \Delta P_{DE} - \Delta P_{EF} = 489.5 - 37.2 - 143.9 - 141.0 = 167.4 \text{ kPa}$$

Because the pressure is less than 185 kPa, the industrial customer is not likely to be satisfied. Minor losses in the system were not accounted for, so the pressure is likely to be even lower when the system is running at the full demand specified. It is left to the student to suggest modifications to the system to accommodate the customer (Problem 4.4.1). Also note that the pressure at *F* could have been determined by subtracting head losses from the total head at *A* and converting the pressure head to pressure.

The procedure outlined above for the Hardy-Cross method is valid if all the inflows entering the network are known. In practice, this occurs when there is only one source of inflow. In this case, the inflow rate is equal to the sum of the known withdrawal rates at all the junctions. However, if the flow is supplied to the network from two or more sources, as shown in Figure 4.10 (a), the inflow rates entering the network will not be known a priori. Therefore, we need to add *inflow path* calculations to the procedure. The number of inflow paths to be considered is equal to the number of inflow sources minus one. In Figure 4.10 (a), there are two reservoirs supplying flow to the network. Therefore, only one inflow path needs to be considered. This can be any path connecting the two reservoirs. For example, one can choose inflow path *ABCDG* in Figure 4.10 (a). There are several other possibilities like *ABFEDG*, *GDCFBA*, and so on. The results will not be affected by the choice of the inflow path.

Once an inflow path is selected, the path calculations are carried out in a manner similar to the loop calculations. Using the subscript *p* (path) to denote the flows in the same direction as the inflow path followed and *cp* (counterpath) to denote the flows in the opposite direction, the discharge correction ΔQ is calculated as

$$\Delta Q = \frac{(\sum h_{fp} - \sum h_{fcp}) + H_d - H_u}{2\left(\sum \frac{h_{fp}}{Q_p} + \sum \frac{h_{fcp}}{Q_{cp}}\right)} \tag{4.18a}$$

where H_u and H_d are the total heads at the beginning (upstream) point of the inflow path and at the end (downstream) point of the path. For path *ABCDG* in Figure 4.10a, $H_u = H_A$ and $H_d = H_G$. A positive value of ΔQ indicates that losses in the direction of the path are dominating. Therefore, the correction would be applied in the counterpath direction. In other words, the flows along the inflow path direction would be decreased in absolute value, and those in the opposite direction would be increased by ΔQ .

Equation 4.18a can be used in conjunction with the Manning equation (3.28), as well as the Darcy-Weisbach equation (3.16). However, when the Hazen-Williams equation (3.27) is used, the equation is reformulated as

$$\Delta Q = \frac{(\sum h_{fp} - \sum h_{fcp}) + H_d - H_u}{1.85\left(\sum \frac{h_{fp}}{Q_p} + \sum \frac{h_{fcp}}{Q_{cp}}\right)} \tag{4.18b}$$

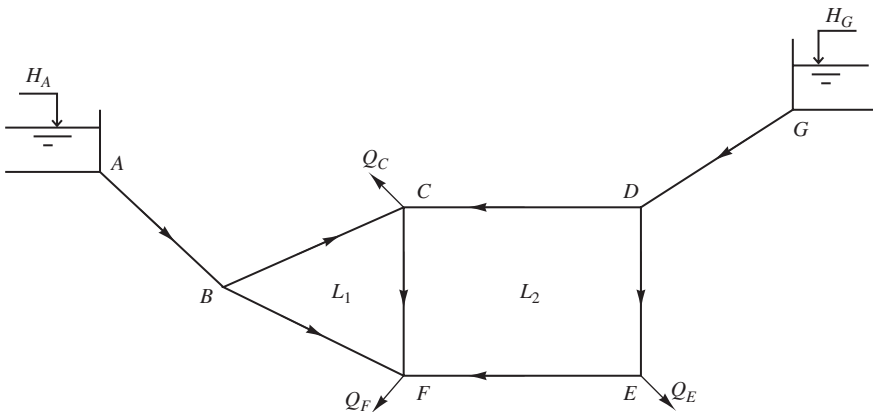


Figure 4.10 (a) Pipe network with two reservoir sources

Example 4.9

Consider the pipe network shown in Figure 4.10 (a), which contains two reservoir sources. Suppose $H_A = 85$ m, $H_G = 102$ m, $Q_C = 0.10$ m³/sec, $Q_F = 0.25$ m³/sec, and $Q_E = 0.10$ m³/sec. The pipe and junction characteristics are tabulated below. Also tabulated are the initial estimates of the flow rates in all the pipes. The flow directions are shown in Figure 4.10 (a). Determine the discharge in each pipe and the pressure head at each junction.

Pipe	Length (m)	Diameter (m)	e/D	f	K (sec ² /m ⁵)	Q (m ³ /sec)	Junction	Elev. (m)
AB	300	0.30	0.00087	0.019	194	0.200	A	48
BC	350	0.20	0.00130	0.021	1,900	0.100	B	46
BF	350	0.20	0.00130	0.021	1,900	0.100	C	43
CF	125	0.20	0.00130	0.021	678	0.050	D	48
DC	300	0.20	0.00130	0.021	1,630	0.050	E	44
EF	300	0.20	0.00130	0.021	1,630	0.100	F	48
DE	125	0.20	0.00130	0.021	678	0.200	G	60
GD	250	0.25	0.00104	0.020	423	0.250		

Solution

Using the estimated flow rates, head losses are found in each pipe by using Equation 4.12, one loop at a time. Equation 4.17a is then used to determine a flow correction and thus improve the flow estimate. The same procedure is applied to all the remaining loops, and Equation 4.18a is applied to inflow path ABCDG connecting reservoir A to G. Then the cycle repeats itself. The process is ended when the flow corrections become acceptably small. At this point, conservation of mass is satisfied for each junction and the loss of head around each loop is the same for counterclockwise and clockwise flow (conservation of energy).

We will proceed through the calculations by using a series of tables; explanations will follow each table. Having said all that, let us commence with loop 1 (L_1).

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
1	BC	0.100	1,900	19.00	190.00	0.098
	CF	0.050	678	1.70	33.90	0.048
	BF	(0.100)	1,900	(19.00)	(190.00)	(0.102)

The flows listed in column 3 are the original estimates. Flows in loop 1 that are counterclockwise are placed in parentheses. Head losses are computed from:

$$h_f = KQ^2$$

The flow correction in m³/sec is found using Equation 4.17a:

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(19.00 + 1.70) - (19.00)}{2[(190.0 + 33.9) + (190.0)]} = 0.002 \text{ m}^3/\text{sec}$$

The positive sign on the flow adjustment indicates that clockwise head losses dominate ($\Sigma h_{fcc} < \Sigma h_{fc}$). Therefore, the flow correction of 0.002 m³/sec is applied in the counterclockwise direction [column 7 (“New Q ”)]. This will help to equalize the losses in the next iteration. Now we continue with loop 2 (L_2).

Loop	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
2	DE	0.200	678	27.12	135.60	0.154
	EF	0.100	1,630	16.30	163.00	0.054
	DC	(0.050)	1,630	(4.08)	(81.50)	(0.096)
	CF	(0.048)	678	(1.56)	(32.54)	(0.094)

Because pipe *CF* is shared by loops 1 and 2, the revised flow from the calculation in loop 1 is used here. Note that in loop 1 the flow in *CF* is clockwise; in loop 2 the flow in *CF* is counterclockwise. The flow correction is found to be

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(27.12 + 16.30) - (4.08 + 1.56)}{2[(135.60 + 163.00) + (81.50 + 32.54)]} = + 0.046 \text{ m}^3/\text{sec}$$

Clockwise flow dominates the loss so the correction of 0.046 m³/sec should be added in the counterclockwise direction.

We complete the first iteration by correcting flows along inflow path *ABCDG*. Note that the flows in *DC* and *GD* are in a direction opposite to the direction of the inflow path chosen and are placed in parentheses.

Inflow Path	Pipe	<i>Q</i> (m ³ /sec)	<i>K</i> (sec ² /m ⁵)	<i>h_f</i> (m)	<i>h_f/Q</i> (sec/m ²)	New <i>Q</i> (m ³ /sec)
ABCDG	<i>AB</i>	0.200	194	7.76	38.80	0.198
	<i>BC</i>	0.098	1,900	18.25	186.20	0.096
	<i>DC</i>	(0.096)	1,630	(15.02)	(156.48)	(0.098)
	<i>GD</i>	(0.250)	423	(26.44)	(105.75)	(0.252)

By using Equation 4.18a,

$$\Delta Q = \frac{\Sigma h_{fp} - \Sigma h_{fcp} + H_G - H_A}{2[\Sigma(h_{fp}/Q_p) + \Sigma(h_{fcp}/Q_{cp})]} = \frac{(7.76 + 18.25) - (15.02 + 26.44) + 102 - 85}{2[(38.80 + 186.20) + (156.48 + 105.75)]} = 0.002 \text{ m}^3/\text{sec}$$

We find that head losses dominate along the designated inflow path, so the flow correction is added in the counterpath direction. The first iteration cycle is now complete.

We will proceed to the second iteration cycle using the most recent flow rate calculated for each pipe. The calculations for loops 1 and 2 and the path *ABCDG* for the second iteration cycle are as follows:

Loop	Pipe	<i>Q</i> (m ³ /sec)	<i>K</i> (sec ² /m ⁵)	<i>h_f</i> (m)	<i>h_f/Q</i> (sec/m ²)	New <i>Q</i> (m ³ /sec)
1	<i>BC</i>	0.096	1,900	17.51	182.40	0.092
	<i>CF</i>	0.094	678	5.99	63.73	0.090
	<i>BF</i>	(0.102)	1,900	(19.77)	(193.80)	(0.106)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(17.51 + 5.99) - (19.77)}{2[(182.4 + 63.73) + (193.8)]} = 0.004 \text{ m}^3/\text{sec}$$

Loop	Pipe	<i>Q</i> (m ³ /sec)	<i>K</i> (sec ² /m ⁵)	<i>h_f</i> (m)	<i>h_f/Q</i> (sec/m ²)	New <i>Q</i> (m ³ /sec)
2	<i>DE</i>	0.154	678	16.08	104.41	0.154
	<i>EF</i>	0.054	1,630	4.75	88.02	0.054
	<i>DC</i>	(0.098)	1,630	(15.65)	(159.74)	(0.098)
	<i>CF</i>	(0.090)	678	(5.49)	(61.02)	(0.090)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})]} = \frac{(16.08 + 4.75) - (15.65 + 5.49)}{2[(104.41 + 88.02) + (159.74 + 61.02)]} = 0.000 \text{ m}^3/\text{sec}$$

Inflow Path	Pipe	Q (m ³ /sec)	K (sec ² /m ⁵)	h_f (m)	h_f/Q (sec/m ²)	New Q (m ³ /sec)
ABCDG	AB	0.198	194	7.61	38.41	0.200
	BC	0.092	1,900	16.08	174.80	0.094
	DC	(0.098)	1,630	(15.65)	(159.74)	(0.096)
	GD	(0.252)	423	(26.86)	(106.60)	(0.250)

$$\Delta Q = \frac{\Sigma h_{fp} - \Sigma h_{fcp} + H_G - H_A}{2[\Sigma(h_{fp}/Q_p) + \Sigma(h_{fcp}/Q_{cp})]} = \frac{(7.61 + 16.08) - (15.65 + 26.86) + 102 - 85}{2[(38.41 + 174.80) + (159.74 + 106.60)]} = -0.002 \text{ m}^3/\text{sec}$$

Similar calculations are carried out for one more iteration cycle, at which point all of the corrections become negligible. The final results are tabulated below and shown in Figure 4.10 (b). Also tabulated are the total head and the pressure head values calculated for all of the nodes. Once the pipe discharges are found, the energy equation is used to calculate the total heads. For example,

$$H_B = H_A - h_{fAB} = 85.00 - 7.76 = 77.24 \text{ m}$$

and

$$H_C = H_B - h_{fbc} = 77.24 - 16.79 = 60.45 \text{ m}$$

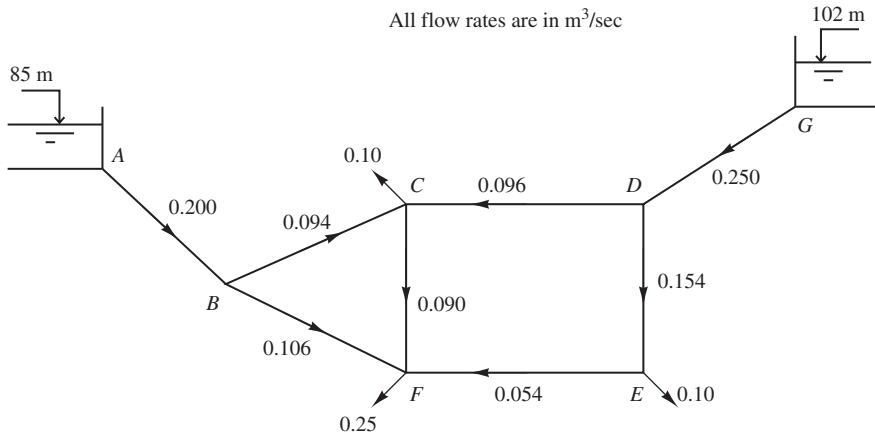


Figure 4.10 (b) Results of Example 4.9

Note that various paths can be used to determine the total head at a given node. For example, H_C can also be calculated as $H_C = H_A - h_{fAB} - h_{fBF} + h_{fCF}$. The results obtained from the different paths should be the same except for round-off errors. The pressure head at a node is equal to the total head minus the elevation of the node.

Pipe	Q (m ³ /sec)	h_f (m)	Junction	Elevation (m)	Total Head (m)	Pressure Head (m)
AB	0.200	7.76	A	48.00	85.00	37.00
BC	0.094	16.79	B	46.00	77.24	31.24
BF	0.106	21.34	C	43.00	60.45	17.45
CF	0.090	5.49	D	48.00	75.56	27.56
DC	0.096	15.02	E	44.00	59.48	15.48
EF	0.054	4.75	F	48.00	55.90	7.90
DE	0.154	16.08	G	60.00	102.00	42.00
GD	0.250	26.44				

Classroom Computer Exercise—Pipe Networks

Obtain or write computer software that is appropriate for solving pipe networks. Examples are EPANET (public domain from the U.S. Environmental Protection Agency), WaterCAD and WaterGEMS (proprietary from Haestad Methods-Bentley), and KYPipe (proprietary from KYPipe LLC). Or you can write your own spreadsheet program. Answer the following questions by performing a computer analysis of the pipe network described in Example 4.8 and its modifications.

- (a) Before using the computer software, what data do you anticipate the software will need to analyze the pipe network in Example 4.8?
 - (b) Now use the computer software to analyze Example 4.8. Enter the data requested by the software and perform a network of analysis. Compare flow rates generated by the computer model to those in the example problem. Why are the solutions not exactly the same? (*Note:* Some computer models require a pipe material and then assign an “ f ” value based on the pipe material and the Reynolds number. You may have to “manipulate” the model to get it to match the “ f ” values in Example 4.8.)
 - (c) What would happen to the flow rate in pipe EF if the friction factor was reduced? What would happen to the pressure at F ? Write down your answers, and then reduce “ f ” in EF from 0.021 to 0.014 and perform a new network analysis. List the original and new flow rate in pipe EF and the original and new pressure at node F . (*Hint:* The program may not let you change the friction factor directly but may allow a change in the pipe material or roughness value. You may have to assume complete turbulence on the Moody diagram and back into the roughness value or pipe material you need to reduce the friction factor in pipe EF to 0.014.) Once you have completed the analysis, restore pipe EF to its original friction factor and proceed to the next question.
 - (d) What would happen to the flow rate in pipe HF if the diameter was doubled? What would happen to the pressure at F ? Estimate the magnitude of these changes and write them down. Now double the diameter and analyze the network. Did you reason correctly? List the original and new flow rate in pipe HF and the original and new pressure at node F . Now restore HF to its original size and proceed to the next question.
 - (e) What would happen to the pressure at F if the demand for water at that point increased by 50 liters/sec? Estimate the magnitude of these changes and write them down. Now increase the demand for water at F and perform a new network analysis. Did you reason correctly? List the original and new pressure at node F . Now restore the demand at node F to its original value and proceed to the next question.
 - (f) What would happen to the flow rate in pipe EF if a new pipe were added to the system from node G to halfway between nodes A and D ? What would happen to the pressure at F ? Add this new pipe with the same characteristics as pipe DE and perform a new network analysis. Did you reason correctly? Now restore the network to its original configuration.
 - (g) Perform any other changes that your instructor requests.
-

4.4.2 The Newton Method

The Newton method is an appropriate and convenient procedure for analyzing pipe networks containing a large number of pipes and loops. In general the Newton iteration method was developed to solve a set of N simultaneous equations, F_i , written as

$$F_i[Q_1, Q_2, \dots, Q_i, \dots, Q_N] = 0$$

where $i = 1$ to N , and Q_i are the N unknowns. Computation for the iterative procedure begins by assigning a set of trial values to the unknowns Q_i for $i = 1$ to N . Substitution of these trial values into the N equations will yield the residuals F_1, F_2, \dots, F_N . These residuals are likely to be different from zero because the trial values assigned to the unknowns are probably not the actual solutions. New values for Q_i for $i = 1$ to N for the next iteration are estimated to make the residuals approach zero. We accomplish this by calculating corrections ΔQ_i for $i = 1$ to N such that the total differentials of the functions F_i are equal to the negative of the calculated residuals. In matrix form,

$$\begin{bmatrix} \frac{\partial F_1}{\partial Q_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial Q_3} & \cdots & \frac{\partial F_1}{\partial Q_{N-2}} & \frac{\partial F_1}{\partial Q_{N-1}} & \frac{\partial F_1}{\partial Q_N} \\ \frac{\partial F_2}{\partial Q_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial Q_3} & \cdots & \frac{\partial F_2}{\partial Q_{N-2}} & \frac{\partial F_2}{\partial Q_{N-1}} & \frac{\partial F_2}{\partial Q_N} \\ \frac{\partial F_3}{\partial Q_1} & \frac{\partial F_3}{\partial Q_2} & \frac{\partial F_3}{\partial Q_3} & \cdots & \frac{\partial F_3}{\partial Q_{N-2}} & \frac{\partial F_3}{\partial Q_{N-1}} & \frac{\partial F_3}{\partial Q_N} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ \frac{\partial F_{N-1}}{\partial Q_1} & \frac{\partial F_{N-1}}{\partial Q_2} & \frac{\partial F_{N-1}}{\partial Q_3} & \cdots & \frac{\partial F_{N-1}}{\partial Q_{N-2}} & \frac{\partial F_{N-1}}{\partial Q_{N-1}} & \frac{\partial F_{N-1}}{\partial Q_N} \\ \frac{\partial F_N}{\partial Q_1} & \frac{\partial F_N}{\partial Q_2} & \frac{\partial F_N}{\partial Q_3} & \cdots & \frac{\partial F_N}{\partial Q_{N-2}} & \frac{\partial F_N}{\partial Q_{N-1}} & \frac{\partial F_N}{\partial Q_N} \end{bmatrix} \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_{N-1} \\ \Delta Q_N \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ -F_3 \\ \vdots \\ -F_{N-1} \\ -F_N \end{bmatrix} \quad (4.19)$$

The solution of Equation 4.19 by any matrix inversion method provides the corrections to the trial values of Q_i for the next iteration. Thus, in equation form

$$(Q_i)_{k+1} = (Q_i)_k + (\Delta Q_i)_k$$

where k and $(k + 1)$ indicate consecutive iteration numbers. This procedure is repeated until the corrections are reduced to acceptable magnitudes. The number of iterations required to achieve the correct solution depends on how close the initial trial values are to the correct solution. If the initial guesses are quite different from the actual results, then the procedure may not converge.

The initial trial values of Q_i do not need to satisfy mass balance at the junctions in the Newton method. This is a major advantage over the Hardy-Cross method, especially when large pipe networks are considered. Also, the equations are formulated based on the flow directions initially chosen. A positive result for a flow rate will indicate that the direction initially chosen is correct. A negative value will indicate that the flow in that particular pipe is in the direction opposite to the direction initially guessed. Because the flow rates can take positive and negative values in this formulation, the friction loss is expressed as

$$h_f = KQ|Q|^{m-1}$$

to ensure that changes in the head are consistent with the flow directions.

The application of the Newton method to a pipe network analysis problem can best be shown through an example.

Example 4.10

Analyze the pipe network of Example 4.9 using the Newton method.

Solution

We will assign numbers to the junctions as shown in Figure 4.10 (c). For instance junction *B* of Example 4.9 is designated as J_1 . In Newton method applications, a junction means points where two or more pipes join. There are eight pipes in the network. Let us designate the flow rates in these pipes by Q_i with $i = 1, 2, \dots, 8$. For example the flow rate in pipe *AB* is designated Q_1 with flow direction *A* to *B*. The solution is formulated using the flow directions chosen.

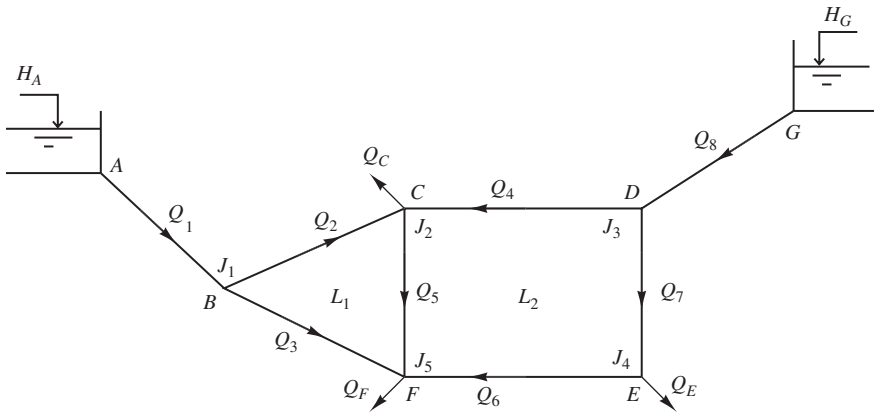


Figure 4.10 (c)

We will first write the junction equations. The correct values of the flow rates are those that will make the right-hand sides of these equations equal to zero since the sum of all the flow rates entering and leaving a junction must be zero.

$$\begin{aligned}
 F_1 &= -Q_1 + Q_2 + Q_3 \\
 F_2 &= -Q_2 - Q_4 + Q_5 + Q_C \\
 F_3 &= Q_4 + Q_7 - Q_8 \\
 F_4 &= Q_6 - Q_7 + Q_E \\
 F_5 &= -Q_3 - Q_5 - Q_6 + Q_F
 \end{aligned}$$

Next, noting that the sum of friction losses around a closed loop must be zero, we will write the loop equations for loops 1 and 2, respectively, as

$$\begin{aligned}
 F_6 &= K_2 Q_2 |Q_2| - K_3 Q_3 |Q_3| + K_5 Q_5 |Q_5| \\
 F_7 &= -K_4 Q_4 |Q_4| - K_5 Q_5 |Q_5| + K_6 Q_6 |Q_6| + K_7 Q_7 |Q_7|
 \end{aligned}$$

Again the correct values of the flow rates will make the right-hand sides of these equations zero.

Finally, the inflow path equation between reservoirs *A* and *G* is written as

$$F_8 = H_A - K_1 Q_1 |Q_1| - K_2 Q_2 |Q_2| + K_4 Q_4 |Q_4| + K_8 Q_8 |Q_8| - H_G$$

Many members of the coefficient matrix are zero because only a few Q 's appear in each equation. The nonzero values are evaluated as

$$\begin{array}{lll}
 \frac{\partial F_1}{\partial Q_1} = -1 & \frac{\partial F_1}{\partial Q_2} = 1 & \frac{\partial F_1}{\partial Q_3} = 1 \\
 \frac{\partial F_2}{\partial Q_2} = -1 & \frac{\partial F_2}{\partial Q_4} = -1 & \frac{\partial F_2}{\partial Q_5} = 1 \\
 \frac{\partial F_3}{\partial Q_4} = 1 & \frac{\partial F_3}{\partial Q_7} = 1 & \frac{\partial F_3}{\partial Q_8} = -1 \\
 \frac{\partial F_4}{\partial Q_6} = 1 & \frac{\partial F_4}{\partial Q_7} = -1 & \\
 \frac{\partial F_5}{\partial Q_3} = -1 & \frac{\partial F_5}{\partial Q_5} = -1 & \frac{\partial F_5}{\partial Q_6} = -1 \\
 \frac{\partial F_6}{\partial Q_2} = 2K_2Q_2 & \frac{\partial F_6}{\partial Q_3} = -2K_3Q_3 & \frac{\partial F_6}{\partial Q_5} = 2K_5Q_5 \\
 \frac{\partial F_7}{\partial Q_4} = -2K_4Q_4 & \frac{\partial F_7}{\partial Q_5} = -2K_5Q_5 & \frac{\partial F_7}{\partial Q_6} = 2K_6Q_6 & \frac{\partial F_7}{\partial Q_7} = 2K_7Q_7 \\
 \frac{\partial F_8}{\partial Q_1} = -2K_1Q_1 & \frac{\partial F_8}{\partial Q_2} = -2K_2Q_2 & \frac{\partial F_8}{\partial Q_4} = 2K_4Q_4 & \frac{\partial F_8}{\partial Q_8} = 2K_8Q_8
 \end{array}$$

The initial (trial) flow rates selected for the pipes are $Q_1 = 0.20 \text{ m}^3/\text{sec}$, $Q_2 = 0.50 \text{ m}^3/\text{sec}$, $Q_3 = 0.10 \text{ m}^3/\text{sec}$, $Q_4 = 0.05 \text{ m}^3/\text{sec}$, $Q_5 = 0.50 \text{ m}^3/\text{sec}$, $Q_6 = 0.10 \text{ m}^3/\text{sec}$, $Q_7 = 0.30 \text{ m}^3/\text{sec}$, and $Q_8 = 0.25 \text{ m}^3/\text{sec}$. The flow directions are designated in Figure 4.10c. Substituting these values into the equations formulated above for this example, we obtain

$$\begin{bmatrix}
 -1.0 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
 0.0 & -1.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & -1.0 \\
 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & -1.0 & 0.0 \\
 0.0 & 0.0 & -1.0 & 0.0 & -1.0 & -1.0 & 0.0 & 0.0 \\
 0.0 & 1900.0 & -380.0 & 0.0 & 678.0 & 0.0 & 0.0 & 0.0 \\
 0.0 & 0.0 & 0.0 & -163.0 & -678.0 & 326.0 & 406.8 & 0.0 \\
 -77.6 & -1900.0 & 0.0 & 163.0 & 0.0 & 0.0 & 0.0 & 211.5
 \end{bmatrix}
 \begin{bmatrix}
 \Delta Q_1 \\
 \Delta Q_2 \\
 \Delta Q_3 \\
 \Delta Q_4 \\
 \Delta Q_5 \\
 \Delta Q_6 \\
 \Delta Q_7 \\
 \Delta Q_8
 \end{bmatrix}
 =
 \begin{bmatrix}
 -0.4000 \\
 -0.0500 \\
 -0.1000 \\
 0.1000 \\
 0.4500 \\
 -625.5000 \\
 96.2550 \\
 469.2475
 \end{bmatrix}$$

Solving this matrix equation using a computer program, we obtain the discharge corrections as $\Delta Q_1 = 0.0419 \text{ m}^3/\text{sec}$, $\Delta Q_2 = -0.2513 \text{ m}^3/\text{sec}$, $\Delta Q_3 = -0.1068 \text{ m}^3/\text{sec}$, $\Delta Q_4 = 0.0233 \text{ m}^3/\text{sec}$, $\Delta Q_5 = -0.2780 \text{ m}^3/\text{sec}$, $\Delta Q_6 = -0.0652 \text{ m}^3/\text{sec}$, $\Delta Q_7 = -0.1652 \text{ m}^3/\text{sec}$, and $\Delta Q_8 = -0.0419 \text{ m}^3/\text{sec}$. Thus, for the second iteration we will use $Q_1 = 0.2419 \text{ m}^3/\text{sec}$, $Q_2 = 0.2487 \text{ m}^3/\text{sec}$, $Q_3 = -0.0068 \text{ m}^3/\text{sec}$, $Q_4 = 0.0733 \text{ m}^3/\text{sec}$, $Q_5 = 0.2220 \text{ m}^3/\text{sec}$, $Q_6 = 0.0348 \text{ m}^3/\text{sec}$, $Q_7 = 0.1348 \text{ m}^3/\text{sec}$, and $Q_8 = 0.2081 \text{ m}^3/\text{sec}$. The same procedure will be repeated until the all the corrections become negligible. The table below summarizes the Q values obtained by the iteration process.

Flow Rates (m^3/sec)								
Iteration Number	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Initial	0.2000	0.5000	0.1000	0.0500	0.5000	0.1000	0.3000	0.2500
1	0.2419	0.2487	-0.0068	0.0733	0.2220	0.0348	0.1348	0.2081
2	0.2511	0.1226	0.1286	0.0821	0.1047	0.0168	0.1168	0.1989
3	0.1989	0.0938	0.1051	0.0925	0.0863	0.0585	0.1585	0.2511
4	0.2007	0.0932	0.1075	0.0961	0.0894	0.0532	0.1532	0.2493
5	0.2008	0.0933	0.1075	0.0961	0.0894	0.0531	0.1531	0.2492
6	0.2008	0.0933	0.1075	0.0961	0.0894	0.0531	0.1531	0.2492

The results are obtained in six iterations. These results are essentially the same as those of Example 4.9 except for rounding off. The resulting total heads are $H_A = 85$ m, $H_B = 77.18$ m, $H_C = 60.65$ m, $H_D = 75.72$ m, $H_E = 59.83$ m, $H_F = 55.23$ m, and $H_G = 102.00$ m. The resulting pressure heads at nodes A, B, C, D, E, F, and G, respectively are 37.00 m, 31.18 m, 17.65 m, 27.72 m, 15.83 m, 7.23 m, and 42.00 m. Again, these results are practically the same as Example 4.9. The discrepancies are from round-off errors.

4.5 Water Hammer Phenomenon in Pipelines

A sudden change of flow rate in a large pipeline (caused by valve closure, pump shutoff, etc.) may affect a large mass of water moving inside the pipe. The force resulting from changing the speed of the water mass could cause a pressure rise in the pipe with a magnitude several times greater than the normal static pressure in the pipe. This phenomenon is commonly known as the *water hammer phenomenon*. The excessive pressure may fracture the pipe walls or cause other damage to the pipeline system. The possible occurrence of water hammer, its magnitude, and the propagation of the pressure wave must be carefully investigated in connection with the pipeline design.

The sudden change of pressure from valve closure may be viewed as the consequence of the force developed in the pipe necessary to stop the flowing water column. The column has a total mass m and is changing its velocity at the rate of dV/dt . According to Newton’s second law of motion,

$$F = m \frac{dV}{dt} \tag{4.20}$$

If the velocity of the entire water column could be reduced to zero instantly, Equation 4.20 would become

$$F = \frac{m(V_0 - 0)}{0} = \frac{mV_0}{0} = \infty$$

The resulting force (hence, pressure) would be infinite. Fortunately, such an instantaneous change is impossible because a mechanical valve requires a certain amount of time to complete a closure operation. In addition, neither the pipe walls nor the water column involved are perfectly rigid under large pressure. The elasticity of both the pipe walls and the water column play very important roles in the water hammer phenomenon.

To examine the water hammer phenomenon more thoroughly, consider a pipe of length L with an inside diameter D , a wall thickness e , and a modulus of elasticity E_p . Furthermore, assume that water is flowing from a reservoir through the pipe and a valve is at the end of the pipe as depicted in Figure 4.11 (a). Assuming losses (including friction) are negligible, the energy grade line is depicted as a horizontal line. Immediately following valve closure, the water in close proximity to the valve is brought to rest. The sudden change of velocity in the water mass causes a local pressure increase. As a result of this pressure increase, the water column in this section is somewhat compressed, and the pipe walls expand slightly from the corresponding increase of stress in the walls. Both of these phenomena help provide a little extra volume, allowing water to enter the section continuously until it comes to a complete stop.

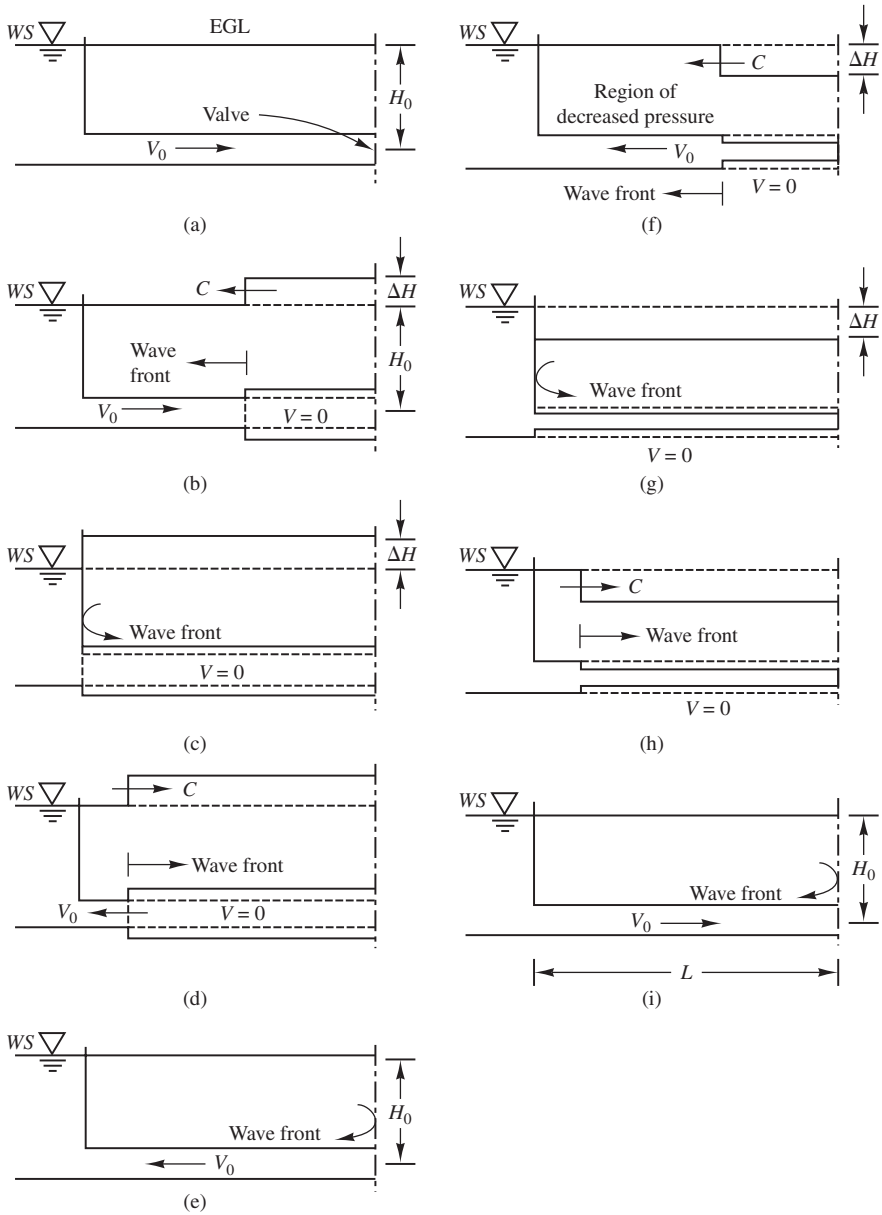


Figure 4.11 Propagation of water hammer pressure waves (friction in pipe neglected).
 (a) Steady-state condition prior to valve movement. (b) Transient conditions at $t < L/C$.
 (c) Transient conditions at $t = L/C$. (d) Transient conditions at $L/C < t < 2L/C$.
 (e) Transient conditions at $t = 2L/C$. (f) Transient conditions at $2L/C < t < 3L/C$.
 (g) Transient conditions at $t = 3L/C$. (h) Transient conditions at $3L/C < t < 4L/C$.
 (i) Transient conditions at $t = 4L/C$. Note: After $t = 4L/C$, the cycle repeats and continues indefinitely if the friction in the pipe is zero. The symbol \odot or \ominus is used to denote the direction of reflection of the wave front.

The next section immediately upstream is involved in the same procedure an instant later. In this manner, a wave of increased pressure propagates up the pipe toward the reservoir as shown in Figure 4.11 (b). When this *pressure wave* reaches the upstream reservoir, the entire pipe is expanded, and the water column within is compressed by the increased pressure. At this very instant, the entire water column within the pipe comes to a complete halt.

This transient state cannot be maintained because the EGL in the pipe is much higher than the EGL of the open reservoir. Because energy differences create flow, the halted water in the pipe flows back into the reservoir as soon as the pressure wave reaches the reservoir. This process starts at the reservoir end of the pipe, and a decreased pressure wave travels downstream toward the valve, as shown in Figure 4.11 (d). During this period, the water behind the wave front moves in the upstream direction as the pipe continuously contracts and the column decompresses. The time required for the pressure wave to return to the valve is $2L/C$, where C is the speed of the wave traveling through the pipe. It is also known as *celerity*.

The speed of pressure wave travel in a pipe depends on the modulus of elasticity of water E_b , and the modulus of elasticity of the pipe wall material E_p . The relationship may be expressed as

$$C = \sqrt{\frac{E_c}{\rho}} \tag{4.21}$$

where E_c is the composite modulus of elasticity of the water-pipe system and ρ is the density of water. E_c is a function of the elasticity of the pipe walls and the elasticity of the fluid within. It may be calculated by the following relationship:

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{Dk}{E_p e} \tag{4.22a}$$

The modulus of elasticity of water, E_b , and the density of water are given in Chapter 1 (and the front jacket of the book). The modulus of elasticity of various common pipe materials is listed in Table 4.1; k is a constant depending on the method of pipeline anchoring, and e is the thickness of the pipe walls. Typical k values are

$k = (1 - \epsilon^2)$ for pipes anchored at both ends against longitudinal movement,

$k = \left(\frac{5}{4} - \epsilon\right)$ for pipes free to move longitudinally (negligible stresses), and

$k = (1 - 0.5\epsilon)$ for pipes with expansion joints,

TABLE 4.1 Modulus of Elasticity (E_p) of Common Pipe Materials

Pipe Material	E_p (N/m ²)	E_p (psi)
Aluminum	7.0×10^{10}	1.0×10^7
Brass, bronze	9.0×10^{10}	1.3×10^7
Concrete, reinforced	1.6×10^{11}	2.5×10^7
Copper	9.7×10^{10}	1.4×10^7
Glass	7.0×10^{10}	1.0×10^7
Iron, cast	1.1×10^{11}	1.6×10^7
Iron, ductile	1.6×10^{11}	2.3×10^7
Lead	3.1×10^8	4.5×10^4
Lucite	2.8×10^8	4.0×10^4
Rubber, vulcanized	1.4×10^{10}	2.0×10^6
Steel	1.9×10^{11}	2.8×10^7

where ε is the *Poisson's ratio* of the pipe wall material. For common pipe materials, $\varepsilon = 0.25$ is often used.

If the longitudinal stress in a pipe can be neglected—that is, $k = \left(\frac{5}{4} - \varepsilon\right) = 1.0$ —Equation 4.22a can be simplified to

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{D}{E_p e} \quad (4.22b)$$

Figure 4.11 (e) shows that by the time the decreased pressure wave arrives at the valve, the entire column of water within the pipe is in motion in the upstream direction. This motion cannot pull any more water from beyond the already closed valve and is stopped when the wave arrives at the valve. The inertia of this moving water mass causes the pressure at the valve to drop below the normal static pressure. A third oscillation period begins as a wave of negative pressure propagates up the pipe toward the reservoir as shown in Figure 4.11 (f). At the instant the negative pressure reaches the reservoir, the water column within the pipe again comes to a complete standstill, and the EGL of the pipe is less than the EGL at the reservoir [Figure 4.11 (g)]. Because of this energy difference, water flows into the pipe starting a fourth period of oscillation.

The fourth period is marked by a wave of normal static pressure moving downstream toward the valve as in Figure 4.11 (h). The water mass behind the wave front also moves in the downstream direction. This fourth-period wave arrives at the valve at time $4L/C$, the entire pipe returns to the original EGL, and the water in the pipe is moving in the downstream direction. For an instant, the conditions throughout the pipe are somewhat similar to the conditions at the time of valve closure (the beginning of the first-period wave), except that the water velocity in the pipe has been reduced. This is a result of energy losses to heat from friction and the viscoelastic behavior of the pipe walls and the water column.

Another cycle begins instantly. The four sequential waves travel up and down the pipe in exactly the same manner as the first cycle described above, except that the corresponding pressure waves are smaller in magnitude. The pressure-wave oscillation continues with each set of waves successively diminishing until finally the waves die out completely.

As mentioned previously, the closure of a valve usually requires a certain period of time t to complete. If t is less than $2L/C$ (valve closure is completed before the first pressure wave returns to the valve), the resulting rise in pressure should be the same as that of instantaneous closure. However, if t is greater than $2L/C$, then the first pressure wave returns to the valve before the valve is completely closed. The returned negative pressure wave can offset the pressure rise resulting from the final closure of the valve.

Knowing the maximum pressure rise created by the water hammer phenomenon is critical for the safe and reliable design of many pipeline systems. The appropriate design equations are based on fundamental principles and are derived as follows.

Consider a pipe with a rapidly closing valve ($t \leq 2L/C$); the extra volume of water (ΔVol) that enters the pipe during the first period ($t = L/C$) [Figure 4.11 (c)] is

$$\Delta\text{Vol} = V_0 A \left(\frac{L}{C}\right) \quad (4.23)$$

where V_0 is the initial velocity of water flowing in the pipe and A is the pipe cross-sectional area. The resulting pressure rise ΔP is related to this extra volume by

$$\Delta P = E_c \left(\frac{\Delta\text{Vol}}{\text{Vol}}\right) = \frac{E_c (\Delta\text{Vol})}{AL} \quad (4.24)$$

where V_0 is the original volume of the water column in the pipe and E_c is the composite modulus of elasticity as defined by Equation 4.22a. Substituting Equation 4.23 into Equation 4.24, we may write

$$\Delta P = \frac{E_c}{AL} \left[V_0 A \left(\frac{L}{C} \right) \right] = \frac{E_c V_0}{C} \tag{4.25a}$$

As the pressure wave propagates upstream along the pipe at speed C , the water behind the wave front is immediately brought to a stop from the initial velocity of V_0 . The total mass of water involved in this sudden change of speed from V_0 to zero in time Δt is $m = \rho AC \Delta t$. Applying Newton’s second law to this mass, we have

$$\Delta P(A) = m \frac{\Delta V}{\Delta t} = \rho AC \Delta t \frac{(V_0 - 0)}{\Delta t} = \rho AC V_0$$

or

$$\Delta P = \rho C V_0 \tag{4.25b}$$

(Note: Equation 4.21 can be derived from Equation 4.25b; see Problem 4.5.10.) Solving Equation 4.25b for C and substituting for C in Equation 4.25a, we have

$$\Delta P = E_c V_0 \frac{\rho V_0}{\Delta P}$$

or

$$\Delta P = V_0 \sqrt{\rho E_c} \tag{4.25c}$$

Also,

$$\Delta H = \frac{\Delta P}{\rho g} = \frac{V_0}{g} \sqrt{\frac{E_c}{\rho}} = \frac{V_0}{g} C \tag{4.26}$$

where ΔH is the pressure head rise caused by water hammer. These equations are only applicable for rapid valve closure ($t \leq 2L/C$).

For valve closures that are not rapid (i.e., $t > 2L/C$), the pressure rise previously discussed (ΔP) will not develop fully because the reflected negative wave arriving at the valve will reduce the pressure rise. For these slow valve closures, the maximum water hammer pressure may be calculated by the Allievi equation,* which is expressed as

$$\Delta P = P_0 \left(\frac{N}{2} + \sqrt{\frac{N^2}{4} + N} \right) \tag{4.27}$$

where P_0 is the static-state pressure in the pipe, and

$$N = \left(\frac{\rho L V_0}{P_0 t} \right)^2$$

* L. Allievi, “The Theory of Water Hammer” (translated by E. E. Halmos), *Trans. ASME* (1929).

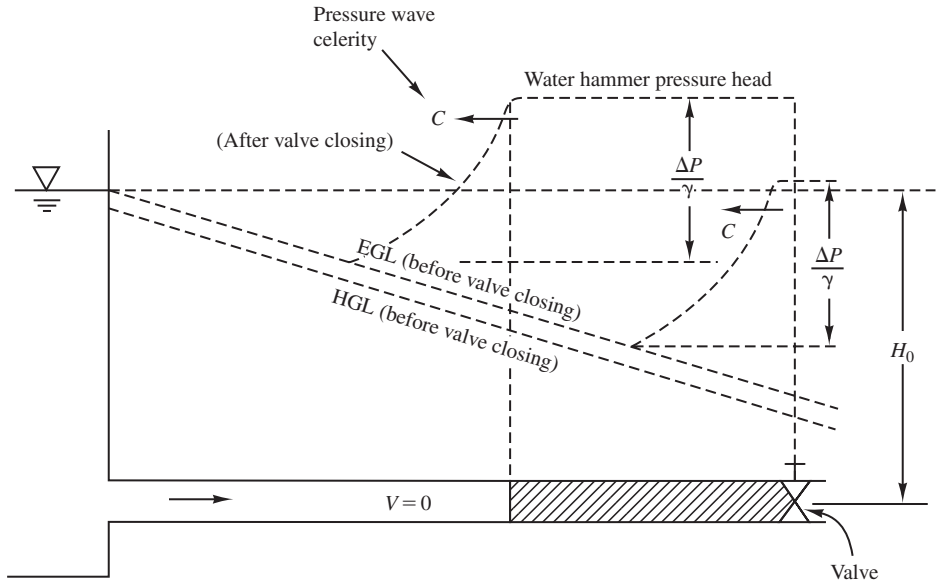


Figure 4.12 Water hammer pressure in a pipeline

Before applying the water hammer equations to pipe flow problems, the energy grade line and the hydraulic grade line for the pipe system under steady flow conditions must be determined, as shown in Figure 4.12. As the pressure wave travels up the pipeline, energy is being stored in the form of pressure in the pipe behind the wave front. Maximum pressure is reached when the wave front arrives at the reservoir,

$$P_{\max} = \gamma H_0 + \Delta P \tag{4.28}$$

where H_0 is the total head before the valve closure, as indicated by the water surface elevation in the reservoir. The location immediately downstream of the reservoir is generally the most vulnerable to pipe and joint damage because the initial pressure is larger here than in the rest of the pipeline.

Example 4.11

A steel pipe 5,000 ft long laid on a uniform slope has an 18-in. diameter and a 2-in. wall thickness. The pipe carries water from a reservoir and discharges it into the air at an elevation 150 ft below the reservoir free surface. A valve installed at the downstream end of the pipe permits a flow rate of 25 cfs. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Assume the longitudinal stresses in the pipeline are negligible.

Solution

From Equation 4.22b,

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{D}{E_p e}$$

where $E_b = 3.2 \times 10^5$ psi (from Chapter 1 or the front jacket of the book) and $E_p = 2.8 \times 10^7$ psi (Table 4.1). The above equation may thus be written as

$$\frac{1}{E_c} = \frac{1}{3.2 \times 10^5} + \frac{18}{(2.8 \times 10^7)2.0}$$

Hence,

$$E_c = 2.90 \times 10^5 \text{ psi}$$

From Equation 4.21 we may obtain the speed of wave propagation along the pipe as

$$C = \sqrt{\frac{E_c}{\rho}} = \sqrt{\frac{2.90 \times 10^5(144)}{1.94}} = 4,640 \text{ ft/sec}$$

The time required for the wave to return to the valve is

$$t = \frac{2L}{C} = \frac{2(5,000)}{4,640} = 2.16 \text{ sec}$$

Because the valve closes in 1.4 sec (< 2.16 sec), rapid-valve-closure equations may be applied. Therefore, water velocity in the pipe before valve closure is

$$V_0 = \frac{25}{\frac{\pi}{4}(1.5)^2} = 14.1 \text{ ft/sec}$$

and the maximum water hammer pressure at the valve can be calculated using Equation 4.25b as

$$\Delta P = \rho CV_0 = 1.94(4,640)(14.1) = 1.27 \times 10^5 \text{ lb/ft}^2 (881 \text{ psi})$$

Example 4.12

A ductile-iron pipe with a 20-cm diameter and 15-mm thick walls is carrying water when the outlet is suddenly closed. If the design discharge is 40 L/sec, calculate the pressure head rise caused by water hammer if

- (a) the pipe wall is rigid,
- (b) the pipe is free to move longitudinally (negligible stresses), and
- (c) the pipeline has expansion joints throughout its length.

Solution

$$A = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$

hence,

$$V_0 = \frac{Q}{A} = \frac{0.04}{0.0314} = 1.27 \text{ m/sec}$$

- (a) For rigid pipe walls, $Dk/E_{pe} = 0$, Equation 4.22a gives the following relation:

$$\frac{1}{E_c} = \frac{1}{E_b} \quad \text{or} \quad E_c = E_b = 2.2 \times 10^9 \text{ N/m}^2$$

From Equation 4.21, we can calculate the speed of pressure wave:

$$C = \sqrt{\frac{E_c}{\rho}} = \sqrt{\frac{2.2 \times 10^9}{998}} = 1,480 \text{ m/sec}$$

From Equation 4.26, we can calculate the pressure head rise caused by water hammer as

$$\Delta H = \frac{V_0 C}{g} = \frac{1.27(1,480)}{9.81} = 192 \text{ m (H}_2\text{O)}$$

- (b) For pipes with no longitudinal stress, $k = 1$, we may use Equation 4.22b:

$$E_c = \frac{1}{\left(\frac{1}{E_b} + \frac{D}{E_{pe}}\right)} = \frac{1}{\left(\frac{1}{2.2 \times 10^9} + \frac{0.2}{(1.6 \times 10^{11})(0.015)}\right)} = 1.86 \times 10^9$$

and

$$C = \sqrt{\frac{E_c}{\rho}} = 1,370 \text{ m/sec}$$

Hence, the pressure head rise caused by the water hammer can be calculated as

$$\Delta H = \frac{V_0 C}{g} = \frac{1.27(1,370)}{9.81} = 177 \text{ m (H}_2\text{O)}$$

Note that once the pipe is free to move longitudinally [as opposed to the rigid pipe considered in part (a)], some of the pressure energy is absorbed by the expanding pipe, and the wave speed is reduced. This in turn reduces the magnitude of the head and pressure rise associated with the water hammer.

- (c) For pipes with expansion joints, $k = (1 - 0.5 \times 0.25) = 0.875$. From Equation 4.22a,

$$E_c = \frac{1}{\left(\frac{1}{E_b} + \frac{Dk}{E_{pe}}\right)} = \frac{1}{\left(\frac{1}{2.2 \times 10^9} + \frac{(0.2)(0.875)}{(1.6 \times 10^{11})(0.015)}\right)} = 1.90 \times 10^9$$

and

$$C = \sqrt{\frac{E_c}{\rho}} = 1,380 \text{ m/sec}$$

Again, we can calculate the pressure head rise caused by water hammer as

$$\Delta H = \frac{V_0 C}{g} = \frac{1.27 \cdot 1,380}{9.81} = 179 \text{ m (H}_2\text{O)}$$

which is essentially the same as case (b) (a pipe free to move longitudinally).

In water hammer analysis, the time history of pressure oscillation in a pipeline is instructive. Because of the friction between the oscillatory water mass and the pipe wall, the pressure–time pattern is modified, and the oscillation gradually dies out. A typical pressure oscillation is shown in Figure 4.13.

In reality, a valve cannot be closed instantaneously. The time required for closure of a valve is a certain period, t_c . The water hammer pressure increases gradually with the rate of closure of the valve. The typical valve closure curve is shown in Figure 4.14.

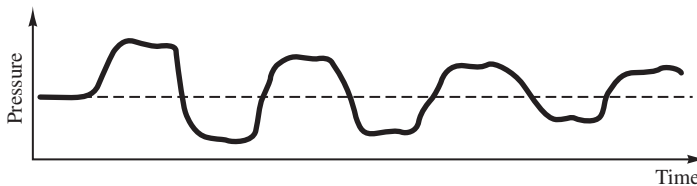


Figure 4.13 Friction effects on water hammer's pressure–time pattern

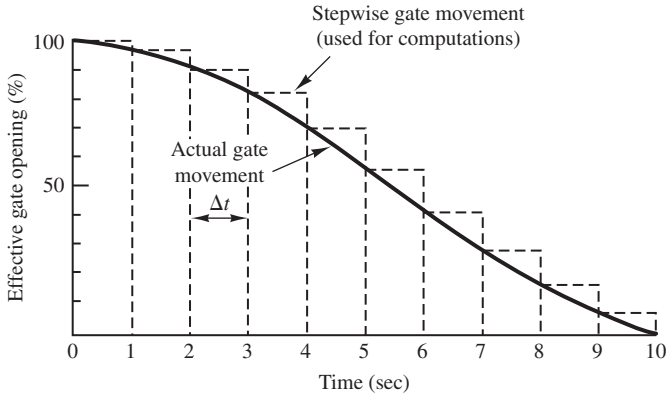


Figure 4.14 Typical valve closure curve

If t_c is smaller than the time required for the wave front to make a round trip along the pipeline and return to the valve site ($t_c < 2L/C$), the operation is defined as *rapid closure*. The water hammer (or shock) pressure will reach its maximum value. The computation of a rapid closure operation is the same as that of an instantaneous closure. To keep water hammer pressure within acceptable limits, valves are commonly designed with closure times considerably greater than $2L/C$. For slow closure operation ($t_c > 2L/C$), the pressure wave returns to the valve site before the closure is completed. A certain amount of water continuously passes through the valve when the pressure wave returns. As a result, the pressure wave pattern will be altered. A complete treatment of the water hammer phenomenon, with consideration of friction and slow valve closure operation, may be found in Chaudhry (1987)* and Popescu et al. (2003).[†]

4.6 Surge Tanks

There are many ways to eliminate the detrimental impacts of water hammer in pipelines. One method is slow valve closure, which was discussed in the last section. Other methods that are effective include relief valves (or diverters) and surge tanks. Relief valves rely on the water hammer pressure to open a valve and divert a large portion of the flow for a short period of time. Although relief valves may be a simple solution to the problem, it results in a waste of water.

The inclusion of a surge tank near the control station (Figure 4.15) in a pipeline will harness the forces produced when a mass of water is slowed or stopped. A surge tank is defined as a stand pipe or storage reservoir placed at the downstream end of a long pipeline to prevent sudden pressure increases (from rapid valve closure) or sudden pressure decreases (from rapid valve opening). When a valve is being closed, the large mass of water moving in a long pipeline takes time to adjust accordingly. The difference in flow between the pipeline and that allowed to pass the closing valve causes a rise of water level in the surge tank. As the water rises above the level of the reservoir, an energy imbalance is produced so that the water in the pipeline flows back toward the reservoir and the water level in the surge tank drops. The cycle is repeated with *mass oscillation* of water in the pipeline and the surge tank until it is gradually damped out by friction.

* M. Hanif Chaudhry, *Applied Hydraulic Transients*, 2nd ed. (New York: Van Nostrand Reinhold, 1987).

[†] M. Popescu, D. Arsenie, and P. Vlase, *Applied Hydraulic Transients: For Hydropower Plants and Pumping Stations* (London: Taylor & Francis, 2003).

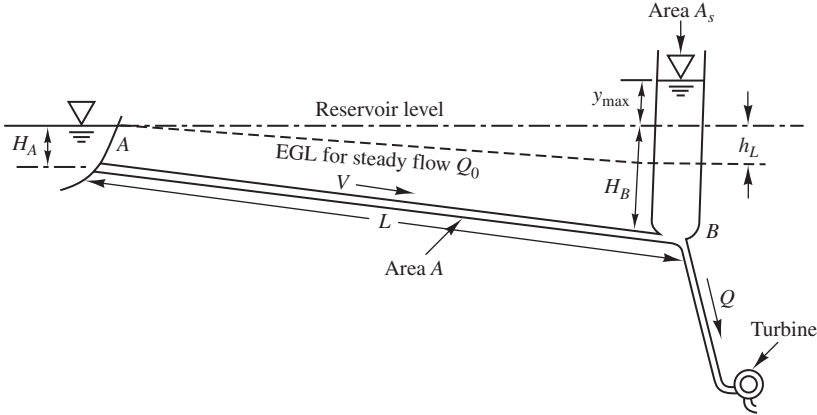


Figure 4.15 Surge tank

Newton’s second law can be applied to analyze the effect of the surge tank on the water column AB , between the two ends of the pipeline. At any time during the closure or opening of the valve, the acceleration of water mass is always equal to the forces acting on it; that is,

$$\begin{aligned} \rho LA \frac{dV}{dt} = & \text{(the pressure force on the column at A)} \\ & + \text{(the weight component of the column in the direction of the pipeline)} \\ & - \text{(the pressure force acting on the column at B)} \\ & \pm \text{(friction losses)} \end{aligned}$$

The pressure force at A is the result of the elevation difference between the water surface in the reservoir and the pipeline inlet, modified by the entrance loss. The pressure force acting on the column at B depends on the elevation of the water surface in the surge tank, also modified by the losses occurring at the entry (may be a restrictive throttle) to the tank. Hence,

$$\begin{aligned} \rho LA \frac{dV}{dt} = \rho gA [& (H_A \pm \text{entrance loss}) + (H_B - H_A) \\ & - (H_B + y \pm \text{throttle loss}) \pm \text{(pipeline losses)}] \end{aligned} \quad (4.29)$$

The sign of the pipeline losses depends on the direction of the flow. The losses always occur in the direction of the flow.

If we introduce the modulus form, $h_L = K_f V |V|$ and $H_T = K_T U |U|$, where

$$U = \frac{dy}{dt} \quad (4.30)$$

and represents the upward velocity of the water surface in the tank, the sign of the losses would always be correct. Here K_f is the pipeline friction factor $K_f = fL/(2gD)$ and h_L is the total head loss in the pipeline between A and B . H_T is the throttle loss.

Substituting these values into Equation 4.29 and simplifying, we have the dynamic equation for the surge tank:

$$\frac{L}{g} \frac{dV}{dt} + y + K_f V |V| + K_T U |U| = 0 \tag{4.31}$$

In addition, the continuity condition at *B* must be satisfied

$$VA = UA_s + Q \tag{4.32}$$

where *Q* is the discharge allowed to pass the closing valve at any given time *t*.

The combination of Equations 4.30 through 4.32 produces second-order differential equations that can only be solved explicitly for special cases. A special solution may be obtained by what is frequently called the *logarithmic method*.* The method provides simple theoretical analysis of surge heights that are close to those observed in practice if the cross-sectional area *A_s* remains constant.

The solution for a simple (unrestricted) constant-area surge tank (Figure 4.15) may be expressed as

$$\frac{y_{\max} + h_L}{\beta} = \ln\left(\frac{\beta}{\beta - y_{\max}}\right) \tag{4.33}$$

where *β* is the damping factor, which is defined as

$$\beta = \frac{LA}{2gK_f A_s} \tag{4.34}$$

Equation 4.33 is an implicit equation and may be solved for the surge height (*y*) by successive approximations or by computer algebra software (e.g., Mathcad, Maple, or Mathematica) as demonstrated in the following example.

Example 4.13

A simple surge tank 8.00 m in diameter is located at the downstream end of a 1,500-m-long pipe, 2.20 m in diameter. The head loss between the upstream reservoir and the surge tank is 15.1 m when the flow rate is 20.0 m³/sec. Determine the maximum elevation of the water in the surge tank if a valve downstream suddenly closes.

Solution

For a smooth entrance where the head loss may be neglected, we may write

$$h_L \cong h_f = K_f V^2$$

or

$$K_f = \frac{h_L}{V^2} = \frac{15.1}{(5.26)^2} = 0.546 \text{ sec}^2/\text{m}$$

and the damping factor, from Equation 4.34, is

$$\beta = \frac{LA}{2gK_f A_s} = \frac{(1,500)(3.80)}{2(9.81)(0.546)(50.3)} = 10.6 \text{ m}$$

* John Pickford, *Analysis of Water Surge* (New York: Gordon and Beach, 1969), pp. 111–124.

By applying Equation 4.33,

$$\frac{y_{\max} + 15.1}{10.6} = \ln \left(\frac{10.6}{10.6 - y_{\max}} \right)$$

The solution is obtained by an iterative process.

y_{\max}	LHS	RHS
9.50	2.32	2.27
9.60	2.33	2.36
9.57	2.33	2.33

The maximum elevation of the water in the surge tank is 9.57 m over the reservoir level. The same solution is obtained using calculators or software that solve implicit equations.

PROBLEMS (SECTION 4.1)

- 4.1.1.** Examine the EGL and HGL in Figure 4.1 and explain the following:
- the location of the EGL at the reservoirs
 - the drop in the EGL moving from reservoir A into pipe 1
 - the slope of the EGL in pipe 1
 - the separation distance between the EGL and the HGL
 - the drop in the EGL moving from pipe 1 to pipe 2
 - the steeper slope of the EGL in pipe 2 (steeper than pipe 1)
 - the drop in the EGL moving from pipe 2 to reservoir B
- 4.1.2.** Sketch the energy grade line and the hydraulic grade line for the pipeline shown in Figure P4.1.2. Consider all the losses and changes of velocity and pressure heads.

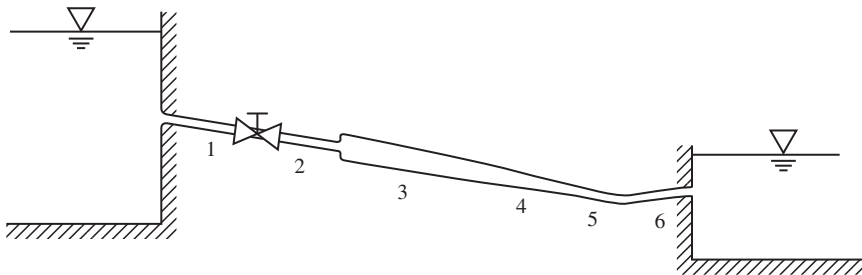


Figure P4.1.2

- 4.1.3.** Often the iterative procedure encountered in pipeline problems can be shortened by assuming complete turbulence in the pipe (if e/D is available) to obtain a preliminary friction factor. This assumption is often valid for water-transmission systems because the viscosity of water is low and the velocities or pipe diameters are large, which yields high N_R values. Refer to the Moody diagram and determine three to five combinations of velocities and pipe size that would produce complete turbulence at a friction factor of 0.02 (assuming water at 20°C).

- 4.1.4.** At a water-treatment plant, water (68°F) flows from tank A to tank B at a rate of 0.50 cfs through a 3-in.-diameter, cast-iron pipe that is 200 feet long. Determine the difference in water surface elevation between tanks (open to the atmosphere) if there are two bends ($R/D = 2.0$) and a fully open gate valve in the pipeline.
- 4.1.5.** Determine the elevation of the upstream reservoir (A) in Figure P4.1.2 if the downstream reservoir (B) is at 750 m, the flow rate (water at 20°C) through the smooth concrete pipeline is 1.2 m³/sec, given the following:
- | | |
|--|------------------------------------|
| pipes 1 and 2: 100 m long ($D = 0.5$ m) | valves 1–2: fully open globe valve |
| expansion 2–3: $D = 0.5$ m to 1.0 m | pipe 3: 100 m long ($D = 1.0$ m) |
| confusor 4: loss coefficient of 0.3 | pipe 5: 50 m long ($D = 0.5$ m) |
| bend 5–6: loss coefficient of 0.2 | pipe 6: 50 m long ($D = 0.5$ m) |
- 4.1.6.** The 40-m-long, 4-in. commercial steel pipe connects reservoirs A and B as shown in Figure P4.1.6. Determine the pressure at each point designated in the figure if the water flow is 10.1 liters/sec (20°C), reservoir A is subjected to a pressure of 9.79 kPa (gauge), all valves are fully open, and bend losses are negligible.

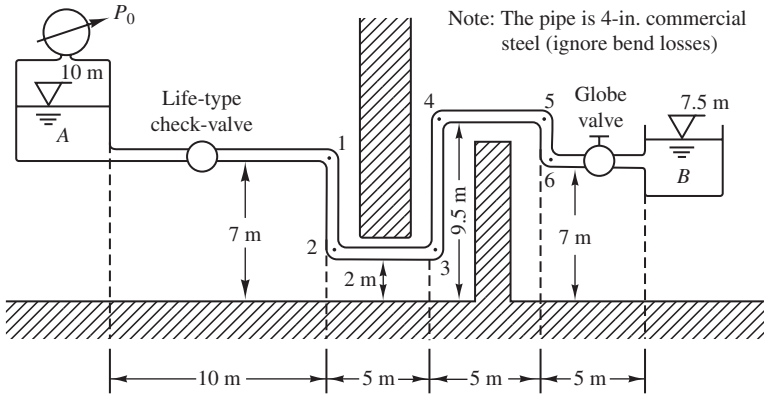


Figure P4.1.6

- 4.1.7.** The 40-m-long, 4-in. diameter commercial steel pipe connects reservoirs A and B as shown in Figure P4.1.6. The water temperature is 20°C. Determine the flow rate (liters/sec) if reservoir A is subjected to atmospheric pressure, the globe valve is fully open, and bend losses are negligible.
- 4.1.8.** A 40-cm straight pipe is used to carry water at 20°C from reservoir A to reservoir B that is 0.7 km away. There is an elevation difference of 9 m between the two reservoirs. Determine the discharge for the following pipes: (a) commercial steel, (b) cast iron, and (c) smooth concrete. Determine the percentage of flow gain if the highest capacity pipe material were chosen instead of the lowest-capacity pipe material.
- 4.1.9.** Water flows from tank A to tank B (Figure P4.1.9), and the water surface elevation difference is 60 feet. Assume the water temperature is 68°F, the pipe material is cast iron, and the pipeline has the following characteristics:
- | | |
|-------------|---------------------------------|
| large pipe: | 1,000 ft long ($D = 16$ in.) |
| bends: | four ($R/D = 4$ in large pipe) |
| small pipe: | 1,000 ft long ($D = 8$ in.) |
| expansion: | sudden |

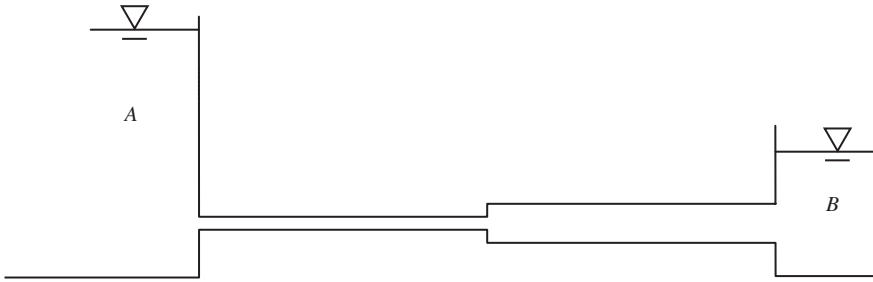


Figure P4.1.9

Determine the system's existing flow rate. What percentage increase in the flow would occur if the 8-in. line were completely replaced with the 16-in. pipeline?

- 4.1.10.** An irrigation company must transport $5.71 \times 10^{-2} \text{ m}^3/\text{sec}$ of water (20°C) from reservoir A to reservoir B. The reservoirs are separated by 600 m and have an elevation difference of 18.4 m. Determine the diameter pipe required if the relative roughness of the pipe material is 0.36 mm. (Include minor losses.) Would the solution differ if minor losses were neglected?
- 4.1.11.** A 75-ft pipe must transport 2.5 cfs of 40°F water from a head tank to a cooling pond. The elevation difference between the tank and the pond is 4.6 ft. Determine the size of the commercial steel pipe that is required. Assume square-edged connections and include a globe valve in the pipe.
- 4.1.12.** It is necessary to deliver 5 L/min of a water-glycerol solution (sp gr. = 1.1; $\nu = 1.03 \times 10^{-5} \text{ m}^2/\text{sec}$) under a pressure head of 50 mm Hg. A glass tube is used ($e = 0.003 \text{ mm}$). Determine the tube diameter if the tube length is 2.5 m. Assume the pressure head (50 mm Hg) is needed to overcome the friction loss in the horizontal tube; no other losses are considered.
- 4.1.13.** The 40-m-long, 4-in. commercial steel pipe connects reservoirs A and B as shown in Figure P4.1.6. If the pressure at point 1 is 39.3 kPa, what is the pressure P_0 at reservoir A? Assume the water is at 20°C , all valves are fully open, and bend losses are negligible.
- 4.1.14.** All the pipes in Figure P4.1.14 have a Hazen-Williams coefficient of 100. Pipe AB is 3,000 ft long and has a diameter of 2.0 ft. Pipe BC_1 is 2,800 ft long with a diameter of 1.0 ft, and pipe BC_2 has a length of 3,000 ft and a diameter of 1.5 ft. Pipe CD is 2,500 ft long with a diameter of 2.0 ft. The water surface elevation of reservoir A is 230 ft (H_A) and reservoir D (H_D) is 100 ft. Determine the discharge in each pipe and the total head at points B and C if $Q_B = 0$ and $Q_C = 0$. Ignore minor losses.

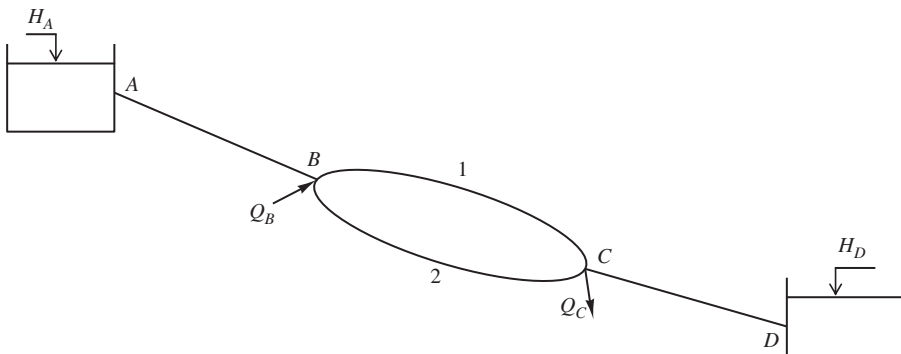


Figure P4.1.14

4.1.15. Redo Problem 4.1.14 with $Q_B = 8$ cfs and $Q_C = 8$ cfs. Should the discharge in AB in this case be greater or less than that of Problem 4.1.14? Why?

(SECTION 4.2)

4.2.1. Water flows in a new 20-cm, 300-m-long ductile iron pipe between reservoirs *A* and *B*, as shown in Figure P4.2.1. The pipe is elevated at *S*, which is 150 m downstream from reservoir *A*. The water surface in reservoir *B* is 25 m below the water surface in reservoir *A*. If $\Delta s = 7.0$ m, is cavitation a concern?

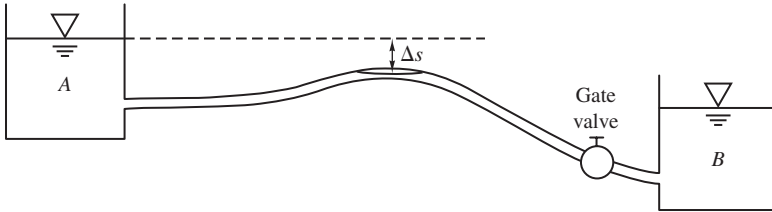


Figure P4.2.1

4.2.2. A 12-cm-diameter tube is 13 m long and used to siphon water from the reservoir and discharge it into the air, as shown in Figure P4.2.2. If the total head loss between the intake end of the tube and the summit, *S*, is 0.8 m and between *S* and the discharge end is 1.8 m, what is the discharge of the tube and pressure at *S*?

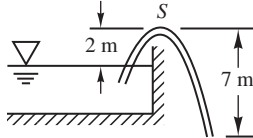


Figure P4.2.2

4.2.3. A siphon spillway, as shown in Figure P4.2.3, is 200 feet long with a diameter of 2 feet. It is used to discharge water (68°F) to a downstream reservoir 50 ft below the upstream reservoir. The friction losses in the rough concrete siphon are evenly distributed throughout its length. If the summit of the siphon is 5 feet above the upper reservoir and 60 feet from the siphon entrance, is cavitation a concern?

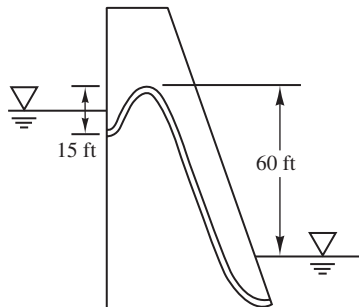


Figure P4.2.3

- 4.2.4. Do all siphons encounter negative pressure at their summits? Prove your answer by using an energy grade line and a hydraulic grade line sketch on Figure P4.2.2.
- 4.2.5. A confuser is installed in a 40-cm pipe. Immediately upstream from the confuser the water pressure is $84,000 \text{ N/m}^2$. Determine the minimum diameter of the confuser outlet that will keep the pressure head at the outlet above -8 m when the flow rate is 440 L/sec . (Note: The -8 m of gauge pressure is the threshold for dissolved gases in the water to begin vaporizing and disrupting the flow.)
- 4.2.6. A pump draws water from reservoir *A* and lifts it to a higher reservoir *B*, as shown in Figure P4.2.6. The head loss from *A* to the pump is four times the velocity head in the 10-cm pipe, and the head loss from the pump to *B* is seven times the velocity head. The pressure head at the inlet of the pump is -6 m . Compute the pressure head the pump must deliver. Why is the pressure head the pump must deliver greater than 50 m , the elevation difference between the two reservoirs? Draw the EGL and HGL.

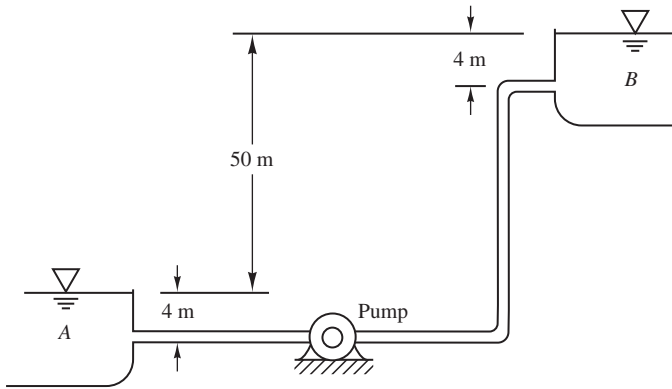


Figure P4.2.6

- 4.2.7. A pump is installed in a 100-m pipeline to lift water at 20°C from reservoir *A* to reservoir *B* (see Figure P4.2.6). The pipe is rough concrete with a diameter of 80 cm . The design discharge is $5 \text{ m}^3/\text{sec}$. Determine the maximum distance from reservoir *A* that the pump could be installed without encountering cavitation problems.
- 4.2.8. A pump installed at an elevation of 10 ft delivers 8 cfs of water (68°F) through a horizontal pipe system to a pressurized tank. The water surface elevation in the receiving tank is 20 ft , and the pressure at the top of the tank is 32.3 psi . The ductile iron pipe has a 15-in. -diameter on the suction side of the pump and a 12-in. -diameter (and 130 ft length) on the discharge side of the pump. If the pump delivers 111 ft of head, what is the pressure head on the discharge side of the pump (in psi)? Is cavitation a concern on the suction side of the pump? Sketch the system and draw the HGL and EGL.
- 4.2.9. An emergency pump is installed in the pipeline system of Example 4.4 at a distance of 500 m from reservoir *A*. The pump is used to boost the flow rate when needed. Determine the pressure head the pump must add to the pipeline to double the flow rate.
- 4.2.10. Water flows in a new 20-cm -diameter, 300-m -long ductile iron pipe between reservoirs *A* and *B*, as shown in Figure P4.2.1. The pipe is elevated at *S*, which is 150 m downstream from reservoir *A*. The water surface difference between the two reservoirs is 25 m . Determine the head that should be provided by a booster pump 100 m downstream from reservoir *A* if Δs is 3 m and the pressure head at that location must be kept above -6 m because of cavitation concerns. Sketch the EGL and the HGL for the pipeline.
- 4.2.11. A 40-m -long, 4-in. commercial steel pipe connects reservoirs *A* and *B* as shown in Figure P4.1.6. Determine the minimum pressure (P_0) that would keep the pressure head throughout the pipe positive. Assume all valves are fully open, bend losses are negligible, and the water temperature is 20°C .

(SECTION 4.3)

4.3.1. When solving a three-reservoir problem, it is advantageous to set the total energy elevation, P , at the junction to match exactly with the elevation of the middle reservoir for the first iteration.

- (a) What is the advantage of doing this? If the answer is not immediately apparent, trace back through the solution of Example 4.6 by using this assumption.
- (b) Use a computer algebra software system or a spreadsheet program to formulate the classic three-reservoir problem and verify its accuracy using Example 4.6. [Hint: If using a spreadsheet, use the Von Kármán equation (Equation 3.23) for the initial estimate of the Darcy-Weisbach friction factor assuming complete turbulence. Then use the Swamee-Jain equation* (Equation 3.24a, shown below) to solve for the friction factor once the N_R value is known.]

$$f = \frac{0.25}{\left[\log\left(\frac{e}{3.7D} + \frac{5.74}{N_R^{0.9}} \right) \right]^2}$$

(Retrieved from http://en.wikipedia.org/wiki/Swamee-Jain_equation.)

4.3.2. Determine flow rates in the branching pipe system depicted in Figure P4.3.2 given the following water surface (WS) elevation and pipe data (lengths and diameters):

WS1 = 5,200 ft	L1 = 6,000 ft	D1 = 4 ft
WS2 = 5,150 ft	L2 = 2,000 ft	D2 = 3 ft
WS3 = 5,100 ft	L3 = 8,000 ft	D3 = 5 ft

All of the pipes are lined ductile iron (DIP, $e = 0.0004$ ft), and the temperature of the water is 68°F. Also determine the elevation of the junction (J) if the pressure head (P/γ) at the junction measured by a piezometer (height from P to J) is 30 ft.

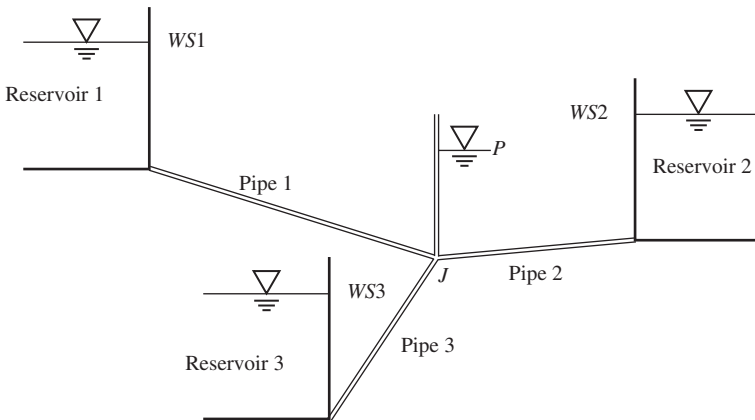


Figure P4.3.2

4.3.3. Determine flow rates in the branching pipe system depicted in Figure P4.3.2 given the following WS elevation and pipe data (lengths and diameters):

WS1 = 2,100 m	L1 = 5,000 m	D1 = 1.0 m
WS2 = 2,080 m	L2 = 4,000 m	D2 = 0.3 m
WS3 = 2,060 m	L3 = 5,000 m	D3 = 1.0 m

All of the pipes are commercial steel ($e = 0.045$ mm), and the temperature of the water is 20°C . Also determine the pressure head (P/γ) at the junction (height from J to P) if the elevation of the junction (J) is 2,070 m. Finally, is it possible to estimate the velocity head at the junction? (Recall that the velocity head at the junction was assumed to be negligible in branching pipe problems.)

- 4.3.4.** Solve Problem 4.3.2 using the Hazen-Williams equation ($C_{HW} = 140$ for lined ductile iron) for friction losses instead of the Darcy-Weisbach equation.
- 4.3.5.** Solve Problem 4.3.2 using the Manning equation ($n = 0.011$ for ductile iron) for friction losses instead of the Darcy-Weisbach equation.
- 4.3.6.** The highest reservoir in a three-reservoir branching system is inaccessible after a mountain storm. Determine the surface elevation of this reservoir given the following water surface elevation and pipe data (lengths and diameters):

WS1 = ?	$L1 = 2,000$ m	$D1 = 0.30$ m
WS2 = 4,080 m	$L2 = 1,000$ m	$D2 = 0.20$ m
WS3 = 4,060 m	$L3 = 3,000$ m	$D3 = 0.50$ m

All of the pipes are rough concrete ($e = 0.6$ mm), so complete turbulence is assumed. The actual elevation of the junction (J) is 4,072, and the pressure at the junction is 127 kPa.

- 4.3.7.** A long pipeline carries 75.0 cfs of water from reservoir 1 to junction J, where it is distributed into pipes 2 and 3 and transported to reservoirs 2 and 3. Determine the surface elevation of reservoir 3, given the following WS elevation and pipe data (lengths and diameters):

WS1 = 3,200 ft	$L1 = 8,000$ ft	$D1 = 3.0$ ft
WS2 = 3,130 ft	$L2 = 2,000$ ft	$D2 = 2.5$ ft
WS3 = ?	$L3 = 3,000$ m	$D3 = 2.0$ ft

All of the pipes are made of PVC and have a Hazen-Williams coefficient of 150.

- 4.3.8.** Two rooftop cisterns supply a tropical bungalow with shower water. The water surface in the uppermost cistern (A) is 8 m above the ground, and the water surface in the lower cistern (B) is 7 m above the ground. They both supply water to a junction below the lowest cistern's water surface through 3-cm PVC tubes ($n = 0.011$). Each tube is 2 m long. A 5-m supply line leads from the junction through another 3-cm PVC tube to a pan shower (essentially a reservoir with holes in the bottom) 3 m above the ground. What flow rate can be expected in the shower in liters per second?

(SECTION 4.4)

- 4.4.1.** Refer to Example 4.8 to answer the following questions.
- Determine the pressure at junction F by accounting for pressure drops in pipes AB , BC , CH , and HF . (Recall that in Example 4.8 we arrived at the pressure at F by a different sequence of pressure drops.) Comment on your answer.
 - Where is the lowest total energy in the system? How can you determine this by inspection (without calculations)?
 - In Example 4.8, it was determined that the pressure at F would be too low to satisfy the customer. What system changes could be made to raise the pressure at junction F ? (*Hint*: Examine the components of the Darcy-Weisbach equation, which was used to compute the head loss.)
 - Obtain computer software appropriate for solving pipe network problems (e.g., the EPANET, WaterCAD or WaterGEMS, or KYPipe programs noted earlier in this chapter) or write your own spreadsheet program and determine whether your suggestions for increasing the pressure

at F work. Check flows and pressures for the existing system first to determine if you are entering the data correctly.

- 4.4.2. The total discharge from A to B in Figure P4.4.2 is 50.0 cfs (ft^3/sec). Pipe 1 is 4,000 ft long with a diameter of 1.5 ft, and pipe 2 is 3,000 ft long with a diameter of 2 ft. Using (a) Hardy-Cross principles and (b) the method of equivalent pipes, determine the head loss between A and B and the flow rate in each pipe if concrete (average) pipes are used at 68°F . Ignore minor losses.

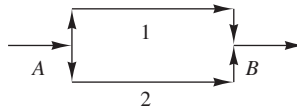


Figure P4.4.2

- 4.4.3. The total discharge from A to B in Figure P4.4.2 is 12 liters/sec. Pipe 1 is 25 m long with a diameter of 4 cm, and pipe 2 is 30 m long with a diameter of 5 cm. Using (a) Hardy-Cross principles and (b) the method of equivalent pipes, determine the head loss between A and B and the flow rate in each pipe if cast-iron pipes are used at 10°C . Assume that bend losses are important and that $K_b = 0.2$.
- 4.4.4. An industrial water-distribution system is schematically shown in Figure P4.4.4. The demands on the system are currently at junctions D ($0.550 \text{ m}^3/\text{sec}$) and E ($0.450 \text{ m}^3/\text{sec}$). Water enters the system at junction A from a storage tank (surface elevation of 355.0 m). All pipes are concrete ($e = 0.36 \text{ mm}$) with lengths and diameters provided in the table below. In addition, the elevations of the junctions are given in the table below. Calculate the flow rate in each pipe (initial estimated flows are provided). Also determine if the pressure at each junction exceeds 185 kPa, a requirement of the water company by the industrial park.

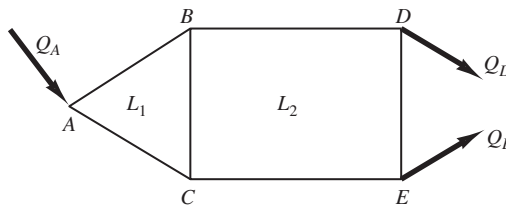


Figure P4.4.4

Pipe	Flow (m^3/sec)	Length (m)	Diameter (m)	e/D	Junction	Elev. (m)
AB	0.500	300	0.45	0.00080	A	355.0
AC	0.500	300	0.45	0.00080	B	315.5
BD	0.530	400	0.40	0.00090	C	313.8
CE	0.470	400	0.40	0.00090	D	313.3
CB	0.030	300	0.20	0.00180	E	314.1
ED	0.020	300	0.20	0.00180		

- 4.4.5. Solve Problem 4.4.4 using the Hazen-Williams equation instead of the Darcy-Weisbach for friction losses. Let $C_{HW} = 120$ for the concrete pipes.
- 4.4.6. The three-loop water-distribution system in Example 4.8 is not functioning effectively. The demand for water at junction F is being met but not at the pressure required by the industrial customer. The water company has decided to increase the diameter of one pipe in the network by 5 cm. Determine which pipe should be replaced to have the greatest impact on the pressure in the system, specifically the pressure at node F. (*Hint:* Examine the output table of Example 4.8 for head losses, flow rates, and pipe sizes. One pipe stands out as the best choice, although there is a second pipe that is only slightly worse.) Replace the pipe of your choice (increase the diameter by 5 cm) and determine the pressure increase at junction F. Pipe network analysis software would be useful.
- 4.4.7. A three-loop water-distribution system is depicted in Figure P4.4.7. The demands on the system are currently at junctions C (6.00 cfs), D (8.00 cfs) and E (11.0 cfs). Water enters the system at junction A from a storage tank with a pressure of 45 psi. Using the pipe network data in the table below, calculate the flow rate in each pipe (initial estimated flows are provided). Also determine the water pressure at each junction (30 psi is required by the customers). Pipe network analysis software would be useful.

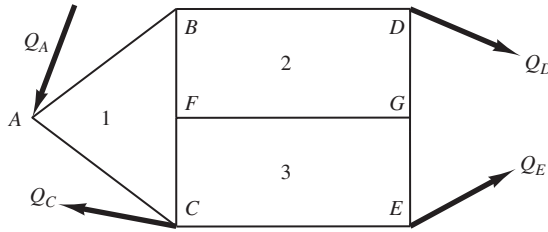


Figure P4.4.7

Pipe	Flow (ft ³ /sec)	Length (ft)	Diameter (ft)	C_{HW}	Junction	Elev. (ft)
AB	11.00	600	1.50	120	A	325.0
AC	14.00	600	1.50	120	B	328.5
BD	7.00	800	1.25	120	C	325.8
CE	7.00	800	1.25	120	D	338.8
BF	4.00	400	1.00	120	E	330.8
CF	1.00	400	1.00	120	F	332.7
FG	5.00	800	1.25	120	G	334.8
GD	1.00	400	1.00	120		
GE	4.00	400	1.00	120		

- 4.4.8. The two-loop water-distribution system in Example 4.9 is not functioning effectively. The demand for water at junction F is being met but not at the pressure required by the industrial customer. (The industrial customer would like to have a pressure head of 14 m on the water delivered.) The water company has decided to increase the diameter of one pipe in the network by 5 cm. Determine which pipe should be replaced to have the greatest impact on the pressure in the system, specifically the pressure at node F. (*Hint:* Examine the output table of Example 4.9 for head losses, flow rates, and pipe sizes. One pipe stands out as the best choice, although there is a second pipe that is only slightly worse.) Replace the pipe of your choice (increase the diameter by 5 cm) and determine the pressure increase at junction F. Pipe network analysis software would be useful.

- 4.4.9. Verify that Equation 4.17b is the proper flow correction equation when the Hazen-Williams formula is used for friction head loss instead of the Darcy-Weisbach formula (i.e., derive Equation 4.17b).
- 4.4.10. Using computer software, determine the flow rate (water at 10°C) and head loss in each cast-iron pipe in the network shown in Figure P4.4.10. The demands on the system are currently at junctions C (0.030 m³/sec), D (0.250 m³/sec), and H (0.120 m³/sec). Water enters the system at junctions A (0.100 m³/sec) and F (0.300 m³/sec). The lengths and diameters of the pipes in the network are provided in the table below.

Pipe	Length (m)	Diameter (m)
AB	1,200	0.50
FA	1,800	0.40
BC	1,200	0.10
BD	900	0.30
DE	1,200	0.30
EC	900	0.10
FG	1,200	0.60
GD	900	0.40
GH	1,200	0.30
EH	900	0.20

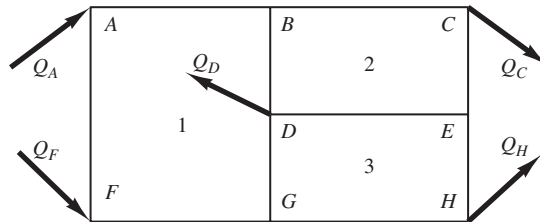


Figure P4.4.10

- 4.4.11. For Problem 4.4.10, use computer software and the Hazen-Williams equation instead of the Darcy-Weisbach equation to solve for the flows.
- 4.4.12. A planning study is being proposed for the pipe network of Example 4.10. In particular, the water company wants to determine the impact of increasing the outflow at junction F from 0.25 m³/sec to 0.30 m³/sec. Although there is an adequate supply of water to meet this demand, there is a concern that the resulting pressure heads may become too low. Using the Newton method and appropriate computer software, determine if the pipe network can accommodate this increase satisfactorily if a minimum of 10.5 m of pressure head is required at every node.
- 4.4.13. Using the Newton method and appropriate computer software, analyze the pipe network in Figure P4.4.13 if $H_A = 190$ ft, $H_E = 160$ ft, $H_G = 200$ ft, $Q_B = 6.0$ cfs, $Q_C = 6.0$ cfs, $Q_D = 6$ cfs, and $Q_F = 12.0$ cfs. Suppose the $K = 1.0$ sec²/ft⁵ for pipes 1, 7, and 8 and 3.0 sec²/ft⁵ for the other pipes. Use initial estimates of $Q_1 = 10$ cfs, $Q_2 = 1.0$ cfs, $Q_3 = 2.0$ cfs, $Q_4 = 1.0$ cfs, $Q_5 = 10$ cfs, $Q_6 = 4.0$ cfs, $Q_7 = 10.0$ cfs, and $Q_8 = 10.0$ cfs.

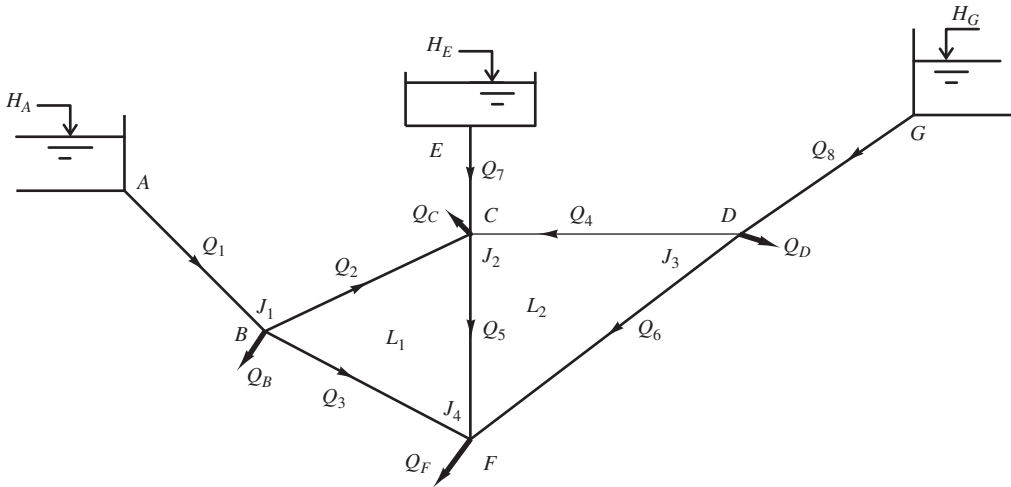


Figure P4.4.13

(SECTION 4.5)

- 4.5.1.** The pressure head rise caused by water hammer can be evaluated by using Equation 4.26. Review the derivation and answer the following questions.
- What concepts (fundamental principles) are used in the derivation?
 - What limitations are placed on the use of the equation?
- 4.5.2.** A 500-m long pipeline carries oil (sp. gr. = 0.85) from a storage tank to the hold of an oil tanker. The 0.5-m-diameter steel pipe has expansion joints and a wall thickness of 2.5 cm. The normal discharge rate is 1.45 m³/sec, but it can be controlled by a valve at the end of the pipeline. The surface of the oil in the storage tank is 19.5 m above the outlet of the pipe. Determine the maximum valve closure time that would put it into the category of a *rapid closure*.
- 4.5.3.** A 2,400-ft-long, 2-ft-diameter pipeline conveys water from a hilltop reservoir to an industrial site. The pipe is made of ductile iron, has an outside diameter of 2.25 feet, and has expansion joints. The flow rate is 30 cfs. Determine the maximum water hammer pressure (in psi) that is likely to occur if the downstream flow valve is closed in 1.05 seconds. Also determine the water hammer reduction (in psi) if a diverter is added that reduces the flow rate from 30 cfs to 10 cfs almost instantly on valve closure.
- 4.5.4.** A horizontal pipe 30 cm in diameter and 420 m long has a wall thickness of 1 cm. The pipe is commercial steel and carries water from a reservoir to a level 100 m below and discharges into the air. A rotary valve is installed at the downstream end. Calculate the maximum water hammer pressure that can be expected on the valve if it closes in a 0.5-sec period (neglect longitudinal stresses). Also determine the total (maximum) pressure the pipeline will be exposed to during the water hammer phenomenon.
- 4.5.5.** A 0.5-m-diameter concrete pipe (5.0 cm wall thickness with rigid pipe walls) carries water 600 m before discharging it into another reservoir. The surface elevation of the downstream reservoir is 55 m lower than the supply reservoir. A gate valve just upstream of the lower reservoir controls the flow rate. If the valve is closed in 0.65 sec, what is the maximum water hammer pressure?
- 4.5.6.** An emergency drawdown structure for a reservoir consists of a 1,000-ft long, 1.0-ft-diameter, steel (commercial) pipe with a wall thickness of 0.5 inches. A gate valve is located at the end of the pipe. If the valve is closed suddenly, determine the maximum water hammer pressure that will

develop in the pipeline. The pipe is free to move longitudinally, and the water level in the reservoir is 98.4 feet above the outlet.

- 4.5.7.** A pipeline is being designed to withstand a total maximum pressure of $2.13 \times 10^6 \text{ N/m}^2$. The 20-cm pipeline is ductile iron and conveys water at 40 L/sec. Determine the required thickness of the pipe wall if the operational head on the pipeline is 40 m and also subject to water hammer if the flow control valve on the downstream end is closed suddenly. Assume that the longitudinal stresses will be negligible when the pipe is installed.
- 4.5.8.** A 700-m-long, 2.0-m-diameter steel penstock conveys water from a reservoir to a turbine. The reservoir water surface is 150 m above the turbine, and the flow rate is $77.9 \text{ m}^3/\text{sec}$. A gate valve is installed at the downstream end of the pipe. Determine the wall thickness to avoid damage to the pipeline if the gate valve is closed rapidly. Use this equation ($PD = 2\tau e$) to determine the allowable pressure the pipe can withstand based on hoop stress theory with $\tau = 1.1 \times 10^8 \text{ N/m}^2$. Neglect longitudinal stresses and assume that the operational pressure is minimal compared to the maximum water hammer pressure.
- 4.5.9.** Determine the wall thickness in Problem 4.5.8 if the valve closes in 60 sec and the pipe walls are assumed to be rigid.
- 4.5.10.** Derive Equation 4.21 from Equation 4.25b.

(SECTION 4.6)

- 4.6.1.** By using logic, sketches, and relevant design equations, answer the following questions about surge tanks.
- (a) Does a surge tank eliminate elevated pressures because of water hammer in the entire pipeline? If not, what portions of the pipeline will still be subject to some increase in pressure? Refer to Figure 4.15.
- (b) What concepts (fundamental principles) are used in the derivation of Equation 4.31?
- (c) What limitations are placed on the use of Equation 4.31?
- 4.6.2.** Review Example 4.13. Determine the size of the surge tank required if the allowable water surface rise is 7.50 m.
- 4.6.3.** A 425-m-long, commercial steel pipe with a 0.90 m diameter carries irrigation water between a reservoir and a distribution junction. The maximum flow is $2.81 \text{ m}^3/\text{sec}$. A simple surge tank is installed just upstream from the control valve to protect the pipeline from water hammer damage. Compute the maximum water rise if the surge tank is 2 m in diameter.
- 4.6.4.** Water flows from a supply reservoir (surface elevation, 450 ft, MSL) through a 2,500-ft horizontal pipeline at a rate of 350 cfs. A surge tank is required to be installed in the pipeline before a gate valve. The pipeline is made of smooth concrete and has a 6-ft diameter. Determine the required height of a 20-ft diameter surge tank if the pipeline is 50 ft lower than the supply reservoir.
- 4.6.5.** Determine the minimum diameter of the surge tank in Problem 4.6.3. if the allowable water surface rise is 5 m over the supply reservoir water level.
- 4.6.6.** A court case involving pipeline damage hinges on knowing the flow rate at the time of valve closure. A simple surge tank was operational in a 1,500-m-long pipeline to protect a turbine, but the flow gauge had malfunctioned. The pipeline is 2 m in diameter and made of rough concrete. If a 5-m rise was measured in a 10-m-diameter surge tank, what was the flow rate in the pipeline when the water was suddenly stopped? (*Hint:* Assume complete turbulence in the pipeline.)

PROJECT PROBLEM

- 4.6.7.** From a reservoir 1,200 m away, a 15-cm-diameter main pipe supplies water to six multistory buildings in an industrial park. The reservoir is 80 m above the datum elevation. The positions of

the buildings are shown in Figure P4.6.7. The height and the water demand of each building are as follows:

Buildings	A	B	C	D	E	F
Height (m)	9.4	8.1	3.2	6.0	9.6	4.5
Water Demand (L/sec)	5.0	6.0	3.5	8.8	8.0	10.0

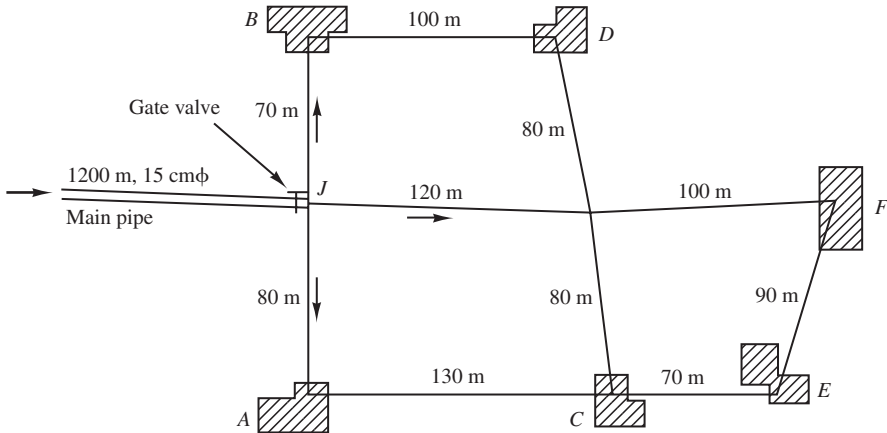


Figure P4.6.7

If commercial steel pipes are used for the network (downstream from the junction *J*), what is the correct size of each pipeline? A gate valve ($K = 0.15$ when fully opened) is installed in the main pipe immediately upstream from the junction *J*. Determine the material to be used for the main pipe. Determine the water hammer pressure if the valve is suddenly closed. What must the minimum wall thickness of the pipe be to withstand the pressure?

5



Water Pumps

Water pumps are devices designed to convert mechanical energy to hydraulic energy. In general, water pumps can be classified into two basic categories:

1. turbo-hydraulic pumps,
2. positive-displacement pumps.

Turbo-hydraulic pumps move fluids with a rotating vane or another moving fluid. Analysis of turbo-hydraulic pumps involves fundamental principles of hydraulics. The most common types of turbo-hydraulic pumps are centrifugal pumps, propeller pumps, and jet pumps. *Positive-displacement pumps* move fluids strictly by precise machine displacements such as a gear system rotating within a closed housing (screw pumps) or a piston moving in a sealed cylinder (reciprocal pumps). Analysis of positive-displacement pumps involves purely mechanical concepts and does not require detailed knowledge of hydraulics. This chapter will only treat the first category, which constitutes most of the water pumps used in modern hydraulic engineering systems.

5.1 Centrifugal (Radial Flow) Pumps

The fundamental principle of the centrifugal pump was first demonstrated by Demour in 1730. The concept involves a simple “pump” consisting of two straight pipes joined to form a tee, as shown in Figure 5.1. The tee is primed (filled with water), and the lower end of the

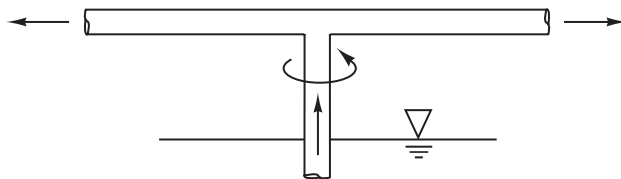


Figure 5.1 Demour's centrifugal pump

tee is submerged. Then the horizontal arms are rotated with sufficient velocity to propel the water from the ends of the tee (i.e., normal acceleration). The exiting water reduces the pressure at the ends of the tee (i.e., creating suction); enough to overcome the friction head loss of the moving water and the position head difference between the ends of the tee and the supply reservoir.

Modern centrifugal pumps are constructed with this same hydraulic principle but with new configurations designed to improve the efficiency. Modern centrifugal pumps basically consist of two parts:

1. the rotating element, which is commonly called the *impeller*; and
2. the *housing*, which encloses the rotating element and seals the pressurized liquid inside.

The power required by the pump is supplied by a motor connected to the shaft of the impeller. The rotary motion of the impeller creates a centrifugal force that enables the liquid to enter the pump at the low-pressure region near the center (*eye*) of the impeller and to move along the direction of the impeller vanes toward the higher-pressure region near the outside of the housing surrounding the impeller, as shown in Figure 5.2 (a). The housing is designed with a gradually expanding spiral shape so that the entering liquid is led toward the discharge pipe with minimum loss [Figure 5.2 (b)]. In essence, the mechanical energy of the pump is converted into pressure energy in the liquid.

The theory of centrifugal pumps is based on the *principle of angular momentum conservation*. Physically, the term *momentum*, which usually refers to linear momentum, is defined as the product of a mass and its velocity, or

$$\text{momentum} = (\text{mass}) (\text{velocity})$$

The angular momentum (or moment of momentum) with respect to a fixed axis of rotation can thus be defined as the moment of the linear momentum with respect to the axis:

$$\begin{aligned} \text{angular momentum} &= (\text{radius}) (\text{momentum}) \\ &= (\text{radius}) (\text{mass}) (\text{velocity}) \end{aligned}$$

The principle of conservation of angular momentum requires that *the time rate of change of angular momentum in a body of fluid be equal to the torque resulting from the external force acting on the body*. This relationship may be expressed as

$$\text{torque} = \frac{(\text{radius}) (\text{mass}) (\text{velocity})}{\text{time}} = (\text{radius}) \rho \left(\frac{\text{volume}}{\text{time}} \right) (\text{velocity})$$

The diagram in Figure 5.3 can be used to analyze this relationship.

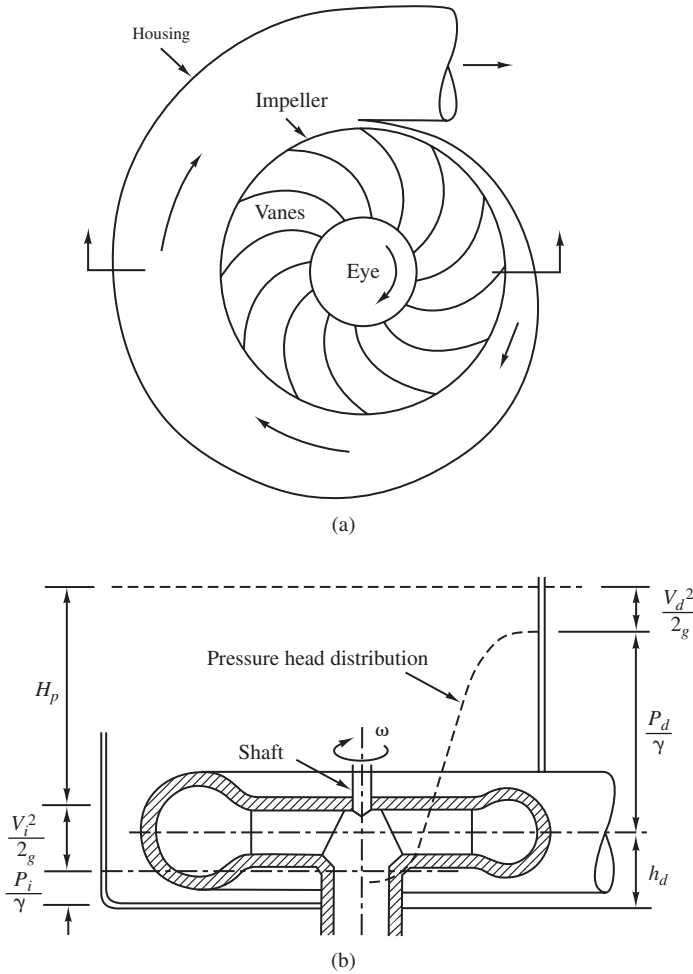


Figure 5.2 (a, b) Cross sections of a centrifugal pump

The angular momentum (or moment of momentum) for a small fluid mass per unit time (ρdQ) is

$$(\rho dQ)(V \cos \alpha)(r)$$

where $V \cos \alpha$ is the tangential component of the absolute velocity depicted in Figure 5.3. For the total fluid mass that enters the pump per unit time, the angular momentum can be evaluated by the following integral:

$$\rho \int_Q r V \cos \alpha dQ$$

The torque applied to a pump impeller must equal the difference of angular momentum at the inlet and outlet of the impeller. It may be expressed as follows:

$$\rho \int_Q r_o V_o \cos \alpha_o dQ - \rho \int_Q r_i V_i \cos \alpha_i dQ \tag{5.1}$$

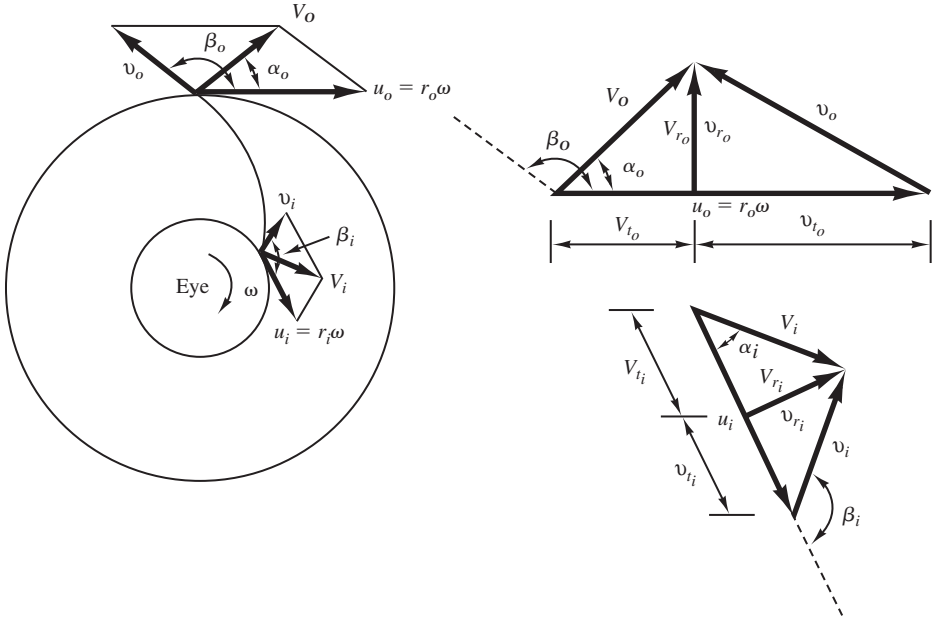


Figure 5.3 Velocity vector diagram; inlet side on the bottom and outlet side on the top
 (Note: u is the speed of the impeller vane ($u = r\omega$); v is the relative velocity of the liquid with respect to the vane; V is the absolute velocity of the liquid, a vector sum of u and v . β_o is the vane angle at the exit, β_i is the vane angle at the entrance, $r = r_i$ is the radius of the impeller eye at the entrance, and $r = r_o$ is the radius of the impeller at the exit.)

For steady flow and uniform conditions around the pump impeller, $r_o V_o \cos \alpha_o dQ$ and $r_i V_i \cos \alpha_i dQ$ have constant values. Equation 5.1 may be simplified to

$$\rho Q(r_o V_o \cos \alpha_o - r_i V_i \cos \alpha_i) \tag{5.2}$$

Let ω be the angular velocity of the impeller. The power input to the pump (P_i in bold to differentiate it from pressure, P) can be computed as

$$P_i = \omega T = \rho Q \omega (r_o V_o \cos \alpha_o - r_i V_i \cos \alpha_i) \tag{5.3}$$

The output power of a pump is usually expressed in terms of the pump discharge and the total energy head that the pump imparts to the liquid (H_p). As previously discussed, the energy head of a fluid can be usually expressed as the sum of the three forms of hydraulic energy head:

1. kinetic ($V^2/2g$),
2. pressure (P/γ), and
3. elevation (h).

Referring to Figure 5.2 (b), we can see that the total energy head that the pump imparts to the liquid is

$$H_p = \frac{V_d^2 - V_i^2}{2g} + \frac{P_d - P_i}{\gamma} + h_d$$

(Refer to Section 4.2 for an alternative way of determining H_p when a pump operates between two reservoirs.) The *pump output power* may be expressed as

$$P_o = \gamma Q H_p \tag{5.4}$$

The *polar vector diagram* (Figure 5.3) is generally used in analyzing the vane geometry and its relationship to the flow. As designated previously, the subscripts i and o are used, respectively, for the inlet and exit flow conditions; u represents the peripheral or tangential velocity of the impeller or vane speed; v represents the water velocity relative to the vane blade (hence, in the direction of the blade); and V is the absolute water velocity. V_t is the tangential component of the absolute velocity, and V_r is the radial component. Theoretically, the energy loss at the inlet reaches its minimum value when water enters the impeller without whirl. This is achieved when the impeller is operated at such a speed that the absolute water velocity at the inlet is in the radial direction.

The efficiency of a centrifugal pump depends largely on the particular design of the vane blades and the pump housing. It also depends on the conditions under which the pump operates. The efficiency of a pump is defined by the ratio of the output power to the input power of the pump:

$$e_p = P_o/P_i = (\gamma Q H_p)/(\omega T) \tag{5.5}$$

A hydraulic pump is usually driven by a motor. The efficiency of the motor is defined as the ratio of the power applied to the pump by the motor (P_i) to the power input to the motor (P_m):

$$e_m = P_i/P_m \tag{5.6}$$

The *overall efficiency* of the pump system is thus

$$e = e_p e_m = (P_o/P_i) (P_i/P_m) = P_o/P_m \tag{5.7}$$

or

$$P_o = e P_m \tag{5.8}$$

Efficiency values are always less than unity because of friction and other energy losses that occur in the system.

In Figure 5.2, the total energy head at the entrance to the pump is represented by

$$H_i = \frac{P_i}{\gamma} + \frac{V_i^2}{2g}$$

and the total energy head at the discharge location is

$$H_d = h_d + \frac{P_d}{\gamma} + \frac{V_d^2}{2g}$$

The difference between the two is the amount of energy that the pump imparts to the liquid:

$$H_p = H_d - H_i = \left(h_d + \frac{P_d}{\gamma} + \frac{V_d^2}{2g} \right) - \left(\frac{P_i}{\gamma} + \frac{V_i^2}{2g} \right) \tag{5.9}$$

Example 5.1

A centrifugal pump has the following characteristics: $r_i = 12$ cm, $r_o = 40$ cm $\beta_i = 118^\circ$, $\beta_o = 140^\circ$. The width of the impeller vanes is 10 cm and is uniform throughout. At the angular speed of 550 rpm, the pump delivers 0.98 m³/sec of water between two reservoirs with a 25-m elevation difference. If a 500-kW motor is used to drive the centrifugal pump, what is the efficiency of the pump and the overall efficiency of the system at this stage of operation?

Solution

The peripheral (tangential) speeds of the vanes at the entrance and at the exit of the impeller are, respectively,

$$u_i = \omega r_i = 2\pi \frac{550}{60} (0.12 \text{ m}) = 6.91 \text{ m/sec}$$

$$u_o = \omega r_o = 2\pi \frac{550}{60} (0.40 \text{ m}) = 23.0 \text{ m/sec}$$

and the radial velocity of the water may be obtained by applying the continuity equation: $Q = A_i V_{r_i} = A_o V_{r_o}$, where $A_i = 2\pi r_i B$ and $A_o = 2\pi r_o B$. (Note: B is the width of the impeller vane which directly impacts the flow rate.) Therefore,

$$V_{r_i} = \frac{Q}{A_i} = \frac{Q}{2\pi r_i B} = \frac{0.98}{2\pi (0.12) (0.1)} = 13.0 \text{ m/sec}$$

$$V_{r_o} = \frac{Q}{A_o} = \frac{Q}{2\pi r_o B} = \frac{0.98}{2\pi (0.4) (0.1)} = 3.90 \text{ m/sec}$$

and from Figure 5.3, we see that $V_{r_i} = v_{r_i}$ and $V_{r_o} = v_{r_o}$. Given the information now available, we can construct vector diagrams specific to this pump. The vector diagrams shown in Figure 5.3 represent the inlet side (bottom) and the outlet side (top) of the impeller. Each of the three major vectors (u , v , and V) is composed of components. The components of v and V are radial and tangential components and form right triangles. Utilizing the computed values for u_i , u_o , V_{r_i} , V_{r_o} , v_{r_i} , and v_{r_o} , and the vector diagrams, the remainder of the vectors and angles can be computed:

$$v_{t_i} = \frac{v_{r_i}}{\tan \beta_i} = \frac{13.0}{\tan 118^\circ} = -6.91 \text{ m/sec}$$

$$v_{t_o} = \frac{v_{r_o}}{\tan \beta_o} = \frac{3.90}{\tan 140^\circ} = -4.65 \text{ m/sec}$$

and

$$V_i = \sqrt{V_{r_i}^2 + (u_i + v_{t_i})^2} = \sqrt{(13.0)^2 + (0.00)^2} = 13.0 \text{ m/sec}$$

$$\alpha_i = \tan^{-1} \frac{V_{r_i}}{(u_i + v_{t_i})} = \tan^{-1} \left(\frac{13.0}{0.00} \right) = 90^\circ$$

Thus, $\cos \alpha_i = 0$ (Note that the absolute water velocity is completely in the radial direction, which minimizes the energy loss at the inlet). Continuing with the vector analysis,

$$V_o = \sqrt{V_{r_o}^2 + (u_o + v_{t_o})^2} = \sqrt{(3.90)^2 + (18.4)^2} = 18.8 \text{ m/sec}$$

$$\alpha_o = \tan^{-1} \frac{V_{r_o}}{(u_o + v_{t_o})} = \tan^{-1} \left(\frac{3.90}{18.4} \right) = 12.0^\circ$$

Thus, $\cos \alpha_o = 0.978$.

Applying Equation 5.3, we get

$$P_i = \rho Q \omega (r_o V_o \cos \alpha_o - r_i V_i \cos \alpha_i)$$

$$P_i = (998) (0.98) \left(2\pi \frac{550}{60} \right) [(0.40) (18.8) (0.978) - 0] = 414,000 \text{ watts}$$

$$P_i = 414 \text{ kW} \quad (\text{Note: } 1 \text{ N} \cdot \text{m/sec} = 1 \text{ watt})$$

Applying Equation 5.4 and assuming the only energy head added by the pump is the elevation (neglect losses in Equation 4.2), we get $H_p = H_R - H_S$

$$P_o = \gamma Q H_p = (9.79 \text{ kN/m}^3)(0.98 \text{ m}^3/\text{sec})(25 \text{ m}) = 240 \text{ kW}$$

From Equation 5.5, the efficiency of the pump is

$$e_p = P_o/P_i = (240)/(414) = 0.580 \text{ (58.0\%)}$$

From Equation 5.7, the overall efficiency of the system is

$$e = e_p e_m = (P_o/P_i) (P_i/P_m) = (0.580)(414/500) = 0.480 \text{ (48.0\%)}$$

5.2 Propeller (Axial Flow) Pumps

A rigorous mathematical analysis for designing propellers based strictly on the energy–momentum relationship is not available. However, the application of the basic *principle of impulse momentum* provides a simple means of describing their operation.

Linear impulse is defined as the integral of the product of the force and the time, dt from t' to t'' , during which the force acts on the body:

$$I = \int_{t'}^{t''} F dt$$

If a constant force is involved during the time period, T , then the impulse may be simplified to

$$(\text{impulse}) = (\text{force}) (\text{time})$$

The principle of impulse momentum requires that *the linear impulse of a force (or force system) acting on a body during a time interval be equal to the change in linear momentum in the body during that time.*

$$(\text{force})(\text{time}) = (\text{mass}) (\text{velocity change})$$

or

$$(\text{force}) = \frac{(\text{mass}) (\text{velocity change})}{(\text{time})} \tag{5.10}$$

The relationship may be applied to a body of fluid in steady motion by taking a control volume between any two sections as shown in Figure 5.4. The force represents all forces acting on the control volume. The factor, (mass)/(time), can be expressed as the mass involved per unit time (i.e., mass flow rate) or

$$\frac{(\text{mass})}{(\text{time})} = \frac{(\text{density}) (\text{volume})}{(\text{time})} = (\text{density}) (\text{discharge}) = \rho Q$$

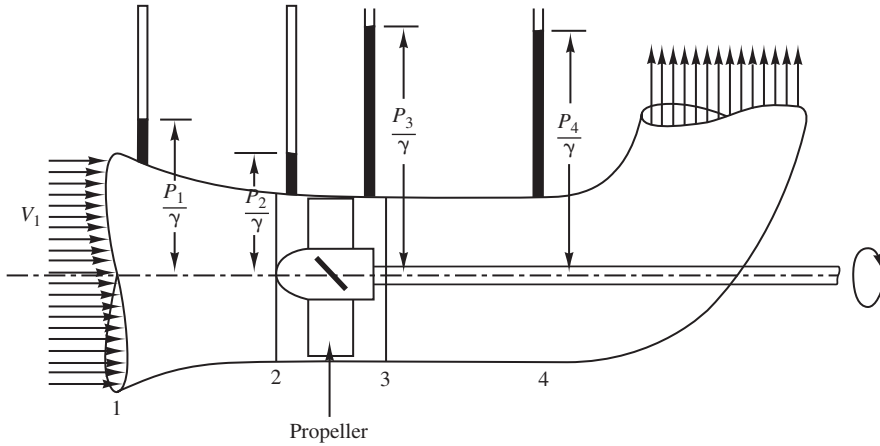


Figure 5.4 Propeller pump

and the velocity change is therefore the change of the fluid velocity between the two ends of the control volume:

$$(\text{velocity change}) = V_f - V_i$$

Substituting the above relationships into Equation 5.10, we have

$$\Sigma F = \rho Q(V_f - V_i) \tag{5.11}$$

Figure 5.4 schematically shows a propeller pump installed in a horizontal position. Four sections are selected along the pump’s flow conduit to demonstrate how the system can be readily analyzed by the principle of impulse momentum.

As the fluid moves from section 1 to section 2, the velocity increases and the pressure drops according to the Bernoulli principle of energy balance:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

Between sections 2 and 3, energy is added to the fluid by the propeller. The energy is added to the fluid in the form of pressure head, which results in a higher pressure immediately downstream from the propeller. Farther downstream at the exit end of the pump (section 4), the flow condition is more stable, and a slight drop in pressure head may result from both the head loss between sections 3 and 4 and a slight increase in mean stream velocity.

Applying the impulse momentum relationship, Equation 5.11, between sections 1 and 4, we may write the following equation:

$$P_1A_1 + F - P_4A_4 = \rho Q(V_4 - V_1) \tag{5.12}$$

where F is the force exerted on the fluid by the propeller. The right-hand side of Equation 5.12 drops out when the pump is installed in a flow conduit of uniform diameter, yielding

$$F = (P_4 - P_1)A$$

In this case, the force imparted by the pump is totally used to generate pressure. Ignoring losses and applying the Bernoulli principle between sections 1 and 2 results in

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \tag{5.13}$$

and between sections 3 and 4, we may write

$$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} = \frac{P_4}{\gamma} + \frac{V_4^2}{2g} \tag{5.14}$$

Subtracting Equation 5.13 from Equation 5.14 and noting that $V_2 = V_3$ for the same cross-sectional area, we have

$$\frac{P_3 - P_2}{\gamma} = \left(\frac{P_4}{\gamma} + \frac{V_4^2}{2g} \right) - \left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g} \right) = H_p \tag{5.15}$$

where H_p is the total energy head imparted to the fluid by the pump. The total power output from the pump may be expressed as

$$P_o = \gamma Q H_p = Q(P_3 - P_2) \tag{5.16}$$

The efficiency of the pump may be computed by the ratio of the output power of the pump to the input power from the motor.

Propeller pumps are generally used for low-head (under 12 m), high-capacity (above 20 L/sec) applications. However, more than one set of propeller blades may be mounted on the same axis of rotation in a common housing to form a *multistage propeller pump*, as shown in Figure 5.5. In this configuration, propeller pumps are capable of delivering a large quantity of

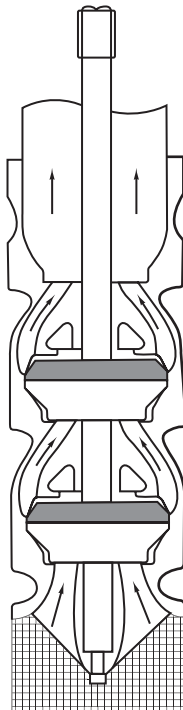


Figure 5.5 Multistage propeller pump

water over a great elevation difference. They are usually designed for self-priming operations and are used most frequently for pumping deepwater wells.

Example 5.2

A 10-ft-diameter propeller pump is installed to deliver a large quantity of water between two reservoirs with a water surface elevation difference of 8.5 ft. The shaft power supplied to the pump is 2,000 hp. The pump operates at 80-percent efficiency. Determine the discharge rate and the pressure just upstream of the pump if the pressure just downstream is 12 psi. Assume the pipe size remains uniform throughout.

Solution

The energy imparted to the flow by the pump is given in Equation 5.5:

$$P_o = e_p P_i = 0.8 (2,000 \text{ hp}) = 1,600 \text{ hp} = 8.80 \times 10^5 \text{ ft}\cdot\text{lb}/\text{sec}$$

Assuming that friction losses are negligible for this short pipe results in

$$P_o = \gamma Q H_p = \gamma Q \left[h + \Sigma K \left(\frac{V^2}{2g} \right) \right]$$

For $K_e = 0.5$ (entrance coefficient) and $K_d = 1.0$ (exit coefficient), we have

$$P_o = \gamma Q H_p = \gamma Q \left[h + 1.5 \left(\frac{Q^2}{2gA^2} \right) \right]$$

and

$$8.80 \times 10^5 \text{ ft}\cdot\text{lb}/\text{sec} = 62.3Q \left[8.5 + 1.5 \left(\frac{Q^2}{2g(25\pi)^2} \right) \right]$$

Solving the above equation yields

$$Q = 1,090 \text{ cfs}$$

Using Equation 5.16, the pressure just upstream of the pump is found to be

$$P_o = Q(P_3 - P_2)$$

$$8.80 \times 10^5 \text{ ft}\cdot\text{lb}/\text{sec} = 1,090 \text{ ft}^3/\text{sec} (P_3 - 12)144$$

$$P_3 = 17.6 \text{ psi}$$

5.3 Jet (Mixed-Flow) Pumps

Jet pumps capitalize on the energy contained in a high-pressure stream of fluid. The pressurized fluid ejects from a nozzle at high speed into a pipeline, transferring its energy to the fluid requiring delivery, as shown in Figure 5.6. Jet pumps are usually used in combination with a centrifugal pump, which supplies the high-pressure stream, and can be used to lift liquid in deep wells. The pumps are usually compact in size and light in weight. They are sometimes used in construction for dewatering the work site. Because the energy loss during the mixing procedure is significant, the efficiency of a jet pump is normally very low (rarely more than 25 percent).

A jet pump can also be installed as a booster pump in series with a centrifugal pump. The jet pump may be built into the casing of the centrifugal pump suction line to boost the water surface elevation at the inlet of the centrifugal pump as shown schematically in Figure 5.7. This arrangement avoids any unnecessary installation of moving parts in the well casing, which is usually buried deep below the ground surface.

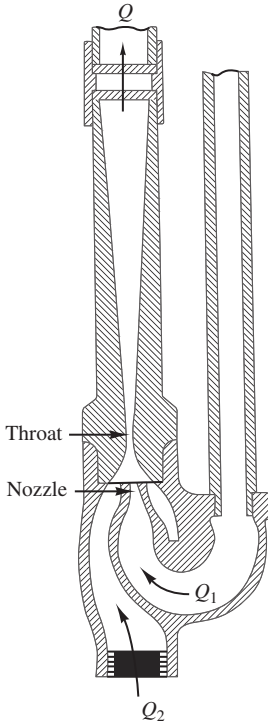


Figure 5.6 Jet pump

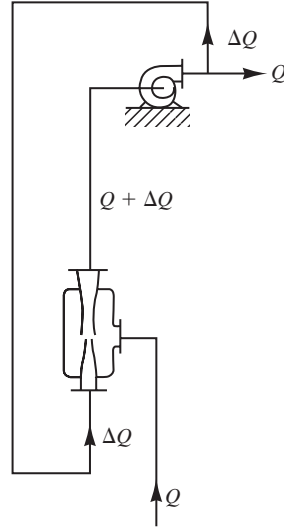


Figure 5.7 Jet pump as a booster

5.4 Centrifugal Pump Characteristic Curves

Pump characteristic curves (or *performance curves*), which are produced and supplied by reputable manufacturers, are graphical representations of a pump’s expected operational performance. These manufacturers will test their pumps in the laboratory, and even field verify the results, to ascertain pump operational performance. Different formats are used by different pump manufacturers. However, these curves generally display the variation of the *pump head*, the *brake horsepower*, and the *efficiency* with the pump’s generated flow rate. Figure 5.8 depicts typical pump characteristic curves for a centrifugal (radial flow) pump. Similar curves are available for axial flow and mixed-flow pumps, although the shapes of their curves are generally different. The *pump head* is the energy head added to the flow by the pump. The *brake horsepower* is the power input required by the pump in power units, and the *efficiency* is the ratio of the power output to the power input. The pump head at zero discharge is called the *shutoff head*. The discharge corresponding to the maximum efficiency is called the *rated capacity*. For variable speed pumps, some manufacturers display the characteristics at various speeds in the same figure.

The characteristics of a given pump vary with the rotational speed. However, if the characteristics are known for one rotational speed, then the characteristics for any other rotational speed with the same impeller size can be obtained using the affinity laws (Section 5.10):

$$\frac{Q_2}{Q_1} = \frac{N_{r2}}{N_{r1}} \tag{5.17a}$$

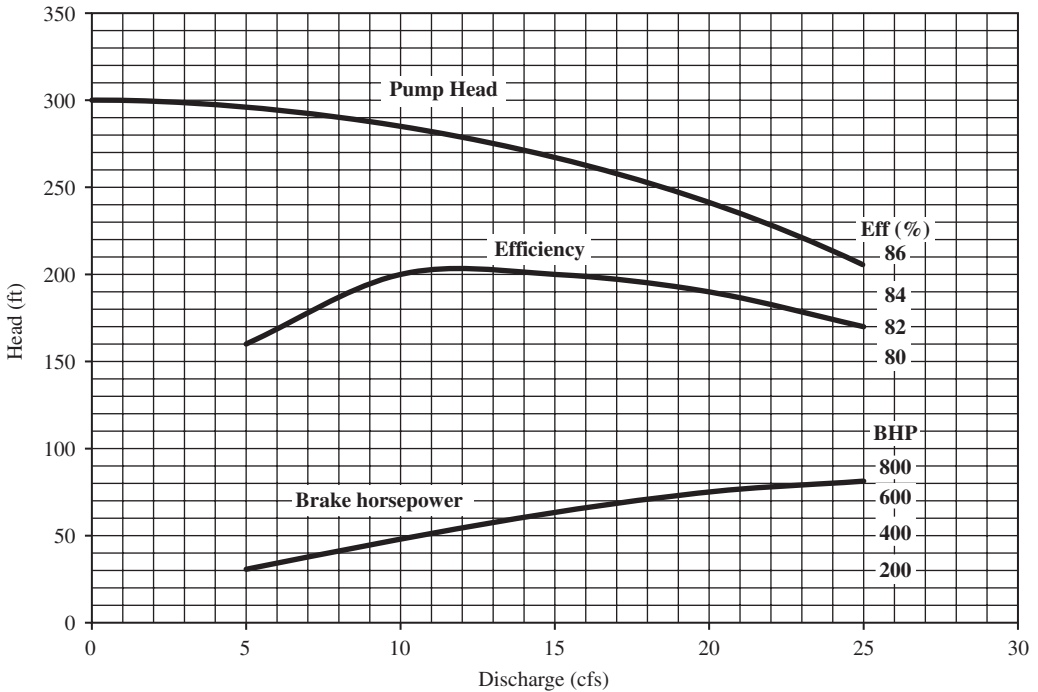


Figure 5.8 Typical pump characteristic curves

$$\frac{H_{p2}}{H_{p1}} = \left(\frac{N_{r2}}{N_{r1}}\right)^2 \tag{5.17b}$$

$$\frac{BHP_2}{BHP_1} = \left(\frac{N_{r2}}{N_{r1}}\right)^3 \tag{5.17c}$$

in which Q = discharge, H_p = pump head, BHP = brake horse power, and N_r = rotational speed. The efficiency curve is not affected significantly by the rotational speed.

5.5 Single Pump and Pipeline Analysis

A single pump placed in a pipeline to move water from one reservoir to another reservoir or to a demand point represents the most common pump-application scenario. Determining the flow rate that is produced in these pump-pipeline systems requires knowledge of both pump operation and pipeline hydraulics.

Consider the pump-pipeline system shown in Figure 5.9. Suppose pipe characteristics, pump characteristics, and the upstream and downstream water surface elevations are given, but the flow rate is unknown. To analyze this system for the flow rate, neglecting the minor losses, we can write the energy equation as

$$E_A + H_p = E_B + h_f \tag{5.18a}$$

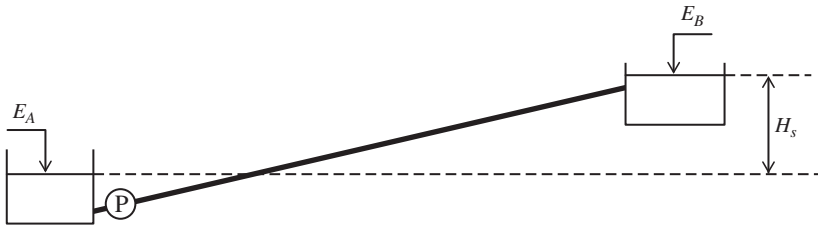


Figure 5.9 Single pump and pipeline

or

$$H_p = (E_B - E_A) + h_f \tag{5.18b}$$

where H_p is the required pump head and E_A and E_B are the water surface elevations of the two reservoirs. This expression can be explained as follows. Part of the energy added to the flow by the pump is expended in raising water from elevation E_A to E_B , and part of it is expended to overcome the flow resistance. With $H_s = E_B - E_A =$ elevation rise (static head),

$$H_p = H_s + H_f \tag{5.19}$$

Equation 5.19 can be analyzed further to determine the pump flow rate. Note that in this equation, H_s is constant, whereas h_f depends on Q . If more flow is pushed through the system, then more friction losses result and a greater pump head is required. Thus, the right-hand side of this equation can be computed for various flow rates; in so doing, it is referred to as the *system head*, denoted by H_{SH} . Substituting any friction loss equation (e.g., Darcy-Weisbach, Hazen-Williams, Manning, etc.) for h_f will yield a relationship between H_{SH} and Q . A plot of this relationship is called the *system head curve*. However, our objective is to determine the actual flow rate in the pipeline with the existing pump, not to determine hypothetical pump heads required for different flow rates. To determine the actual flow rate, we can superimpose a plot of the existing pump's characteristic curve (H_p and Q) on the system head curve. The intersection of the two curves (often called the *match point*) represents the flow rate of that particular pump operating in that particular pipe system. In essence, two equations in two unknowns are being solved graphically.* Once the discharge is determined, we can calculate the velocity and other flow characteristics as well as the energy grade line. The following example problem will help to clarify the solution process.

Example 5.3

Consider the pump-pipeline system shown in Figure 5.9. The reservoir water surface elevations are known: $E_A = 100$ ft and $E_B = 220$ ft. The 2.0-ft-diameter pipe connecting the two reservoirs has a length of 12,800 ft. and a Hazen-Williams coefficient (C_{HW}) of 100.

- (a) The pump characteristics are known (columns 1 and 2 in the following table) and are plotted in Figure 5.10 (a). Determine the discharge in the pipeline, the velocity of flow, and the energy grade line.
- (b) Suppose the pump characteristics given in part (a) are at a rotational speed of 2,000 rpm. Determine the discharge in the pipeline and the pump head if the pump runs at 2,200 rpm.

* A graphical solution is required because the H_p versus Q relationship for the pump is generally available in graphical form. If it is available or can be cast in equation form, then the pump characteristic equation is solved simultaneously with the system head equation to produce an identical solution.

Q (cfs)	H_p (ft)	h_f (ft)	H_s (ft)	H_{SH} (ft)
0	300.0	0.0	120.0	120.0
5	295.5	8.1	120.0	128.1
10	282.0	29.2	120.0	149.2
15	259.5	61.9	120.0	181.9
20	225.5	105.4	120.0	225.4
25	187.5	159.3	120.0	279.3
30	138.0	223.2	120.0	343.2
35	79.5	296.8	120.0	416.8

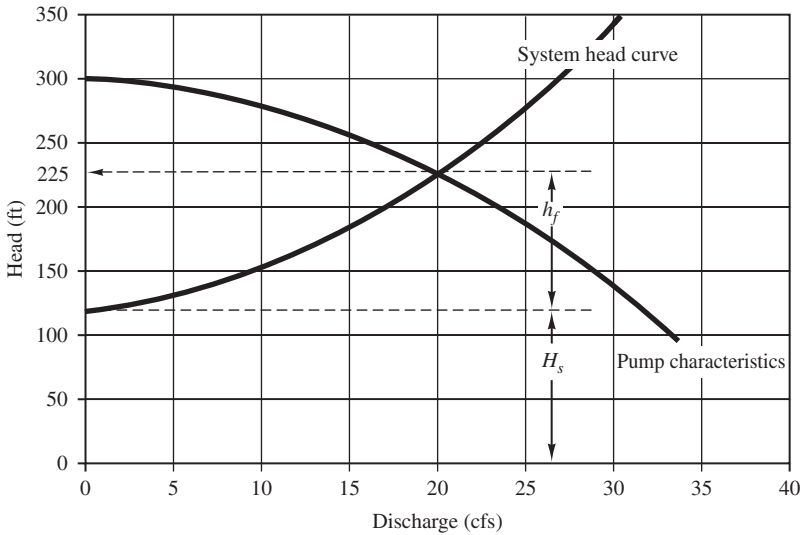


Figure 5.10 (a) Single pump and pipeline analysis

Solution

- (a) For this system, $H_s = E_B - E_A = 220 - 100 = 120$ ft. We will use the Hazen-Williams friction formula to calculate the losses resulting from friction. With reference to Table 3.4, the Hazen-Williams formula in the British unit system can be expressed as

$$h_f = KQ^{1.85}$$

where

$$K = \frac{4.73L}{D^{4.87} C_{HW}^{1.85}} = \frac{4.73(12,800)}{(2)^{4.87} (100)^{1.85}} = 0.413 \text{ sec}^{1.85}/\text{ft}^{4.55}$$

The friction loss and the system head are calculated for various values of Q as summarized in the preceding table. The system head is then plotted as in Figure 5.10 (a). The intersection of the system head curve and the pump characteristics curve yields $Q = 20$ cfs and $H_p \approx 225$ ft. We can also read from the preceding table that the friction loss is about 105 ft .

The velocity is found as

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{20}{\pi(2.0)^2/4} = 6.37 \text{ ft/sec}$$

The energy head just before the pump is $E_A = 100$ ft (ignoring losses on the suction side of the pump) and just after the pump is $E_A + H_p = 100 + 225 = 325$ ft. The energy head decreases linearly along the pipe to $E_B = 220$ ft at reservoir B. (Note: These problems can be solved quickly and accurately with spreadsheets.)

- (b) To obtain the pump characteristics at 2,200 rpm, we use Equations 5.17a and 5.17b with $N_{r1} = 2,000$ rpm, $N_{r2} = 2,200$ rpm, $N_{r2}/N_{r1} = 1.10$, and $(N_{r2}/N_{r1})_2 = 1.21$. The calculations are summarized in the following table.

$N_{r1} = 2,000$ rpm		$N_{r2} = 2,200$ rpm	
Q_1 (cfs)	H_{p1} (ft)	Q_2 (cfs)	H_{p2} (ft)
0.0	300.0	0.0	363.0
5.0	295.5	5.5	357.6
10.0	282.0	11.0	341.2
15.0	259.5	16.5	314.0
20.0	225.5	22.0	272.9
25.0	187.5	27.5	226.9
30.0	138.0	33.0	167.0
35.0	79.5	38.5	96.2

The values of Q_2 are obtained by multiplying the Q_1 values by 1.10. Likewise, the values of H_{p2} are obtained by multiplying the H_{p1} values by 1.21. A plot of Q_2 versus H_{p2} will give the pump characteristics curve at 2,200 rpm as displayed in Figure 5.10 (b). The system head curve will be the same as in part (a). The point of intersection of the two curves yields $Q = 23.4$ cfs, and $H_p = 261$ ft.

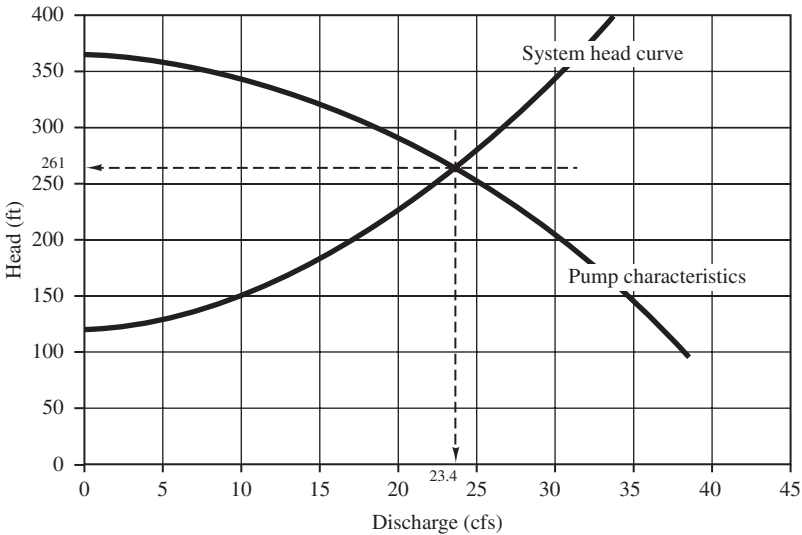


Figure 5.10 (b) Single pump and pipeline analysis at a different rotational speed

5.6 Pumps in Parallel or in Series

As discussed in Section 5.1, the efficiency of a pump varies with the discharge rate of the pump and the total head overcome by the pump. The optimum efficiency of a pump can be obtained only over a limited range of operation (i.e., discharges and total heads). Therefore, it is often

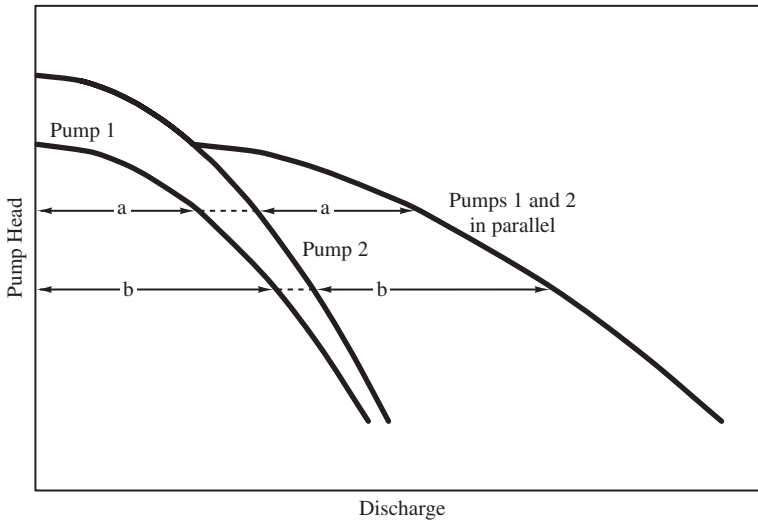


Figure 5.11 Pump characteristics for two pumps in parallel

advantageous to install several pumps in parallel or series configurations in pumping stations to efficiently operate over a broad range of expected flow rates and required system heads.

When two pumps are installed in parallel, neglecting the minor losses in their resident branch lines, the energy head added to the flow by the two pumps must be the same to satisfy the energy equation of the resident pipeline system. The discharge through the two pumps will be different, however, unless the two pumps are identical. The total discharge will split between the two pumps such that $H_{p1} = H_{p2}$ where H_{p1} and H_{p2} denote the pump heads for the first and second pumps, respectively. To obtain the pump characteristic curves for two-pump parallel configurations, we add the flow rates (abscissa) of the characteristic curves of the individual pumps for each value of pump head as illustrated in Figure 5.11. If the two pumps are identical, we simply double the discharge value for each value of pump head. However, this does not mean that the actual discharge will be doubled when two pumps are operated in parallel (see Example 5.4). The same principles are applied to determine pump system characteristic curves when more than two pumps are installed in parallel.

When two pumps are installed in series, the discharge through the two pumps must be the same. However, the pump heads will be different unless the pumps are identical. To obtain the pump characteristic curves for two-pump series configurations, we add the pump heads (ordinates) of the individual characteristic curves for each value of pump flow as shown in Figure 5.12. If the two pumps are identical, we simply double the head values for each value of pump flow. However, this does not mean that the actual head will be doubled when two pumps are operated in series (see Example 5.4). The same principles are applied to determine pump system characteristic curves when more than two pumps are installed in series.

Pump combinations add flow and head flexibility to pump-pipeline systems while maintaining high operational efficiency. For example, when flow requirements are highly variable, several pumps can be connected in parallel and switched on and off to meet the variable demand. Again note that two identical pumps operating in parallel may not double the discharge in a pipeline because the total head loss in a pipeline is proportional to the second power of discharge: $H_p \propto Q^2$. The

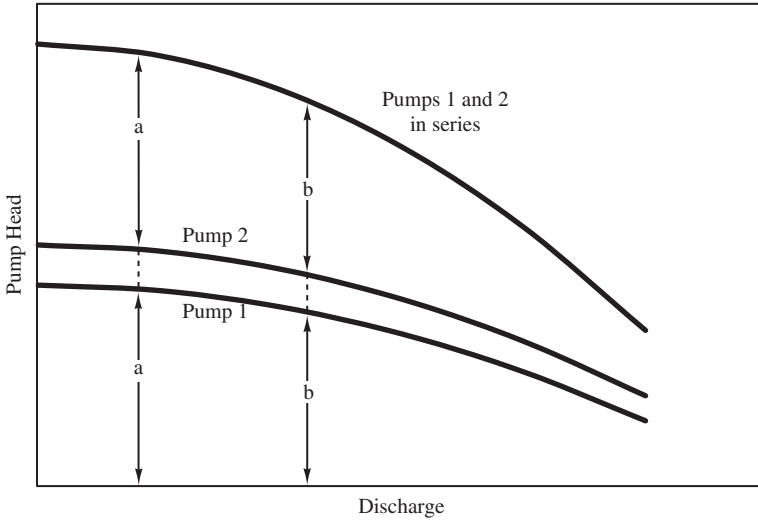


Figure 5.12 Pump characteristics for two pumps in series

additional resistance in the pipeline will cause a reduction in the total discharge. Curve *B* in Figure 5.13 schematically shows the operation of two identical pumps in parallel. The joint discharge of the two pumps is always less than twice the discharge of a single pump.

Pump combinations can also add flexibility when pump head requirements change. For example, in pipeline installations where pipeline losses or the elevation rise are variable, pumps can be connected in series and switched on and off to meet the variable head demand. Curve *C* in Figure 5.13 schematically shows the operation of two identical pumps connected in series.

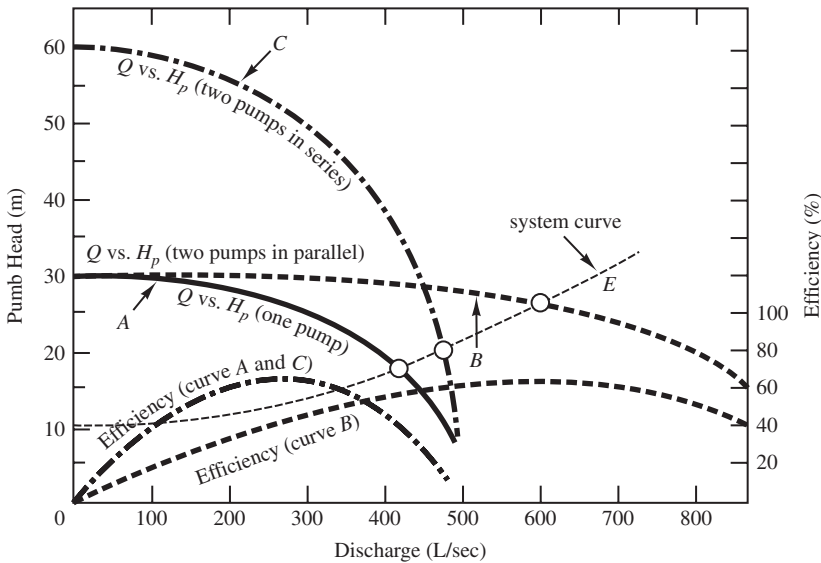


Figure 5.13 Typical performance curves of two pumps connected in parallel *B* and in series *C*

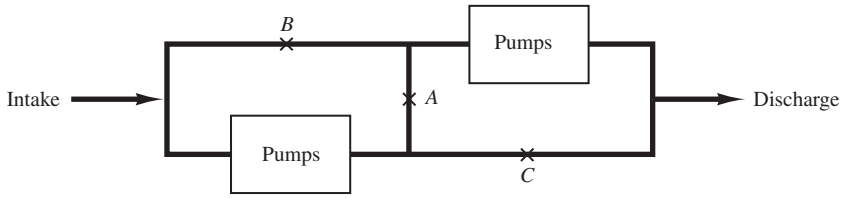


Figure 5.14 Schematic of pump operation either in series or in parallel

The efficiency of two (or more) identical pumps operating in parallel or in series is essentially the same as that of the single pump based on discharge. The installation can be arranged with a separate motor for each pump or with one motor operating two (or more) pumps. Multipump installations can be designed to perform in either series or parallel operations with the same set of pumps. Figure 5.14 is a typical schematic of such an installation. For series operations, valve *A* is opened and valves *B* and *C* are closed; for parallel operations, valve *A* is closed and valves *B* and *C* are open.

Example 5.4

Two reservoirs are connected by a 300-m-long, asphalt-lined, cast-iron pipeline, 40 cm in diameter. The minor losses include the entrance, the exit, and a gate valve. The elevation difference between the reservoirs is 10 m, and the water temperature is 10°C. Determine the discharge, head, and the efficiency using (a) one pump, (b) two pumps in series, and (c) two pumps in parallel. Use the pump with characteristics depicted in Figure 5.13.

Solution

To deliver the water, the pump system must provide a total energy head (H_{SH}) of

$$H_{SH} = H_s + \left(f \frac{L}{D} + \Sigma K \right) \frac{V^2}{2g} = 10 + (750f + 1.65) \frac{V^2}{2g}$$

based on Equation 5.19 with minor losses considered. Using $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{sec}$ (Table 1.3) and $e/D = 0.0003$, the following values are computed for a range of discharges within which each pump system may expect to operate. A Q versus H_{SH} curve is constructed based on the values computed in the following table (curve *E*, Figure 5.13).

Q (L/sec)	V (m/sec)	N_R	f	H_{SH} (m)
0	0	—	—	10.0
100	0.80	2.44×10^5	0.0175	10.5
300	2.39	7.30×10^5	0.0160	14.0
500	3.98	1.22×10^6	0.0155	20.7
700	5.57	1.70×10^6	0.0155	31.0

From curve *E* in Figure 5.13 (using a more refined grid), we obtain the following.

- (a) For one pump:

$$Q \approx 420 \text{ L/sec}, H_p \approx 18 \text{ m}, \text{ and } e_p \approx 40\%$$

- (b) For two pumps in series:

$$Q \approx 470 \text{ L/sec}, H_p \approx 20 \text{ m}, \text{ and } e_p \approx 15\%$$

(c) For two pumps in parallel:

$$Q \approx 590 \text{ L/sec}, H_p \approx 26 \text{ m}, \text{ and } e_p \approx 62\%$$

Note: Based on the system requirements, two pumps in parallel are the best choice for operation at high efficiency.

5.7 Pumps and Branching Pipes

Consider the simple branching pipe system shown in Figure 5.15 in which a single pump delivers flow from reservoir A to two reservoirs (B and C) through pipes 1 and 2. The system heads, H_{SH1} and H_{SH2} , respectively, for pipes 1 and 2 can be expressed as

$$H_{SH1} = H_{s1} + h_{f1}$$

$$H_{SH2} = H_{s2} + h_{f2}$$

where $H_{s1} = E_B - E_A$ and $H_{s2} = E_C - E_A$. The total discharge will be split between the two pipes such that $H_{SH1} = H_{SH2} = H_p$. These three heads must be equal to satisfy the energy equation for both pipes. When the two pipes are considered together as a single system, the system head curve of the combined system can be obtained by adding the flow rates (abscissa) of the individual system head curves for each value of head.

To analyze a branching pipe system for flow rates, we plot the individual and the combined system head curves as well as the pump characteristic curve. The point of intersection of the pump characteristic curve and the combined system head curve will yield the total discharge and the pump head. The discharge in each pipe is obtained from the individual system head curves at the value of the pump head. As a check, the sum of the two discharges should be equal to the total system discharge already determined. The following example problems will clarify the procedure.

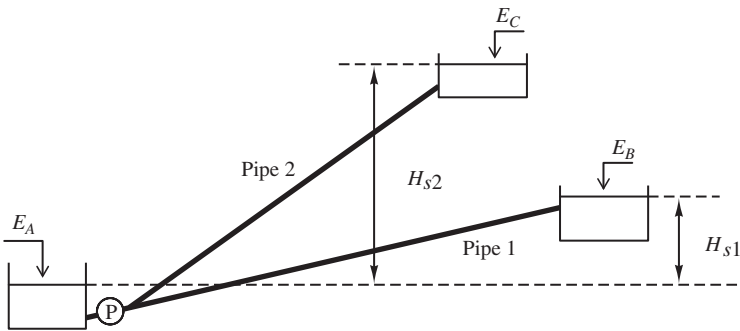


Figure 5.15 Single pump and two pipes

Example 5.5

In Figure 5.15, $E_A = 110 \text{ ft}$, $E_B = 120 \text{ ft}$, and $E_C = 140 \text{ ft}$. Both pipes have a Darcy-Weisbach friction factor of 0.02. Pipe 1 is 10,000 ft long and has a diameter of 2.5 ft. Pipe 2 is 15,000 ft long and has a diameter of 2.5 ft. The pump characteristics are given in the following table and plotted in Figure 5.16. Determine the discharge in each pipe.

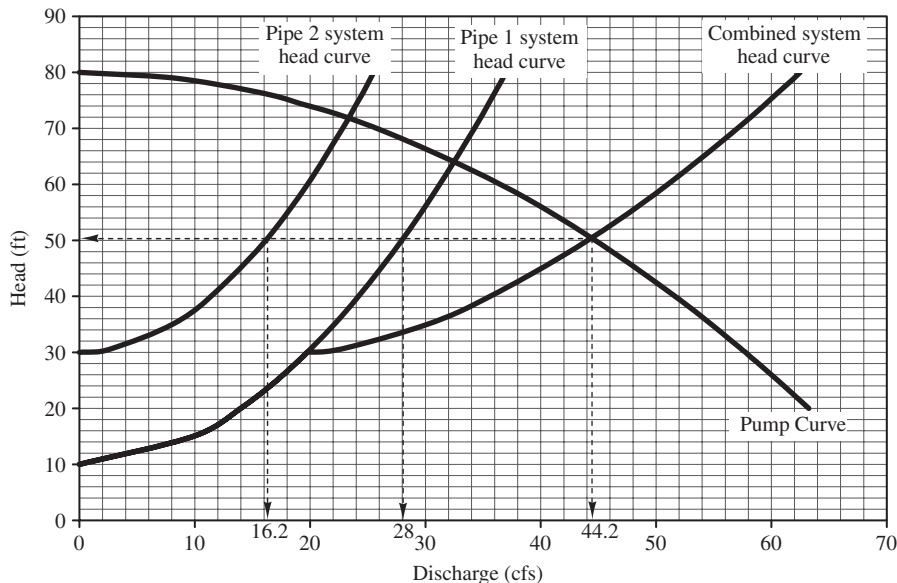


Figure 5.16 Graphical solution for Example 5.5

Q (cfs)	0	10	20	30	40	50	60
H_p (ft)	80	78.5	74	66.5	56	42.5	26

Solution

For this system, $H_{s1} = 120 - 110 = 10$ ft and $H_{s2} = 140 - 110 = 30$ ft. With reference to Table 3.4, the Darcy-Weisbach friction formula can be written as

$$h_f = KQ^2, \text{ where } K = \frac{0.025fL}{D^5}$$

For pipe 1,

$$K = \frac{0.025(0.02)(10,000)}{2.5^5} = 0.0512 \text{ sec}^2/\text{ft}^5$$

and for pipe 2

$$K = \frac{0.025(0.02)(15,000)}{2.5^5} = 0.0768 \text{ sec}^2/\text{ft}^5$$

Then the system head curves for pipes 1 and 2, respectively, are calculated as

$$H_{SH1} = 10 + 0.0512Q_1^2$$

$$H_{SH2} = 30 + 0.0768Q_2^2$$

by assigning various values to Q , solving the above equations for H_{SH} , and plotting them in Figure 5.16. The combined system head curve is obtained by adding the discharges on these curves for the same head. The point intersection of the combined system head curve with the pump characteristics curve yields a total discharge of 44.2 cfs and a pump head of 50.1 ft. For this pump head, we read

$Q_1 = 28.0$ cfs and $Q_2 = 16.2$ cfs from the respective system head curves. The reader should verify these results by checking if the energy equation is satisfied separately for pipes 1 and 2.

Example 5.6

Consider the pump and the branching pipe system shown in Figure 5.17. The reservoir water elevations and the characteristics of pipes 1 and 2 are the same as in Example 5.5. Pipe 3 has a Darcy-Weisbach friction factor of 0.02, a diameter of 3 ft, and a length of 5,000 ft. Determine the discharge in each pipe.

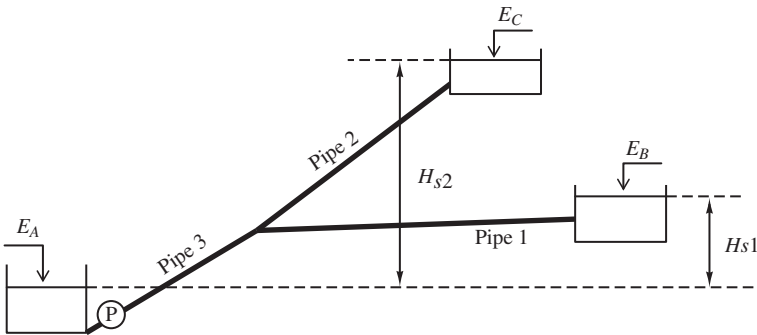


Figure 5.17 Branching pipe system of Example 5.6

Solution

We can solve this example in the same way as Example 5.5 if we can incorporate the friction losses in pipe 3. The easiest way is to subtract the losses in pipe 3 from the heads of the pump curve for respective discharges.

For pipe 3,

$$K = \frac{0.025(0.02)(5,000)}{3^5} = 0.0103$$

Then the head loss pipe 3 is calculated as

$$h_f = 0.0103Q^2$$

Calculating the head loss for the tabulated values of Q and subtracting from the respective H_p values will yield the net H_p values.

Q (cfs)	0	10	20	30	40	50
H_p (ft)	80.0	78.5	74.0	66.5	56.0	42.5
h_f (ft)	0.0	1.0	4.1	9.3	16.5	25.8
net H_p (ft)	80.0	77.5	69.9	57.2	39.5	16.7

We can now solve the problem just like in Example 5.5, except that we now plot the net pump heads as shown in Figure 5.18. The intersection of the combined system head curve with the net pump characteristics curve (match point) yields a total discharge of 38.2 cfs and a net pump head of 42.8 ft. For this net pump head, we read $Q_1 = 25.3$ cfs and $Q_2 = 12.9$ cfs from the respective system head curves. The actual

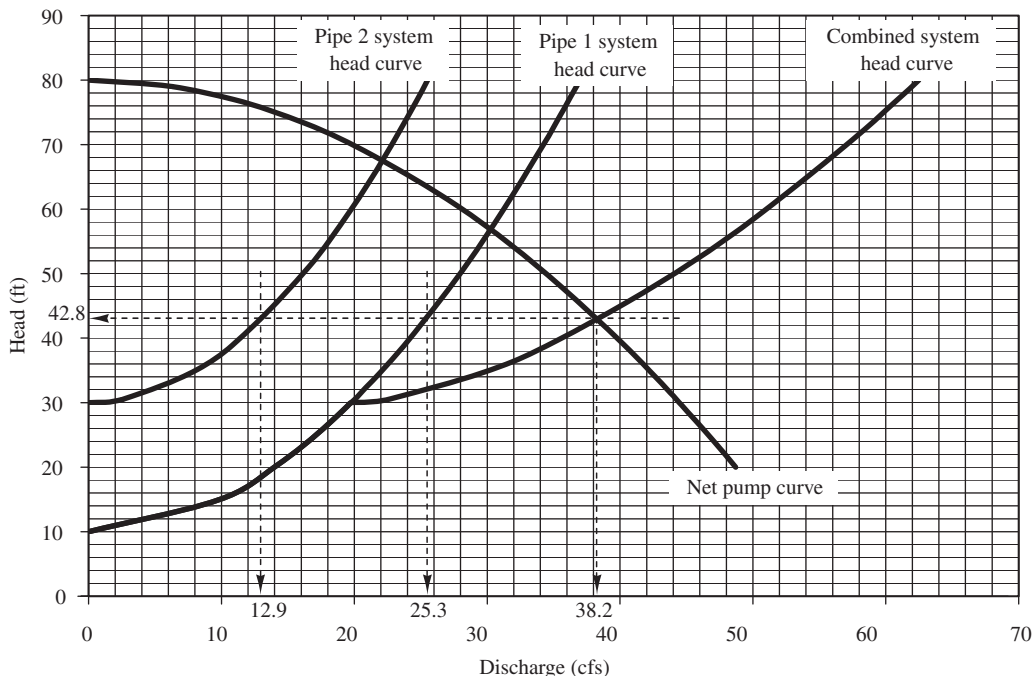


Figure 5.18 Graphical solution for Example 5.6

pump head is $42.8 + 0.0103 (38.2)^2 = 57.8$ ft. The reader should verify these results by checking if the energy equation is satisfied separately for the flow paths along pipes 1 and 2.

5.8 Pumps and Pipe Networks

Pumps are an integral part of many pipe networks. The Hardy-Cross and Newton methods, introduced in Chapter 4, can be modified easily to analyze pipe networks that contain pumps. The modification is applied to the energy equation for the path (or loop) containing the pump. It now must include the head added to the flow by the pump. To facilitate this, the pump characteristics are expressed in a polynomial form such as

$$H_p = a - bQ|Q| - cQ$$

The coefficients a , b , and c are fitting parameters. They can be determined by using three data points from the pump characteristics curve.

Example 5.7

The pipe system shown in Figure 5.19 is identical to that of Examples 4.9 and 4.10. However, the demand has increased at junction F ($Q_F = 0.30 \text{ m}^3/\text{sec}$, not $0.25 \text{ m}^3/\text{sec}$), which necessitates adding a pump to the system just downstream of reservoir A. The pump characteristics can be expressed as

$$H_p = 30 - 50Q^2 - 5Q$$

where H_p is in m and Q is in m^3/sec . Determine the discharge in each pipe using the same initial values as in Example 4.10.

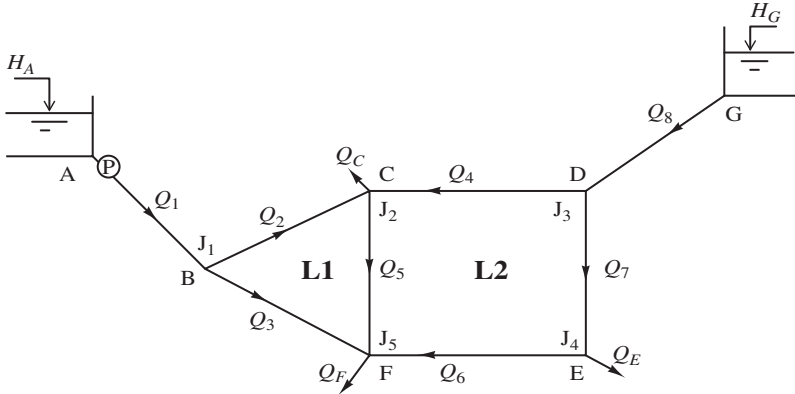


Figure 5.19 Pipe network for Example 5.7

Solution

We will use the same equations as in Example 4.10 except

$$F_8 = H_A + a - bQ_1|Q_1| - cQ_1 - K_1Q_1|Q_1| - K_2Q_2|Q_2| + K_4Q_4|Q_4| + K_8Q_8|Q_8| - H_G$$

and

$$\frac{\partial F_8}{\partial Q_1} = -2bQ_1 - c - 2K_1Q_1$$

where $a = 30$ m, $b = 50 \text{ sec}^2/m^5$, and $c = 5 \text{ sec}/m^2$.

The results are obtained in five iterations as summarized in the following table.

(m ³ /sec)								
Iteration Number	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Initial	0.2000	0.5000	0.1000	0.0500	0.5000	0.1000	0.3000	0.2500
1	0.2994	0.2577	0.0417	0.0659	0.2236	0.0347	0.1347	0.2006
2	0.2955	0.1464	0.1492	0.0760	0.1223	0.0285	0.1285	0.2045
3	0.2647	0.1253	0.1394	0.0829	0.1081	0.0524	0.1524	0.2353
4	0.2650	0.1245	0.1405	0.0843	0.1088	0.0507	0.1707	0.2350
5	0.2650	0.1245	0.1405	0.0843	0.1088	0.0507	0.1507	0.2350

The resulting energy heads are $H_A = 85.0$ m, $H_B = 96.5$ m, $H_C = 67.1$ m, $H_D = 78.6$ m, $H_E = 63.2$ m, $H_F = 59.0$ m, and $H_G = 102.0$ m. Also, the energy head added to flow by the pump is 25.2 m.

5.9 Cavitation in Water Pumps

One of the important considerations in pump installation design is the relative elevation between the pump and the water surface in the supply reservoir. Whenever a pump is positioned above the supply reservoir, the water in the suction line is under pressure lower than atmospheric. The phenomenon of

cavitation becomes a potential danger whenever the water pressure at any location in the pumping system drops substantially below atmospheric pressure. To make matters worse, water enters into the suction line through a *strainer* that is designed to keep out trash. This additional energy loss at the entrance reduces pressure even further.

A common site of cavitation is near the tips of the impeller vanes where the velocity is very high. In regions of high velocities much of the pressure energy is converted to kinetic energy. This is added to the elevation difference between the pump and the supply reservoir, h_p , and to the inevitable energy loss in the pipeline between the reservoir and the pump, h_L . Those three items all contribute to the *total suction head*, H_S , in a pumping installation as shown schematically in Figure 5.20.

The value of H_S must be kept within a limit so that the pressure at every location in the pump is always above the vapor pressure of water; otherwise, the water will be vaporized and cavitation will occur. The vaporized water forms small vapor bubbles in the flow. These bubbles collapse when they reach the region of higher pressure in the pump. Violent vibrations may result from the collapse of vapor bubbles in water. Successive bubble breakup with considerable impact force may cause high local stresses on the metal surface of the vane blades and the housing. These stresses can cause surface pitting and will rapidly damage the pump.

To prevent cavitation, the pump should be installed at an elevation so that the total suction head (H_S) is less than the difference between the atmospheric head and the water vapor pressure head, or

$$H_S < \left(\frac{P_{\text{atm}}}{\gamma} - \frac{P_{\text{vapor}}}{\gamma} \right)$$

The maximum velocity near the tip of the impeller vanes is not assessable by users. Pump manufacturers usually provide a value known commercially as the *net positive suction head* (NPSH), or H'_S . (The NPSH can be displayed as part of the pump characteristic curves, as shown in Figure 5.24 in the section on pump selection.) NPSH represents the pressure drop between the eye of the pump and the tip of the impeller vanes. With the value of NPSH given, the maximum pump elevation above the supply reservoir can be easily determined by accounting for all of the energy components that make up H_S in Figure 5.20. The resulting expression is

$$h_p \leq \left(\frac{P_{\text{atm}}}{\gamma} - \frac{P_{\text{vapor}}}{\gamma} \right) - \left(H'_S + \frac{V^2}{2g} + h_L \right) \quad (5.20)$$

where h_L is the total loss of energy in the suction side of the pump. It usually includes the entrance loss at the strainer, the friction loss in the pipe, and other minor losses.

Another parameter commonly used for expressing cavitation potential in a pump is the *cavitation parameter*, σ , which is defined as

$$\sigma = \frac{H'_S}{H_p} \quad (5.21)$$

where H_p is the total head developed by the pump and the numerator is the NPSH. The increase in velocity through the impeller vanes is accounted for in the parameter σ . The value of σ for each type of pump is usually furnished by the manufacturer and is based on pump test data.

Applying Equation 5.21 to the relationships depicted in Figure 5.20, we may write

$$H'_S = \sigma H_p = \frac{P_{\text{atm}}}{\gamma} - \frac{P_{\text{vapor}}}{\gamma} - \left(\frac{V_i^2}{2g} + h_p + h_L \right) \quad (5.22)$$

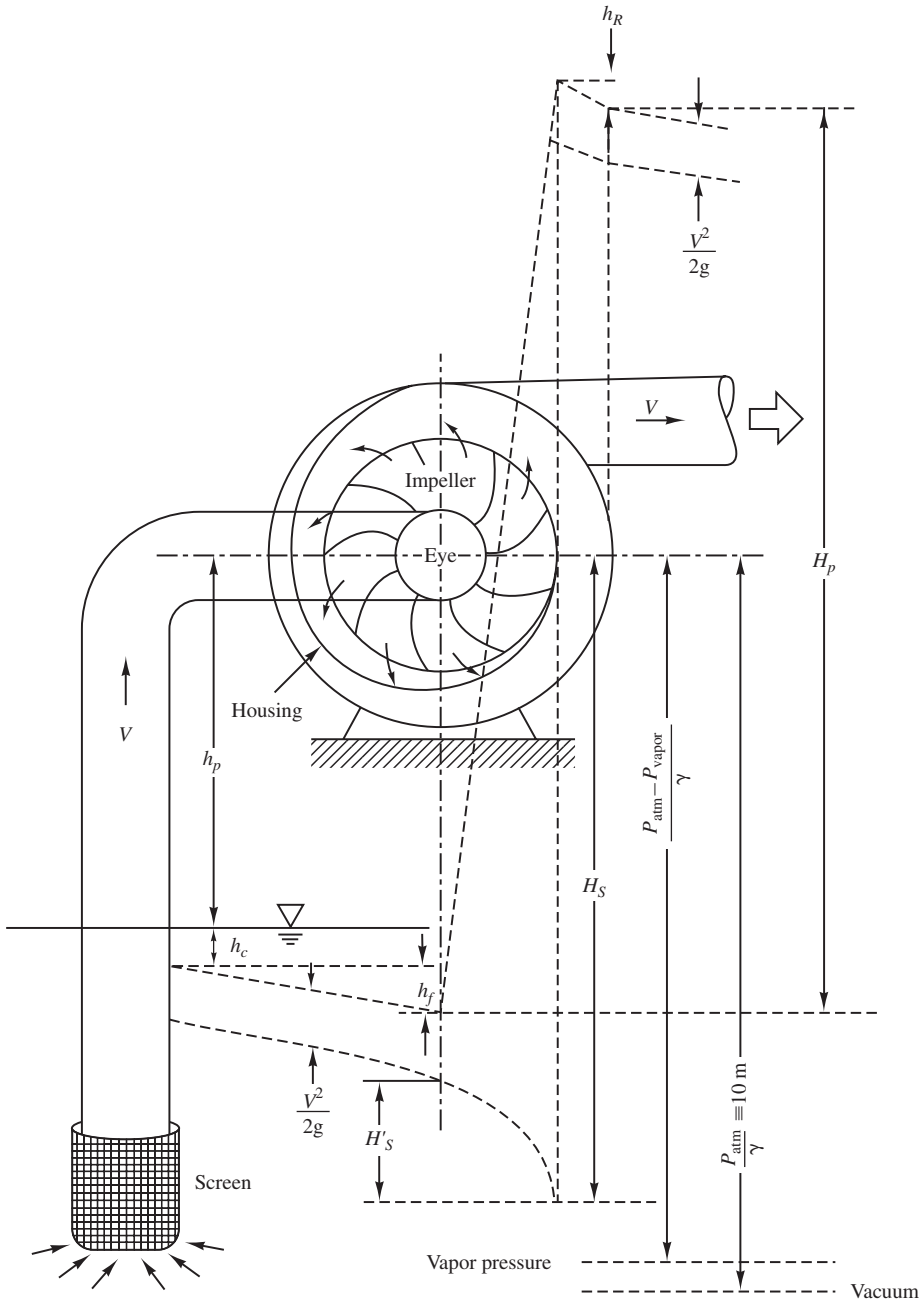


Figure 5.20 Energy and pressure relationship in a centrifugal pump

where V_i is the speed of the water at the entrance to the impeller. Rearranging Equation 5.22, we get

$$h_p = \frac{P_{atm}}{\gamma} - \frac{P_{vapor}}{\gamma} - \frac{V_i^2}{2g} - h_L - \sigma H_p \quad (5.23)$$

which defines the maximum allowable elevation of the pump intake (entrance to the impeller) above the surface of the supply reservoir. The losses (h_L) are on the suction side of the pump. If the value determined by Equation 5.23 is negative, then the pump must be placed at an elevation below the water surface elevation in the supply reservoir.

Example 5.8

A pump is installed in a 15-cm, 300-m-long pipeline to pump $0.060 \text{ m}^3/\text{sec}$ of water at 20°C . The elevation difference between the supply reservoir and the receiving reservoir is 25 m. The pump has an 18-cm impeller intake diameter, a cavitation parameter of $\sigma = 0.12$, and experiences a total head loss of 1.3 m on the suction side. Determine the maximum allowable distance between the pump intake and the water surface elevation in the supply tank. Assume the pipeline has $C_{HW} = 120$.

Solution

The friction loss in the pipeline can be determined from Equation 3.31 and Table 3.4 as

$$h_f = KQ^m = [(10.7 L)/(D^{4.87} C^{1.85})]Q^{1.85}$$

$$h_f = [10.7 (300)/\{(0.15)^{4.87} (120)^{1.85}\}](0.06)^{1.85} = 25.8 \text{ m}$$

The only minor loss in the discharge line is the exit (discharge) head loss, where $K_d = 1.0$.

The velocities in the intake pipe and in the main pipeline are, respectively,

$$V_i = \frac{Q}{A_i} = \frac{0.06}{\pi(0.09)^2} = 2.36 \text{ m/sec}$$

$$V_d = \frac{Q}{A_d} = \frac{0.06}{\pi(0.075)^2} = 3.40 \text{ m/sec}$$

The total pump head can be determined by applying the energy equation as

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 + H_p = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$$

where subscripts 1 and 2 refer to the reservoirs at the supply end and delivery end, respectively. At the surface of the reservoirs, we may write $V_1 \cong V_2 \cong 0$ and $P_1 = P_2 = P_{\text{atm}}$, therefore

$$H_p = (h_2 - h_1) + h_L = 25 + \left(1.3 + 25.8 + (1) \frac{(3.40)^2}{2g}\right) = 52.7 \text{ m}$$

The vapor pressure is found in Table 1.1:

$$P_{\text{vapor}} = 0.023042 \text{ bar} = 2,335 \text{ N/m}^2$$

whereas

$$P_{\text{atm}} = 1 \text{ bar} = 101,400 \text{ N/m}^2$$

Finally, by applying Equation 5.23, we obtain the maximum allowable height of the pump above the supply reservoir:

$$h_p = \frac{P_{\text{atm}}}{\gamma} - \frac{P_{\text{vapor}}}{\gamma} - \frac{V_i^2}{2g} - \Sigma h_{L_s} - \sigma H_p$$

$$= \frac{101,400}{9,790} - \frac{2,335}{9,790} - \frac{(2.36)^2}{2(9.81)} - 1.3 - (0.12)(52.7) = 2.21 \text{ m}$$

5.10 Specific Speed and Pump Similarity

The selection of a pump for a particular purpose is based on the required discharge rate and the head against which the discharge is delivered. To raise a large quantity of water over a relatively small elevation (e.g., removing water from an irrigation canal onto a crop field), a high-capacity, low-stage pump is required. To pump a relatively small quantity of water against great heights (e.g., supplying water to a high-rise building), a low-capacity, high-stage pump is required. The designs of these two pumps are very different.

Generally speaking, impellers of a relatively large radius and narrow flow passages transfer more kinetic energy from the pump into pressure head in the flow stream than impellers of smaller radius and large flow passages. Pumps designed with geometry that allows water to exit the impeller in a radial direction impart more centrifugal acceleration to the flow than those that allow water to exit axially or at an angle. Thus, the relative geometry of the impeller and the pump housing determine the performance and the field application of a specific pump.

Dimensional analysis, a computational procedure that is described in Chapter 10, shows that centrifugal pumps built with identical proportions but different sizes have similar dynamic performance characteristics that are consolidated into one number called a *shape number*. The shape number of a particular pump design is a dimensionless number defined as

$$S = \frac{\omega \sqrt{Q}}{(gH_p)^{3/4}} \quad (5.24)$$

where ω is the angular velocity of the impeller in radians per second, Q is discharge of the pump in cubic meters per second, g is the gravitational acceleration in meters per second squared, and H_p is the total pump head in meters.

In engineering practice, however, the dimensionless shape number is not commonly used. Instead, most commercial pumps are specified by the term *specific speed*. The specific speed of a specific pump design (i.e., impeller type and geometry) can be defined in two different ways. Some manufacturers define the specific speed of a specific pump design as the speed an impeller would turn if reduced enough in size to deliver a unit discharge at unit head. This way, the specific speed may be expressed as

$$N_s = \frac{\omega \sqrt{Q}}{H_p^{3/4}} \quad (5.25)$$

Other manufacturers define the specific speed of a specific pump design as the speed an impeller would turn if reduced enough in size to produce a unit of power at unit head. This way, the specific speed is expressed as

$$N_s = \frac{\omega \sqrt{P_i}}{H_p^{5/4}} \quad (5.26)$$

Most of the commercial pumps manufactured in the United States are currently specified with U.S. conventional units: gallons per minute (gpm), brake horsepower (bhp), feet (ft), and revolutions per minute (rpm). In SI units, cubic meter per second, kilowatts, meters, and radians per second are usually used in the computations. The conversions of specific speed among the U.S., English, metric, and SI units are provided in Table 5.1.

TABLE 5.1 Conversion of Specific Speed

Units	Discharge Units	Head Units	Pump Speed	Equation	Symbol	Conversion	
United States	U.S. gal/min	ft	rev/min	(5.25)	N_{s1}	$N_{s1} = 45.6 S$	$N_{s1} = 51.6 N_{s3}$
English	Imp. gal/min	ft	rev/min	(5.25)	N_{s2}	$N_{s2} = 37.9 S$	$N_{s2} = 43.0 N_{s3}$
Metric	m ³ /sec	m	rev/min	(5.25)	N_{s3}	$N_{s3} = 0.882 S$	$N_{s3} = 0.019 N_{s1}$
SI	m ³ /sec	m	rad/sec	(5.24)	S	$S = 0.022 N_{s1}$	$S = 1.134 N_{s3}$

Note: $g = 9.81 \text{ m/sec}^2 = 32.2 \text{ ft/sec}^2$

Normally, the specific speed is defined as the optimum point of operational efficiency. In practice, pumps with high specific speeds are generally used for large discharges at low-pressure heads, whereas pumps with low specific speeds are used to deliver small discharges at high-pressure heads. Centrifugal pumps with identical geometric proportions but different sizes have the same specific speed. Specific speed varies with impeller type. Its relationship to discharge and pump efficiency is shown in Figure 5.21.

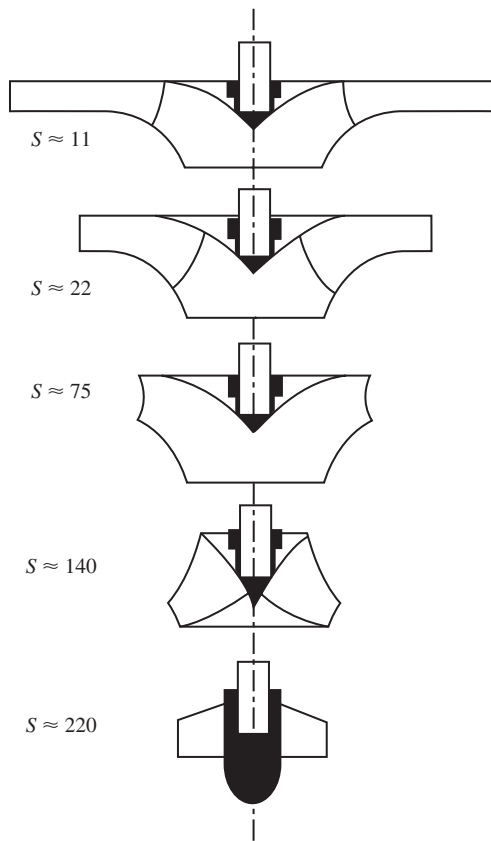


Figure 5.21 Relative impeller shapes and the approximate values of shape numbers, S , as defined in Table 5.1

Example 5.9

A centrifugal water pump operating at its optimum efficiency delivers 2.5 m³/sec over a height of 20 m. The pump has a 36-cm diameter impeller and rotates at 300 rad/sec. Compute the specific speed of the pump in terms of (a) discharge and (b) power if the maximum efficiency of the pump is 80 percent.

Solution

The given conditions are $Q = 2.5 \text{ m}^3/\text{sec}$, $H_p = 20 \text{ m}$, and $\omega = 300 \text{ rad/sec}$.

Applying Equation 5.25, we get

$$N_s = \frac{300\sqrt{2.5}}{(20)^{3/4}} = 50$$

At 80 percent efficiency, the shaft power is

$$P_i = (\gamma QH_p)/e_p = [(9,790)(2.5)(20)]/0.80 = 6.12 \times 10^5 \text{ W (612 kW)}$$

Applying Equation 5.26, we get

$$N_s = \frac{300\sqrt{612}}{(20)^{5/4}} = 175$$

Example 5.10

The impeller of the pump in Example 5.9 has a diameter of 0.36 m. What diameter should the impeller of a geometrically similar pump be for it to deliver one-half of the water discharge at the same head? What is the speed of the pump?

Solution

Applying Equation 5.25, from Example 5.9, we have

$$N_s = \frac{\omega\sqrt{\frac{1}{2}(2.5)}}{(20)^{3/4}} = 50$$

$$\omega = \frac{50(20)^{3/4}}{(1.25)^{1/2}} = 423 \text{ rad/sec}$$

By definition, two pumps are geometrically similar if they have the same ratio of water discharge velocity to vane tip peripheral speed. When this is the case, the following relationship will be satisfied:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \quad (a)$$

Hence,

$$\frac{2.5}{300(0.36)^3} = \frac{1.25}{423(D_2)^3}$$

The diameter: $D_2 = 0.255 \text{ m} = 25.5 \text{ cm}$.

5.11 Selection of a Pump

There are many different types of water pumps, and hydraulic engineers are faced with the task of choosing the proper pump for a particular application. However, certain types of pumps are more suited than others depending on the discharge, head, and power requirements of the pump. For the major pump types discussed in this chapter, the approximate ranges of application are indicated in Figure 5.22.

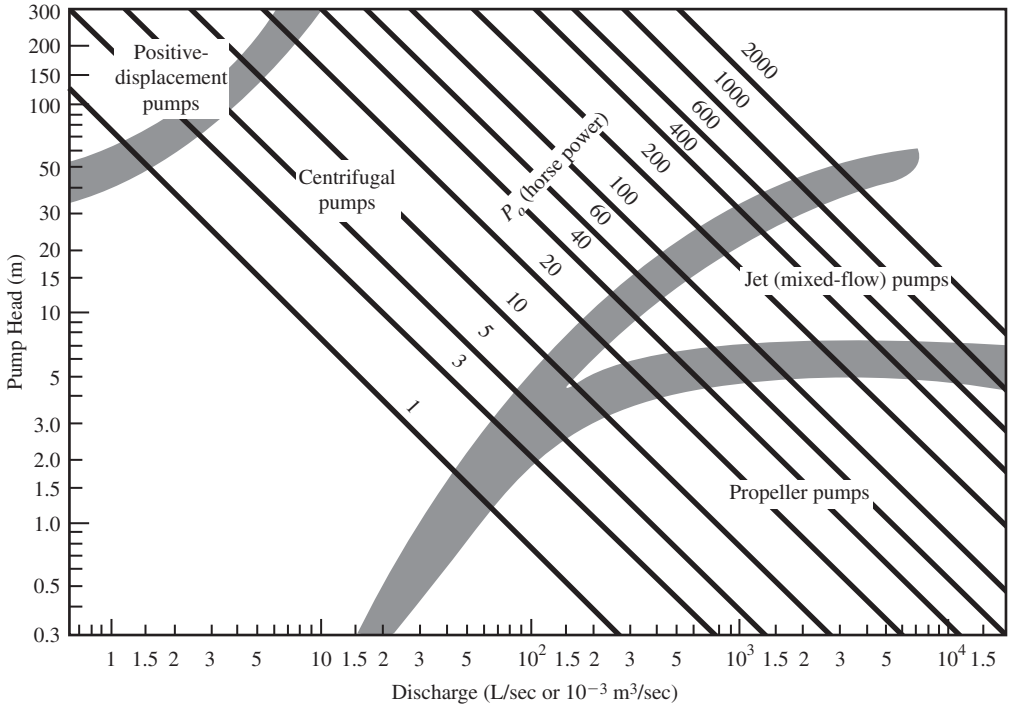


Figure 5.22 Discharge, head, and power requirements of different types of pumps

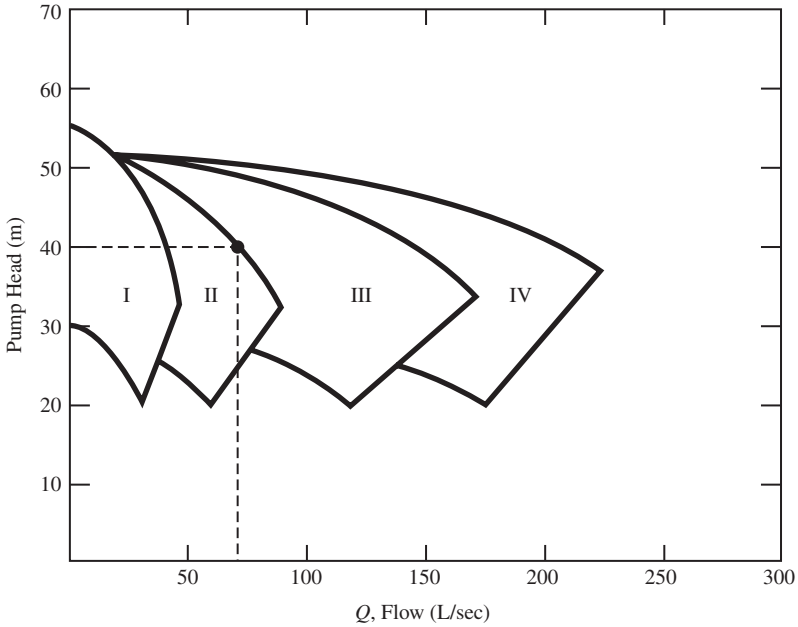


Figure 5.23 Pump model selection chart

The total head that a pump must produce to deliver the requisite flow rate is the sum of the required elevation rise and the head losses incurred in the system. Because friction loss and minor losses in the pipeline depend on the velocity of water in the pipe (Chapter 3), the total head loss is a function of the flow rate. For a given pipeline system (including the pump), a unique system head curve can be plotted by computing the head losses for a range of discharges. The process was discussed in detail in Section 5.5.

In selecting a particular pump for a given application, the design conditions are specified and the appropriate pump model is chosen (e.g., Figure 5.23; Pump I, II, III, or IV). The system head curve is then plotted on the pump performance curve (e.g., Figure 5.24) provided by the manufacturer. The intersection of the two curves, called the match point (*M*), indicates the actual operating conditions. The selection process is demonstrated in the following example.

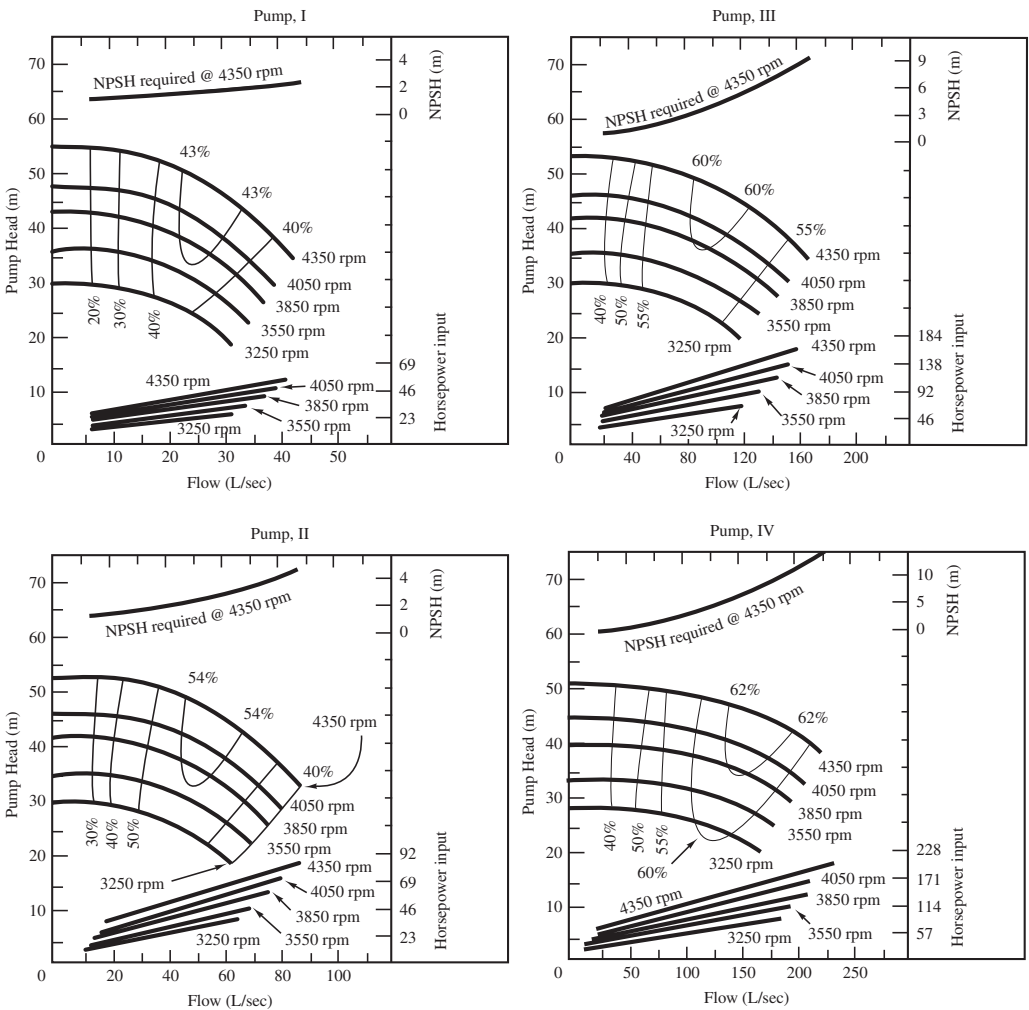


Figure 5.24 Characteristic curves for several pump models

Example 5.11

A pump will be used to deliver a discharge of 70 L/sec of water between two reservoirs 1,000 m apart with an elevation difference of 20 m. Commercial steel pipes 20-cm in diameter are used for the project. Select the proper pump and determine the operating conditions for the pump based on the pump selection chart (Figure 5.23) and the pump characteristics curves (Figure 5.24), both provided by the manufacturer.

Solution

For commercial steel pipe, the roughness height, $e = 0.045$ mm (Table 3.1). The flow velocity in the pipe is

$$V = \frac{Q}{A} = \frac{0.070 \text{ m}^3/\text{sec}}{\frac{\pi}{4}(0.2 \text{ m})^2} = 2.23 \text{ m/sec}$$

and the corresponding Reynolds number at 20°C is

$$N_R = \frac{VD}{\nu} = \frac{(2.23 \text{ m/sec})(0.2 \text{ m})}{1 \times 10^{-6} \text{ m}^2/\text{sec}} = 4.5 \times 10^5$$

and

$$e/D = 0.045 \text{ mm}/200 \text{ mm} = 2.3 \times 10^{-4} = 0.00023$$

The friction coefficient can be obtained from the Moody diagram (Figure 3.8) ($f = 0.016$).

The pipe friction losses are then

$$h_f = f \left(\frac{L}{D} \right) \frac{V^2}{2g} = 0.016 \left(\frac{1000}{0.2} \right) \left(\frac{(2.23)^2}{2(9.81)} \right) = 20.3 \text{ m}$$

The total head (neglecting minor losses) that the pump must work against is

$$\begin{aligned} H_{SH} &= (\Delta \text{ elevation}) + (\text{friction loss}) \\ &= 20.0 \text{ m} + 20.3 \text{ m} = 40.3 \text{ m} \end{aligned}$$

From the pump-selection chart provided by the manufacturers (e.g., Figure 5.23), pumps II and III may be used for the project. The system head curve should be determined before making a selection.

Q (L/s)	V (m/s)	N_R	f	h_f	H_{SH}
50	1.59	3.2×10^5	0.0165	10.7	30.7
60	1.91	3.8×10^5	0.0160	14.9	34.9
80	2.55	5.1×10^5	0.0155	25.6	45.6

Values of H_{SH} versus Q (system head curve) are graphed on the pump characteristic (performance) curves shown in Figure 5.24. Superimposing this system curve over the characteristics of pumps II and III as provided by the manufacturer (Figure 5.24), we find that the possibilities are as follows.

First selection: Pump II at 4,350 rpm,

$$Q = 70 \text{ L/sec}, \quad H_p = 40.3 \text{ m}$$

hence,

$$P_i = 71 \text{ hp and efficiency} = 52\%$$

Second selection: Pump III at 3,850 rpm,

$$Q = 68 \text{ L/sec}, \quad H_p = 39 \text{ m}$$

hence,

$$P_i = 61 \text{ hp and efficiency} = 58\%$$

Third selection: Pump III at 4,050 rpm,

$$Q = 73 \text{ L/sec, } H_p = 42 \text{ m}$$

hence,

$$P_i = 70 \text{ hp and efficiency} = 59\%$$

Hydraulically, the final selection should be pump II at 4,350 rpm because it best fits the given conditions. However, one may notice that the second selection (pump III at 3,850 rpm) and the third selection (pump III at 4,050 rpm) also fit the conditions rather closely with higher pump efficiencies. In this case, the selection would best be made based on the considerations of the cost of the pump versus the cost of electricity.

PROBLEMS (SECTION 5.1)

- 5.1.1.** A centrifugal pump is installed in a pipeline between two reservoirs. The pump is required to produce a flow rate of 2,500 gpm (gallons per minute) in moving water from the lower reservoir to the upper reservoir. The water surface elevations of the two reservoirs are separated by 104 feet, and the pump operates at an overall efficiency of 78.5 percent. Determine the required power input (in kW) to the motor. (Assume that pipeline losses are negligible.)
- 5.1.2.** A pump is needed to quickly drain a small pond before the earth dam fails. Water must be pumped over the top of the dam, which is about 2 m above the water surface. The only pump available is an old 10-cm diameter propeller pump. The power requirement for the motor is 1,000 watts, and the pump-motor combination has a low efficiency of 50 percent. How many centimeters will the lake go down in the initial 24-hour period if the lake has a surface area of 5,000 square meters? (*Note:* Both friction losses and minor losses are insignificant when compared to the position head the pump must overcome.)
- 5.1.3.** A pump is installed in a 100-m pipeline to raise water 20 m from reservoir A to reservoir B. The pipe is rough concrete with a diameter of 80 cm. The design discharge is 2.06 m³/sec. Determine the overall efficiency of the pump system if the power requirement for the motor is 800 kW.
- 5.1.4.** Answer the following questions about pumps.
- (a) Referring to Figure 4.3, balance energy between positions 1 and 4. Solve for H_p and describe what the energy head added by the pump accomplishes physically in the system.
 - (b) Referring to Figure 4.3, balance energy between positions 2 and 3. Solve for H_p and describe what the energy head added by the pump accomplishes. Assume the pipe diameters at 2 and 3 are the same.
 - (c) What concepts (first principles) are used in deriving the power input to a centrifugal pump (Equation 5.3)?
- 5.1.5.** A pump impeller has an outside radius of 50 cm, an inside radius of 15 cm, and vanes with a uniform opening (width) of 20 cm. When the impeller is rotated at an angular speed of 450 rpm, water exits from the impeller with an absolute velocity of 45 m/sec. The angle of the exiting water is 55 degrees measured from a radial line emanating from the center of the impeller outward. Determine the torque generated by the exiting flow.
- 5.1.6.** A centrifugal water pump runs at 1,800 rpm and has an outside radius of 12 in. ($\beta_o = 170^\circ$), an inside radius of 4 in. ($\beta_i = 160^\circ$), and an impeller thickness (width) of 2 in. at $r = r_i$ and $\frac{3}{4}$ in. width at $r = r_o$. Determine the pump flow rate for a shockless entry (i.e., $\alpha_i = 90^\circ$) and the exit angle α_o .

- 5.1.7.** A centrifugal pump has the following specifications: a uniform impeller thickness of 4 in., an inlet radius of 1 ft, an outlet radius of 2.5 ft, $\beta_i = 120^\circ$, and $\beta_o = 135^\circ$. The pump delivers a flow rate of 70 cfs (ft^3/sec) in overcoming a head of 33 ft. If the pump rotates at such a speed that no tangential velocity component exists at the inlet (shockless entry), what is the rotational speed (in rpm) of the pump? Also calculate the power input (in hp) to the pump.
- 5.1.8.** A centrifugal pump is being tested in a laboratory. The inside and outside radii of the impellers are 7.5 cm and 15.0 cm, respectively. The width of the impeller vanes (or flow opening height) varies from 5.0 cm at intake to 3.0 cm at outflow. If the measured flow rate is 55 liters/sec, what is the pump speed (in rpm) and the power input to the pump (in kW)? Assume a shockless entry ($\alpha_i = 90^\circ$), an exit flow angle of 22.4° , and $\beta_i = 150^\circ$.

(SECTION 5.5)

- 5.5.1.** In the pump-pipeline system of Example 5.3, the discharge is controlled by a flow-throttling valve installed just upstream of the pump. What is the required head loss from this valve if the desired discharge in the pipe is 15 cfs? Would this be an efficient system? The pump characteristics are the same as in Example 5.3 and are shown in the following table to facilitate plotting.

Q (cfs)	H_p (ft)
0	300.0
5	295.5
10	282.0
15	259.5
20	225.5
25	187.5
30	138.0
35	79.5

- 5.5.2.** In the pump-pipeline system of Example 5.3, the discharge is controlled by a flow-throttling valve installed just upstream of the pump. Determine the discharge and pump head if the head loss from this valve in feet can be expressed as $0.1Q^2$, where Q is in cfs. The pump characteristics are tabulated in Problem 5.5.1.
- 5.5.3.** Consider a pump-pipeline system that delivers water from reservoir A to B with $E_A = 45.5$ m and $E_B = 52.9$ m. The pipe has a length of $L = 3,050$ m, diameter of $D = 0.50$ m, and a Darcy-Weisbach friction factor of $f = 0.02$. Minor losses include an inlet, exit, and swing-type check valve. The pump characteristics are shown in the following table. Determine the flow rate and velocity in the pipeline.

Q (m^3/s)	H_p (m)
0.00	91.4
0.15	89.8
0.30	85.1
0.45	77.2
0.60	65.9
0.75	52.6
0.90	36.3
1.05	15.7

5.5.4. A centrifugal pump delivers water through a 40-cm-diameter, 1,000-m-long commercial steel pipe from reservoir A to B with $E_A = 920.5$ m and $E_B = 935.5$ m. Friction losses vary according to the Moody diagram (alternatively, the Swamee-Jain equation, 3.24a). Neglecting minor losses, determine the discharge in the pump-pipeline system using the pump performance characteristics in the following table.

Q (L/sec)	H_p (m)
0	30.0
100	29.5
200	28.0
300	25.0
400	19.0
500	4.0

5.5.5. Water is being pumped from a supply reservoir to an elevated holding tank. The elevation gain is 14.9 m, and the length of the ductile iron supply pipe ($f = 0.019$) connecting the reservoirs is 22.4 m. The pipe is 5.0 cm in diameter, and the performance curve of the pump is given by $H_p = 23.9 - 7.59Q^2$, where H_p is in meters and Q is in liters per second. (The equation is valid for flows less than or equal to 1.5 liters per second.) Using this pump, what flow do you expect in the pipeline if minor losses are ignored? What pump head is required for this flow?

5.5.6. In the transmission system shown in Figure P5.5.6, $E_A = 100$ ft and $E_D = 150$ ft. The Darcy-Weisbach friction factor is 0.02 for all the pipes. Pipe AB is 4,000 ft long with a diameter of 3 ft, and pipe CD is 1,140 ft long with a diameter of 3 ft. The branch BC1 has a length of 100 ft and a diameter of 2 ft, and branch BC2 is 500 ft long with a diameter of 1 ft. The pump characteristics are shown in the following table. Determine the system flow rate and the discharge in branches 1 and 2.

Q (cfs)	H_p (ft)
0	60
10	55
20	47
30	37
40	23
50	7

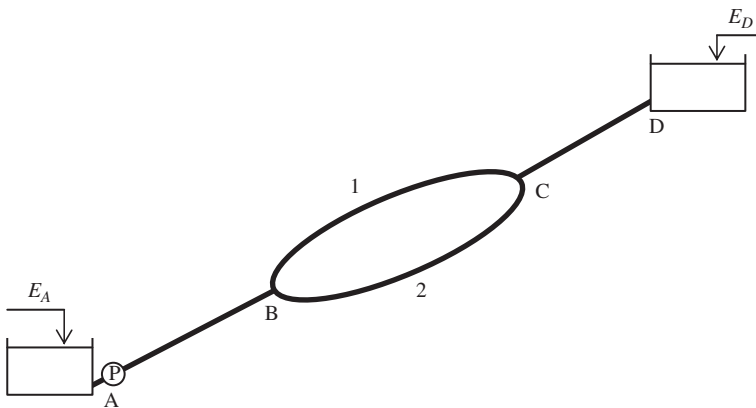


Figure P5.5.6

(SECTION 5.6)

5.6.1. The following table provides the results of a pump performance test.

Discharge (gpm) ^a	0	200	400	600	800	1,000
Dynamic head (ft)	150	145	135	120	90	50

^aA common flow unit for pumps is gallons per minute (gpm).

- (a) Plot the pump characteristic (performance) curve.
 (b) Plot the characteristic curve for two pumps in series.
 (c) Plot the characteristic curve for two pumps in parallel.
 (d) What pump configuration would work for a required flow of 1,700 gpm, which must overcome a head of 80 ft?
 (e) What pump configuration would work for a required flow of 1,700 gpm, which must overcome a head of 160 ft?
- 5.6.2. Two identical pumps have the characteristic curves shown in Figure 5.13, which are shown in the following table. The pumps are connected in series and deliver water through a 40-cm-diameter, 1,000-m-long commercial steel pipe into a reservoir where the water level is 25 m above the pump. Neglecting minor losses, determine (a) the discharge in the system if only one pump is used, and (b) the discharge when the system contains two pumps connected in series. [*Hint*: Use a spreadsheet with the Swamee-Jain equation (3.24a) applied to determine the friction factor.]

Q (L/sec)	H_p (m)
0	30.0
100	29.5
200	28.0
300	25.0
400	19.0
500	4.0

- 5.6.3. Two identical pumps are used in parallel to move water through a pipeline from reservoir A to reservoir B where $E_A = 772$ ft and $E_B = 878$ ft. The 2.0-ft-diameter pipe connecting the two reservoirs has a length of 4,860 ft. and $C_{HW} = 100$. The pump characteristics are provided in the following table. Determine the discharge in the pipeline, the velocity of flow, and the pump head. Would the solution change if minor losses were considered?

Q (cfs)	H_p (ft)
0	300.0
5	295.5
10	282.0
15	259.5
20	225.5
25	187.5
30	138.0
35	79.5

- 5.6.4. Consider a pump-pipeline system that delivers water from reservoir A to B with $E_A = 45.5$ m and $E_B = 52.9$ m. The pipe has a length of $L = 3,050$ m, diameter of $D = 0.50$ m, and a Darcy-Weisbach friction factor of $f = 0.02$. Minor losses include an inlet, exit, and swing-type check

valve. The pump characteristics are shown in the following table. When a single pump is used in the pipeline, the flow rate is $0.595 \text{ m}^3/\text{sec}$ with a pump head of about 66.3 m . Determine the flow rate in the pipeline if two identical pumps are used in a series combination. Also, determine the flow rate in the pipeline if two identical pumps are used in a parallel combination.

$Q \text{ (m}^3/\text{s)}$	$H_p \text{ (m)}$
0.00	91.4
0.15	89.8
0.30	85.1
0.45	77.2
0.60	65.9
0.75	52.6
0.90	36.3
1.05	15.7

(SECTION 5.7)

5.7.1. In Figure 5.15, $E_A = 10 \text{ m}$, $E_B = 16 \text{ m}$, and $E_C = 22 \text{ m}$. Both pipes have a Darcy-Weisbach friction factor of 0.02 . Pipe 1 is $1,000 \text{ m}$ long and has a diameter of 1.0 m . Pipe 2 is $3,000 \text{ m}$ long and has a diameter of 1.0 m . The pump characteristics are shown in the following table. Determine the discharge in each pipe.

$Q \text{ (m}^3/\text{sec)}$	0	1	2	3	4	5	6	7
$H_p \text{ (m)}$	30.0	29.5	28.0	25.5	22.0	17.5	12.0	5.0

5.7.2. In Figure P5.7.2, each pipe has the following properties: $D = 3 \text{ ft}$, $L = 5,000 \text{ ft}$, and $f = 0.02$. The two pumps are identical; the pump characteristics are shown in the following table. The water surface elevations are 80.0 ft in reservoir A and 94.0 ft in reservoir D. The discharge in pipe BC is 20 cfs in the direction from B to C. Determine (a) the discharge in AB and BD, (b) the water surface elevation in reservoir C, and (c) the head added to the flow by each pump.

$Q \text{ (cfs)}$	0	10	20	30	40	50	60
$H_p \text{ (ft)}$	70	68	64	57	47	35	23

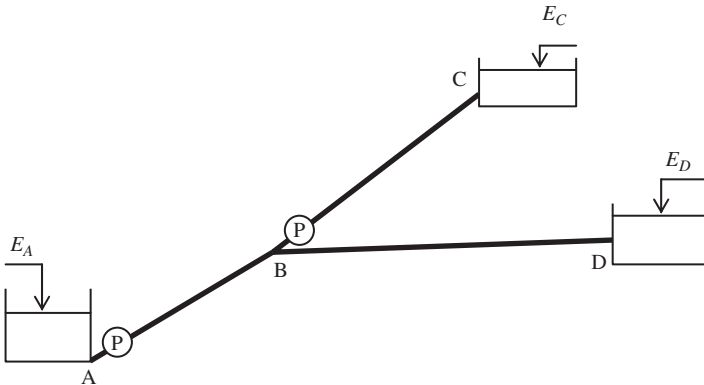


Figure P5.7.2

5.7.3. Determine the discharge in each pipe in Figure P5.7.3 if $E_A = 100$ ft, $E_B = 80$ ft, and $E_C = 120$ ft. The pipe and pump characteristics are shown in the following tables.

Pipe Characteristics			
Pipe	L (ft)	D (ft)	f
1	8,000	2	0.02
2	9,000	2	0.02
3	15,000	2.5	0.02

Pump Characteristics		
Q (cfs)	H_{p1} (ft)	H_{p2} (ft)
0.0	200.0	150.0
10.0	195.0	148.0
15.0	188.8	145.5
20.0	180.0	142.0
25.0	168.8	137.5
30.0	155.0	132.0
40.0	120.0	118.0
50.0	75.0	100.0

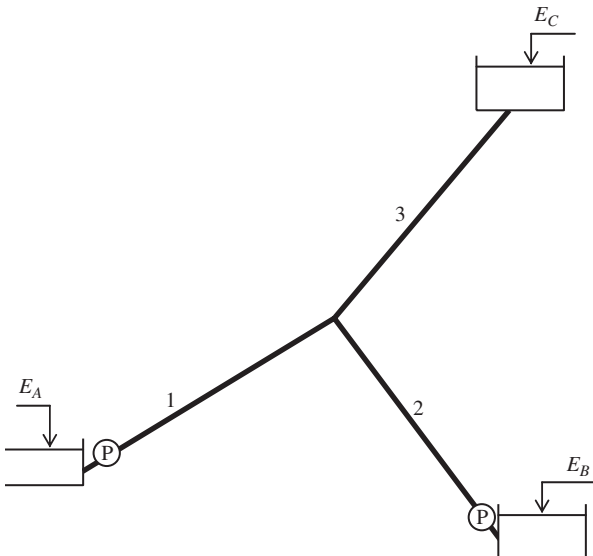


Figure P5.7.3

(SECTION 5.9)

- 5.9.1. A pump delivers 6.0 cfs of 68°F water to a holding tank 65 ft above the supply reservoir. The suction side possesses a strainer ($K_s = 2.5$), a foot valve ($K_v = 0.1$), and 35 ft of cast-iron pipe, 10 in. in diameter. Determine the allowable height the pump can be placed above the supply reservoir to avoid cavitation if the NPSH is 15 ft. (Note: The strainer incorporates the entrance loss.)
- 5.9.2. A pump delivers 30°C water from a supply reservoir to an elevated tank at a rate of 120 liters per second. The elevation difference between the reservoirs is 45 m, and the supply line is 150 m long (ductile-iron pipe) and 35 cm in diameter. The suction line is 10 m of the 150 m length. Minor loss coefficients in the suction line total 3.7. If the pump is installed 1 m to 3 m above the supply reservoir (depending on fluctuations in the water level), will the installation be susceptible to cavitation? The net positive suction head of the pump is rated at 6 m.

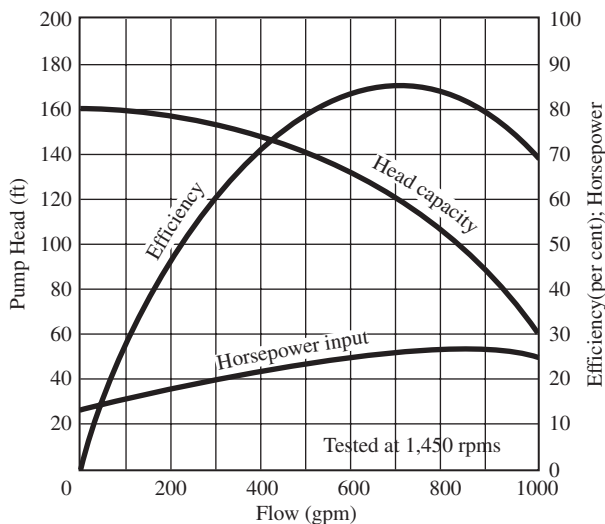
- 5.9.3.** A pump delivers water at 10°C between a reservoir and a water tank 20 m higher. The suction side consists of a strainer ($k_s = 2.5$), three 90° bends ($R/D = 2$), and 10 m of ductile-iron pipe, 25 cm in diameter. The discharge side includes a 160-m-long, ductile-iron pipe, 20 cm in diameter, and a gate valve. The friction factor for the pipeline is 0.02, the net positive suction head is 7.5 m, and the design discharge is 170 liters/sec. Determine the allowable elevation difference between the pump and the reservoir water surface to avoid cavitation.
- 5.9.4.** A pump is installed in a 250-m pipeline to raise 20°C water 55 m from a supply reservoir to an elevated tank. The pipe is rough concrete with a diameter of 80 cm, and the design discharge is 2.19 m³/s. The pump is placed outside the supply reservoir (0.9 m below the water surface in the tank) and has a cavitation parameter of 0.15. Determine the maximum distance in meters that the pump could be installed away from the supply reservoir (i.e., the allowable length of the suction line) without encountering cavitation problems. The only minor loss coefficient on the suction line is an entrance loss of 0.5.
- 5.9.5.** Factory tests indicated that a cavitation parameter for a particular pump is $\sigma = 0.075$. The pump is installed to pump 60°C water at sea level. If the total head loss between the inlet and the suction side of the pump is 0.5 m, determine the allowable level of the pump intake relative to the supply reservoir water surface elevation if cavitation is to be avoided. The discharge is 0.04 m³/sec, and the pipe system is the one used in Example 5.8.
- 5.9.6.** The efficiency of a pump will drop suddenly if cavitation takes place in the pump. This phenomenon is observed in a particular pump (with $\sigma = 0.08$) operating at sea level, and the pump delivers 0.42 m³/sec of water at 40°C. Determine the sum of the gauge pressure head and the velocity head at the inlet (i.e., the sum, not the individual components). The total head delivered by the pump is 85 m, and the suction pipe diameter is 30 cm.

(SECTION 5.10)

- 5.10.1.** Answer the following questions about specific speed and pump similarity.
- Using both U.S. and SI units, show that the shape number given in Equation 5.24 is dimensionless.
 - Is specific speed a dimensionless number (Equations 5.25 and 5.26)?
 - Is it possible to derive Equation 5.25 from Equation 5.26 based on the relationship between power and flow rate? Will the specific speed defined by the two different relationships be identical?
 - In Example 5.10, Equation (a) was used to determine the impeller diameter. Derive this equation based on the premise that geometrically similar pumps have the same ratio of water discharge velocity to vane tip peripheral speed.
- 5.10.2.** A geometrically similar pump of the same design as in Example 5.9 has an impeller diameter of 72 cm and the same efficiency when operated at 1,720 rpm (revolutions per minute). Determine the pump head and shaft power required to operate the pump if the discharge is 12.7 m³/sec.
- 5.10.3.** The specific speed of a pump is rated at 68.6 (based on unit discharge) and 240 (based on unit power). In a specific operation, the flow rate is 0.15 m³/sec when the pump is operating at 1,800 rpm. Determine the efficiency of the pump.
- 5.10.4.** A pump is required for a U.S. field application with the following specifications: a flow rate of 12.5 cfs (ft³/sec) against a head of 95 ft. To design the pump, a model is built with a 6-in. diameter impeller and tested under optimum conditions. The test results show that at a speed of 1,150 rpm the pump requires 3.1 hp to discharge 1 cfs against a head of 18 ft. Determine the power requirement, diameter, and speed of a geometrically similar pump for the field.
- 5.10.5.** The design of a centrifugal water pump is studied by a 1/10 scale model in a hydraulic laboratory. At the optimum efficiency of 89 percent the model delivers 75.3 L/sec of water against a 10-m head at 4,500 rpm. If the prototype pump has a rotational speed of 2,250 rpm, what is the discharge and efficiency required to operate the pump under this condition?

(SECTION 5.11)

- 5.11.1.** Referring to Example 5.11, plot the system head curve with a spreadsheet program. On the same graph, plot the pump characteristic curve for pump III at a speed of 3,850 rpm. Now answer the following questions.
- What is the shape of the system curve? Why does it take this shape?
 - What is the shape of the characteristic curve? Why does it take this shape?
 - What flow rate produces the highest total head on the pump characteristic curve? What physical meaning does this have?
 - What is the intersection point of the two curves? Does it meet the design conditions?
- 5.11.2.** A pump is required to deliver $0.125 \text{ m}^3/\text{sec}$ of water (20°C) from reservoir A to reservoir B (water surface elevations of 385.7 m and 402.5 m, respectively). The pipeline (concrete) is 300 m long with a diameter of 0.20 m and contains five bends ($R/D = 6$) and two gate valves. Determine the appropriate pump from Figure 5.24 along with the working conditions.
- 5.11.3.** A pump is required to deliver 20°C water to an elevated storage tank. The water must be raised 44 m, and a 150-m-long, ductile-iron pipe, 35 cm in diameter, is to be used. Determine the appropriate pump speed (based on the highest efficiency obtainable) and the operating conditions if pump III in Figure 5.24 is used. The suction line is 10 m of the 150-m length, minor loss coefficients in the suction line total 3.7, and the discharge line contains only an exit loss.
- 5.11.4.** Select two different pumps (model and rotational speed) from Figure 5.24 that are capable of supplying water (20°C) to a reservoir at a flow rate of 30 L/sec. The water must be raised 20 m, and the distance between the supply and receiving reservoirs is 100 m. A ball check valve is used with 15-cm diameter galvanized iron pipeline. Determine the working conditions and the pump efficiencies.
- 5.11.5.** Determine the working conditions (H_p , Q , e , and P_i) of a pump capable of moving water (68°F) from reservoir A to reservoir B ($E_A = 102 \text{ ft}$ and $E_B = 180 \text{ ft}$). The 1.0-ft-diameter pipe connecting the two reservoirs has a length of 8,700 ft, and $C_{HW} = 100$. The pump characteristics are available in Figure P5.11.5. Also verify that the pump efficiency obtained from the pump characteristic curves (Figure P5.11.5) corresponds with the pump efficiency obtained from efficiency equations.

**Figure P5.11.5**

5.11.6. A project requires a pump that will operate with a minimum discharge of 20 L/sec against an elevation head of 40 m. The distance between the supply and delivery points is 150 m. A ball check valve will be used in the system that consists of commercial steel pipes. Determine the most economical pipe diameter for the pipeline and select a single pump from Figure 5.24 (including its operating conditions) if the total cost can be expressed as

$$C = d^{1.75} + 0.75P + 18$$

where d is the pipe diameter in centimeters and P is the pump horsepower input.

5.11.7. A pump station is required to deliver 20°C water from a reservoir to an elevated storage tank at a minimum required discharge of 300 L/sec. The difference in elevations is 15 m, and a 1,500-m-long, wrought-iron pipe that is 40 cm in diameter is used. Select the pump(s) from the set given in Figure 5.24. Determine the number of pumps, the configuration (series or parallel), discharge, total head, and efficiency at which the pumps operate. Ignore minor losses.

PROJECT PROBLEM

5.11.8. A pumping system is designed to pump water from a 6-m-deep supply reservoir to a water tower 40 m above the ground. The system consists of a pump (or a combination of pumps), a 20-m long pipeline with one elbow on the suction side of the pump, a 60-m long pipeline, a gate valve, a check valve, and two elbows on the delivery side of the pump. The system is designed to pump 420 L/sec of water 350 days a year. Select a pump (or pumps) based on the characteristics provided in Figures 5.23 and 5.24 and a pipe size for optimum economy. Assume $C_{HW} = 100$ for all pipe sizes listed in the following table, and all elbows are 90° ($R/D = 2.0$). Power cost is \$0.04/kW-hr. Motor efficiency is 85 percent for all sizes listed.

Pumps	Cost (\$)	Motor (hp)	Cost (\$)
I	700	60	200
II	800	95	250
III	900	180	300
IV	1,020	250	340

Appurtenances	Size/Cost		
	20 cm (\$)	25 cm (\$)	30 cm (\$)
Pipe (10 m)	120	150	180
Elbow	15	25	35
Gate valve	60	90	120
Check valve	80	105	130

6



Water Flow in Open Channels

Open-channel flow differs from pipe flow in one important aspect. Pipe flow fills the entire conduit, so the flow boundaries are fixed by the conduit geometry. In addition, pipe flow possesses hydraulic pressure that varies from one section to another along the pipeline. Open-channel flow has a free surface that adjusts itself depending on the flow conditions. The free surface is subject to atmospheric pressure, which remains relatively constant throughout the entire length of the channel. Therefore, open-channel flow is driven by the component of the gravitational force along the channel slope. Notice that the channel slope will appear in all the open-channel flow equations, whereas the pipe flow equations include only the slope of the energy grade line.

In Figure 6.1, open-channel flow is schematically compared to pipe flow. Figure 6.1(a) shows a pipe flow segment with two open-ended vertical tubes (piezometers) installed through the pipe wall at an upstream section, 1, and a downstream section, 2. The water level in each tube represents the pressure head (P/γ) in the pipe at the section. A line connecting the water levels in the two tubes represents the *hydraulic grade line* (HGL) between these sections. The velocity head at each section is represented in the familiar form, $V^2/2g$, where V is the mean velocity, $V = Q/A$, at the section. The total energy head at any section is equal to the sum of the elevation (potential) head (h), the pressure head (P/γ), and the velocity head ($V^2/2g$). A line connecting the total energy head at the two sections is called the *energy grade line* (EGL). The amount of energy lost when water flows from section 1 to section 2 is indicated by h_L .

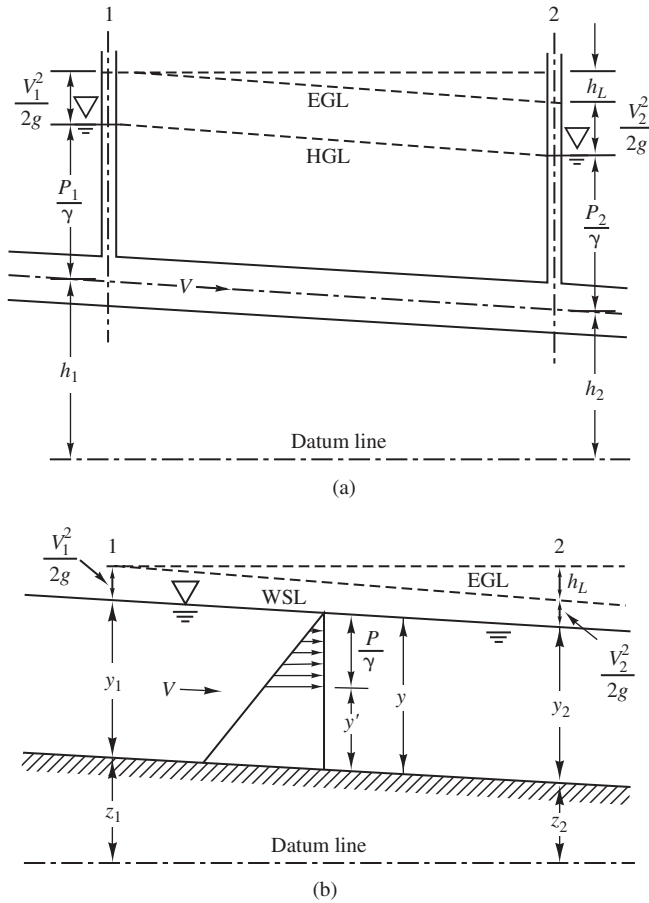


Figure 6.1 Comparison of: (a) pipe flow and (b) open-channel flow

Figure 6.1(b) shows an open-channel flow segment. The free water surface is subjected to only atmospheric pressure, which is commonly referred to as the *zero pressure reference* in hydraulic engineering practice. The pressure distribution at any section is directly proportional to the depth measured from the free water surface. In this case, the water surface line corresponds to the hydraulic grade line in pipe flow.

To solve open-channel flow problems, we must seek the interdependent relationships between the slope of the channel bottom, the discharge, the water depth, and other channel characteristics. The basic geometric and hydraulic definitions used to describe open-channel flow through a channel section are:

- | | |
|--------------------------|---|
| Discharge (Q) | Volume of water passing through a flow section per unit time |
| Flow area (A) | Cross-sectional area of the flow |
| Average velocity (V) | Discharge divided by the flow area: $V = Q/A$ |
| Flow depth (y) | Vertical distance from the channel bottom to the free surface |
| Top width (T) | Width of the channel section at the free surface |

Wetted perimeter (P)	Contact length of the water and the channel at a cross section
Hydraulic depth (D)	Flow area divided by the top width: $D = A/T$
Hydraulic radius (R_h)	Flow area divided by the wetted perimeter: $R_h = A/P$
Bottom slope (S_0)	Longitudinal slope of the channel bottom
Side slope (m)	Slope of channel sides defined as 1 vertical over m horizontal
Bottom width (b)	Width of the channel section at the bottom

Table 6.1 depicts the cross-sectional characteristics for various types of channel sections and their geometric and hydraulic relationships.

6.1 Open-Channel Flow Classifications

Open-channel flow may be classified by space and time criteria.

Based on the space criterion, an open channel characterizes *uniform flow* if the water depth remains the same throughout a length of channel reach at a given time. Uniform flow is more likely to occur in *prismatic channels*, channels where the cross-sectional area and bottom slope do not change over the channel reach. An open channel characterizes *varied flow* if the water depth or the discharge change along the length of the channel. Varied flow can further be classified as *gradually varied flow* or *rapidly varied flow*, depending on whether the changes in the flow depth are gradual or abrupt. Examples of uniform, gradually varied, and rapidly varied flows are shown in Figure 6.2 (a), where flow enters a channel with a mild slope from underneath a sluice gate. The flow will reach its minimum depth immediately after the sluice gate and will become gradually varied further downstream. The depth will then change rapidly through a hydraulic jump and will remain constant afterward.

Based on the time criterion, open-channel flow may be classified into two categories: *steady flow* and *unsteady flow*. In steady flow, the discharge and water depth at any section in the

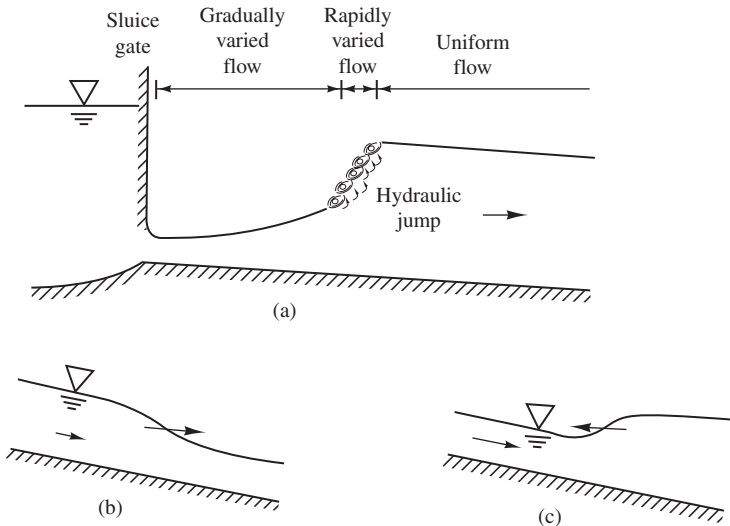
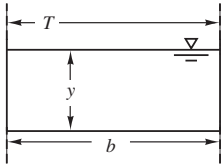
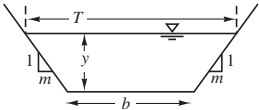
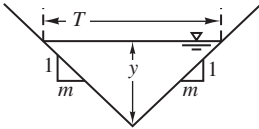
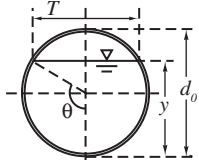


Figure 6.2 Classifications of open-channel flow: (a) gradually varied flow (GVF), rapidly varied flow (RVF), and uniform flow (UF); (b) unsteady varied flow; (c) unsteady varied flow

TABLE 6.1 Cross-Sectional Relationships for Open-Channel Flow

Section Type	Area (A)	Wetted perimeter (P)	Hydraulic Radius (R_h)	Top Width (T)	Hydraulic Depth (D)
Rectangular 	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y
Trapezoidal 	$(b + my)y$	$b + 2y\sqrt{1 + m^2}$	$\frac{(b + my)y}{b + 2y\sqrt{1 + m^2}}$	$b + 2my$	$\frac{(b + my)y}{b + 2my}$
Triangular 	my^2	$2y\sqrt{1 + m^2}$	$\frac{my}{2\sqrt{1 + m^2}}$	$2my$	$\frac{y}{2}$
Circular (θ is in radians) 	$\frac{1}{8}(2\theta - \sin 2\theta)d_0^2$	θd_0	$\frac{1}{4}\left(1 - \frac{\sin 2\theta}{2\theta}\right)d_0$	$(\sin \theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{2\theta - \sin 2\theta}{\sin \theta}\right)d_0$

Source: V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

reach do not change with time during the period of interest. In unsteady flow, the discharge and the water depth at any section in the reach change with time.

Uniform flows in open channels are mostly steady; unsteady uniform flows are very rare in nature. Varied open-channel flow may be either steady or unsteady. A flood wave [Figure 6.2 (b)] and a tidal surge [Figure 6.2 (c)] are examples of *varied unsteady flows*.

6.2 Uniform Flow in Open Channels

Uniform flow in an open channel must satisfy the following conditions:

1. The water depth, flow area, discharge, and the velocity distribution must remain unchanged in all sections of the entire channel reach.
2. The EGL, the water surface, and the channel bottom must be parallel to each other.

Based on the second condition, the slopes of these lines are the same,

$$S_e = S_{W.S.} = S_0$$

as shown in Figure 6.3.

Water in an open channel can reach the state of uniform flow only if no acceleration (or deceleration) takes place between sections. This is only possible when the gravity force component and the resistance to the flow are equal and opposite in direction in the reach. Therefore, for a uniform open channel, a *free-body diagram* can be taken between two adjacent sections (the *control volume*) to show the balance of the force components of gravity and resistance (Figure 6.3).

The forces acting on the free body in the direction of the flow include:

1. the hydrostatic pressure forces, F_1 and F_2 , acting on the control volume;
2. the weight of the water body in the reach, W , which has a component, $W \sin \theta$, in the direction of the flow; and
3. the resistance force, F_f , exerted by the channel (bottom and sides) on the flow.

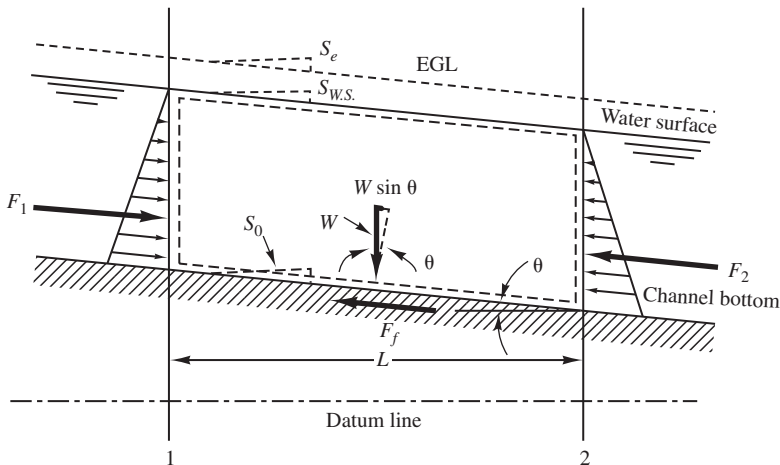


Figure 6.3 Force components in uniform open-channel flow

A summation of all these force components in the direction of the channel yields

$$F_1 + W \sin \theta - F_2 - F_f = 0 \quad (6.1a)$$

This equation can be further simplified because there is no change in water depth in uniform flow. Therefore, the hydrostatic forces at the two ends of the control volume must be equal, $F_1 = F_2$. The total weight of the water body is

$$W = \gamma AL$$

where γ is the unit weight of water, A is the cross-sectional area normal to the flow, and L is the length of the reach. In most open channels, the channel slopes are small and the approximation, $\sin \theta = \tan \theta = S_0$, is made. The gravity force component may thus be expressed as

$$W \sin \theta = \gamma ALS_0 \quad (6.1b)$$

The resistance force exerted by the channel boundaries may be expressed in terms of resisting force per unit area (i.e., shear stress) multiplied by the total channel bed area that is in contact with the flowing water. This channel contact area is the product of the wetted perimeter (P) and the length of the channel reach (L).

In 1769, French engineer Antoine Chezy assumed that the resisting force per unit area of the channel bed is proportional to the square of the mean velocity, KV^2 , where K is a constant of proportionality. The total resistance force may thus be written as

$$F_f = \tau_0 PL = KV^2 PL \quad (6.1c)$$

where τ_0 is the resisting force per unit area of the channel bed, also known as the *wall shear stress*.

Substituting Equation 6.1b, and Equation 6.1c into Equation 6.1a, we have

$$\gamma ALS_0 = KV^2 PL$$

or

$$V = \sqrt{\left(\frac{\gamma}{K}\right)\left(\frac{A}{P}\right)S_0}$$

In this equation, $A/P = R_h$, and $\sqrt{\gamma/K}$ may be represented by a constant, C . For uniform flow, $S_0 = S_e$, the above equation may thus be simplified to

$$V = C\sqrt{R_h S_e} \quad (6.2)$$

in which R_h is the *hydraulic radius* of the channel cross section. The hydraulic radius is defined as the water area divided by the wetted perimeter for all shapes of open-channel cross sections.

Equation 6.2 is the well-known *Chezy's formula* for open-channel flow. Chezy's formula is probably the first formula derived for uniform flow. The constant C is commonly known as *Chezy's resistance factor*; it was found to vary in relation to both the conditions of the channel and the flow.

Over the past two and a half centuries, many attempts have been made to determine the value of Chezy's C . The simplest relationship, and the one most widely applied in the United States, derives from the work of an Irish engineer, Robert Manning (1891 and 1895).* Using the

* Robert Manning, "On the Flow of Water in Open Channels and Pipes," *Transactions, Institution of Civil Engineering of Ireland*, 10 (1891), 161–207; 24 (1895), 179–207.

analysis performed on his own experimental data and on those of others, Manning derived the following empirical relation:

$$C = \frac{1}{n} R_h^{1/6} \quad (6.3)$$

in which n is known as *Manning's coefficient of channel roughness*. Some typical values of Manning's coefficients are given in Table 6.2.

Substituting Equation 6.3 into Equation 6.2, we have *Manning's equation*:

$$V = \frac{k_M}{n} R_h^{2/3} S_e^{1/2} \quad (6.4)$$

where $k_M = 1.00 \text{ m}^{1/3}/\text{s} = 1.49 \text{ ft}^{1/3}/\text{s}$ is a unit conversion factor. This will allow the use of the same n -values in different unit systems. Manning's equation may be used for gradually varied flow using the EGL slope (S_e) and uniform flow using the bottom slope ($S_0 = S_e$ for uniform flow). In terms of the discharge (Q) and the flow area (A), the equation is written as

$$Q = AV = \frac{k_M}{n} AR_h^{2/3} S_e^{1/2} \quad (6.5)$$

Setting $k_M = 1$ in the SI unit system, these equations become

$$V = \frac{1}{n} R_h^{2/3} S_e^{1/2} \quad (6.4a)$$

and

$$Q = AV = \frac{1}{n} AR_h^{2/3} S_e^{1/2} \quad (6.5a)$$

TABLE 6.2 Typical Values of Manning's n

Channel Surface	n
Glass, PVC, HDPE	0.010
Smooth steel, metals	0.012
Concrete	0.013
Asphalt	0.015
Corrugated metal	0.024
Earth excavation, clean	0.022–0.026
Earth excavation, gravel and cobbles	0.025–0.035
Earth excavation, some weeds	0.025–0.035
Natural channels, clean and straight	0.025–0.035
Natural channels, stones or weeds	0.030–0.040
Riprap lined channel	0.035–0.045
Natural channels, clean and winding	0.035–0.045
Natural channels, winding, pools, shoals	0.045–0.055
Natural channels, weeds, debris, deep pools	0.050–0.080
Mountain streams, gravel and cobbles	0.030–0.050
Mountain streams, cobbles and boulders	0.050–0.070

where V has units of m/sec, R_h is given in m, S_e in m/m, A is given in m^2 , and Q is given in m^3/sec . On the right-hand side of this equation, the water area (A) and the hydraulic radius (R_h) are both functions of water depth (y), which is known as the *uniform depth* or *normal depth* (y_n) when the flow is uniform.

Setting $k_M = 1.49$ in the BG system, Manning’s equation is written as

$$V = \frac{1.49}{n} R_h^{2/3} S_e^{1/2} \tag{6.4b}$$

or

$$Q = \frac{1.49}{n} A R_h^{2/3} S_e^{1/2} \tag{6.5b}$$

where V is in ft/sec, Q is in ft^3/sec (or cfs), A in ft^2 , R_h in ft, and S_e in ft/ft. The computation of uniform flow may be performed by the use of either Equation 6.4 or Equation 6.5 and basically involves seven variables:

1. the roughness coefficient (n);
2. the channel slope (S_0) (because $S_0 = S_e$ in uniform flow);
3. the channel geometry that includes the water area (A) and
4. the hydraulic radius (R_h);
5. the normal depth (y_n);
6. the normal discharge (Q); and
7. the mean velocity (V).

A successive substitution procedure is generally required when the normal depth is sought. Alternatively, Figure 6.4 (a) can be used to determine the normal depth in trapezoidal and rectangular channels. Likewise, Figure 6.4 (b) can be used to determine the normal depth in circular channels.

Example 6.1

A 3-m-wide rectangular irrigation channel carries a discharge of $25.3 \text{ m}^3/\text{sec}$ at a uniform depth of 1.2 m. Determine the slope of the channel if Manning’s coefficient is $n = 0.022$.

Solution

For a rectangular channel, the wetted perimeter and the hydraulic radius are

$$\begin{aligned} A &= by = (3)(1.2) = 3.6 \text{ m}^2 \\ P &= b + 2y = 5.4 \text{ m} \\ R_h &= \frac{A}{P} = \frac{3.6}{5.4} = \frac{2}{3} = 0.667 \text{ m} \end{aligned}$$

Equation 6.5a can be rewritten as

$$S_0 = S_e = \left(\frac{Qn}{AR_h^{2/3}} \right)^2 = 0.041$$

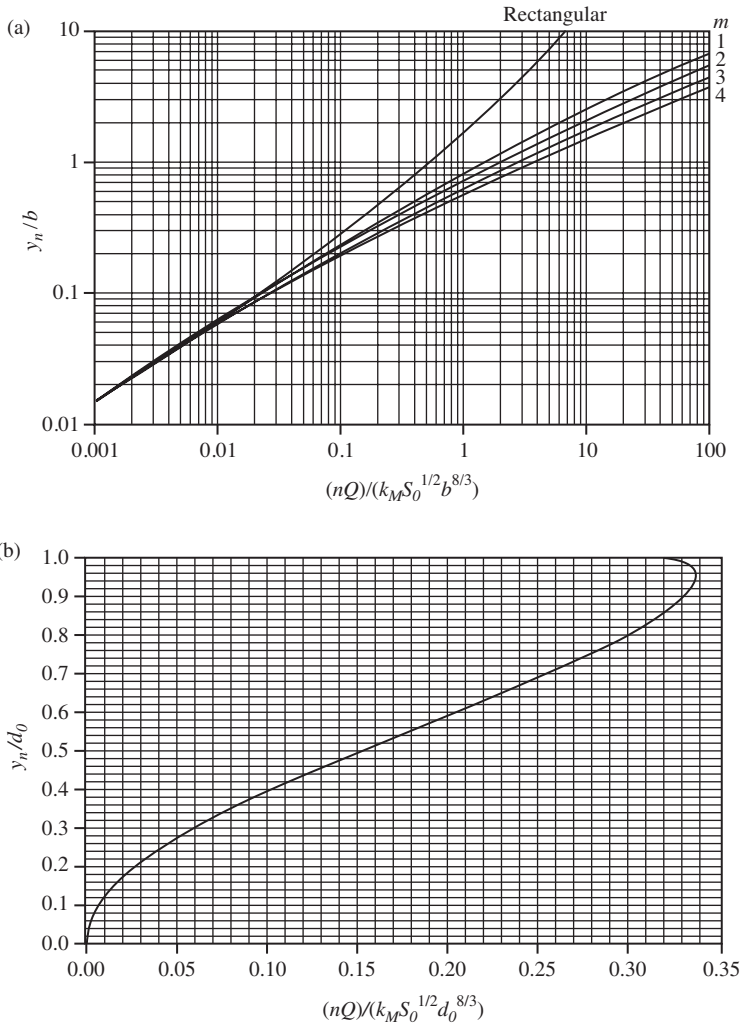


Figure 6.4 Normal depth solution procedure: (a) trapezoidal channels ($m =$ side slope) and (b) circular channels ($d_0 =$ diameter)

Example 6.2

A 6-ft-diameter, concrete pipe is flowing with a free surface (i.e., not under pressure). If the pipe is laid on a slope of 0.001 and carries a uniform flow at a depth of 4 feet (y in Table 6.1), what is the discharge?

Solution

Based on Table 6.1, the value of $\theta = 90^\circ + \alpha$ (in degrees) where $\alpha = \sin^{-1}(1 \text{ ft}/3 \text{ ft}) = 19.5^\circ$.

Thus, $\theta = 90^\circ + 19.5^\circ = 109.5^\circ$; and in radians, $\theta = (109.5^\circ/360^\circ)(2\pi) = 0.608 \pi$ radians.

The area of the circular section is

$$A = \frac{1}{8}(2\theta - \sin 2\theta)d_0^2 = 1/8[2(0.608 \pi) - \sin 2(0.608 \pi)](6\text{ft})^2 = 20.0 \text{ ft}^2$$

The wetted perimeter is

$$P = \theta d_o = (0.608 \pi)(6 \text{ ft}) = 11.5 \text{ ft}$$

Then the hydraulic radius is

$$R_h = A/P = (20.0 \text{ ft}^2)/(11.5 \text{ ft}) = 1.74 \text{ ft}$$

Substituting the above values into Equation 6.5b with $S_0 = S_e$ (uniform flow) and $n = 0.013$ (Table 6.2) yields

$$Q = \frac{1.49}{n} AR_h^{2/3} S_o^{1/2} = \frac{1.49}{0.013} (20)(1.74)^{2/3} (0.001)^{1/2}$$

$$Q = 105 \text{ ft}^3/\text{sec} \text{ (or cfs)}$$

Example 6.3

If the discharge in the channel in Example 6.1 is increased to $40 \text{ m}^3/\text{sec}$, what is the normal depth of the flow?

Solution

The geometric parameters are

Area: $A = by = 3y$

Wetted Perimeter: $P = b + 2y = 3 + 2y$

Hydraulic Radius: $R_h = \frac{A}{P} = \frac{3y}{3 + 2y}$

Substituting these values in Equation 6.5 with $S_0 = S_e$ (uniform flow) yields

$$Q = \frac{1}{n} AR_h^{2/3} S_o^{1/2}$$

$$40 = \frac{1}{0.022} (3y) \left(\frac{3y}{3 + 2y} \right)^{2/3} (0.041)^{1/2}$$

or

$$AR_h^{2/3} = (3y) \left(\frac{3y}{3 + 2y} \right)^{2/3} = \frac{(0.022)(40)}{(0.041)^{1/2}} = 4.346$$

Solving by successive substitution, we find that

$$y = y_n = 1.69 \text{ m}$$

Alternatively, we can use Figure 6.4 (a):

$$\frac{nQ}{(1.0)S_o^{1/2}b^{8/3}} = \frac{(0.022)(40)}{(1.0)(0.041)^{1/2}(3)^{8/3}} = 0.23$$

Then, from the figure, $y_n/b = 0.56$ and $y_n = (0.56)(3.0) = 1.68 \text{ m}$.

Note: Uniform depth computations that involve implicit equations can be solved by some programmable calculators, computer algebra software (e.g., Mathcad, Maple, or Mathematica), spreadsheet programs, and computer software designed specifically for the task (both proprietary and freeware; try a Google search on the Internet).

**Classroom Computer Exercise—Normal Depth
in Open Channels**

Review Example 6.3. Obtain or write software appropriate for solving normal depth in open channels. (See the note at the end of Example 6.3 and the book's preface for suggestions.) Answer the following questions by performing a computer analysis of the open channel described in Example 6.3 and its modifications.

- (a) Before using the software, what data do you anticipate it will need to compute normal depth for the channel in Example 6.3?
 - (b) Now enter the data requested by the software and perform a normal depth analysis. Compare your computer result with the answer listed for Example 6.3. Is there any discrepancy? Comments?
 - (c) What would happen to the flow rate in the channel if the depth was unchanged but the bottom slope was doubled? Estimate the magnitude of the change. Double the slope and perform a new analysis on the computer. Did you reason correctly? Now restore the slope and flow rate to their original values.
 - (d) What would happen to the flow depth if the flow rate remained unchanged but the channel was lined with concrete ($n = 0.013$)? Estimate the magnitude of the depth change. Change the roughness value and perform a new normal depth analysis on your computer. Did you reason correctly? Now restore the roughness and flow depth to their original value.
 - (e) Trapezoidal geometry is necessary if a channel is unlined. [For bank stability, the side slopes may be limited to 1(V):3(H).] Because of easement requirements, the unlined channel ($n = 0.022$) will be cheaper if the top width does not exceed 6 m. Using the computer software, determine the top width of the channel assuming the bottom width remains 3 m and the discharge remains $40 \text{ m}^3/\text{sec}$. Also, determine the side slope that would produce a top width that is exactly 6 m.
 - (f) Is it possible to design a triangular channel with the software? Explain.
 - (g) Perform any other changes your instructor requests.
-

6.3 Hydraulic Efficiency of Open-Channel Sections

The Manning uniform flow equations (6.4 and 6.5) show that for the same cross-sectional area (A) and channel slope (S_0), the channel section with a larger hydraulic radius (R_h) delivers a larger discharge. It is a section of higher *hydraulic efficiency*. Because the hydraulic radius is equal to the water cross-sectional area divided by the wetted perimeter, for a given cross-sectional area, the channel section with the least wetted perimeter is the *best hydraulic section*.

Among all open-channel shapes, the semicircle has the least perimeter for a given area, so it is the most hydraulically efficient of all sections. A channel with a semicircular cross section, however, has sides that are curved and almost vertical at the water surface level, which makes the channel expensive to construct (excavation and forming) and difficult to maintain (bank stability). In practice, semicircular sections are only used when pipes are appropriate or in smaller flumes of prefabricated materials.

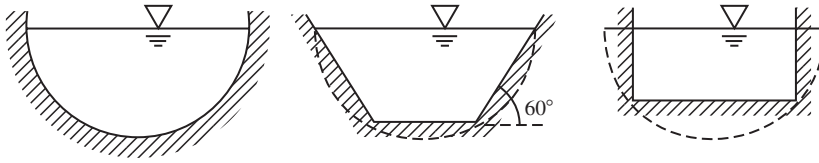


Figure 6.5 Hydraulically efficient sections

For large channels, trapezoidal sections are most commonly used. The most efficient trapezoidal section is a half hexagon, which can be inscribed into a semicircle with its center at the free water surface and 60° angles on the sides. Another commonly used channel section is the rectangular section. The most efficient rectangular section is the half-square section, which can also be inscribed into a semicircle with the center of the circle at the free water surface. The hydraulically efficient semicircular, half-hexagon, and half-square sections are shown in Figure 6.5.

The concept of hydraulically efficient sections is only valid when the channel is lined with stabilized, nonerodible materials. Ideally, a channel should be designed for the best hydraulic efficiency, but it should be modified for practicality and construction cost. It should be noted that although the best hydraulic section offers the least water area for a given discharge, it does not necessarily have the lowest excavation cost. A half-hexagon section, for example, is a best hydraulic section only when the water surface reaches the level of the bank top. This section is not suitable for general applications because a sufficient distance above the water surface must be provided to prevent waves or fluctuations of water surface from overflowing the sides. The vertical distance from the designed water surface to the top of the channel banks is known as the *freeboard* of the channel. Freeboard and other channel design issues are further discussed in Section 6.9.

Example 6.4

Prove that the best hydraulic trapezoidal section is a half-hexagon.

Solution

The water cross-sectional area (A) and the wetted perimeter (P) of a trapezoidal section are

$$A = by + my^2 \quad (1)$$

and

$$P = b + 2y\sqrt{1 + m^2} \quad (2)$$

From Equation (1), $b = A/y - my$. This relationship is substituted into Equation (2):

$$P = \frac{A}{y} - my + 2y\sqrt{1 + m^2}$$

Now consider both A and m constant and let the first derivative of P with respect to y equal zero to obtain the minimum value of P :

$$\frac{dP}{dy} = -\frac{A}{y^2} - m + 2\sqrt{1 + m^2} = 0$$

Substituting A from Equation (1), we get

$$\frac{by + my^2}{y^2} = 2\sqrt{1 + m^2} - m$$

or

$$b = 2y(\sqrt{1 + m^2} - m) \quad (3)$$

Note that this equation provides a relationship between the flow depth and the channel bottom for an efficient section if the side slope m is fixed at a predetermined value. The most efficient section is obtained as follows if m can vary.

By definition, the hydraulic radius, R_h , may be expressed as

$$R_h = \frac{A}{P} = \frac{by + my^2}{b + 2y\sqrt{1 + m^2}}$$

Substituting the value of b from Equation (3) into the above equation and simplifying, we have

$$R_h = \frac{y}{2}$$

It shows that the best hydraulic trapezoidal section has a hydraulic radius equal to one-half of the water depth. Substituting Equation (3) into Equation (2) and solving for P , we have

$$P = 2y(2\sqrt{1 + m^2} - m) \quad (4)$$

To determine the value of m that makes P the least, the first derivative of P is taken with respect to m . Equating it to zero and simplifying, we have

$$m = \frac{\sqrt{3}}{3} = \cot 60^\circ \quad (5)$$

and thus,

$$b = 2y\left(\sqrt{1 + \frac{1}{3}} - \frac{\sqrt{3}}{3}\right) = 2\frac{\sqrt{3}}{3}y \quad \text{or} \quad y = \frac{\sqrt{3}}{2}b = b \sin 60^\circ$$

This means that the section is a half-hexagon, as depicted in Figure 6.6.

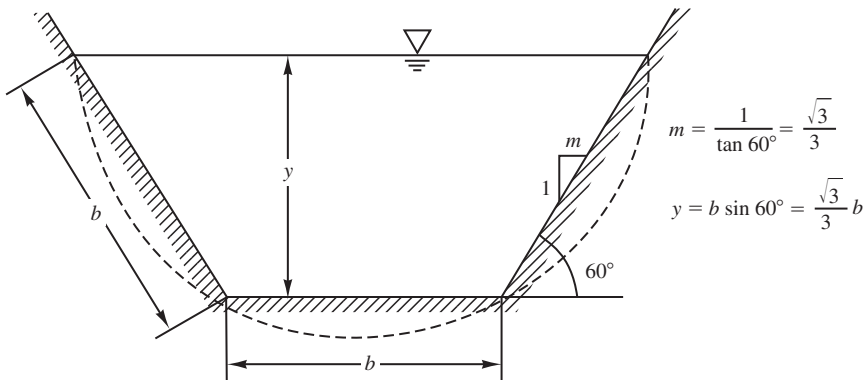


Figure 6.6 Best hydraulic trapezoidal section

6.4 Energy Principles in Open-Channel Flow

The energy principles derived from pressure flow in pipes are generally applicable to open-channel flow. The energy contained in a unit weight of water flowing in an open channel may also be measured in the three basic forms:

1. kinetic energy,
2. pressure energy, and
3. elevation (potential) energy above a certain energy datum line.

Kinetic energy in any section of an open channel is expressed in the form of $V^2/2g$, where V is the mean velocity defined by the discharge divided by the water area (i.e., $V = Q/A$) in the section. The actual velocity of water flowing in an open-channel section varies in different parts of the section. The velocities near the channel bed are retarded because of friction, and reach a maximum near the water surface in the center part of the channel. The distribution of velocities in a cross section results in a different value of kinetic energy for different parts of the cross section. An average value of the kinetic energy in a cross section of open channel may be expressed in terms of the mean velocity as $\alpha(V^2/2g)$, where α is known as the *energy coefficient*. The value of α depends on the actual velocity distribution in a particular channel section. Its value is always greater than unity. The ordinary range of α lies between 1.05 for uniformly distributed velocities and 1.20 for highly varied velocities in a section. In simple analysis, however, the velocity heads (kinetic energy heads) in an open channel are taken as $V^2/2g$ by assuming α equal to unity as an approximation.

Because open-channel flow always has a free surface that is exposed to the atmosphere, the pressure on the free surface is constant and commonly taken as a zero pressure reference. Pressure energy in open-channel flow is usually computed with reference to the free surface. If the free surface in a channel approximates a straight-line slope, the pressure head at any submerged point A is equal to the vertical distance measured from the free surface to the point. Therefore, the water depth (y) at a given cross section is commonly used to represent the pressure head: $p/\gamma = y$. However, if the water is flowing over a vertical curve, such as a spillway or a weir, the centrifugal force produced by the fluid mass flowing over the curved path may cause a marked difference in pressure from a depth measurement alone. When water flows over a convex path [Figure 6.7 (a)], the centrifugal force acts in the direction opposite to the gravity force and the pressure is less than

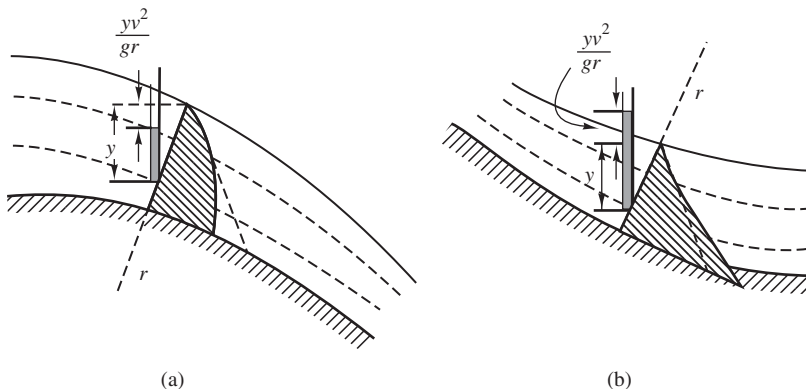


Figure 6.7 Flow over curved surfaces: (a) convex surface and (b) concave surface

that of the water depth, by mv^2/r , where m is the mass of water column immediately above a unit area and v^2/r is the centrifugal acceleration of the water mass flowing along a path with radius of curvature (r). The resulting pressure head is

$$\frac{p}{\gamma} = y - \frac{yv^2}{gr} \quad (6.6a)$$

When water flows over a concave path [Figure 6.7 (b)], the centrifugal force is in the same direction as the gravity force, and the pressure is greater than that represented by the water depth. The resulting pressure head is

$$\frac{p}{\gamma} = y + \frac{yv^2}{gr} \quad (6.6b)$$

where γ is the unit weight of water, y is the depth measured from the free water surface to the point of interest, v is the velocity at the point, and r is the radius of curvature of the curved flow path.

The elevation (potential) energy head in open-channel flow is measured with respect to a selected horizontal datum line. The vertical distance measured from the datum to the channel bottom (z) is commonly taken as the elevation energy head at the section.

Therefore, the total energy head at any section in an open channel is generally expressed as

$$H = \frac{V^2}{2g} + y + z \quad (6.7)$$

Specific energy in a channel section is defined as the energy head measured with respect to the channel bottom at the section. According to Equation 6.7, the specific energy at any section is

$$E = \frac{V^2}{2g} + y \quad (6.8)$$

or the specific energy at any section in an open channel is equal to the sum of the velocity head and the water depth at the section.

Given the water area (A) and the discharge (Q) at a particular section, Equation 6.8 may be rewritten as

$$E = \frac{Q^2}{2gA^2} + y \quad (6.9)$$

Thus, for a given discharge Q , the specific energy at any section is a function of the depth of the flow only.

When the depth of the flow, y , is plotted against the specific energy for a given discharge at a given section, a *specific energy curve* is obtained (Figure 6.8). The specific energy curve has two limbs: AC and CB . The lower limb always approaches the horizontal axis toward the right and the upper limb approaches (asymptotically) a 45°-line that passes through the origin. At any point on the specific energy curve, the ordinate represents the depth of the flow at the section, and the abscissa represents the corresponding specific energy. Usually, the same scales are used for both the ordinate and the abscissa.

In general, a family of similar curves may be plotted for various values of discharge at a given section. For higher discharge, the curve moves to the right: $A'C'B'$. For lower discharge, the curve moves to the left: $A''C''B''$.

The vertex C on a specific energy curve represents the depth (y_c) at which the discharge Q may be delivered through the section at minimum energy (E_c). This depth is commonly known as

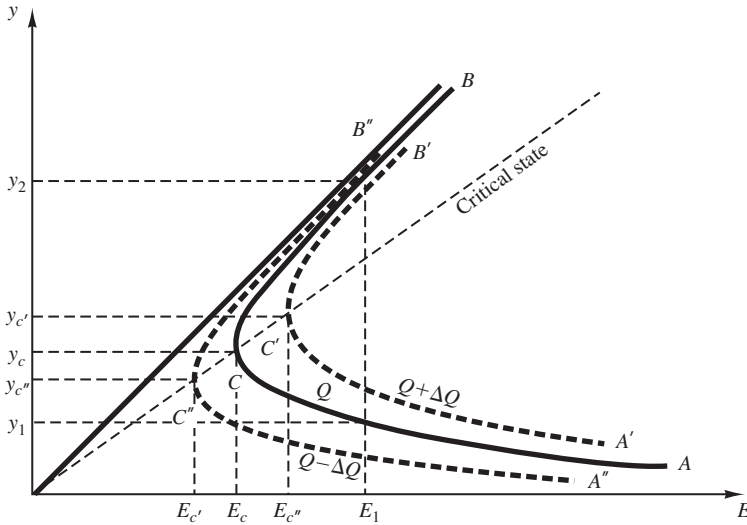


Figure 6.8 Specific energy curves of different discharges at a given channel section

the *critical depth* for the discharge Q at the given section. The corresponding flow in the section is known as the *critical flow*. At a smaller depth, the same discharge can be delivered only by a higher velocity and a higher specific energy. The state of rapid and shallow flow through a section is known as *supercritical flow* or *rapid flow*. At a larger depth, the same discharge may be delivered through the section with a smaller velocity and a higher specific energy than at critical depth. This tranquil, high-stage flow is known as *subcritical flow*.

For a given value of specific energy, say E_1 , the discharge may pass through the channel section at either depth y_1 (supercritical flow) or y_2 (subcritical flow), as shown in Figure 6.8. These two depths, y_1 and y_2 , are commonly known as *alternate depths*.

At the critical state the specific energy of the flow takes a minimum value. This value can be computed by equating the first derivative of the specific energy with respect to the water depth to zero:

$$\frac{dE}{dy} = \frac{d}{dy} \left(\frac{Q^2}{2gA^2} + y \right) = -\frac{Q^2}{gA^3} \frac{dA}{dy} + 1 = 0$$

The differential water area (dA/dy) near the free surface is $dA/dy = T$, where T is the *top width* of the channel section. Hence,

$$-\frac{Q^2 T}{gA^3} + 1 = 0 \tag{6.10a}$$

An important parameter for open-channel flow is defined by $A/T = D$, which is known as the *hydraulic depth* of the section. For rectangular cross sections, the hydraulic depth is equal to the depth of the flow. The above equation may thus be simplified to

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gDA^2} = 1 - \frac{V^2}{gD} = 0 \tag{6.10b}$$

or

$$\frac{V}{\sqrt{gD}} = 1 \quad (6.11)$$

The quantity V/\sqrt{gD} is dimensionless. It can be derived as the ratio of the inertial force in the flow to the gravity force in the flow (see Chapter 10 for detailed discussion). This ratio may be interpreted physically as the ratio between the mean flow velocity (V) and the speed of a small gravity (disturbance) wave traveling over the water surface. It is known as the *Froude number* (N_F):

$$N_F = \frac{V}{\sqrt{gD}} \quad (6.12)$$

When the Froude number is equal to unity, as indicated by Equation 6.11, $V = \sqrt{gD}$, the speed of the surface (disturbance) wave and that of the flow is the same. The flow is in the critical state. When the Froude number is less than unity, $V < \sqrt{gD}$, the flow velocity is smaller than the speed of a disturbance wave traveling on the water surface. The flow is classified as subcritical. When the Froude number is greater than unity, $V > \sqrt{gD}$, the flow is classified as supercritical.

From Equation 6.10, we may also write (for critical flow),

$$\frac{Q^2}{g} = \frac{A^3}{T} = DA^2 \quad (6.13)$$

In a rectangular channel, $D = y$ and $A = by$. Therefore,

$$\frac{Q^2}{g} = y^3 b^2$$

Because this relation is derived from the critical flow conditions stated above, $y = y_c$, which is the critical depth, and

$$y_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{q^2}{g}} \quad (6.14)$$

where $q = Q/b$, is the discharge per unit width of the channel.

For trapezoidal and circular channels, an explicit equation such as Equation 6.14 is not available, and a successive substitution procedure is required to solve Equation 6.13 for critical depth. Alternatively, Figures 6.9 (a) and 6.9 (b) can be used to determine critical depth in trapezoidal and circular channels, respectively. For an open channel of any sectional shape, the critical depth is always a function of the channel discharge and does not vary with the channel slope.

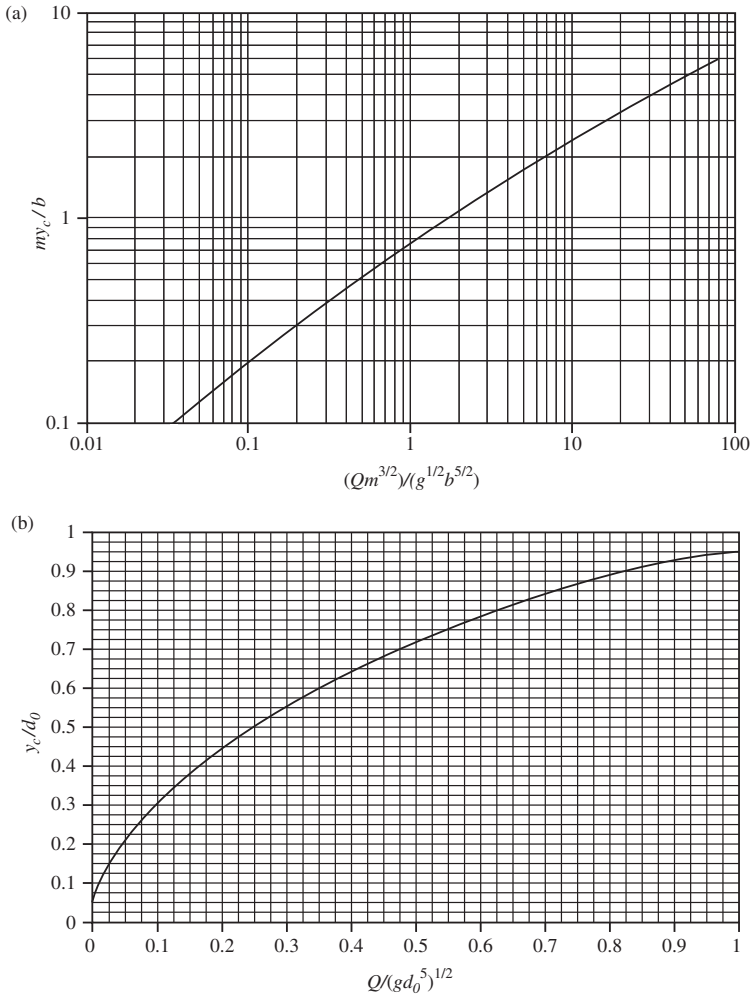


Figure 6.9 Critical depth solution procedure: (a) trapezoidal channels and (b) circular channels

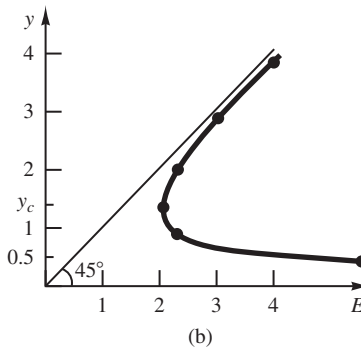
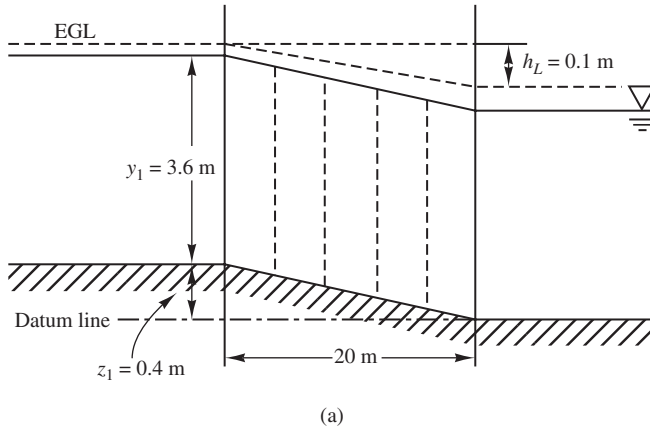
Example 6.5

A hydraulic transition is designed to connect two rectangular channels of the same width by a sloped floor, as shown in Figure 6.10 (a). Assume the channel is 3 m wide and is carrying a discharge of 15 m³/sec at 3.6 m depth. Also assume a 0.1 m energy loss uniformly distributed through the transition. Determine the water surface profile in the transition.

Solution

The specific energy curve can be constructed based on the given discharge and the sectional geometry by using the following relationship from Equation 6.9:

$$E = \frac{Q^2}{2gA^2} + y = \frac{(15)^2}{2(9.81)(3y)^2} + y = \frac{1.27}{y^2} + y$$



$$y_c = \sqrt[3]{\frac{5^2}{g}} = 1.37 \text{ m}$$

$$E_c(\text{m}) = 2.05 \text{ m}$$

y (m)	E (m)
0.5	5.60
1.0	2.27
2.0	2.32
3.0	2.32
3.0	3.14
4.0	4.07

Figure 6.10 Hydraulic transition

At the inlet to the transition, the velocity is V_i :

$$V_i = \frac{Q}{A_i} = \frac{15}{(3.6)(3)} = 1.39 \text{ m/sec}$$

where A_i is the water area at the inlet and the velocity head is

$$\frac{V_i^2}{2g} = \frac{(1.39)^2}{2(9.81)} = 0.10 \text{ m}$$

The total energy head at the inlet as measured with respect to the datum line is

$$H_i = \frac{V_i^2}{2g} + y_i + z_i = 0.10 + 3.60 + 0.40 = 4.10 \text{ m}$$

The top horizontal line in Figure 6.10 (a) shows this energy level.

At the exit of the transition, the total energy available is reduced by 0.1 m, as indicated by the EGL in Figure 6.10 (a):

$$H_e = \frac{V_e^2}{2g} + y_e + z_e = H_i - 0.1 = 4.00 \text{ m}$$

E_e is the specific energy measured with respect to the channel bottom:

$$E_e = H_e = 4.00 \text{ m}$$

This value is applied to the specific energy curve shown in Figure 6.10 (b) to obtain the water depth at the exit section.

Water surface elevations at four other sections (4.00 m, 8.00 m, 12.00 m, and 16.00 m from the entrance section) are computed by using the same method. The results for all six sections are shown in the following table.

Section	Inlet	4.00 m	8.00 m	12.00 m	16.00 m	Exit
Specific Energy, E (m)	3.70	3.76	3.82	3.88	3.94	4.00
Water Depth, y (m)	3.60	3.67	3.73	3.79	3.86	3.92

Example 6.6

A trapezoidal channel has a bottom width of 5 m and side slopes $m = 2$. If the flow rate is $20 \text{ m}^3/\text{sec}$, what is the critical depth?

Solution

By using Equation 6.13 and Table 6.1:

$$\frac{Q^2}{g} = DA^2 = \frac{A^3}{T} = \frac{[(b + my)y]^3}{b + 2my}$$

or

$$\frac{20^2}{9.81} = 40.8 = \frac{[(5 + 2y)y]^3}{5 + 2(2)y}$$

By successive substitution, we obtain $y = y_c = 1.02 \text{ m}$. Alternatively, using Figure 6.9:

$$\frac{Qm^{3/2}}{g^{1/2}b^{5/2}} = \frac{(20)(2)^{3/2}}{(9.81)^{1/2}(5)^{5/2}} = 0.323$$

From Figure 6.9 (a), we obtain $my_c/b = 0.41$. Therefore, $y_c = (0.41)(5)/2 = 1.03 \text{ m}$.

Note: Critical depth computations that involve implicit equations can be solved by some programmable calculators, computer algebra software (e.g., Mathcad, Maple, or Mathematica), spreadsheet programs, and computer software designed specifically for the task (both proprietary and freeware; try a topical search on the Internet).

6.5 Hydraulic Jumps

Hydraulic jumps can occur naturally in open channels but are more common in constructed structures such as energy dissipation (or hydraulic jump) basins. They are the result of an abrupt reduction in flow velocity by means of a sudden increase in water depth in the downstream direction. Most energy dissipation basins are rectangular in cross section, so this book limits the discussion of hydraulic jumps to rectangular channels.

Hydraulic jumps convert a high-velocity supercritical flow (upstream) into a low-velocity subcritical flow (downstream). Correspondingly, a low-stage supercritical depth (y_1) is changed to a high-stage subcritical depth (y_2); these are known, respectively, as the *initial depth* and the *sequent depth* of a hydraulic jump (Figure 6.11). In the region of the hydraulic jump, the characteristic rolling

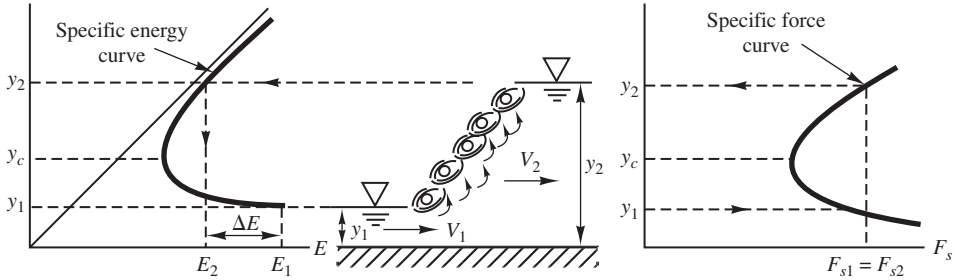


Figure 6.11 Hydraulic jump

water surface and turbulence of water can be seen. These violent motions are accompanied by a significant loss of energy head through the jump. Given the discharge in a particular channel, the amount of energy head loss through a jump (ΔE) can be determined by simply measuring the initial and sequent depths and using the specific energy curve shown in Figure 6.11. Predicting the sequent depth by estimating the energy loss, however, is impractical because it is difficult to determine the energy loss through a jump. The relationship between the initial depth and the sequent depth in a hydraulic jump may be determined by considering the balance of forces and momentum immediately before and after the jump.

Consider a control volume in the vicinity of a jump, as shown in Figure 6.11. The balance between the hydrostatic forces and the momentum flux through section 1 and section 2, per unit width of the channel, may be expressed as

$$F_1 - F_2 = \rho q(V_2 - V_1) \tag{6.15}$$

where q is discharge per unit width of the channel. Substituting the following quantities

$$F_1 = \frac{\gamma}{2}y_1^2, \quad F_2 = \frac{\gamma}{2}y_2^2, \quad V_1 = \frac{q}{y_1}, \quad V_2 = \frac{q}{y_2}$$

into Equation 6.15 and simplifying, we get

$$\frac{q^2}{g} = y_1y_2\left(\frac{y_1 + y_2}{2}\right) \tag{6.16}$$

This equation may be rearranged into a more convenient form as follows:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8N_{F_1}^2} - 1\right) \tag{6.17}$$

where N_{F_1} is the Froude number of the approaching flow:

$$N_{F_1} = \frac{V_1}{\sqrt{gy_1}} \tag{6.18}$$

Example 6.7

A 10-ft-wide rectangular channel carries 500 cfs of water at a 2-ft depth before entering a jump. Compute the downstream water depth and the critical depth.

Solution

The discharge per unit width is

$$q = \frac{500}{10} = 50 \text{ ft}^3/\text{sec} \cdot \text{ft}$$

Using Equation 6.14, the critical depth is

$$y_c = \sqrt[3]{\frac{50^2}{32.2}} = 4.27 \text{ ft}$$

The approaching velocity is

$$V_1 = \frac{q}{y_1} = \frac{50}{2} = 25 \text{ ft/sec}$$

The Froude number for the approaching flow can be computed by using this velocity and the initial depth $y_1 = 2.0$:

$$N_{F_1} = \frac{V_1}{\sqrt{gy_1}} = 3.12$$

Substituting this value into Equation 6.17 gives

$$\frac{y_2}{2.0} = \frac{1}{2}(\sqrt{1 + 8(3.12)^2} - 1)$$

and solving for the sequent depth yields:

$$y_2 = 7.88 \text{ ft}$$

Equation 6.15 may also be arranged as

$$F_1 + \rho qV_1 = F_2 + \rho qV_2$$

where

$$F_s = F + \rho qV \tag{6.19}$$

The quantity F_s is known as the *specific force* per unit width of the channel. For a given discharge, the specific force is a function of the water depth at a given section. When F_s is plotted against the water depth, the resulting curve is similar to a specific energy curve with a vertex that appears at the critical depth. A typical specific force curve is shown in Figure 6.11.

A hydraulic jump usually takes place in a rather short reach in a channel. Therefore, it is reasonable to assume that through a hydraulic jump the specific forces immediately before and after a jump are approximately the same. The value of F_s can be computed from the given conditions of the approaching flow. If we apply this value to the specific force curve in Figure 6.11, we can draw a vertical line that gives both the initial and sequent depths of a jump.

The energy head loss through the hydraulic jump (ΔE) may then be estimated by applying the definition

$$\begin{aligned} \Delta E &= \left(\frac{V_1^2}{2g} + y_1 \right) - \left(\frac{V_2^2}{2g} + y_2 \right) \\ &= \frac{1}{2g}(V_1^2 - V_2^2) + (y_1 - y_2) = \frac{q^2}{2g} \left(\frac{1}{y_1^2} - \frac{1}{y_2^2} \right) + (y_1 - y_2) \end{aligned}$$

Substituting Equation 6.16 into the above equation and simplifying, we get

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} \quad (6.20)$$

Example 6.8

A long, rectangular open channel 3 m wide carries a discharge of 15 m³/sec. The channel slope is 0.004, and the Manning's coefficient is 0.01. At a certain point in the channel, flow reaches normal depth.

- Determine the flow classification. Is it supercritical or subcritical?
- If a hydraulic jump takes place at this depth, what is the sequent depth?
- Estimate the energy head loss through the jump.

Solution

- The critical depth is calculated using Equation 6.14 and $y_c = 1.37$ m. The normal depth of this channel can be determined by the Manning equation (Equation 6.5):

$$Q = \frac{1}{n} A_1 R_{h1}^{2/3} S^{1/2}$$

where

$$A = y_1 b, \quad R_h = \frac{A_1}{P_1} = \frac{y_1 b}{2y_1 + b}, \quad b = 3 \text{ m}$$

We have

$$15 = \frac{1}{0.01} (3y_1) \left(\frac{3y_1}{2y_1 + 3} \right)^{2/3} (0.004)^{1/2}$$

Solving the equation for y_1 , we obtain

$$y_1 = 1.08 \text{ m}, \quad V_1 = \frac{15}{3y_1} = 4.63 \text{ m/sec}$$

and

$$N_{F_1} = \frac{V_1}{\sqrt{gy_1}} = 1.42$$

Because $N_{F_1} > 1$, the flow is supercritical.

- Applying Equation 6.17, we get

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8N_{F_1}^2} - 1 \right) = 1.57y_1 = 1.70 \text{ m}$$

- The head loss can be estimated by using Equation 6.20:

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(0.62)^3}{4(1.70)(1.08)} = 0.032 \text{ m}$$

6.6 Gradually Varied Flow

Gradually varied flow in open channels differs from uniform flow and rapidly varied flow (hydraulic jumps, flow through a streamlined transition, etc.) in that the change in water depth in the channel takes place very gradually with distance.

In uniform flow, the water depth remains a constant value known as the *normal depth* (or *uniform depth*). The energy grade line is parallel to the water surface and the channel bottom. The velocity distribution also remains unchanged throughout the reach. Thus, the computation of only one water depth is sufficient for the entire reach.

In rapidly varied flow such as a hydraulic jump, rapid changes in water depth take place in a short distance. A significant change in water velocities is associated with the rapid variation of water cross-sectional area. At this high rate of flow deceleration, the energy loss is inevitably high. The computation of water depths using the energy principles is not reliable. In this case, computations can only be carried out by applying the momentum principles (i.e., Equation 6.15).

In gradually varied flow, velocity changes take place very gradually with distance so that the effects of acceleration on the flow between two adjacent sections are negligible. Thus, computation of the *water surface profile*, defined as depth changes along the channel length, can be carried out strictly on energy considerations.

The total energy head at any section in an open channel, as defined in Equation 6.7, is restated here as

$$H = \frac{V^2}{2g} + y + z = \frac{Q^2}{2gA^2} + y + z$$

To compute the water surface profile, we must first obtain the variation of the total energy head along the channel. Differentiating H with respect to the channel distance x , we obtain the energy gradient in the direction of the flow:

$$\frac{dH}{dx} = \frac{-Q^2}{gA^3} \frac{dA}{dx} + \frac{dy}{dx} + \frac{dz}{dx} = -\frac{Q^2T}{gA^3} \frac{dy}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

where $dA = T(dy)$. Rearranging the equation gives

$$\frac{dy}{dx} = \frac{\frac{dH}{dx} - \frac{dz}{dx}}{1 - \frac{Q^2T}{gA^3}} \tag{6.21}$$

The term dH/dx is the slope of the energy grade line. It is always a negative quantity because the total energy head reduces in the direction of the flow, or $S_e = -dH/dx$. Similarly, the term dz/dx is the slope of the channel bed. It is negative when the elevation of the channel bed reduces in the direction of the flow; it is positive when the elevation of the channel bed increases in the direction of the flow. In general, we may write $S_0 = -dz/dx$.

The energy slope in gradually varied flow between two adjacent sections may also be approximated by using a uniform flow formula. For simplicity, the derivation will be demonstrated with a wide rectangular channel section where $A = by$, $Q = bq$, and $R_h = A/P = by/(b + 2y) \cong y$ (for wide rectangular channels because $b \gg y$).

Using the Manning formula (Equation 6.5), we get

$$S_e = -\frac{dH}{dx} = \frac{n^2Q^2}{R_h^{4/3}A^2} = \frac{n^2Q^2}{b^2y^{10/3}} \tag{6.22}$$

The slope of the channel bed may also be expressed in similar terms if uniform flow were assumed to take place in the channel. Because the slope of the channel bed is equal to the energy slope in uniform flow, the hypothetical uniform flow conditions are designated with the subscript n . We have

$$S_0 = -\frac{dz}{dx} = \left(\frac{n^2 Q^2}{b^2 y^{10/3}} \right)_n \quad (6.23)$$

From Equation 6.14 for rectangular channels,

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{Q^2}{gb^2}}$$

or

$$Q^2 = gy_c^3 b^2 = \frac{gA_c^3}{b} \quad (6.24)$$

Substituting Equations 6.22, 6.23, and 6.24 into Equation 6.21, we have

$$\frac{dy}{dx} = \frac{S_0 \left[1 - \left(\frac{y_n}{y} \right)^{10/3} \right]}{\left[1 - \left(\frac{y_c}{y} \right)^3 \right]} \quad (6.25a)$$

For nonrectangular channels, Equation 6.24a can be generalized as

$$\frac{dy}{dx} = \frac{S_0 \left[1 - \left(\frac{y_n}{y} \right)^N \right]}{\left[1 - \left(\frac{y_c}{y} \right)^M \right]} \quad (6.25b)$$

where the exponents M and N depend on the cross-sectional shape and the flow conditions as given by Chow.*

This form of the *gradually varied flow equation* is very useful for a qualitative analysis, which helps to understand the gradually varied flow classifications covered in the next sections. Other forms are often used to compute water surface profiles. Physically, the term dy/dx represents the slope of the water surface with respect to the bottom of the channel. For $dy/dx = 0$, the water depth remains constant throughout the reach or the special case of uniform flow. For $dy/dx < 0$, the water depth decreases in the direction of the flow. For $dy/dx > 0$, the water depth increases in the direction of the flow. Solutions of this equation under different conditions will yield the various water surface profiles that occur in open channels.

6.7 Classifications of Gradually Varied Flow

In analyzing gradually varied flow, the role of critical depth, y_c , is very important. When open-channel flow approaches critical depth ($y = y_c$), the denominator of Equation 6.25 approaches zero and the value of dy/dx approaches infinity. The water surface becomes very steep. This is

* V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

seen at hydraulic jumps or at a water surface entering a steep channel from a mild channel or a lake. The latter case provides a unique one-to-one relationship between the discharge and the water depth in a channel and is known as a *control section* in open-channel flow.

Depending on the channel slope, geometry, roughness, and discharge, open channels may be classified into five categories:

1. steep channels,
2. critical channels,
3. mild channels,
4. horizontal channels, and
5. adverse channels.

The classification depends on the flow conditions in the channel as indicated by the relative positions of normal depth (y_n) and critical depth (y_c) calculated for each particular channel. The criteria are as follows:

Steep channels:	$y_n/y_c < 1.0$	or	$y_n < y_c$
Critical channels:	$y_n/y_c = 1.0$	or	$y_n = y_c$
Mild channels:	$y_n/y_c > 1.0$	or	$y_n > y_c$
Horizontal channels:	$S_0 = 0$		
Adverse channels:	$S_0 < 0$		

A further classification of water surface profile curves depends on the actual water depth and its relationship to the critical and normal depths. The ratios of y/y_c and y/y_n may be used in the analysis, where y is the actual water depth at any section of interest in the channel.

If both y/y_c and y/y_n are greater than 1.0, then the water surface profile curve is above both the critical depth line and the normal depth line in the channel, as depicted in Figure 6.12. The curve is designated as a type 1 curve. There are S-1, C-1, and M-1 curves for steep, critical, and mild channels, respectively.

If the water depth (y) is between the normal depth and the critical depth, the curves are designated as type 2 curves. There are S-2, M-2, H-2, and A-2 curves. The type 2 curve does not exist in critical channels. In critical channels, normal depth is equal to critical depth. Thus, no depth of flow can come between the two.

If the water depth is less than both y_c and y_n , then the water surface profile curves are type 3. There are S-3, C-3, M-3, H-3, and A-3 curves. Each of these water surface profile curves is listed and shown schematically in Figure 6.12. Examples of physical occurrences in open channels are also given.

Certain important characteristics of water surface profile curves can be demonstrated from direct analysis of the gradually varied flow equation (Equation 6.25). By making substitutions into Equation 6.25, we note the following:

1. For type 1 curves, $y/y_c > 1$ and $y/y_n > 1$. Thus, the value of dy/dx is positive, indicating that water depth increases in the direction of the flow.
2. For type 2 curves, the value of dy/dx is negative. The water depth decreases in the direction of the flow.
3. For type 3 curves, the value of dy/dx is again positive. The water depth increases in the direction of the flow.

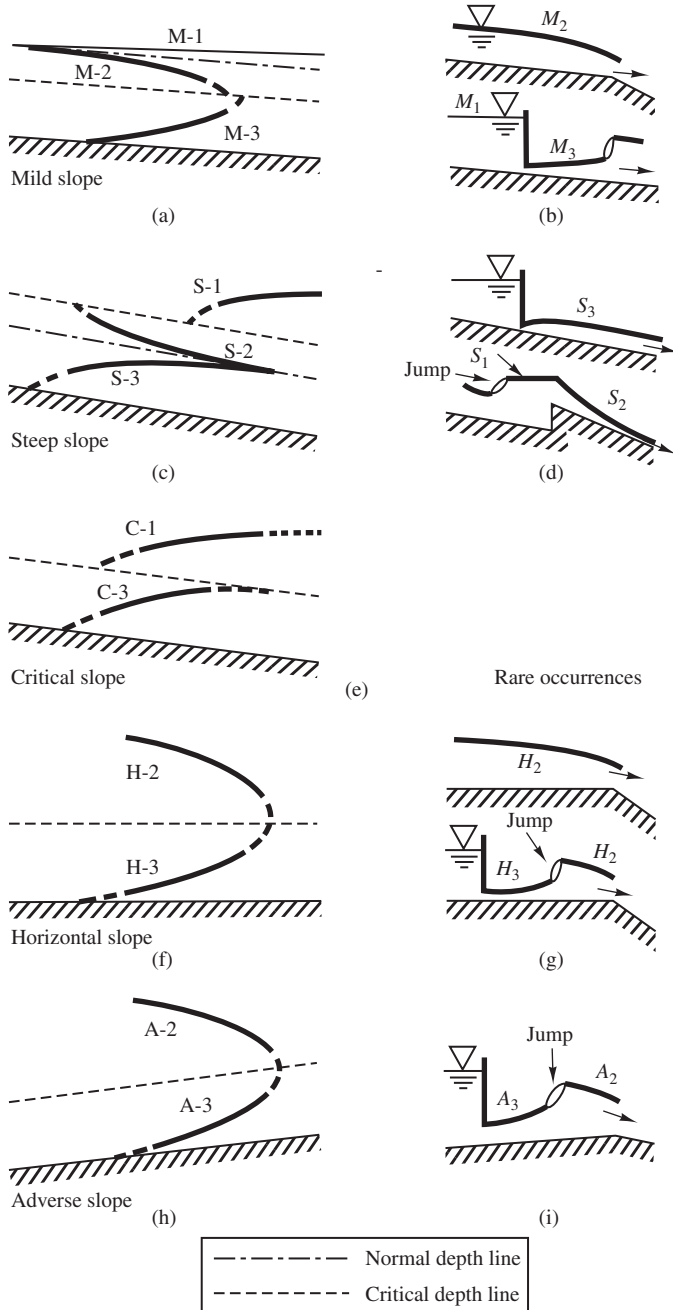


Figure 6.12 Classifications of gradually varied flow

TABLE 6.3 Characteristics of Water Surface Profile Curves

Channel	Symbol	Type	Slope	Depth	Curve
Mild	M	1	$S_0 > 0$	$y > y_n > y_c$	M-1
Mild	M	2	$S_0 > 0$	$y_n > y > y_c$	M-2
Mild	M	3	$S_0 > 0$	$y_n > y_c > y$	M-3
Critical	C	1	$S_0 > 0$	$y > y_n = y_c$	C-1
Critical	C	3	$S_0 > 0$	$y_n = y_c > y$	C-3
Steep	S	1	$S_0 > 0$	$y > y_c > y_n$	S-1
Steep	S	2	$S_0 > 0$	$y_c > y > y_n$	S-2
Steep	S	3	$S_0 > 0$	$y_c > y_n > y$	S-3
Horizontal	H	2	$S_0 = 0$	$y > y_c$	H-2
Horizontal	H	3	$S_0 = 0$	$y_c > y$	H-3
Adverse	A	2	$S_0 < 0$	$y > y_c$	A-2
Adverse	A	3	$S_0 < 0$	$y_c > y$	A-3

- When the actual water depth approaches critical depth, $y = y_c$, Equation 6.25 yields $dy/dx = \infty$, indicating that the slope of the water surface profile curve is theoretically vertical. Likewise, as y approaches y_n , dy/dx approaches zero, indicating that the water surface profile approaches the normal depth line asymptotically.
- A few types of water surface profile curves never approach a horizontal line (S-2, S-3, M-2, M-3, C-3, H-3, and A-3). Others approach a horizontal line asymptotically, except for the C-1 curve, which is horizontal throughout the channel reach. Because $y_n = y_c$ in a critical channel, Equation 6.25 yields $dy/dx = S_0$, indicating the water depth increases at the same rate as the channel bed elevation decreases, which, theoretically, results in a horizontal water surface profile.

In channels where $y < y_c$, the velocity of water flow is greater than that of the disturbance wave. For this reason, flow conditions in the downstream channel will not affect those upstream. The change of water depth resulting from any channel disturbance propagates only in the downstream direction. Thus, computation of the water surface profile should be carried out in the downstream direction (M-3, S-2, S-3, C-3, H-3, and A-3).

In channels where $y > y_c$, the speed of wave propagation is greater than the velocity of water flow. Any disturbance in the downstream channel can travel upstream and affect the flow conditions upstream as well as downstream. Any change of water depth in the downstream channel propagates upstream and may also change the water depth in the upstream channel. Thus, computation of the water surface profile should be carried out in the upstream direction (M-1, M-2, S-1, C-1, H-2, and A-2).

At the break of a channel from mild to steep slope or a significant drop of the channel bottom, critical depth is assumed to take place in the immediate vicinity of the brink. At this point, a definite depth–discharge relationship can be obtained (i.e., *control section*) and is frequently used as a starting point for water surface profile computations.

Table 6.3 provides a summary of the water surface profile curves.

6.8 Computation of Water Surface Profiles

Water surface profiles for gradually varied flow may be computed by using Equation 6.25. The computation normally begins at a section where the relationship between the water surface elevation (or flow depth) and the discharge is known. These sections are commonly known as

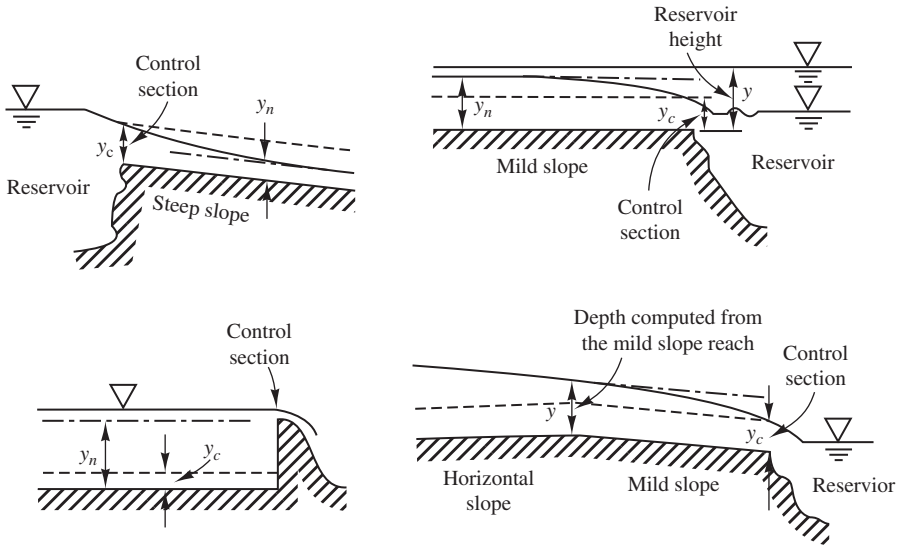


Figure 6.13 Control sections in open channels

control sections (or mathematically, boundary conditions). A few examples of common control sections in open channels are depicted in Figure 6.13. Locations where uniform flow occurs can also be viewed as a control section because the Manning equation describes a flow depth–discharge relationship. Uniform flow (i.e., flow at normal depth) tends to occur in the absence of or far away from other control sections and where the stream slope and cross section are relatively constant.

A successive computational procedure based on an energy balance is used to obtain the water surface elevation at the next section, either upstream or downstream from the control section. The distance between sections is critical because the water surface will be represented by a straight line. Thus, if the depth of flow is changing quickly over short distances, adjacent sections should be closely spaced to represent accurately the water surface profile. The step-by-step procedure is carried out in the downstream direction for rapid (supercritical) flows and in the upstream direction for tranquil (subcritical) flows.

6.8.1 Standard Step Method

The standard step method is presented in this section to calculate gradually varied flow water surface profiles. The method employs a finite difference solution scheme to solve the differential, gradually varied flow equation (Equation 6.25). It is the most common algorithm used in computer software packages that solve gradually varied flow profiles. For example, it is the primary algorithm in the widely used HEC-RAS program developed by the U.S. Army Corps of Engineers. For less common methods of computation, the reader is referred to the classic textbook by Ven T. Chow.*

* V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

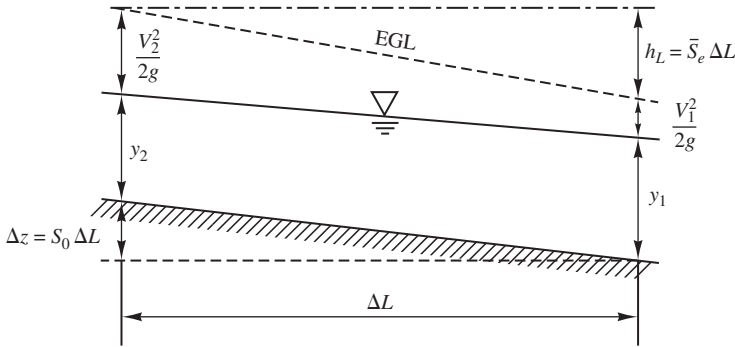


Figure 6.14 Energy relationships in a water surface profile

The standard step method is derived directly from an energy balance between two adjacent cross sections (Figure 6.14) that are separated by a sufficiently short distance so that the water surface can be approximated by a straight line. The energy relation between the two sections may be written as

$$\frac{V_2^2}{2g} + y_2 + \Delta z = \frac{V_1^2}{2g} + y_1 + h_L \tag{6.26a}$$

where Δz is the elevation difference in the channel bottom and h_L is the energy head loss between the two sections, as shown in Figure 6.14.

Equation 6.26a may be rewritten as

$$\left(z_2 + y_2 + \frac{V_2^2}{2g} \right) = \left(z_1 + y_1 + \frac{V_1^2}{2g} \right) + \bar{S}_e \Delta L \tag{6.26b}$$

or

$$E'_2 = E'_1 + \text{losses} \tag{6.26c}$$

where z is the position head (channel bottom elevation with respect to some datum) and E' is the total energy head (position + depth + velocity). It is important to note that, in Equation 6.26, the sections 1 and 2 represent downstream and upstream sections, respectively. If the sections are numbered differently, the losses should always be added to the downstream side.

The computation procedure yields the correct depth at a cross section that is a distance ΔL away from a section with a known depth. Computations begin at a control section and progress upstream (subcritical flow) or downstream (supercritical flow). For subcritical flow, the water surface profile is occasionally called a backwater curve because the process moves from downstream to upstream. Likewise, the profile for supercritical flow is occasionally called a front-water curve.

Equation 6.26b cannot be solved directly for the unknown depth (e.g., y_2) because V_2 and \bar{S}_e depend on y_2 . Therefore, an iterative procedure is required using successive

approximations of y_2 until the downstream and upstream energies balance (or come within an acceptable range). The energy slope (S_e) can be computed by applying the Manning equation, in either SI units

$$S_e = \frac{n^2 V^2}{R_h^{4/3}} \quad (6.27a)$$

or BG units

$$S_e = \frac{n^2 V^2}{2.22 R_h^{4/3}} \quad (6.27b)$$

where \bar{S}_e is the average of the energy (EGL) slopes at the upstream and downstream sections. A tabulated computation procedure is recommended as illustrated in the example problems to follow.

The astute reader may ask why ΔL is not solved for in Equation 6.26b. By assigning a depth to the next section instead of assuming the depth, the equation could be used to determine the distance between sections and avoid the iterative process altogether. This is a legitimate solution procedure called the *direct step method*, but it only works for *prismatic channels* (channels of uniform slope and cross section). When water surface profiles are sought in natural stream channels that are nonprismatic, cross sections on these streams are field surveyed or obtained from geographic information system maps at predetermined locations, which establishes the distance between sections. Then the standard step method is used to assess depth of flow at these sections. Fortunately, water surface profiles are generally solved with the aid of computer software, which takes the drudgery out of the iterative process

6.8.2 Direct Step Method

In the direct step method, the gradually varied flow equations are rearranged to determine the distance (ΔL) explicitly between two selected flow depths. This method is applicable to prismatic channels only because the same cross-sectional geometric relationships are used for all the sections along the channel.

Replacing sections 1 and 2 with U and D , respectively, and noting that $S_0 = (z_U - z_D)/\Delta L = \Delta z/\Delta L$ Equation 6.26b is rearranged as

$$\Delta L = \frac{(y_D + \frac{V_D^2}{2g}) - (y_U + \frac{V_U^2}{2g})}{S_0 - \bar{S}_e} = \frac{E_D - E_U}{S_0 - \bar{S}_e} \quad (6.26d)$$

where $E = y + V^2/2g$ is *specific energy*. In Equation 6.26d, U and D represent upstream and downstream sections, respectively. For subcritical flow, the computations begin at the downstream end and progress upstream. In this case y_D and E_D would be known. An appropriate value for y_U is selected and the associated E_U is calculated. Then ΔL is determined by using Equation 6.26d. For supercritical flow, the computations begin at the upstream end and progress downstream. In this case, y_U and E_U would be known. An appropriate value for y_D is selected, and the associated E_D is calculated. Then ΔL is determined by using Equation 6.26d.

Example 6.9

A grouted-riprap, trapezoidal channel ($n = 0.025$) with a bottom width of 4 meters and side slopes of $m = 1$ carries a discharge $12.5 \text{ m}^3/\text{sec}$ on a 0.001 slope. Compute the backwater curve (upstream water surface profile) created by a low dam that backs water up to a depth of 2 m immediately behind the dam. Specifically, water depths are required at critical diversion points that are located at distances of 188 m, 423 m, 748 m, and 1,675 m upstream of the dam.

Solution

Normal depth for this channel can be calculated by using Equation 6.5 (iterative solution), Figure 6.4 (a), or appropriate computer software. Using Figure 6.4 (a),

$$\frac{nQ}{k_M S_0^{1/2} b^{8/3}} = \frac{(0.025)(12.5)}{(1.00)(0.001)^{1/2} (4)^{8/3}} = 0.245$$

From Figure 6.4 (a), with $m = 1$, we obtain

$$y_n/b = 0.415$$

therefore, $y_n = (4 \text{ m})(0.415) = 1.66 \text{ m}$.

Critical depth for this channel can be calculated by using Equation 6.13 (iterative solution), Figure 6.9 (a), or appropriate computer software. Using Figure 6.9 (a),

$$\frac{Qm^{3/2}}{g^{1/2} b^{5/2}} = \frac{(12.5)(1)^{3/2}}{(9.81)^{1/2} (4)^{5/2}} = 0.125$$

From Figure 6.9 (a), we obtain

$$m y_c/b = 0.230; \text{ therefore, } y_c = (4 \text{ m})(0.230)/1.0 = 0.92 \text{ m}$$

We will first use the standard step method. Water surface profile computations require the use of the Manning equation (Equation 6.27a), which contains the variables R_h and V . Recall from earlier discussions that $R_h = A/P$, where A is the flow area and P is the wetted perimeter, and $V = Q/A$.

The computation procedure displayed in Table 6.4 (a) is used to determine the water surface profile. The depth just upstream from the dam is the control section, designated as section 1. Energy balance computations begin here and progress upstream (backwater) because the flow is subcritical ($y_c < y_n$). The finite difference process is iterative; the depth of flow is assumed at section 2 until the energy at the first two sections match using Equation 6.26b. Once the water depth at section 2 is determined, the depth of flow at section 3 is assumed until the energies at sections 2 and 3 balance. This stepwise procedure continues upstream until the entire water surface profile is developed.

Because the starting depth of 2.00 m is greater than the normal depth and normal depth exceeds critical depth, the profile has an M-1 classification (Figure 6.12). The flow depth will approach normal depth asymptotically as the computations progress upstream, as depicted in Figure 6.13 (c). Once the depth becomes normal, or relatively close, the computation procedure is ended. The first few standard step computations are displayed in Table 6.4 (a); completion of the problem is left to the student in Problem 6.8.6.

Because the channel considered in this example is prismatic, we can also use the direct step method to calculate the water surface profile. Table 6.4 (b) is used to determine the profile by setting up and solving Equation 6.26d. The calculations in the table are self-explanatory. Like the standard step method, the computations begin at the downstream end and progress upstream. For the first channel reach considered, $y_D = 2.00 \text{ m}$ is known, and $y_U = 1.91 \text{ m}$ is a depth we select based on the water surface profile (M-1; depths go down) and to compare with the standard step method solution. Then we calculate the distance between the sections with these two depths. For the next reach, 1.91 m becomes the downstream depth, and we select $y_U = 1.82 \text{ m}$. The results are slightly different from those of the standard step method. The discrepancies result from the iterative nature of the standard step method in which the results depend on the tolerance limit selected.

TABLE 6.4 (a) Water Surface Profile (Backwater) Computations Using the Standard Step Method (Example 6.9)

(1) Section	(2) <i>U/D</i>	(3) <i>y</i> (m)	(4) <i>z</i> (m)	(5) <i>A</i> (m ²)	(6) <i>V</i> (m/sec)	(7) $V^2/2g$ (m)	(8) <i>P</i> (m)	(9) <i>R_h</i> (m)	(10) <i>S_e</i>	(11) <i>S_{e(avg)}</i>	(12) <i>h_L</i> (m)	(13) Total Energy (m)
1	<i>D</i>	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000538	0.1011	2.156
2	<i>U</i>	1.94	0.188	11.52	1.085	0.0600	9.487	1.215	0.000567	($\Delta L = 188$ m)		2.188
<i>Note:</i> The trial depth of 1.94 m is too high; the energy does not balance. Try a lower upstream depth.												
1	<i>D</i>	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000554	0.1042	2.159
2	<i>U</i>	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000600	($\Delta L = 188$ m)		2.160
<i>Note:</i> The trial depth of 1.91 m is correct. Now balance energy between sections 2 and 3.												
2	<i>D</i>	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000673	0.1582	2.319
3	<i>U</i>	1.80	0.423	10.44	1.197	0.0731	9.091	1.148	0.000745	($\Delta L = 235$ m)		2.296
<i>Note:</i> The trial depth of 1.80 m is too low; the energy does not balance. Try a higher upstream depth.												
2	<i>D</i>	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000659	0.1549	2.315
3	<i>U</i>	1.82	0.423	10.59	1.180	0.0710	9.148	1.158	0.000716	($\Delta L = 235$ m)		2.314
<i>Note:</i> The trial depth of 1.82 m is correct. Now balance energy between sections 3 and 4.												

Column (1) Section numbers are arbitrarily designated from downstream to upstream.

Column (2) Sections are designated as either downstream (*D*) or upstream (*U*) to assist in the energy balance.

Column (3) Depth of flow (meters) is known at section 1 and assumed at section 2. Once the energies balance, the depth is now known at section 2, and the depth at section 3 is assumed until the energies at sections 2 and 3 balance.

Column (4) The channel bottom elevation (meters) above some datum (e.g., mean sea level) is given. In this case, the datum is taken as the channel bottom at section 1. The bottom slope and distance interval are used to determine subsequent bottom elevations.

Column (5) Water cross-sectional area (square meters) corresponds to the depth in the trapezoidal cross section.

Column (6) Mean velocity (meters per second) is obtained by dividing the discharge by the area in column 5.

- Column (7) Velocity head (meters).
- Column (8) Wetted perimeter (meters) of the trapezoidal cross section based on the depth of flow.
- Column (9) Hydraulic radius (meters) equal to the area in column 5 divided by the wetted perimeter in column 8.
- Column (10) Energy slope obtained from Manning equation (Equation 6.27a).
- Column (11) Average energy grade line slope of the two sections being balanced.
- Column (12) Energy loss (meters) from friction between the two sections found using $h_L = S_{e(\text{avg})}(\Delta L)$ from Equation 6.26b.
- Column (13) Total energy (meters) must balance in adjacent sections (Equation 6.26b). Energy losses are always added to the downstream section. Also, the energy balance must be very close before proceeding to the next pair of adjacent sections or errors will accumulate in succeeding computations. Thus, even though depths were only required to the nearest 0.01 m, energy heads were calculated to the nearest 0.001 m.

TABLE 6.4 (b) Water Surface Profile (Backwater) Computations Using the Direct Step Method (Example 6.9)

Section	<i>U/D</i>	<i>y</i> (m)	<i>A</i> (m ²)	<i>P</i> (m)	<i>R_h</i> (m)	<i>V</i> (m/sec)	<i>V</i> ² /2 <i>g</i> (m)	<i>E</i> (m)	<i>S_e</i>	ΔL (m)	Distance to Dam (m)
1	<i>D</i>	2.00	12.00	9.657	1.243	1.042	0.0553	2.0553	0.000508		0
2	<i>U</i>	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601	186	186
A distance of 186 m separates the two flow depths (2.00 m and 1.91 m).											
2	<i>D</i>	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601		186
3	<i>U</i>	1.82	10.59	9.148	1.158	1.180	0.0710	1.8910	0.000716	239	425
A distance of 239 m separates the two flow depths (1.91 m and 1.82 m).											

Example 6.10

A rough-concrete trapezoidal channel ($n = 0.022$) with a 3.5-ft bottom width, side slope $m = 2$, and bed slope of 0.012 discharges 185 cfs of fresh water from a reservoir. Determine the water surface profile in the discharge channel to within 2 percent of normal depth.

Solution

Normal depth and critical depth are calculated before solving water surface profiles in order to determine the gradually varied flow classification. Normal depth may be determined using the Manning equation in conjunction with Figure 6.4 (a):

$$\frac{nQ}{k_M S_0^{1/2} b^{8/3}} = \frac{(0.022)(185)}{(1.49)(0.012)^{1/2} (3.5)^{8/3}} = 0.883$$

Then, from the figure, $y_n/b = 0.685$ and $y_n = (0.685)(3.5) = 2.40$ ft.

Alternatively, normal depth can be obtained with the Manning equation and Table 6.1 (using successive substitution) or by using appropriate computer software.

Critical depth can be computed using Equation 6.13 and Table 6.1:

$$\frac{Q^2 T}{g A^3} = \frac{Q^2 (b + 2m y_c)}{g [(b + m y_c) y_c]^3} = \frac{(185)^2 [3.5 + 2(2)y_c]}{32.2 [(3.5 + 2y_c)y_c]^3} = 1$$

By successive substitution (or, alternatively, appropriate computer software), we obtain

$$y_c = 2.76 \text{ ft}$$

Because critical depth exceeds normal depth, the channel is steep with an S-2 classification (Figure 6.12). Water from the reservoir will enter the channel and pass through critical depth as depicted in Figure 6.13 (a). Because the control section is at the entrance to the channel and flow is supercritical ($y_n < y_c$), computations will proceed in the downstream direction (front-water) starting from critical depth at the entrance section and approaching normal depth asymptotically. In an S-2 profile, the water surface elevation changes quickly at first and then approaches normal depth more gradually. Therefore, use five cross sections, including the control section, with separation distances (ΔL) of 2, 5, 10, and 40 feet, respectively, moving downstream. The first few standard step computations are displayed in Table 6.5 (a); completion of the problem is left to the student in Problem 6.8.7.

We can also use the direct step method to calculate the water surface profile in this problem because the channel is prismatic. The calculations are summarized in Table 6.5 (b). Again, the calculations begin from the upstream end and progress downstream. For the first channel reach considered, $y_U = 2.76$ ft is known, and $y_D = 2.66$ ft is a depth we select based on the type of the water surface profile (S-2; depths go down) and to compare with the standard step method solution. We then calculate the distance between the sections with these two depths. For the next reach, 2.66 ft is the upstream depth, and we select $y_D = 2.58$ ft. The discrepancies between the standard step and the direct step method result from the iterative nature of the standard step method in which the results depend on the tolerance limit selected.

TABLE 6.5 (a) Water Surface Profile (Front-water) Computations Using the Standard Step Method (Example 6.10)

(1) Section	(2) y (ft)	(3) z (ft)	(4) A (ft ²)	(5) V (ft/s)	(6) V ² /2g (ft)	(7) R _h (ft)	(8) S _e	(9) S _e	(10) h _L = S _e ΔL (ft)	(11) Total Energy (ft)
1	2.76	10.000	24.90	7.431	0.857	1.571	0.00659		(ΔL = 2)	13.617
2	2.66	9.976	23.46	7.885	0.966	1.524	0.00773	0.00716	0.014	13.616
2	2.66	9.976	23.46	7.885	0.966	1.524	0.00773		(ΔL = 5)	13.602
3	2.58	9.916	22.34	8.280	1.065	1.486	0.00882	0.00827	0.041	13.602

- Column (1) Section numbers are arbitrarily designated from upstream to downstream.
- Column (2) Depth of flow (ft) is known at section 1 and assumed at section 2. Once the energies balance, the depth is now known at section 2, and the depth at section 3 is assumed until the energies at sections 2 and 3 balance. In this table, only the final depths that result in an energy balance are shown.
- Column (3) The channel bottom elevation (ft) above some datum is given. In this case, the datum is 10.0 ft below the channel bottom at section 1. The bottom slope and distance interval are used to determine subsequent bottom elevations.
- Column (4) Water cross-sectional area (ft²) corresponding to the depth. Refer to Table 6.1 for appropriate equation.
- Column (5) Mean velocity (ft/s) obtained by dividing the discharge by the area in column 4.
- Column (6) Velocity head (ft).
- Column (7) Hydraulic radius (ft) corresponding to the depth. Refer to Table 6.1 for appropriate equation.
- Column (8) Energy slope obtained from Manning equation (Equation 6.27b).
- Column (9) Average energy grade line slope of the two sections being balanced.
- Column (10) Energy loss (ft) from friction between the two sections found using the average energy grade line slope.
- Column (11) Total energy (ft) must balance in adjacent sections (Equation 6.26b). Energy losses are always added to the downstream section. Also, the energy balance must be very close before proceeding to the next pair of adjacent sections or errors will accumulate in succeeding computations. Thus, even though depths were only required to the nearest 0.01 ft energy heads were calculated to the nearest 0.001 ft.

TABLE 6.5 (b) Water Surface Profile (Front-Water) Computations Using the Direct Step Method (Example 6.10)

Section	U/D	y (ft)	A (ft ²)	P (ft)	R_h (ft)	V (ft/sec)	$V^2/2g$ (ft)	E (ft)	S_e	ΔL (ft)	Distance to Dam (ft)
1	U	2.76	24.90	15.843	1.571	7.431	0.8575	3.6175	0.00659		0
2	D	2.66	23.46	15.396	1.524	7.885	0.9655	3.6255	0.00773	1.66	1.66
A distance of 1.66 ft separates the two flow depths (2.76 ft and 2.66 ft).											
2	U	2.66	23.46	15.396	1.524	7.885	0.9655	3.6255	0.00773		1.66
3	D	2.58	22.34	15.038	1.486	8.280	1.0646	3.6446	0.00882	5.12	6.78
A distance of 5.12 ft separates the two flow depths (2.66 ft and 2.58 ft).											

Classroom Computer Exercise—Water Surface Profiles

Review Example 6.10. Obtain or write computer software appropriate for determining normal and critical depth and water surface profiles. Spreadsheets can be programmed quickly to perform the task, and the widely used water surface profile model called HEC-RAS (U.S. Army Corps of Engineers) is freely available on the Internet. (See the book's preface for other suggestions.) Answer the following questions by performing a computer analysis of the open channel described in Example 6.10 and modifications thereof.

- (a) Before using the software, what data do you anticipate the software will need to evaluate the water surface profile of Example 6.10? Is it necessary to solve for critical and normal depth? Why?
- (b) Using normal and critical depth software, enter the data requested to determine normal and critical depth. Compare your computer results with the answers listed for Example 6.10. Are there any discrepancies? Comments?
- (c) Using the water surface profile software, enter the data requested to evaluate the entire water surface profile. Compare your computer results with the answers for the first two sections given in Example 6.10. (The channel depths for the remaining two cross sections are not provided in Example 6.10 but are available in the back of the book for Problem 6.8.7.) Are there any discrepancies between the two results? Comments?
- (d) What would happen to the water surface profile if the channel slope were doubled? Will you need to compute a new normal depth? Critical depth? Double the slope and compute the new water surface profile. Did you reason correctly? Now restore the channel to its original slope.
- (e) What would happen to the water surface profile if the flow rate were doubled? Will you need to compute a new normal depth? Critical depth? Double the flow rate and compute the new water surface profile. Did you reason correctly? Now restore the flow rate to its original value.
- (f) Determine the channel slope required to make normal depth equal critical depth. Speculate on the water surface profile that would result if normal depth was greater than critical depth. Refer to Figure 6.13 (a) for assistance in reasoning through this.

Perform any other changes your instructor requests of you.

6.9 Hydraulic Design of Open Channels

Open channels are usually designed for uniform flow or normal conditions. Therefore, uniform flow equations are used in sizing these channels. Designing an open channel involves the selection of channel alignment, channel size and shape, longitudinal slope, and the type of lining material. Normally, we consider several hydraulically feasible alternatives and compare them to determine the most cost-effective alternative. This section focuses on the hydraulic considerations involved in channel design.

The topography of the project site, the available width of right-of-way, and the existing and planned adjacent structures control the channel alignment. The topography also controls the

TABLE 6.6 Stable Side Slopes for Channels

Material	Side Slope ^a (Horizontal:Vertical)
Rock	Nearly Vertical
Muck and peat soils	1/4:1
Stiff clay or earth with concrete lining	1/2:1 to 1:1
Earth with stone lining or earth for large channels	1:1
Firm clay or earth for small ditches	1 1/2:1
Loose, sandy earth	2:1 to 4:1
Sandy loam or porous clay	3:1

^aIf channel slopes are to be mowed, a maximum side slope of 3:1 is recommended.
 Source: Adapted from V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

bottom slope of the channel. Slope stability considerations often govern the selection of the side slopes. The recommended side slopes for different types of channel materials are given in Table 6.6. There may also be limitations on channel depth because of a high water table in the underlying soil or underlying bedrock. Most open channels are designed for subcritical flow. It is important to keep the Froude number sufficiently lower than the critical value of 1.0 under design conditions. If the design Froude number is close to 1.0, there is a possibility that the flow will be unstable and fluctuate between subcritical and supercritical conditions because of variations in the actual discharge.

Channels are often lined to prevent the sides and bottom of the channel from eroding because of the shear stresses caused by the flow. The types of channel linings available can be categorized into two broad groups: rigid and flexible. *Rigid liners* are inflexible—for example, concrete. *Flexible liners* are slightly pliable (with the underlying soil) and self-healing such as gravel, riprap, gabions, and grass liners. The discussion in this section is limited to unlined earthen channels and channels lined with rigid materials. Flexible linings are beyond the scope of this book but are discussed in the literature by Chen and Cotton* and Akan.†

Freeboard is the vertical distance between the top of the channel and the water surface that prevails under the design flow conditions. This distance should be sufficient to allow variations in the water surface because of wind-driven waves, tidal action, occurrence of flows exceeding the design discharge, and other causes. There are no universally accepted rules to determine an acceptable freeboard. In practice, freeboard selection is often a matter of judgment, or it is stipulated as part of the prevailing design standards. For example, the U.S. Bureau of Reclamation recommends that unlined channel freeboard be computed as

$$F = \sqrt{Cy} \tag{6.28}$$

where F = freeboard, y = flow depth, and C = freeboard coefficient. If F and y are in ft, C varies from 1.5 for a channel capacity of 20 cfs to 2.5 for a channel capacity of 3,000 cfs or more. If metric units are used with F and y in meters, C varies from 0.5 for a flow capacity for 0.6 m³/s to 0.76 for a flow capacity of 85 m³/s or more. For lined channels, the Bureau recommends that the curves displayed in Figure 6.15 are used to estimate the height of the bank above the water surface (W.S.) and the height of the lining above the water surface.

* Y. H. Chen and G. K. Cotton, *Design of Roadside Channels with Flexible Linings*, Hydraulic Engineering Circular No. 15, Federal Highway Administration, 1988.

† A. O. Akan, *Open Channel Hydraulics* (New York: Butterworth-Heinemann/Elsevier, 2006).

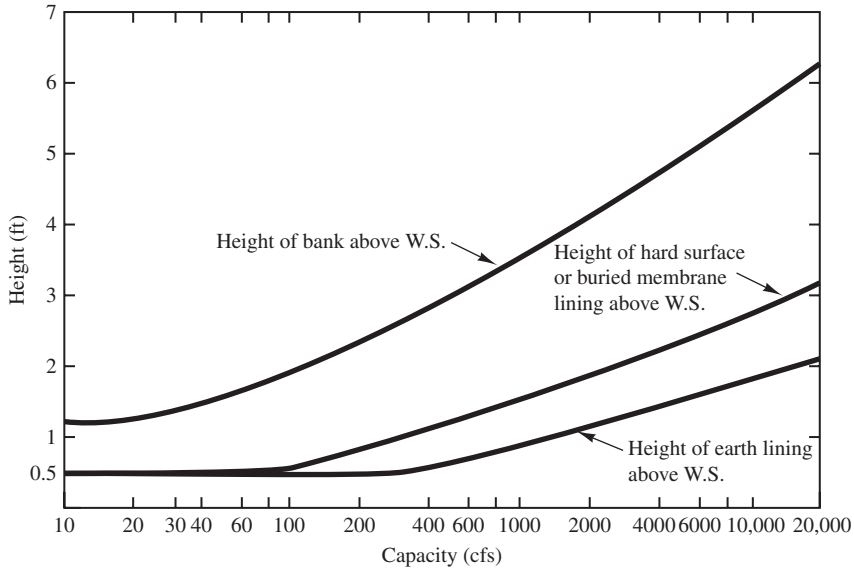


Figure 6.15 Recommended freeboard and height of banks in lined channels.
 Source: U.S. Bureau of Reclamation, *Linings for Irrigation Canals*, 1976.

6.9.1 Unlined Channels

The sides and bottoms of earthen channels are both *erodible*. The main criterion for earthen channel design is that the channel is not eroded under the design flow conditions. There are two approaches to erodible channel design, namely, the *maximum permissible velocity method* and the *tractive force method*. We will discuss the maximum permissible velocity approach because of its simplicity.

This method is based on the assumption that a channel will not erode if the average cross-sectional velocity in the channel does not exceed the *maximum permissible velocity*. Therefore, a channel cross-section is designed so that, under the design flow conditions, the average flow velocity remains below the maximum permissible value. The value of the maximum permissible velocity depends on the type of material into which the channel is excavated and the channel alignment. Table 6.7 presents the maximum permissible velocities for various types of soils. Following Lane,* the values given in Table 6.7 can be reduced by 13 percent for moderately sinuous channels and by 22 percent for very sinuous channels.

In a typical channel design problem, the channel bottom slope (S_0), the design discharge (Q), and the channel material would be given. The procedure to size the channel section would consist of the following steps:

1. For the specified channel material, determine the Manning roughness coefficient from Table 6.2, a stable side slope from Table 6.6, and the maximum permissible velocity from Table 6.7.

* E. W. Lane, "Design of Stable Channels," *Transactions of the American Society of Civil Engineers*, Vol. 120, 1955.

TABLE 6.7 Suggested Maximum Permissible Channel Velocities

Channel Material	V_{\max} (ft/sec)	V_{\max} (m/sec)
Sand and Gravel		
Fine sand	2.0	0.6
Coarse sand	4.0	1.2
Fine gravel ^a	6.0	1.8
Earth		
Sandy silt	2.0	0.6
Silt clay	3.5	1.0
Clay	6.0	1.8

^aApplies to particles with median diameter (D_{50}) less than 0.75 in (20 mm).
 Source: U.S. Army Corps of Engineers. "Hydraulic Design of Flood Control Channels," Engineer Manual, EM 1110-2-1601. Washington, DC: Department of the Army, 1991.

2. Compute the hydraulic radius (R_h) from the Manning equation rearranged as

$$R_h = \left(\frac{n V_{\max}}{k_M \sqrt{S_0}} \right)^{3/2} \tag{6.29}$$

where $k_M = 1.49 \text{ ft}^{1/3}/\text{sec}$ for the conventional U.S. unit system and $1.0 \text{ m}^{1/3}/\text{sec}$ for the metric system.

3. Compute the required flow area from $A = Q/V_{\max}$.
4. Compute the wetted perimeter from $P = A/R_h$.
5. Using the expressions for A and P given in Table 6.1, solve for the flow depth (y) and the bottom width (b) simultaneously.
6. Check the Froude number and ensure that it is not close to unity.
7. Add a freeboard (Equation 6.28) and modify the section for practical purposes.

Example 6.11

An unlined channel to be excavated in stiff clay will convey a design discharge of $Q = 9.0 \text{ m}^3/\text{sec}$ on a slope of $S_0 = 0.0028$. Design the channel dimensions using the maximum permissible velocity method.

Solution

From Table 6.6, $m = 1.0$ for stiff clay; from Table 6.2, use $n = 0.022$ (clean and smooth surface). Also, from Table 6.7, $V_{\max} = 1.8 \text{ m}/\text{sec}$. Using Equation 6.29 with $k_M = 1.00$

$$R_h = \left[\frac{0.022(1.8)}{1.00 \sqrt{0.0028}} \right]^{3/2} = 0.647 \text{ m}$$

Also, $A = Q/V_{\max} = 9.0/1.8 = 5.0 \text{ m}^2$. Hence, $P = A/R_h = 5.0/0.647 = 7.73 \text{ m}$. Now, from expressions given in Table 6.1 and using $m = 1.0$,

$$A = (b + my)y = (b + y)y = 5 \text{ m}^2$$

and

$$P = b + 2y\sqrt{1 + m^2} = b + 2.83 y = 7.73 \text{ m}$$

We now have two equations with two unknowns, y and b . From the second equation, $b = 7.73 - 2.83y$. Substituting this into the first equation and simplifying yields

$$1.83y^2 - 7.73y + 5.00 = 0$$

This equation has two roots, $y = 0.798$ m and 3.43 m. The first root results in a channel width of $b = 7.73 - 2.83(0.798) = 5.47$ m. The second root results in a channel width of $b = 7.73 - 2.83(3.43) = -1.98$ m. Obviously, a negative channel width has no physical meaning. Therefore $y = 0.798$ m will be used.

Next we will check to see if the Froude number is close to the critical value of 1.0. From the expression given for the top width in Table 6.1,

$$T = b + 2my = 5.47 + 2(1)0.798 = 7.07 \text{ m}$$

Then the hydraulic depth becomes $D = A/T = 5.0/7.07 = 0.707$ m, and finally

$$N_F = \frac{V}{\sqrt{gD}} = \frac{1.8}{\sqrt{9.81(0.707)}} = 0.683$$

This value indicates that, under the design flow conditions, the flow will not be near the critical state.

Finally, we will determine a freeboard using Equation 6.28. It is known that C varies from 0.5 for a channel capacity of $0.6 \text{ m}^3/\text{s}$ to 0.76 for a capacity of $85 \text{ m}^3/\text{s}$. Assuming this variation is linear, we determine C as being 0.526 for $Q = 9.0 \text{ m}^3/\text{s}$ by interpolation. Then,

$$F = \sqrt{0.526(0.798)} = 0.648 \text{ m}$$

The total depth for the channel is $y + F = (0.798 + 0.648) = 1.45 \text{ m} \approx 1.5 \text{ m}$ (for practicality in field construction). The bottom width of 5.47 m is increased to 5.5 m for the same reason. The top width of the excavated channel then becomes $b + 2m(y) = 5.5 + 2(1)(1.5) = 8.5 \text{ m}$.

6.9.2 Rigid Boundary Channels

Channels lined with materials such as concrete, asphaltic concrete, soil cement, and grouted riprap are considered to have rigid boundaries. These channels are nonerodible because of the high shear strength of the lining material. In general, there are not any design constraints on the maximum velocity. Therefore, the best hydraulic section concept may be used to size channels with rigid boundaries.

The best hydraulic section concept was discussed in Section 6.3. In summary, the conveyance capacity of a channel section for a given flow area is maximized when the wetted perimeter is minimized. For trapezoidal channel shapes, the best hydraulic section for a fixed side slope (m) is represented by

$$\frac{b}{y} = 2\left(\sqrt{1 + m^2} - m\right) \tag{6.30}$$

The procedure to size a trapezoidal section using the best hydraulic section approach is as follows:

1. Select m and determine n for the specified lining material.
2. Evaluate the ratio b/y from Equation 6.30.

3. Rearrange the Manning formula as

$$y = \frac{[(b/y) + 2\sqrt{1 + m^2}]^{1/4} \left(\frac{Qn}{k_M \sqrt{S_0}} \right)^{3/8}}{[(b/y) + m]^{5/8}} \quad (6.31)$$

and solve for y knowing all the terms on the right-hand side. Then find b .

4. Check the Froude number.

5. Determine the height of lining and the freeboard using Figure 6.15 and modify the section for practical purposes.

Example 6.12

A trapezoidal, concrete-lined channel is required to convey a design discharge of $15 \text{ m}^3/\text{s}$ ec. The channel bottom slope is $S_0 = 0.00095$, and the maximum side slope based on local ordinances is $m = 2.0$. Design the channel dimensions using the best hydraulic section approach.

Solution

From Table 6.2, $n = 0.013$ for concrete. Substituting $m = 2$ into Equation 6.30, we find

$$\frac{b}{y} = 2(\sqrt{1 + 2^2} - 2) = 0.472$$

Next, using Equation 6.31 with $k_M = 1.0$ for the metric unit system,

$$y = \frac{[(0.472) + 2\sqrt{1 + 2^2}]^{1/4} \left[\frac{(15.0)(0.013)}{1.0 \sqrt{0.00095}} \right]^{3/8}}{[(0.472) + 2]^{5/8}} = 1.69 \text{ m}$$

Then, $b = 0.472(1.69) = 0.798 \text{ m}$. For this section,

$$A = (b + my)y = [0.798 + 2(1.69)]1.69 = 7.06 \text{ m}^2,$$

$$T = b + 2my = 0.798 + 2(2)1.69 = 7.56 \text{ m},$$

$$D = A/T = 7.06/7.56 = 0.934 \text{ m},$$

$$V = Q/A = 15.0/7.06 = 2.12 \text{ m/sec, and}$$

$$N_F = V/(gD)^{1/2} = 2.12/[9.81(0.933)]^{1/2} = 0.701.$$

The Froude number is sufficiently lower than the critical value of 1.0.

Finally, from Figure 6.15 (with $Q = 15 \text{ m}^3/\text{s} = 530 \text{ cfs}$), the lining height above the free surface is 1.2 ft (0.37 m). Also, the freeboard (height of bank) above the free surface is 2.9 ft (0.88 m). Thus, the design channel depth is $y + F = (1.69 + 0.88) = 2.57 \text{ m} \approx 2.6 \text{ m}$ (for practicality in field construction). The bottom width of 0.798 m is increased to 0.8 m for the same reason. The top width of the channel is $b + 2m(y) = 0.8 + 2(2)(2.6) = 11.2 \text{ m}$.

PROBLEMS (SECTION 6.1)

- 6.1.1. Using the time and space criterion, classify the following open-channel flow scenarios:
- (a) continuous opening of a gate to let water into a prismatic channel;
 - (b) uniform rainfall over a long, sloped roof top;
 - (c) flow in the roof gutter resulting from (b);

- (d) flow in the above roof gutter when the rainfall intensity increases with time;
- (e) flow in a prismatic shipping canal; and
- (f) flow in a small urban stream because of an intense thunderstorm.

6.1.2. Why is uniform flow rare in natural channels? Is steady flow rare in natural channels (e.g., rivers, streams)? Explain.

(SECTION 6.2)

- 6.2.1. A discharge of 2,200 cfs (ft^3/sec) flows in a trapezoidal channel at a normal depth of 8.0 ft. The channel has a slope of 0.01, a 12-ft bottom width, and side slopes of 1:1 ($H:V$). Determine the Manning coefficient of channel roughness. Also determine the sensitivity range of the roughness coefficient estimate if the flow rate can only be estimated to within ± 200 cfs.
- 6.2.2. A triangular roadside channel has side slopes of 3:1 ($H:V$) and a longitudinal slope of 0.01. Determine the flow rate in the concrete channel if uniform flow is assumed when the top width of the flowing water is 2 meters.
- 6.2.3. Water flows at a depth of 1.83 m in a trapezoidal, riprap-lined channel with a bottom width of 3 m, side slopes of 2:1 ($H:V$), and a bottom slope of 0.005. Assuming uniform flow, what is the discharge in the channel? Check your solution using Figure 6.4 (a).
- 6.2.4. Determine normal depth for a 4-m-wide trapezoidal channel that is carrying a discharge of $49.7 \text{ m}^3/\text{sec}$. The excavated earth channel is well maintained (clean) with a flow slope of 0.2 percent and side slopes of 4:1 ($H:V$). Solve using Figure 6.4 (a) or successive substitution. Check your solution using appropriate computer software.
- 6.2.5. A grass-lined roadside channel ($n = 0.02$) is triangular in shape with 30° side slopes and a bottom slope of 0.006. Determine the normal depth of flow when the discharge is 4 cfs (ft^3/sec). Check your solution using appropriate computer software.
- 6.2.6. A corrugated metal storm water pipe is not flowing full but is discharging $5.83 \text{ m}^3/\text{sec}$. Assuming uniform flow in the 2-m-diameter pipe, determine the flow depth if the 100-m-long pipe goes through a 2-m drop in elevation. Check your solution using Figure 6.4 (b) and appropriate computer software.
- 6.2.7. A triangular highway gutter (Figure P6.2.7) is designed to carry a discharge of $52 \text{ m}^3/\text{min}$ on a channel slope of 0.0016. The gutter is 0.8 m deep with one side vertical and one side sloped at $m:1$ ($H:V$). The channel surface is very smooth and clean earth excavation. Determine the side slope m . Check your solution using appropriate computer software.

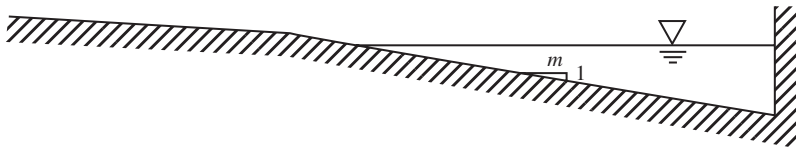


Figure P6.2.7

- 6.2.8. A corrugated metal storm water pipe is being designed to carry a flow rate of 6.0 cfs (ft^3/sec) while flowing half full. If it has a slope of 0.005, determine the diameter of the pipe required. Also determine the pipe size required if full flow was the design condition. Check your solution using Figure 6.4 (b) and appropriate computer software.
- 6.2.9. Uniform flow occurs in a 12-m-wide rectangular channel with a discharge of $100 \text{ m}^3/\text{sec}$. If the normal depth of the flow is 3 m, what will be the new normal depth when the width of the channel contracts to 8 m? Neglect losses. Check your solution using appropriate computer software.

- 6.2.10.** Using appropriate computer software, design a trapezoidal channel and a rectangular channel to convey 100 cfs (ft^3/sec) on a slope of 0.002. Both channels are lined with concrete. Specify width, depth, and side slopes. In both cases, try to obtain channels where the depth is about 60 percent of the bottom width.

(SECTION 6.3)

- 6.3.1.** A flow area of 100 m^2 is required to pass the design flow in a trapezoidal channel (best hydraulic section). Determine the channel depth and bottom width.
- 6.3.2.** Using Example 6.4 as a guide, prove that the best hydraulic rectangular section is a half-square.
- 6.3.3.** An open channel ($n = 0.011$) is to be designed to carry $1.0 \text{ m}^3/\text{sec}$ on a slope of 0.0065. Find the diameter of the best hydraulic section (semicircle).
- 6.3.4.** Show all of the computation steps in progressing from Equation 4 to Equation 5 in Example 6.4.
- 6.3.5.** Determine the side slopes of the best hydraulic (triangular) section.
- 6.3.6.** Design the best hydraulic section (trapezoidal) to convey $150 \text{ ft}^3/\text{sec}$ on a slope of 0.01 in a concrete channel. Check your solution using appropriate computer software.

(SECTION 6.4)

- 6.4.1.** There are two ways of determining the flow classification (subcritical, critical, or supercritical) in an open channel:
- (a) compute the Froude number (if $N_F < 1$, subcritical, if $N_F > 1$, supercritical), or
 - (b) compute critical depth (y_c) and compare it with the depth of flow (y). If $y > y_c$, then flow is subcritical; if $y < y_c$, then flow is supercritical.
- From Example 6.5, compute the flow classification at the transition entrance (where $y = 3.60 \text{ m}$) using both methods.
- 6.4.2.** Determine normal and critical depth for a 4-m-wide rectangular channel with a discharge of $100 \text{ m}^3/\text{sec}$. The channel is made of concrete, and the slope is 1.0 percent. Also determine the flow classification (critical, supercritical, or subcritical) when the channel flows at normal depth.
- 6.4.3.** Water flows in a long, uniform trapezoidal channel lined with brick ($n = 0.013$) at a depth of 4.5 feet. The base width of the channel is 16.0 ft., the side slopes are 3:1 ($H:V$), and the bottom drops a foot for every 1,000 feet of length. Determine the velocity of flow and the flow classification (subcritical, critical, or supercritical). Check your solution using appropriate computer software.
- 6.4.4.** A 60-cm-diameter concrete pipe on a 1:400 slope carries water at a depth of 30 cm. Determine the flow rate and flow classification (subcritical, critical, or supercritical). Check your solution using appropriate computer software.
- 6.4.5.** A 10-ft-wide rectangular channel carries 834 cfs at the depth of 6 ft. What is the specific energy of the channel? Is this flow subcritical or supercritical? If $n = 0.025$, what channel slope must be provided to produce a uniform flow at this depth? Check your solution using appropriate computer software.
- 6.4.6.** A trapezoidal channel carries $100 \text{ m}^3/\text{sec}$ at a depth of 5 m. If the channel has a bottom width of 5 m and 1:1 side slopes, what is the flow classification (subcritical, critical, or supercritical)? What is the specific energy of flow? Also determine the total energy head if the water surface is 50 m above the energy datum. Check your solution using appropriate computer software.
- 6.4.7.** A 40-ft-wide rectangular channel with a bottom slope of $S = 0.0025$ and a Manning's coefficient of $n = 0.035$ is carrying a discharge of 1,750 cfs. Using appropriate computer software (or desk-top methods), determine the normal and critical depths. Also construct a specific energy curve for this discharge.

- 6.4.8.** A trapezoidal channel with a bottom width of 4 m and side slopes of $z = 1.5$ is carrying a discharge of $50 \text{ m}^3/\text{sec}$ at a depth of 3 m. Determine the following:
1. the alternate depth for the same specific energy,
 2. the critical depth, and
 3. the uniform flow depth for a slope of 0.0004 and $n = 0.022$.

Check your solution using appropriate computer software.

- 6.4.9.** A transition is constructed to connect two trapezoidal channels with bottom slopes of 0.001 and 0.0004, respectively. The channels have the same cross-sectional shape, with a bottom width of 3 m, a side slope of $z = 2$, and Manning's coefficient $n = 0.02$. The transition is 20 m long and is designed to carry a discharge of $20 \text{ m}^3/\text{sec}$. Assume an energy loss of 0.02 m is uniformly distributed through the transitional length. Determine the change in the bottom elevations at the two ends of the transition. Assume uniform (normal) depth before and after the transition and use appropriate computer software to assist you.
- 6.4.10.** A hydraulic transition 100 ft long is used to connect two rectangular channels, 12 ft and 6 ft wide, respectively. The design discharge is 500 cfs, $n = 0.013$, and the slopes of the channels are 0.0009 for both. Determine the change in the bottom elevation and the water surface profile in the transition if the energy loss in the transition is 1.5 ft and is uniformly distributed throughout the transition length. Assume uniform (normal) depth before and after the transition; use appropriate computer software to assist you.

(SECTION 6.5)

- 6.5.1.** What is the difference between alternate depths and sequent depths?
- 6.5.2.** When the specific energy for a given discharge is plotted against the depth of the flow at a given section, a *specific energy curve* is obtained. The specific energy curve approaches the abscissa asymptotically for low depths (supercritical flow) and usually approaches a 45° line asymptotically for high depths (subcritical flow). Discuss the physical meaning of these limits.
- 6.5.3.** A hydraulic jump occurs in a rectangular channel that is 7 m wide. The initial depth is 0.8 m, and the sequent depth is 3.1 m. Determine the energy loss and the discharge in the channel. Also determine the Froude number before and after the jump.
- 6.5.4.** A hydraulic jump occurs in 12-ft wide rectangular channel. If the flow rate is 403 cfs and the downstream depth is 5 feet, determine the upstream depth, the loss of energy in the jump, and the Froude number before and after the jump.
- 6.5.5.** Construct the specific energy and specific force curves for a 3-ft-wide rectangular concrete channel (4.5 ft high) that carries 48 cfs.
- 6.5.6.** Plot the specific energy and specific force curves for a 10-m rectangular channel carrying $15 \text{ m}^3/\text{sec}$ discharge. Use 0.2 m-depth increments up to 1.4 m. Also determine the critical depth and minimum specific energy, and discuss how a change in discharge would affect the specific force curve.

(SECTION 6.8)

- 6.8.1.** Identify all of the gradually varied flow classifications depicted in Figure 6.13. All possible classifications are depicted in Figure 6.12.
- 6.8.2.** An obstruction is lodged in a 10-ft wide rectangular channel ($n = 0.022$) that has a bottom slope of 0.005. The depth of the channel just upstream of the obstruction is 5.0 feet. If the flow rate is 325 cfs, what is the channel and flow classification (e.g., S-3, M-2)? Explain your supporting logic.

- 6.8.3.** Water flows at a rate of $22.8 \text{ m}^3/\text{sec}$ in a trapezoidal, riprap-lined channel ($n = 0.04$) with a bottom width of 3 m, side slopes of 2:1 ($H:V$), and a bottom slope of 0.005. At a certain location, the bottom slope increases to 0.022, but the dimensions of the channel remain the same. Determine the channel and flow classification (e.g., M-3, S-2) just upstream of the break in slope. Explain your supporting logic.
- 6.8.4.** Water emerges from under a gate into a trapezoidal, concrete channel. The gate opening is 0.55 m, and the flow rate is $12.6 \text{ m}^3/\text{sec}$. The entry channel has a 1.5-m bottom width, side slopes of 1:1 ($H:V$), and a bottom slope of 0.015. Determine the channel and flow classification (e.g., M-1, S-2) and explain your supporting logic.
- 6.8.5.** At a certain location, the depth of flow in a wide rectangular channel is 0.73 m. The channel discharge is $1.6 \text{ m}^3/\text{sec}$ per unit width, the bottom slope is 0.001, and Manning's coefficient is 0.015. Determine the channel and flow classification (M-1, S-2, etc.) and explain your supporting logic. Will the depth increase or decrease going upstream? Using the standard step method (one step), determine the depth of flow 12 m upstream.
- 6.8.6.** Complete Example 6.9. In that example problem, water depths were required at critical diversion points at distances of 188 m, 423 m, 748 m, and 1,675 m upstream of the dam. Depths were only found at locations 188 m and 423 m upstream. Determine the depths of flow at the other two diversion points using
- the standard step method and
 - the direct step method.
- Has normal depth been reached in the last cross section? (*Note: A spreadsheet program may be helpful.*)
- 6.8.7.** Complete Example 6.10. In that example problem, water depths were required in the discharge channel until the water level was within 2 percent of normal depth. Depths were found at 2 ft and 7 ft downstream [separation distances (ΔL) of 2 ft and 5 ft] but are still needed at distances of 17 ft and 57 ft downstream. Determine the depths of flow at these other two locations using
- the standard step method and
 - the direct step method.
- Has normal depth been reached (within 2 percent) in the last cross section? (*Note: A spreadsheet program may be helpful.*)
- 6.8.8.** A trapezoidal channel has a 5-m bottom width and side slope $m = 1.0$ and discharges $35 \text{ m}^3/\text{sec}$. The smooth, concrete channel (0.011) has a slope of 0.004. Determine the depth of flow 6.0 m upstream from a section that has a measured depth of 1.69 m. But first determine the channel and flow classification (e.g., M-1, S-2) and explain your supporting logic. Will the depth increase or decrease going upstream? Use the standard step method (one step) to determine the depth.
- 6.8.9.** In Figure P6.8.9, a 10-m-wide rectangular channel carries $16.0 \text{ m}^3/\text{sec}$ and has a roughness coefficient of 0.015 and a slope of 0.0016. If a 5-m high dam is placed across the channel raising the water depth to 5.64 m at a location 5.0 m upstream from the dam, what is the channel and flow classification (e.g., M-2, S-1)? Explain your supporting logic. Then determine the water surface profile upstream from the dam by finding the depth of flow at distances of 300 m, 900 m, 1,800 m, and 3,000 m upstream. Use a spreadsheet or appropriate computer software to perform the computations. (See the book's preface for computer software suggestions.)

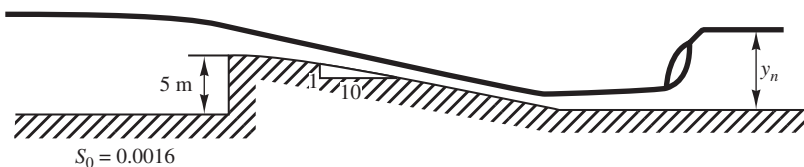


Figure P6.8.9

- 6.8.10.** The downstream side of the dam in Problem 6.8.9 has a slope of 1/10 until it reaches the original channel slope at the bottom (Figure P6.8.9). The 10-m wide rectangular channel carries $16.0 \text{ m}^3/\text{sec}$ and has a roughness coefficient of 0.015. Determine the channel and flow classification (e.g., M-2, S-1) on the downstream side of the dam and explain your supporting logic. Also determine the location and depth of the control section. (*Hint:* Refer to Figures 6.12 and 6.13 after you determine the channel and flow classification.) Then determine the water surface profile on the downstream side of the dam by finding the depth of flow at distances of 0.3 m, 2.0 m, 7.0 m, and 30.0 m downstream from the top of the dam. Use a spreadsheet or appropriate computer software to perform the computations. (See the book's preface for computer software suggestions.)
- 6.8.11.** A diversion gate backs up flow in a 5-ft wide rectangular irrigation channel ($S_0 = 0.001$, $n = 0.015$). The discharge in the channel is 50 cfs. If the depth of flow at the gate is 5 ft, determine the channel and flow classification (e.g., M-2, S-1) upstream of the gate and explain your supporting logic. Then determine the water surface profile by finding the depth of flow at distances of 200 ft, 500 ft, and 1,000 ft upstream of the gate. Use a spreadsheet or appropriate computer software to perform the computations. (See the book's preface for computer software suggestions.)
- 6.8.12.** A long trapezoidal channel with a roughness coefficient of 0.015, bottom width measuring 3.6 m, and $m = 2.0$ carries a flow rate of $44 \text{ m}^3/\text{sec}$. An obstruction is encountered in the channel that raises the water depth to 5.8 m. If the channel slope is 0.001, what is the channel and flow classification (e.g., M-2, S-1) upstream of the obstruction? Explain your supporting logic. Then compute the water surface profile upstream from the obstruction by finding depths at 250 m intervals until the depth is within 2 percent of normal depth. Use a spreadsheet or appropriate computer software to perform the computations. (See the book's preface for computer software suggestions.)
- 6.8.13.** A concrete, trapezoidal channel with a 5-m bottom width and side slopes of 1:1 ($H:V$) discharges $35 \text{ m}^3/\text{sec}$. The bottom slope of the channel is 0.004. A dam is placed across the channel and raises the water level to a depth of 3.4 m. Determine the channel and flow classification (e.g., M-2, S-1) upstream of the dam and explain your supporting logic. Then compute the water surface profile at 50-m intervals until you reach the hydraulic jump. Use a spreadsheet or appropriate computer software to perform the computations. (See the book's preface for computer software suggestions.)

(SECTION 6.9)

- 6.9.1.** An earthen channel will be excavated into sandy soil for which $V_{\max} = 4.0 \text{ ft/s}$, $n = 0.022$, and the recommended $m = 3$. The channel will have a bottom slope of 0.0011 and accommodate a design flow of 303 cfs. Design (size) the channel section.
- 6.9.2.** A sufficiently deep channel excavated into stiff clay (earth excavation, very smooth surface) has side slopes of $m = 1.5$, a bottom slope of $S_0 = 0.001$, and a bottom width of $b = 1.0 \text{ m}$. Can this channel convey $11 \text{ m}^3/\text{sec}$ without being eroded?
- 6.9.3.** A trapezoidal channel lined with concrete is required to carry 342 cfs. The channel will have a bottom slope of $S_0 = 0.001$ and side slopes of $m = 1.5$. Design (size) the channel using the best hydraulic section approach.
- 6.9.4.** The best hydraulic section for the channel described in Problem 6.9.3 turned out to be $y = 4.95 \text{ ft}$ (before freeboard) and $b = 3.00 \text{ ft}$. However, the depth is limited to 3.5 ft because of a high water table at the site. Resize the channel section considering the depth limitation.



Groundwater Hydraulics

Groundwater is found in permeable, water-bearing geologic formations known as aquifers. There are basically two types of aquifers.

1. A *confined aquifer* is a relatively high-permeability, water-bearing formation (e.g., sand or gravels) confined below a layer of very low permeability (e.g., clay).
2. An *unconfined aquifer* is a relatively high-permeability, water-bearing formation with a definable *water table*, a free surface subjected to atmospheric pressure from above and below which the soil is completely saturated.

Figure 7.1 schematically shows several examples of groundwater occurrence in both confined and unconfined aquifer formations.

Movement of groundwater occurs because of a hydraulic gradient or gravitational slope in the same way water movement occurs in pipes or open channels. Hydraulic gradients in aquifers may occur naturally (e.g., sloping water tables) or as a result of artificial means (e.g., well pumps).

The pressure level, or pressure head, in a confined aquifer is represented by the piezometric surface that usually originates from a distant source such as the water table elevation in a recharging location. An artesian spring is formed if the impermeable confining stratum is perforated at a location where the ground surface falls below the piezometric surface. It becomes an artesian well if the ground surface rises above the piezometric surface. The water table of an unconfined aquifer is usually unrelated to the piezometric surface of a confined aquifer in the

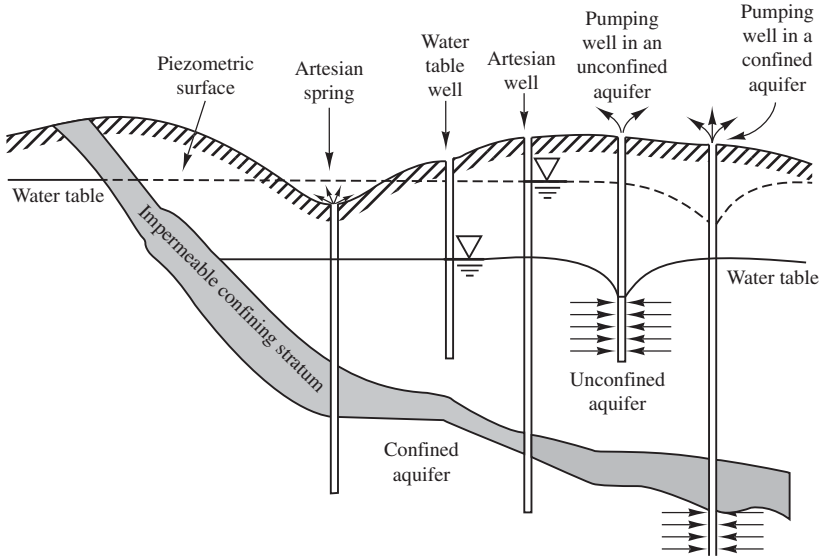


Figure 7.1 Groundwater occurrence in confined and unconfined aquifers

same region, as shown in Figure 7.1. This is because the confined and unconfined aquifers are hydraulically separated by the impermeable confining stratum.

The capacity of a groundwater reservoir in a particular geological formation depends strictly on the percentage of void space in the formation and on the how the *interstitial spaces* are connected. However, the ability to remove the water economically depends on many other factors (e.g., the size and interconnectivity of the pore spaces, the direction of flow) that will be discussed later. Figure 7.2 schematically shows several types of rock formation and their relation to the interstitial space.

The ratio of void space to the total volume of an aquifer (or a representative sample) is known as the *porosity* of the formation, defined as

$$\alpha = \frac{Vol_v}{Vol} \tag{7.1}$$

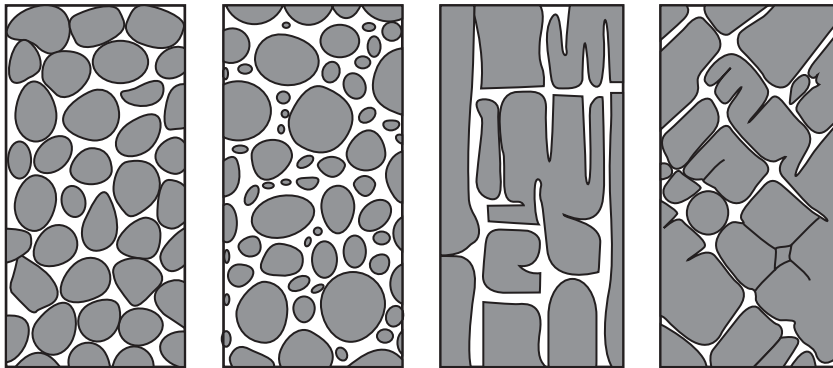


Figure 7.2 Examples of rock texture and interstices

TABLE 7.1 Porosity Range in Common Water-Bearing Formations

Material	α
Clay	0.45–0.55
Silt	0.40–0.50
Medium to coarse mixed sand	0.35–0.40
Fine to medium mixed sand	0.30–0.35
Uniform sand	0.30–0.40
Gravel	0.30–0.40
Gravel and sand mixed	0.20–0.35
Sandstone	0.10–0.20
Limestone	0.01–0.10

where Vol_v is the volume of the void space, and Vol is the total volume of the aquifer (or the sample). Table 7.1 lists the porosity range for common water-bearing formations.

7.1 Movement of Groundwater

Groundwater velocity is proportional to the hydraulic gradient in the direction of the flow. The *apparent velocity* of groundwater movement in a porous medium is governed by *Darcy's law**

$$V = K \frac{dh}{dL} \quad (7.2)$$

where dh/dL is the hydraulic gradient in the direction of the flow path (dL) and K is a proportional-ity constant known as the *coefficient of permeability*, sometimes called the *hydraulic conductivity*. The apparent velocity is defined by the quotient of discharge divided by the total cross-sectional area of the aquifer stratum through which it flows.

The coefficient of permeability depends on the aquifer properties and the properties of the fluid. By dimensional analysis, we may write

$$K = \frac{Cd^2\gamma}{\mu} \quad (7.3)$$

where Cd^2 is a property of the aquifer material, while γ is the specific gravity and μ is the viscosity of the fluid, respectively. The constant C represents the various properties of the aquifer formation that affect the flow other than d , which is a representative dimension proportional to the size of the interstitial space in the aquifer material. The coefficient of permeability can be effectively determined by laboratory experiments or field tests by applying Darcy's equation (Equation 7.2), as will be discussed later. Table 7.2 gives a representative range of the coefficients of permeability for some natural soil formations.

The *seepage velocity* (V_s) is the average speed that water moves between two locations ΔL apart in an aquifer over a time increment Δt . It is different from the *apparent velocity* (V) defined in

* H. Darcy, *Les Fontaines Publiques de la Ville de Dijon* (Paris: V. Dalmont, 1856).

TABLE 7.2 Typical Coefficient of Permeability Ranges for Some Natural Soil Formations

Soils	K (m/sec) ^a
Clays	$<10^{-9}$
Sandy clays	10^{-9} to 10^{-8}
Peat	10^{-9} to 10^{-7}
Silt	10^{-8} to 10^{-7}
Very fine sands	10^{-6} to 10^{-5}
Fine sands	10^{-5} to 10^{-4}
Coarse sands	10^{-4} to 10^{-2}
Sand with gravels	10^{-3} to 10^{-2}
Gravels	$>10^{-2}$

^aFor K in (ft/sec), multiply by 3.28; for K in (gpd/ft²), multiply by 2.12×10^6 .

Darcy’s equation because water can only move through the pore spaces. The appropriate equations are

$$V_s = \Delta L / \Delta t \quad \text{and} \quad V = Q / A$$

Since water can only travel through pore space,

$$V_s = Q / (\alpha A) = V / \alpha$$

where A is the total cross-sectional flow area and (αA) represents just the open (pore) area. Even the seepage velocity is not the *actual velocity* or speed of an individual water molecule as it travels through the pore spaces. The actual distance that water molecules travel between any two locations ΔL apart in the porous medium is not a straight line but a tortuous path, so it must be longer than ΔL . In this chapter, the movement of groundwater is treated from a hydraulic engineer’s point of view; that is, we are primarily interested in the movement of water at the macro level, not the micro level. The fluid mechanical aspect of water particle movement in pores will not be included in this discussion.

For an area A perpendicular to the direction of the water movement in an aquifer, the discharge may be expressed as

$$Q = AV = KA \frac{dh}{dL} \tag{7.4}$$

Laboratory measurement of the coefficient of permeability can be demonstrated by the following example.

Example 7.1

A small sample of an aquifer (uniform sand) is packed in a test cylinder (Figure 7.3) to form a column 30 cm long and 4 cm in diameter. At the outlet of the test cylinder, 21.3 cm³ of water is collected in 2 min. During the testing period, a constant piezometric head difference of $\Delta h = 14.1$ cm is observed. Determine the coefficient of permeability of the aquifer sample.

Solution

The cross-sectional area of the sample volume is

$$A = \frac{\pi}{4}(4)^2 = 12.6 \text{ cm}^2$$

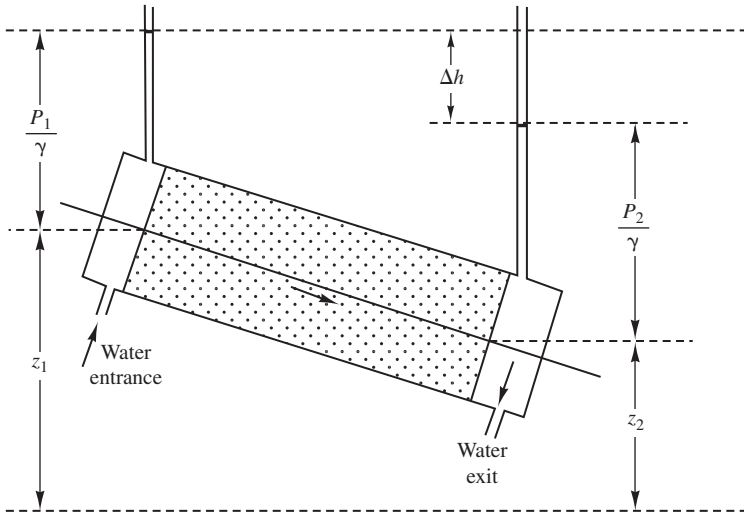


Figure 7.3 Laboratory determination of coefficient of permeability

The hydraulic gradient is equal to the change of energy head (velocity head is negligible in groundwater flow) per unit length of the aquifer measured in the direction of the flow.

$$\frac{dh}{dL} = \frac{14.1}{30} = 0.470$$

The discharge rate is

$$Q = 21.3 \text{ cm}^3/2 \text{ min} = 10.7 \text{ cm}^3/\text{min}$$

Applying Darcy’s law (Equation 7.4), we may calculate

$$K = \frac{Q}{A} \frac{1}{\left(\frac{dh}{dL}\right)} = \frac{10.7}{12.6} \cdot \frac{1}{0.470} = 1.81 \text{ cm}^3/\text{min} \cdot \text{cm}^2 = 3.02 \times 10^{-4} \text{ m/sec}$$

Note that permeability units are actually stated in terms of flow rate (volume/time) per unit area of aquifer. However, this basic unit can be reduced to a form of velocity. Also note that the movement of water is very slow even in sand, which is why the velocity head component of energy head is typically ignored.

Based on Darcy’s law, several different types of permeameters have been developed in the past for laboratory measurements of permeability in small samples of aquifers.* Although laboratory testing is conducted under controlled conditions, such measurements may not be representative of the field permeability because of the small sample size. In addition, when unconsolidated samples are taken from the field and repacked in laboratory permeameters, the texture, porosity, grain orientation, and packing may be markedly disturbed and changed; consequently, permeabilities are modified. Undisturbed samples taken with thin-walled tubes are better but still suffer from some disturbance, wall effects of the sample tube, entrained air, and flow direction (i.e., the flow direction in the field is likely to be different than the laboratory flow

* L. K. Wenzel, *Methods for Determining Permeability of Water-Bearing Materials with Special Reference to Discharging-Well Methods*, U.S.G.S. Water Supply Paper 887 (1942).

direction through the sample). For better reliability, field permeability of aquifers can be determined by well-pumping tests in the field. This method will be discussed in Section 7.4.

7.2 Steady Radial Flow to a Well

Strictly speaking, groundwater flow is three dimensional. However, in most aquifer flow situations the vertical component is negligible. This results from the fact that the horizontal dimensions of aquifers are several orders of magnitude larger than the vertical dimension or the thickness of the aquifer. Therefore, flow in aquifers can be assumed to take place horizontally with velocity components in the x and y directions only. All the aquifer and flow characteristics are assumed to remain constant over the thickness of the aquifer. In accordance with this assumption of *aquifer-type flow*, we can say that the piezometric head is constant at a given vertical section of an aquifer. All the equations in this section and the subsequent sections are derived with the assumption that the aquifer is *homogeneous* (aquifer properties are uniform) and *isotropic* (permeability is independent of flow direction) and that the well completely penetrates the aquifer.

The removal of water from an aquifer by pumping a well results in radial flow in the aquifer to the well. This happens because the pumping action lowers the water table (or the piezometric surface) at the well and forms a region of pressure depression that surrounds the well. At any given distance from the well, the *drawdown* of the water table (or piezometric surface) is defined by the vertical distance measured from the original to the lowered water table (or piezometric surface). Figure 7.4 (a) shows the drawdown curve of the water table in an unconfined aquifer; Figure 7.4 (b) shows the drawdown curve of the piezometric surface in a confined aquifer.

In a homogeneous, isotropic aquifer, the axisymmetric drawdown curve forms a conic geometry commonly known as the *cone of depression*. The outer limit of the cone of depression defines the *area of influence* of the well. The flow is said to be at steady state if the cone of depression does not change with respect to time. For a hydraulic system to be at steady state, the volume of water being added to storage per unit time must be equal to the water being taken from storage.

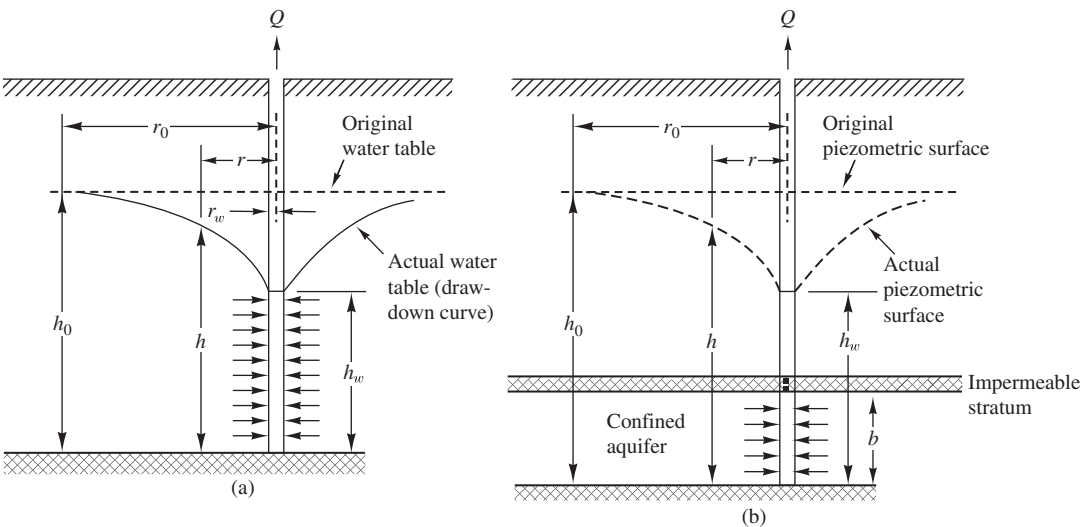


Figure 7.4 Radial flow to a pumping well from (a) an unconfined aquifer and (b) a confined aquifer

When a discharge well pumps water from an aquifer, a steady state would be reached only if the aquifer is recharged at the same rate from a lake or river, rainfall infiltration, leakage from another aquifer, or some other source of water. In the absence of such recharge, a strict steady state is not possible. However, if pumping continues at a constant rate for some time, an approximate steady state will be reached as the changes in the cone of depression become unnoticeable.

7.2.1 Steady Radial Flow in Confined Aquifers

Darcy's law may be directly applied to derive the *radial flow equation* that relates the discharge to the drawdown of the piezometric head in a confined aquifer after a steady state of equilibrium is reached. By using plane polar coordinates with the well as the origin, we find that the discharge flowing through a cylindrical surface at a radius r from the center of the well equals

$$Q = AV = 2\pi rb \left(K \frac{dh}{dr} \right) \quad (7.5)$$

In this equation, b is the thickness of the confined aquifer. Because the flow is at steady state, Q is also equal to the *well discharge*, the flow rate at which the well is pumped. The piezometric head (h) can be measured from any horizontal datum. However, usually it is measured from the bottom of the aquifer. Figure 7.4 (b) depicts all of the variables.

Integrating Equation 7.5 between the boundary conditions at the well ($r = r_w, h = h_w$) and at the *radius of influence* ($r = r_0, h = h_0$) yields

$$Q = 2\pi K b \frac{h_0 - h_w}{\ln\left(\frac{r_0}{r_w}\right)} \quad (7.6)$$

The previous equations apply only to steady flow where the discharge remains a constant value at any distance from the well. In other words, the radius of influence is neither expanding nor contracting. A more general equation for the discharge (steady flow) may be written for any distance (r) as

$$Q = 2\pi K b \frac{h - h_w}{\ln\left(\frac{r}{r_w}\right)} \quad (7.7)$$

Eliminating Q between Equations 7.6 and 7.7 results in

$$h - h_w = (h_0 - h_w) \frac{\ln\left(\frac{r}{r_w}\right)}{\ln\left(\frac{r_0}{r_w}\right)} \quad (7.8)$$

which shows that the piezometric head varies linearly with the logarithm of the distance from the well, regardless of the rate of discharge.

Transmissivity (T , or transmissibility) is a confined aquifer characteristic defined as $T = Kb$. Often the confined flow equations are written in terms of T by replacing Kb with T . Rearranging Equation 7.7 and using the definition of transmissivity, we obtain

$$h = h_w + \frac{Q}{2\pi T} \ln\left(\frac{r}{r_w}\right) \quad (7.9)$$

This equation is useful to determine the piezometric head (h) at any distance r if the head at the well (h_w) is known. If the head is known at a location other than the pumped well—say, at an observation well—then a similar equation can be written as

$$h = h_{ob} + \frac{Q}{2\pi T} \ln\left(\frac{r}{r_{ob}}\right) \tag{7.10}$$

where r_{ob} is the distance between the pumped well and the observation well and h_{ob} is the head at the observation well.

In most groundwater problems, we are more interested in the drawdown rather than the head. The drawdown (s) is defined as $s = h_0 - h$ where h_0 is the undisturbed head. In terms of drawdown, Equation 7.10 becomes

$$s = s_{ob} + \frac{Q}{2\pi T} \ln\left(\frac{r_{ob}}{r}\right) \tag{7.11}$$

Equation 7.11 can be used to determine the drawdown caused by a single pumping well. Because the confined flow equations are linear, we can use the *principle of superposition* to find the drawdown resulting from multiple pumping wells. In other words, the drawdown produced by multiple wells at a particular location is the sum of the drawdowns produced by the individual wells. Suppose, for example, a well located at point A is being pumped at a constant rate Q_A and a well at point B is being pumped at a constant rate Q_B , as shown in Figure 7.5. Suppose there is an observation well at point O at which the observed drawdown is s_{ob} . The drawdown (s) at point C can be calculated using

$$s = s_{ob} + \frac{Q_A}{2\pi T} \ln\left(\frac{r_{Ao}}{r_A}\right) + \frac{Q_B}{2\pi T} \ln\left(\frac{r_{Bo}}{r_B}\right) \tag{7.12a}$$

where r_{Ao} is the distance between pumped well A and the observation well, r_{Bo} is the distance between pumped well B and the observation well, r_A is the distance from pumped well A to the point where s is sought, and r_B is the distance from the pumped well B to the point where s is sought.

If there are M pumping wells and the drawdown (s_{ob}) is known at one observation well, then we can generalize Equation 7.12 to obtain

$$s = s_{ob} + \sum_{i=1}^M \frac{Q_i}{2\pi T} \ln\left(\frac{r_{io}}{r_i}\right) \tag{7.12b}$$

where Q_i is the constant pumping rate of well i , r_{io} is the distance between the pumped well i and the observation well, and r_i is the distance between the pumped well i and the point where the drawdown is sought.

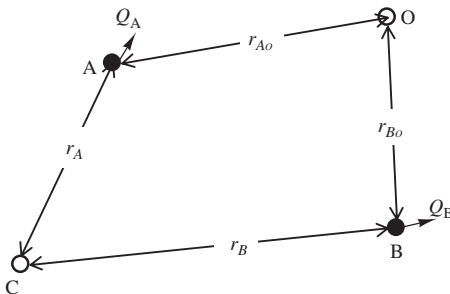


Figure 7.5 Plan view of an aquifer with two pumping wells

Example 7.2

A discharge well that completely penetrates a confined aquifer is pumped at a constant rate of 2,500 m³/day. The aquifer transmissivity is known to be 1,000 m²/day. The steady-state drawdown measured at a distance 60 m from the pumped well is 0.80 m. Determine the drawdown at a distance 150 m from the pumped well.

Solution

By using Equation 7.11,

$$s = s_{ob} + \frac{Q}{2\pi T} \ln\left(\frac{r_{ob}}{r}\right) = 0.80 + \frac{2,500}{2\pi(1,000)} \ln\left(\frac{60}{150}\right) = 0.435 \text{ m}$$

Example 7.3

Two discharge wells (1 and 2) penetrating a confined aquifer are pumped at constant rates of 3,140 m³/day and 942 m³/day, respectively. The aquifer transmissivity is 1,000 m²/day. The steady-state drawdown measured at an observation well is 1.20 m. The observation well is 60 m from well 1 and 100 m from well 2. Determine the drawdown at another point in the aquifer located 200 m from well 1 and 500 m from well 2.

Solution

From Equation 7.12, we can write

$$s = s_{ob} + \frac{Q_1}{2\pi T} \ln\left(\frac{r_{1o}}{r_1}\right) + \frac{Q_2}{2\pi T} \ln\left(\frac{r_{2o}}{r_2}\right)$$

$$s = 1.20 + \frac{3,140}{2\pi(1,000)} \ln\left(\frac{60}{200}\right) + \frac{942}{2\pi(1,000)} \ln\left(\frac{100}{500}\right) = 0.357 \text{ m}$$

7.2.2 Steady Radial Flow in Unconfined Aquifers

Darcy's law may be directly applied to derive the *radial flow equation* that relates the discharge to the drawdown of the water table in an unconfined aquifer after a steady state of equilibrium is reached. By using plane polar coordinates with the well as the origin, we find that the discharge flowing through a cylindrical surface at a radius r from the center of the well equals

$$Q = AV = 2\pi rh \left(K \frac{dh}{dr} \right) \quad (7.13)$$

In this equation, h is measured from the bottom of the aquifer to the water table. Because the flow is at steady state, Q is also equal to the *well discharge*, the flow rate at which the well is pumped. Figure 7.4 (a) depicts all of the variables.

Integrating Equation 7.13 between the boundary conditions at the well ($r = r_w$, $h = h_w$) and at the radius of influence ($r = r_0$, $h = h_0$) yields

$$Q = \pi K \frac{h_0^2 - h_w^2}{\left(\ln \frac{r_0}{r_w} \right)} \quad (7.14)$$

The selection of the radius of influence, r_0 , can be somewhat arbitrary. The variation of Q is rather small for a wide range of r_0 because the influence on the well by the water table at great distances is small. In practice, approximate values of r_0 may be taken between 100 m and 500 m, depending on the nature of the aquifer and the operation of the pump.

We can rearrange Equation 7.14 as

$$h_w^2 = h_0^2 - \frac{Q}{\pi K} \ln\left(\frac{r_0}{r_w}\right) \tag{7.15}$$

A more general equation for any distance r from the pumped well and an observation well at distance r_{ob} from the pumped well can be written as

$$h^2 = h_{ob}^2 - \frac{Q}{\pi K} \ln\left(\frac{r_{ob}}{r}\right) \tag{7.16}$$

The unconfined flow equations are not linear in h . Therefore, superposition of h values is not allowed for multiple wells in an unconfined aquifer. However, the equations are linear in differences in h^2 . Therefore, to find the steady-state h at a point resulting from M pumping wells, we can use

$$h^2 = h_{ob}^2 - \sum_{i=1}^M \frac{Q_i}{\pi K} \ln\left(\frac{r_{io}}{r_i}\right) \tag{7.17}$$

where h_{ob} is the head measured at the observation well, Q_i is the constant pumping rate of well i , r_{io} is the distance between the pumped well i and the observation well, and r_i is the distance between the pumped well i and the point where the drawdown is sought.

Example 7.4

A 95-ft-thick, unconfined aquifer is penetrated by an 8-in.-diameter well that pumps at a rate of 50 gal/min (gpm). The drawdown at the well is 3.5 ft and the radius of influence is 500 ft. Determine the drawdown 80 ft from the well.

Solution

First, we must determine the permeability of the aquifer. Substituting the given conditions into Equation 7.14 and noting that there are 449 gpm in 1 cfs yields

$$(50/449) = \pi K \frac{(95^2 - 91.5^2)}{\ln(500/0.333)}$$

$$K = 3.97 \times 10^{-4} \text{ ft/sec}$$

Now we can use Equation 7.16 and the known head at the well (or alternatively, at the radius of influence) as the observation well data.

$$h^2 = 91.5^2 - \frac{(50/449)}{\pi(3.97 \times 10^{-4})} \ln\left(\frac{0.333}{80}\right); h = 94.1 \text{ ft}$$

Therefore, the drawdown is: $s = h_o - h = 95.0 - 94.1 = 0.9 \text{ ft}$.

Example 7.5

Two discharge wells (1 and 2) penetrating an unconfined aquifer are pumped at constant rates of 3,000 and 500 m³/day, respectively. The steady-state head (i.e., water table height) measured at an observation well is 40.0 m. The observation well is 50 m from well 1 and 64 m from well 2. The water table height measured at a second observation well is 32.9 m, which is located 20 m from well 1 and 23 m from well 2. Determine the aquifer permeability (in m/day).

Solution

From Equation 7.17,

$$h^2 = h_{ob}^2 - \frac{Q_1}{\pi K} \ln\left(\frac{r_{1o}}{r_1}\right) - \frac{Q_2}{\pi K} \ln\left(\frac{r_{2o}}{r_2}\right)$$

$$32.9^2 = 40^2 - \frac{3,000}{\pi K} \ln\left(\frac{50}{20}\right) - \frac{500}{\pi K} \ln\left(\frac{64}{23}\right)$$

Therefore, $K = 2.01$ m/day.

7.3 Unsteady Radial Flow to a Well

Groundwater flow is said to be unsteady if the flow conditions at a given point, such as piezometric head and velocity, are changing with time. These changes are also associated with changes in the volume of water in storage. *Storage coefficient* (S , also referred to as *storage constant* or *storativity*) is an aquifer parameter linking the changes in the volume of water in storage to the changes in the piezometric head. It is a dimensionless number defined as the water yield from a column of aquifer (of unit area) that results from lowering the water table or piezometric surface by a unit height. The upper limit of S is the porosity, although it is impossible to remove all of the water stored in the pores because of capillary forces. Although S may approach the porosity in unconfined aquifers, its value is much lower in confined aquifers because the pores do not drain. Water is removed by compression of the saturated layer and expansion of the groundwater.

The derivations of the unsteady groundwater equations and their solutions are beyond the scope of this book. Therefore, only the results will be presented herein.

7.3.1 Unsteady Radial Flow in Confined Aquifers

With reference to Figure 7.6, unsteady radial flow in a confined aquifer at time t is expressed as*

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + = \frac{S}{T} \frac{\partial h}{\partial t} \quad (7.18)$$

where S is the storage coefficient and T is the transmissivity. Both of these aquifer characteristics are assumed to remain constant with time. An analytical solution can be obtained to this equation using the initial condition $h = h_0$ at $t = 0$ for all r , and the two boundary conditions $h = h_0$ at $r = \infty$, and

$$r \frac{\partial h}{\partial r} = \frac{Q_w}{2 \pi T} \quad \text{for} \quad r = r_w \quad \text{and} \quad r_w \rightarrow 0$$

where Q_w is a constant well discharge starting at $t = 0$ and r_w is the radius of the well. The initial condition implies that the piezometric surface is level at the time pumping first begins. The first boundary condition implies that the aquifer is infinitely large—that is, there are no boundaries such as rivers or lakes affecting the flow. The second boundary condition assumes that the rate of flow entering the well from the aquifer equals the well discharge. This boundary condition also implies the well radius is negligibly small compared to other horizontal dimensions in the aquifer, and it neglects the changes in the volume of water within the well with time.

Theis[†] first presented a solution to the unsteady confined equation as

$$h_0 - h = s = \frac{Q_w}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du \quad (7.19)$$

* W. C. Walton, *Groundwater Resources Evaluation* (New York: McGraw-Hill, 1970).

† C. V. Theis, "This relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage," *Trans. Am. Geophys. Union*, 16 (1935): 519–524.

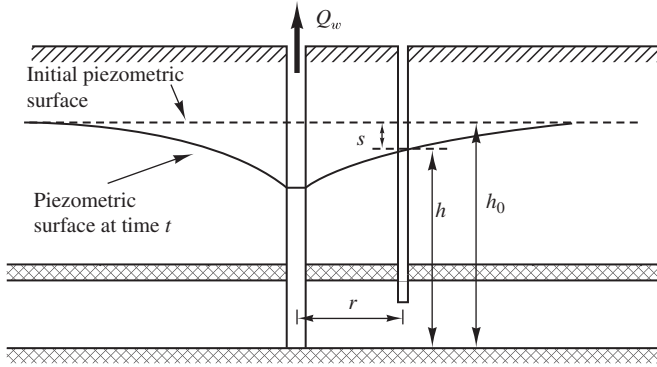


Figure 7.6 Definition sketch for unsteady confined flow

where $h_0 - h = s$ is the drawdown in an observation well at a distance r from the pumping well and u is a dimensionless parameter given by

$$u = \frac{r^2 S}{4Tt} \tag{7.20}$$

In Equation 7.20, t is the time since the pumping began.

The integral in Equation 7.19 is commonly written as $W(u)$ —known as the *well function* of u —and the equation becomes

$$s = \frac{Q_w}{4\pi T} W(u) \tag{7.21}$$

The well function is not directly integratable but may be evaluated by the infinite series

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} \dots \tag{7.22}$$

The values of the well function $W(u)$ are given in Table 7.3 for a wide range of u .

For a reasonably large value of t and a small value of r in Equation 7.20, u becomes small enough that the terms following “ $\ln u$ ” in Equation 7.22 become very small and may be neglected. Thus, when $u < 0.01$, the Theis equation may be modified to the Jacob formulation,* which is written as

$$\begin{aligned} s = h_0 - h &= \frac{Q_w}{4\pi T} \left[-0.5772 - \ln \frac{r^2 S}{4Tt} \right] \\ &= \frac{-2.30 Q_w}{4\pi T} \log_{10} \frac{0.445 r^2 S}{Tt} \end{aligned} \tag{7.23}$$

The confined flow equation and its solution are linear with respect to s , Q_w , and t . This property allows us to utilize the Theis solution with the concept of superposition in many situations beyond those for which the solution was first obtained. Suppose the well discharge is not

* C. E. Jacob, “Drawdown test to determine effective radius of artesian well,” *Trans. ASCE*, 112 (1947): 1047–1070.

TABLE 7.3 Well Function, $W(u)$

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.823	1.223	0.906	0.702	0.560	0.454	0.374	0.311	0.260
$\times 10^{-2}$	4.038	3.355	2.959	2.681	2.468	2.295	2.151	2.027	1.919
$\times 10^{-3}$	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
$\times 10^{-4}$	8.633	7.940	7.535	7.247	7.024	6.842	6.688	6.554	6.437
$\times 10^{-5}$	10.936	10.243	9.837	9.549	9.326	9.144	8.990	8.856	8.739
$\times 10^{-6}$	13.238	12.545	12.140	11.852	11.629	11.447	11.292	11.159	11.041
$\times 10^{-7}$	15.541	14.848	14.442	14.155	13.931	13.749	13.595	13.461	13.344
$\times 10^{-8}$	17.843	17.150	16.745	16.457	16.234	16.052	15.898	15.764	15.646
$\times 10^{-9}$	20.146	19.453	19.047	18.760	18.537	18.354	18.200	18.067	17.949
$\times 10^{-10}$	22.449	21.756	21.350	21.062	20.839	20.657	20.503	20.369	20.251
$\times 10^{-11}$	24.751	24.058	23.653	23.365	23.142	22.959	22.805	22.672	22.554
$\times 10^{-12}$	27.054	26.361	25.955	25.668	25.444	25.262	25.108	24.974	24.857
$\times 10^{-13}$	29.356	28.663	28.258	27.970	27.747	27.565	27.410	27.277	27.159
$\times 10^{-14}$	31.659	30.966	30.560	30.273	30.050	29.867	29.713	29.580	29.462

constant but varies as shown in Figure 7.7. The drawdown s at time t where $t_N > t > t_{N-1}$ can be found as

$$s = \frac{1}{4\pi T} \sum_{k=1}^N (Q_k - Q_{k-1}) \cdot W(u_k) \tag{7.24}$$

where

$$u_k = \frac{r^2 S}{4 T (t - t_{k-1})} \tag{7.25}$$

Note that if we are seeking the drawdown at some point in time after the pumping stopped, $Q_N = 0$ in this formulation. Also, $Q_0 = 0$ and $t_0 = 0$.

Fortunately, the concept of superposition can be used for multiple well situations also. For example, if M wells start to pump simultaneously at $t = 0$ with constant discharges Q_j , $j = 1, 2, \dots, M$ in an infinite aquifer, the total drawdown at a particular location at time t is

$$s = \sum_{j=1}^M s_j = \frac{1}{4\pi T} \sum_{j=1}^M Q_j \cdot W(u_j) \tag{7.26}$$

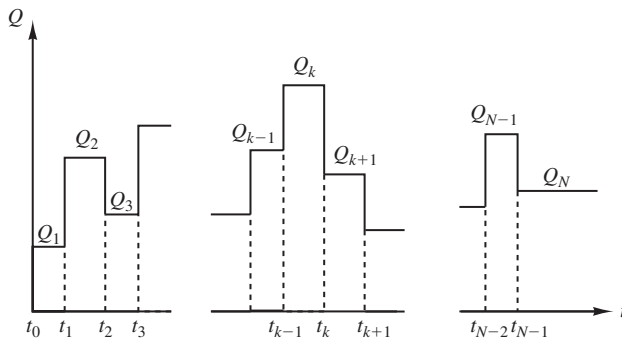


Figure 7.7 Variable pumping rate

where

$$u_j = \frac{r_j^2 S}{4 T t} \tag{7.27}$$

and r_j is the distance between the pumped well j and point where “ s ” is sought.

For M wells starting to pump at different times t_j with constant discharges Q_j in an infinite aquifer, the total drawdown can still be calculated using Equation 7.26. However, for this case

$$u_j = \frac{r_j^2 S}{4 T (t - t_j)} \tag{7.28}$$

If M wells have variable pumping rates as shown in Figure 7.7, the total drawdown can still be calculated using the method of superposition

$$s = \sum_{j=1}^M s_j \tag{7.29}$$

where s_j is the drawdown from well j to be obtained by applying Equation 7.26 to each well separately.

7.3.2 Unsteady Radial Flow in Unconfined Aquifers

The governing mechanism for the release of water from storage in an unconfined aquifer is the gravity drainage of water that once occupied some of the pore space above the cone of depression. However, the gravity drainage does not occur instantaneously. When an unconfined aquifer is pumped, the initial response of the aquifer is similar to that of a confined aquifer. In other words, the release of water is mainly caused by the compressibility of the aquifer skeleton and water. Therefore, the early stages of pumping flow in an unconfined aquifer can be calculated like the flow in a confined aquifer using the Theis solution and Equations 7.20 and 7.21. The storage coefficient to be used would be comparable in magnitude to that of a confined aquifer. However, as gravity drainage commences, the drawdowns will be affected by this mechanism and deviations from the Theis solution will occur. As gravity drainage is fully established at later stages, the behavior of the aquifer will again resemble that of a confined aquifer, provided that the drawdowns are much smaller than the initial thickness of the aquifer. The Theis solution will still apply, but the storage coefficient to be used in the equation will be that of unconfined conditions.

A procedure developed by Neuman* to calculate drawdowns takes into account the delayed gravity drainage. A graphical representation of the Neuman procedure, which was also presented and explained by Mays,† is given in Figure 7.8 where

$$u_a = \frac{r^2 S_a}{4 T t} \tag{7.30}$$

$$u_y = \frac{r^2 S_y}{4 T t} \tag{7.31}$$

$$\eta = \frac{r^2}{h_0^2} \tag{7.32}$$

* S. P. Neumann, “Analysis of pumping test data from anisotropic unconfined aquifers considering delayed gravity response,” *Water Resources Research*, 11 (1975): 329–342.

† L. W. Mays, *Water Resources Engineering* (New York: John Wiley and Sons, Ltd., 2001)

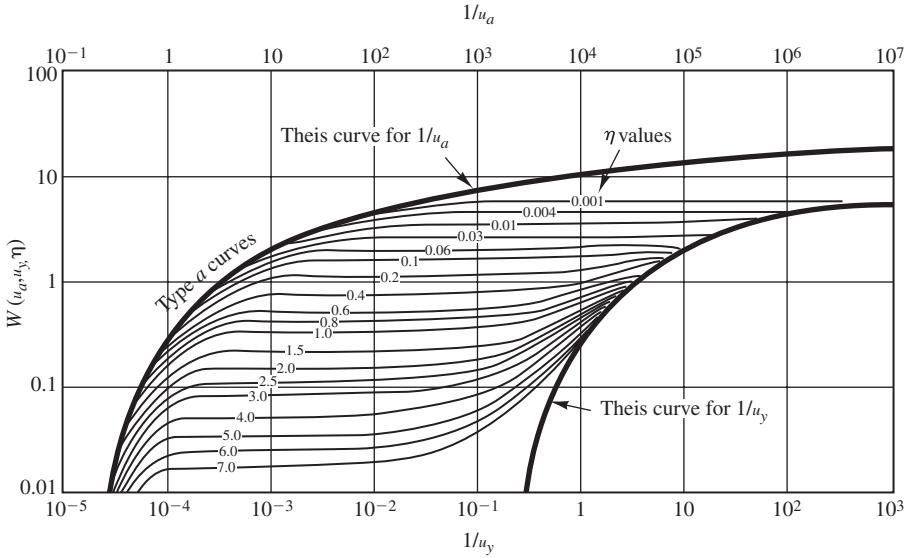


Figure 7.8 Well function for unconfined aquifers

with S_a = efficient early time storage coefficient, S_y = unconfined storage coefficient, $T = Kh_0$, K = permeability coefficient, h_0 = initial water table elevation, and $W(u_a, u_y, \eta) =$ well function for unconfined aquifers with delayed gravity drainage. Equation 7.30 is applicable for small values of time, and it corresponds to those line segments to the left of printed η values in Figure 7.8. Equation 7.31 is applicable for large time values, and it corresponds to the line segments to the right of printed η values. Once the value of the well function is obtained from Figure 7.8, the drawdown is calculated using

$$s = \frac{Q_w}{4 \pi T} W(u_a, u_y, \eta) \tag{7.33}$$

Example 7.6

A high-capacity discharge well fully penetrating a confined aquifer is pumped at constant rate of 5,000 m³/day. The aquifer transmissivity is 1,000 m²/day, and storage coefficient is 0.0004. Determine the drawdown at a distance of 1,500 m from the pumped well after 1.5 days of pumping.

Solution

By using Equation 7.20,

$$u = \frac{r^2 S}{4 T t} = \frac{(1,500)^2 (0.0004)}{4 (1,000) (1.5)} = 0.15$$

From Table 7.3, we obtain $W(u) = 1.523$. Therefore, by using Equation 7.21,

$$s = \frac{Q_w}{4 \pi T} W(u) = \frac{5,000}{4 \pi (1,000)} (1.523) = 0.61 \text{ m}$$

Example 7.7

Suppose in Example 7.6 the pump is shut off after 1.5 days. Determine the drawdown at the same location, 1,500 m from the pumped well, one day after pumping has stopped. Explain the difference between the result of this example and Example 7.6.

Solution

For this problem, we will use the concept of superposition for unsteady well flow because the well discharge is not constant during the study period. This requires the use of Equation 7.24, with $N = 2$, $Q_0 = 0$, $Q_1 = 5,000 \text{ m}^3/\text{day}$, $Q_2 = 0$, $t_0 = 0$, and $t_1 = 1.5$ days.

By using Equation 7.25 for $k = 1$,

$$u_1 = \frac{r^2 S}{4 T (t - t_0)} = \frac{(1,500)^2(0.0004)}{4(1,000)(2.5 - 0)} = 0.090$$

and for $k = 2$,

$$u_2 = \frac{r^2 S}{4 T (t - t_1)} = \frac{(1,500)^2(0.0004)}{4(1,000)(2.5 - 1.5)} = 0.225$$

From Table 7.3, we then obtain $W(u_1) = 1.919$ and $W(u_2) = 1.144$. Now using (Equation 7.24), we obtain

$$s = \frac{1}{4 \pi T} \{ [Q_1 - Q_0] W(u_1) + [Q_2 - Q_1] W(u_2) \}$$

$$s = \frac{1}{4\pi(1,000)} \{ (5,000 - 0)(1.919) + (0 - 5,000)(1.144) \} = 0.31 \text{ m}$$

The drawdown obtained in this example is smaller than the drawdown of Example 7.6. The explanation is that when the pump is shut off at $t = 1.5$ days, the drawdown is 0.61 m. However, at this time, there is a hydraulic gradient towards the well because of the cone of depression formed. Therefore, flow toward the well will continue although pumping has stopped. As a result, the piezometric surface will rise. This is called *aquifer recovery*.

Example 7.8

A discharge well penetrating a confined aquifer will be pumped at a constant rate of $60,000 \text{ ft}^3/\text{day}$. At what constant rate can a second well be pumped so that the drawdown at a critical location 300 ft from the first well and 400 ft from the second well will not exceed 5 ft after two days of pumping? The aquifer transmissivity is $10,000 \text{ ft}^2/\text{day}$, and the storage coefficient is 0.0004.

Solution

For this problem, we will use the concept of superposition for unsteady flow in an aquifer with multiple wells. For the first well, using Equation 7.27 yields

$$u_1 = \frac{r_1^2 S}{4 T t} = \frac{(300)^2(0.0004)}{4(10,000)(2.0)} = 0.00045$$

and from Table 7.3, $W(u_1) = 7.136$. For the second well, using the same equation yields

$$u_2 = \frac{r_2^2 S}{4 T t} = \frac{(400)^2(0.0004)}{4(10,000)(2.0)} = 0.0008$$

and from Table 7.3, $W(u_2) = 6.554$. Now applying Equation 7.26,

$$s = \frac{1}{4\pi T} [Q_1 \cdot W(u_1) + Q_2 \cdot W(u_2)]$$

$$5.0 = \frac{1}{4\pi(10,000)}[60,000(7.136) + Q_2(6.554)]$$

Solving for Q_2 , we obtain $Q_2 = 3.05 \times 10^4 \text{ ft}^3/\text{day}$.

7.4 Field Determination of Aquifer Characteristics

Laboratory tests of soil permeability (Example 7.1) are performed on small samples of soil. Their value in solving engineering problems depends on how well they represent the entire aquifer in the field. When used with consideration of field conditions and careful handling of the samples, the laboratory test methods can be very valuable. Nevertheless, important groundwater projects often require pumping tests in the field to determine an aquifer's hydraulic parameters such as permeability, transmissivity, and storage coefficient. One well is pumped at a known rate and the resulting drawdowns are measured, either in the pumping well itself or in one or more observation wells. The aquifer parameters are then determined from analyses of drawdown data. The basic idea in analyzing the pumping test data is to fit the observed drawdowns to available analytical solutions. The values of the aquifer parameters sought are those providing the best fit between the theoretical and observed results. We sometimes refer to this procedure as the *inverse problem*. Aquifer permeability and transmissivity can be obtained from drawdown data collected under either steady or unsteady flow conditions. The storage coefficient, however, will require unsteady drawdown data.

7.4.1 Equilibrium Test in Confined Aquifers

Pumping tests under equilibrium (steady) conditions are used to determine the transmissivity of confined aquifers. If, as shown in Figure 7.9, the drawdowns, s_1 and s_2 , are respectively measured at two observation wells located at distances r_1 and r_2 from a pumped well under steady-state conditions, we determine the transmissivity (T) by rewriting Equation 7.11 as

$$T = \frac{Q_w \ln(r_1/r_2)}{2\pi (s_2 - s_1)} \quad (7.34)$$

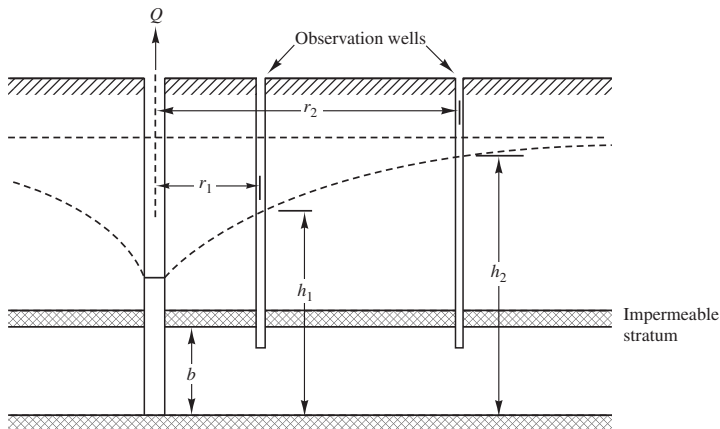


Figure 7.9 Field determination of transmissivity coefficient in confined aquifers

Recall that $s = h_o - h$, where h_o is the piezometric head of the undisturbed aquifer.

In most pumping tests, however, multiple observation wells are used to characterize the aquifer better. For this purpose, we can rewrite Equation 7.11 in terms of common logarithms as

$$T = - \frac{2.30 Q_w}{2 \pi} \frac{\Delta (\log r)}{\Delta s} \tag{7.35}$$

where $\Delta(\log r) = (\log r_2 - \log r_1)$ and $\Delta s = s_2 - s_1$. When more than two observation wells are available, based on the form of Equation 7.11, a plot of s versus r on semilog paper (with s on the linear scale and r on the log scale) produces a straight line. In reality, all the points will not fall exactly on a straight line. We usually draw (estimate) a best-fit straight line, as shown in Figure 7.10, and use the slope of the line to find T . Note that for any log cycle of r , $\Delta(\log r) = 1.0$. Thus, defining $\Delta^*s =$ drop in s per log cycle of r yields

$$T = \frac{2.30 Q_w}{2 \pi (\Delta^*s)} \tag{7.36}$$

Example 7.9

A field test is conducted in a confined aquifer by pumping a constant discharge of 400 ft³/hr from an 8-inch-diameter well. After an approximate steady state is reached, a drawdown of 2.48 ft is measured in the pumped well, and a drawdown of 1.72 ft is measured at 150 ft from the pumped well. Determine the transmissivity for this aquifer.

Solution

Using Equation 7.34 with observation well 1 represented by the pumped well yields

$$T = \frac{Q_w \ln(r_1/r_2)}{2\pi s_2 - s_1} = \frac{400 \ln(0.33/150)}{2\pi (1.72 - 2.48)} = 513 \text{ ft}^2/\text{hr}$$

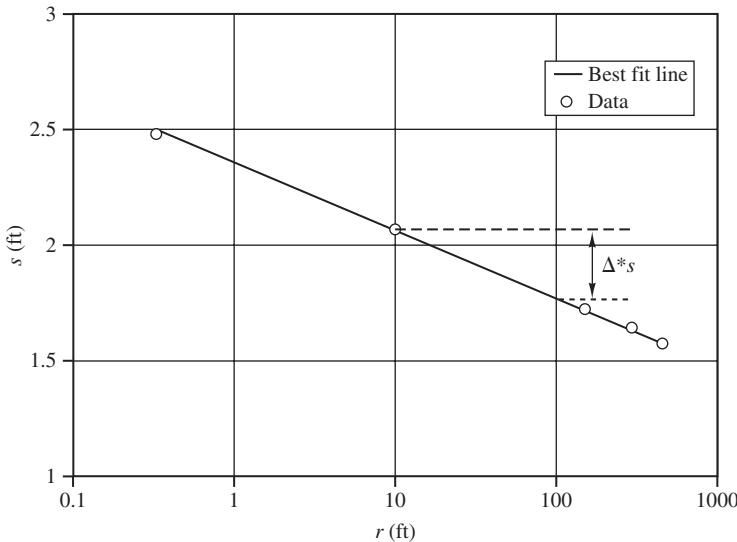


Figure 7.10 Analysis of steady pumping test data for confined aquifers

Example 7.10

Suppose in the case of Example 7.9, three additional observations are available. The drawdowns measured at $r = 10, 300,$ and 450 ft are, respectively, $s = 2.07, 1.64,$ and 1.58 ft. Determine the transmissivity for the aquifer.

Solution

A plot of s versus r is prepared on semilog paper as shown in Figure 7.10. From the best-fit line, we determine $\Delta^*s = 0.29$ ft. Then Equation 7.36 yields the transmissivity.

$$T = \frac{2.30 Q_w}{2 \pi (\Delta^*s)} = \frac{2.30 (400)}{2 \pi (0.29)} = 505 \text{ ft}^2/\text{hr}$$

7.4.2 Equilibrium Test in Unconfined Aquifers

The aquifer permeability coefficient, K , in unconfined aquifers can be effectively measured in the field by well-pumping tests. In addition to the pumped well, the pumping test requires two observation wells which penetrate the aquifer. The observation wells are located at two arbitrary distances r_1 and r_2 from the pumped well, as schematically represented in Figure 7.11. After pumping the well at a constant discharge Q_w for a long period, the water levels in the observation wells, h_1 and h_2 , will reach final equilibrium values. The equilibrium water levels in the observation wells are measured to calculate the aquifer’s permeability coefficient.

For unconfined aquifers, the coefficient can be calculated by integrating Equation 7.13 between the limits of the two observations wells to obtain

$$K = \frac{Q_w}{\pi(h_2^2 - h_1^2)} \ln\left(\frac{r_2}{r_1}\right) \tag{7.37}$$

To determine the coefficient of permeability when multiple observation wells are available, we rewrite Equation 7.37 in terms of common logarithms as

$$K = \frac{2.30 Q_w \Delta(\log r)}{\pi \Delta h^2} \tag{7.38}$$

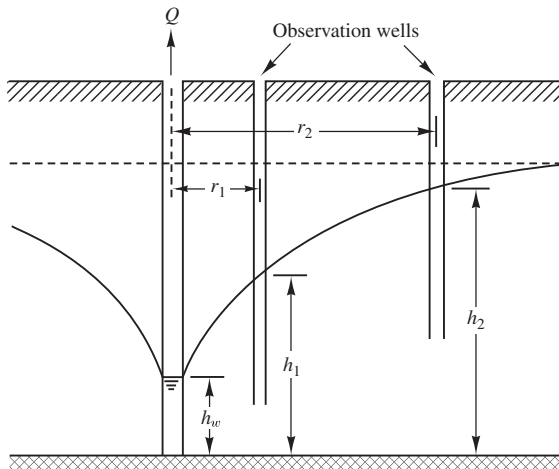


Figure 7.11 Field determination of permeability coefficient in unconfined aquifers

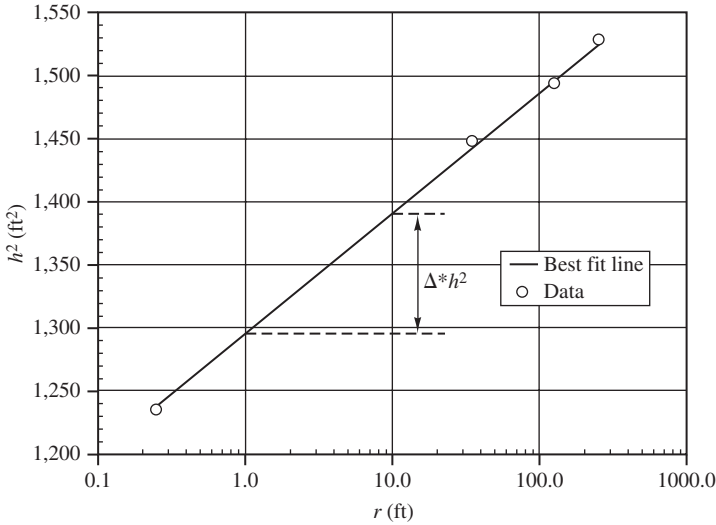


Figure 7.12 Analysis of steady pumping test data for unconfined aquifers

where $\Delta(\log r) = (\log r_2 - \log r_1)$ and $\Delta h^2 = h_2^2 - h_1^2$. When more than two observation wells are available, a plot of h^2 versus r on semilog paper (with h^2 on the linear scale and r on the log scale) produces a straight line. In reality, all the points will not fall exactly on a straight line. We usually draw (estimate) a best-fit straight line, as shown in Figure 7.12, and use the slope of the line to find K . Note that for any log cycle of r , $\Delta(\log r) = 1.0$. Therefore, defining $\Delta^*h^2 =$ increase in h^2 per log cycle of r yields

$$K = \frac{2.30 Q_w}{\pi(\Delta^*h^2)} \tag{7.39}$$

Example 7.11

A well 20 cm in diameter completely penetrates the undisturbed water table of a 30.0 m deep unconfined aquifer. After a long period of pumping at the constant rate of 0.1 m³/sec, the drawdown at distances of 20 m and 50 m from the well are observed to be 4.0 m and 2.5 m, respectively. Determine the coefficient of permeability of the aquifer. What is the drawdown at the pumped well?

Solution

In reference to Figure 7.11, the conditions given are as follows: $Q = 0.1$ m³/sec, $r_1 = 20$ m, $r_2 = 50$ m; also, $h_1 = 30.0$ m - 4.0 m = 26.0 m and $h_2 = 30.0$ m - 2.5 m = 27.5 m. Substituting these values into Equation 7.37, we have

$$K = \frac{0.1}{\pi(27.5^2 - 26.0^2)} \ln\left(\frac{50}{20}\right) = 3.63 \times 10^{-4} \text{ m/sec}$$

The drawdown at the pumped well can be calculated by using the same equation with the calculated value of the coefficient of permeability and the well diameter. At the well, $r = r_w = 0.1$ m and $h = h_w$. Therefore,

$$3.63 \times 10^{-4} = \frac{Q}{\pi(h_1^2 - h_w^2)} \ln\left(\frac{r_1}{r_w}\right) = \frac{0.1}{\pi(26^2 - h_w^2)} \ln\left(\frac{20}{0.1}\right)$$

from which we have

$$h_w = 14.5 \text{ m}$$

The drawdown at the well is $(30 - 14.5) = 15.5 \text{ m}$.

Example 7.12

A field test is conducted in an unconfined aquifer by pumping a constant discharge of $1,300 \text{ ft}^3/\text{hr}$ from a 6-inch well penetrating the aquifer. The undisturbed aquifer thickness is 40 ft. The drawdowns measured at steady state at various locations are tabulated in the first two columns of the following table. Determine the coefficient of permeability.

r (ft)	s (ft)	$h = 40 - s$ (ft)	h^2 (ft)
0.25	4.85	35.15	1,236
35.00	1.95	38.05	1,448
125.00	1.35	38.65	1,494
254.00	0.90	39.10	1,529

Solution

We first calculate the values of h^2 as shown in the preceding table. Then we prepare a plot of h^2 versus r on semilog paper as displayed in Figure 7.12. From the best-fit line we obtain $\Delta^*h^2 = 95 \text{ ft}^2$. Then, by using Equation 7.39,

$$K = \frac{2.30 Q_w}{\pi(\Delta^*h^2)} = \frac{2.30(1,300)}{\pi(95)} = 10.0 \text{ ft/hr}$$

7.4.3 Nonequilibrium Test

Field data collected under steady-state conditions can be used to determine the aquifer permeability or transmissivity, as discussed in the previous sections. The storage coefficient can be determined only if unsteady-state drawdown data are available. As mentioned previously, the procedures for determining aquifer characteristics basically fit the field data to the analytical solutions developed for groundwater flow. Unfortunately, analytical solutions are not available for unsteady flow in unconfined aquifers. Therefore, the procedures discussed herein are limited to confined aquifers. However, for practical purposes, the same procedures can be applied to unconfined aquifers if the drawdowns are very small compared to the thickness of the aquifer. Although various procedures are available to analyze unsteady pumping test data, those based on the Jacobs solution are adopted here because of their simplicity.

The Jacobs solution, previously introduced as Equation 7.23, can be rewritten in terms of common logarithms as

$$s = \frac{2.30 Q_w}{4 \pi T} \left[\log \frac{2.25 T t}{r^2 S} \right] \quad (7.40)$$

or

$$s = \frac{2.30 Q_w}{4 \pi T} \log \frac{2.25 T}{r^2 S} + \frac{2.30 Q_w}{4 \pi T} \log t \quad (7.41)$$

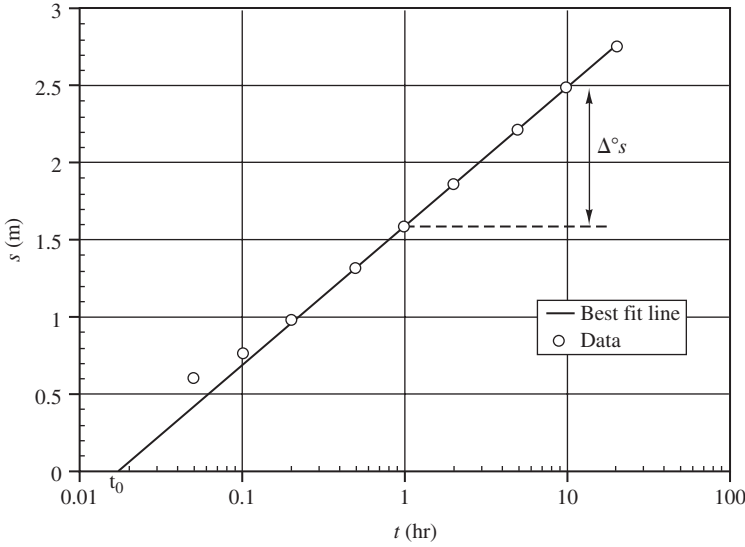


Figure 7.13 Analysis of unsteady pumping test data from a single observation well

This equation implies that a plot of s versus t on semilog paper (with s on the linear axis and t on the log axis) should give a straight line as shown in Figure 7.13. The slope of this line ($\Delta s / \Delta \log t$) is equal to $(2.30Q_w / 4\pi T)$. Therefore

$$T = \frac{2.30 Q_w \Delta \log t}{4 \pi \Delta s}$$

For any log cycle of t , $\Delta \log t = 1.0$. Thus

$$T = \frac{2.30 Q_w}{4 \pi (\Delta^\circ s)} \tag{7.42}$$

where $\Delta^\circ s =$ increase in s per log cycle of t . Also, we can show from (Equation 7.40) that

$$S = \frac{2.25 T t_o}{r^2} \tag{7.43}$$

where $r =$ distance between the pumped well and the observation well, and $t_o =$ the value of the time when the fitted straight line intersects the horizontal axis as shown in Figure 7.13. Equation 7.43 is obtained by simply setting $t = t_o$ for $s = 0$ in Equation 7.40.

When the Jacobs solution was initially introduced as Equation 7.23, it was noted that it was valid only for small values of u (or large values of t). Therefore, the duration of the pumping test should be long enough for the observed drawdowns to satisfy Equation 7.40 for the straight-line analysis to be applicable. Otherwise, the data points will not form a straight line. Sometimes when we plot the data points, we see that the points for larger t values form a straight line whereas the points for smaller t values do not fall on that line. In that event, we simply disregard the small t points as long as we have enough data points to obtain a straight line. The interpretation is that the data points for small t values do not satisfy the Jacob solution, so they will not be used in the analysis.

When drawdown data from more than one observation well are available, the procedure described above can be applied to each well individually. Then we can use the averages of the T

and S values obtained from different wells. Alternatively, we can use all the data at once. For this purpose, Equation 7.40 is rewritten as

$$s = \frac{2.30 Q_w}{4 \pi T} \left[\log \frac{2.25 T / (r^2/t)}{S} \right] \tag{7.44}$$

or

$$s = \frac{2.30 Q_w}{4 \pi T} \log \frac{2.25 T}{S} - \frac{2.30 Q_w}{4 \pi T} \log \frac{r^2}{t} \tag{7.45}$$

As implied by Equation 7.45, a plot of s versus r^2/t on a semilog graph paper (s on the linear axis and r^2/t on the log axis) forms a straight line as shown in Figure 7.14. The slope of this straight line is

$$\frac{\Delta s}{\Delta \log(r^2/t)} = - \frac{2.30 Q_w}{4 \pi T}$$

Solving for T ,

$$T = - \frac{2.30 Q_w}{4 \pi} \frac{\Delta \log(r^2/t)}{\Delta s}$$

For any log cycle of r^2/t , we have $\Delta \log (r^2/t) = 1.0$. Then, defining Δ^+s as the drop in s per log cycle of r^2/t , we obtain

$$T = \frac{2.30 Q_w}{4 \pi (\Delta^+s)} \tag{7.46}$$

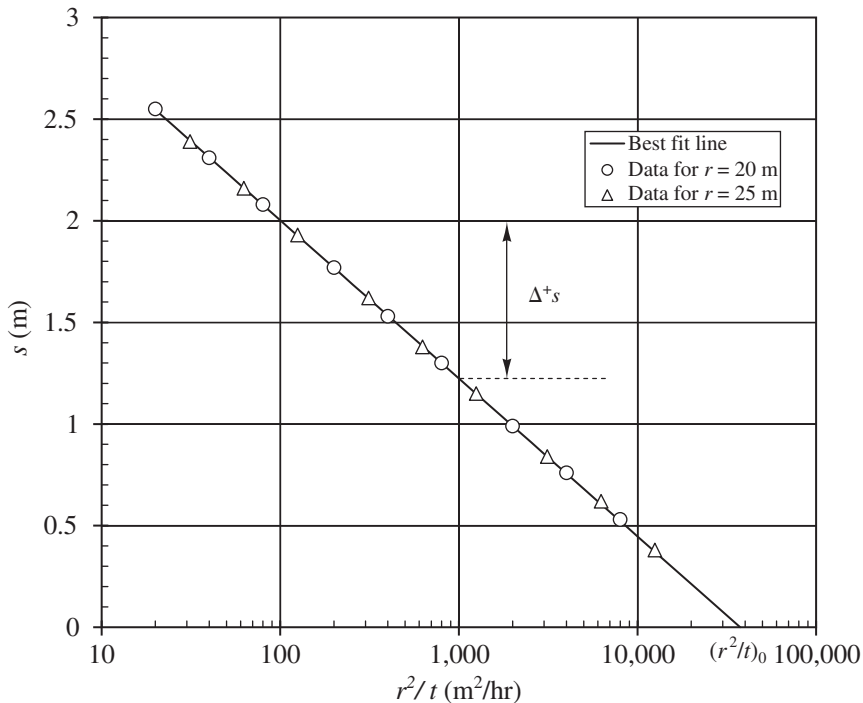


Figure 7.14 Analysis of unsteady pumping test data from multiple observation wells

Also, defining $(r^2/t)_o$ as the value of r^2/t where the straight line intersects the horizontal axis,

$$S = \frac{2.25 T}{(r^2/t)_o} \tag{7.47}$$

Example 7.13

A pumping test was conducted in a confined aquifer using a constant pump discharge of 8.5 m³/hr. The drawdowns are measured at an observation well located 20 m from the pumped well and are shown in the following table. Determine the aquifer transmissivity and storage coefficient.

<i>t</i> (hr)	0.05	0.10	0.20	0.50	1.00	2.00	5.00	10.0	20.0
<i>s</i> (m)	0.60	0.76	0.98	1.32	1.58	1.86	2.21	2.49	2.76

Solution

The observed data are plotted and the best-fit line is drawn as shown in Figure 7.13. From the best-fit line, we obtain $\Delta^{\circ}s = 0.90$ m. Therefore, Equation 7.42 yields

$$T = \frac{2.30 Q_w}{4 \pi (\Delta^{\circ}s)} = \frac{2.30(8.5)}{4\pi(0.90)} = 1.73 \text{ m}^2/\text{hr}$$

Also from Figure 7.13, we obtain $t_o = 0.017$ hr. From Equation 7.43, we then obtain

$$S = \frac{2.25 T t_o}{r^2} = \frac{2.25(1.73)(0.0170)}{(20)^2} = 1.65 \times 10^{-4}$$

Example 7.14

A pumping test was conducted in a confined aquifer using a constant pump discharge of 8.5 m³/hr. The drawdowns measured at two observation wells located 20 m and 25 m from the pumped well are listed in the first three columns of the following table. Analyzing all the available data at once, determine the aquifer transmissivity and storage coefficient.

<i>t</i> (hr)	At <i>r</i> = 20 m		At <i>r</i> = 25 m	
	<i>s</i> (m)	<i>s</i> (m)	<i>r</i> ² / <i>t</i> (m ² /hr)	<i>r</i> ² / <i>t</i> (m ² /hr)
0.05	0.53	0.38	8,000	12,500
0.10	0.76	0.62	4,000	6,250
0.20	0.99	0.84	2,000	3,125
0.50	1.30	1.15	800	1,250
1.00	1.53	1.38	400	625
2.00	1.77	1.62	200	312.5
5.00	2.08	1.93	80	125
10.00	2.31	2.16	40	62.5
20.00	2.55	2.39	20	31.25

Solution

The r^2/t values are first calculated for both observation wells as listed in the last two columns of the table above. Then all the available data are plotted as shown in Figure 7.14 and a best-fit line is

drawn. From this line we obtain $\Delta^+s = 0.78$ m and $(r^2/t)_o = 37,500$ m²/hr. Solving Equations 7.46 and 7.47 yields

$$T = \frac{2.3 Q_w}{4 \pi (\Delta^+s)} = \frac{2.3(8.5)}{4\pi(0.78)} = 1.99 \text{ m}^2/\text{hr}$$

$$S = \frac{2.25 T}{(r^2/t)_o} = \frac{2.25(1.99)}{37,500} = 1.19 \times 10^{-4}$$

7.5 Aquifer Boundaries

Previous discussions on well hydraulics have assumed the impacted aquifers to be uniform (homogeneous and isotropic) and infinite in extent. This resulted in radially symmetrical drawdown patterns. Oftentimes, these drawdown patterns are impacted by aquifer boundaries such as nearby impermeable strata (no flow boundary) and water bodies such as lakes and rivers (constant head boundary). If the aquifer boundary is located within the radius of influence of the pumping well, then the shape of the drawdown curve may be significantly modified; this, in turn, affects the discharge rate as predicted by the radial flow equations.

The solution of aquifer boundary problems can often be simplified by applying the *method of images*. Hydraulic image wells are imaginary sources or sinks, with the same strength (i.e., flow rate) as the original well, placed on the opposite side of the boundary to represent the effect of the boundary. Figure 7.15 depicts the effect of a fully penetrating impermeable boundary on a well located a short distance from it. Note that the impermeable boundary produces a greater drawdown and upsets the radially symmetric pattern. Figure 7.16 portrays the application of the method of images by placing an imaginary well of the same strength (Q) at the same distance from the boundary but on the opposite side. Thus, the original boundary condition is hydraulically replaced by the two-well system in the hypothetically uniform aquifer of infinite extent.

Computations to determine the drawdown curve for wells impacted by fully penetrating boundaries depend on the aquifer type. For confined aquifers, the two wells (i.e., the real well and the image well) depicted in Figure 7.16 hydraulically affect each other in such a way that the

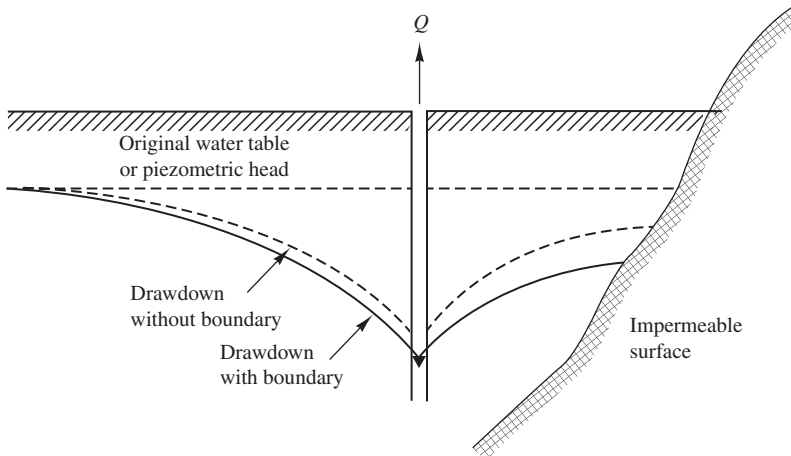


Figure 7.15 Pumping well near a fully penetrating impermeable boundary

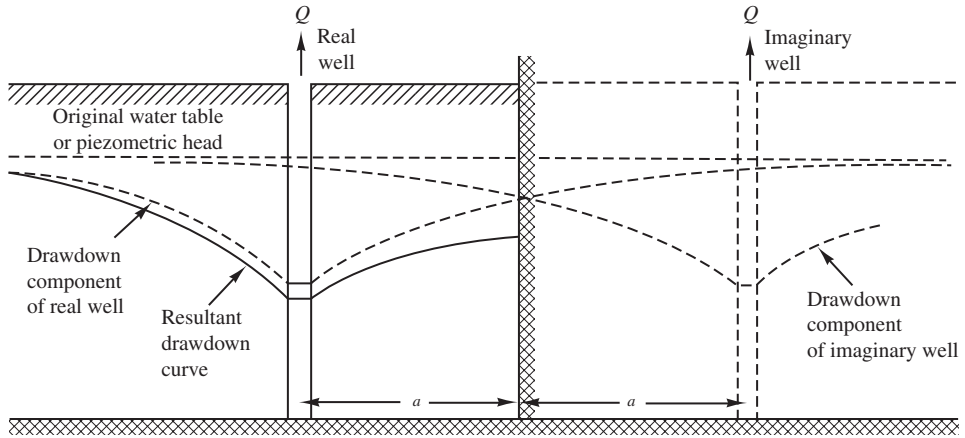


Figure 7.16 Equivalent hydraulic system with imaginary well

drawdown curves can be linearly superimposed. Thus, using the *principle of superposition*, the resultant drawdown curve is found by adding the drawdown curves of the two wells together. Also note that the imaginary well and the real well offset one another at the boundary line creating a horizontal hydraulic gradient ($dh/dr = 0$); as a result, there is no flow across the boundary. Note that unconfined aquifers are not linear in drawdowns but linear in differences in h^2 . Various procedures are available to analyze unconfined aquifer boundaries, but we will concentrate on those for confined flow because of their simplicity.

The presence of a lake, river, or other large body of water in the vicinity of a well increases flow to the well. The effect of a fully penetrating water body on the drawdown is exactly opposite to that of a fully penetrating impermeable boundary. The resulting drawdown, as depicted in Figure 7.17, is less than normal, but the symmetric pattern is still upset. Instead of an imaginary pumping well, the equivalent hydraulic system (Figure 7.18) involves an imaginary *recharge well* placed at an equal distance across the boundary. The recharge well infuses water at a discharge rate Q into the aquifer under positive pressure.

The drawdown curve in confined aquifers resulting from fully penetrating water bodies is obtained by linearly superimposing the drawdown component of the real well and the drawdown

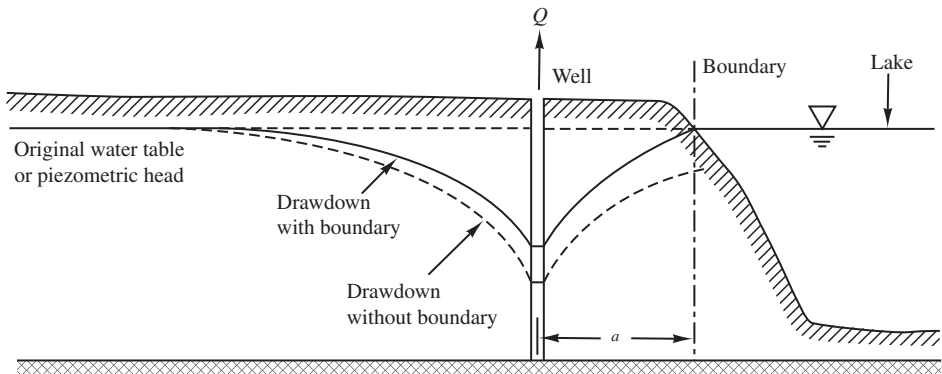


Figure 7.17 Pumping well near a fully penetrating perennial water body

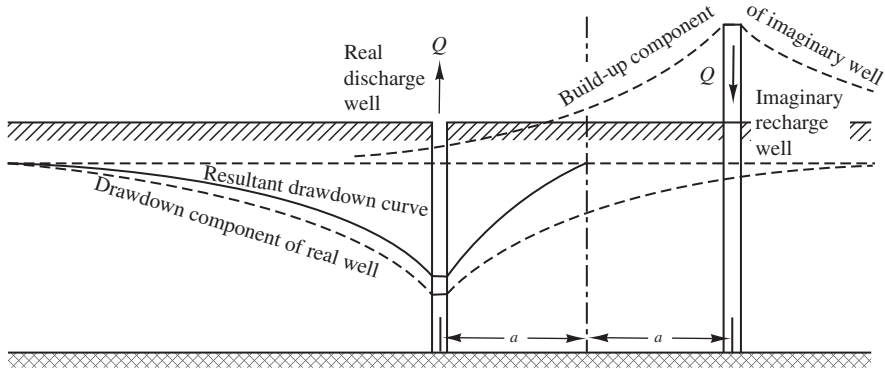


Figure 7.18 Equivalent hydraulic system with imaginary recharge well

component of the image (recharge) well that replaces the water boundary, as shown in Figure 7.18. The resulting drawdown curve of the real well intersects the boundary line at the elevation of the free water surface. The steeper hydraulic gradient causes more water to flow across the boundary line. Thus, much of the water from the well is obtained from the water body rather than from the aquifer.

Replacing aquifer boundaries by an equivalent hydraulic system of image wells can be applied to a variety of groundwater boundary conditions. Figure 7.19(a) shows a discharge well pumping water from an aquifer bounded on two sides by impermeable boundaries. Three imaginary wells are required to provide the equivalent hydraulic system. The imaginary discharge well, I_1 and I_2 , provide the required absence of flow across the boundaries; a third imaginary well, I_3 , is necessary to balance the system. The three image wells all have the same discharge rate (Q) as the real well. All wells are placed at equal distances from the physical boundaries.

Figure 7.19 (b) represents the situation of a discharge well pumping water from an aquifer bounded on one side by an impermeable boundary and on the other side by a perennial stream. The equivalent hydraulic system includes three image wells of equal strength Q . I_1 is a recharge well, and I_2 is a discharge well. A third image well, I_3 , a recharge well like I_1 , is necessary to balance the system.

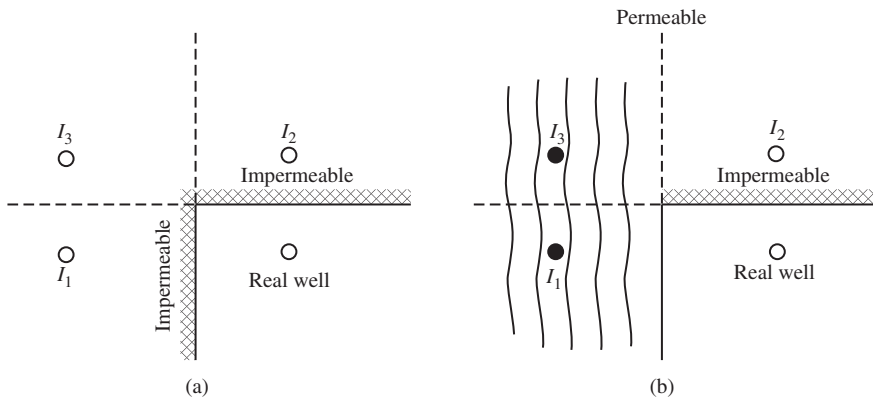


Figure 7.19 Pumping wells near multiple boundaries

Example 7.15

A factory located near a river bank needs to extract a discharge of 1.55 cfs from a confined aquifer ($K = 0.0004$ ft/sec, and $b = 66$ ft). Local authorities require that, at a distance of 100 ft from the bank, the groundwater table may not be lower than one-third of a foot from the normal surface elevation of the river. Determine the minimum distance from the bank that the well can be located.

Solution

As shown in Figures 7.17 and 7.20, a pumping well located near a perennial water body may be hydraulically replaced by an imaginary recharge well of the same strength and at the same distance but on the opposite side of the boundary. The resulting drawdown curve may be obtained by superimposing the real and the imaginary wells and assuming an infinite extent of the aquifer without the boundary. However, this will only work for confined aquifers and fully penetrating water bodies.

Assume P to be the distance between the pumping well and the river bank. Then the drawdown (100 ft from the bank or $P - 100$ ft from the real well and $P + 100$ ft from the imaginary recharge well) is the sum of the piezometric surfaces produced by the pumping well and the recharge well. Using Equation 7.11 for the real well yields

$$s_{real} = s_{ob} + \frac{Q}{2\pi T} \ln\left(\frac{r_{ob}}{r}\right) = s_{P(real)} + \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P}{P - 100}\right)$$

where the observation well is placed at the river and noting that $T = Kb = (0.0004)(66) = 0.0264$ ft²/sec. The same equation for the image well yields

$$s_{image} = s_{ob} + \frac{Q}{2\pi T} \ln\left(\frac{r_{ob}}{r}\right) = s_{P(image)} + \frac{-1.55}{2\pi(0.0264)} \ln\left(\frac{P}{P + 100}\right)$$

where a negative sign is used for the recharge pump rate. Based on the principle of superposition, we will add drawdowns to obtain 0.333 ft at the location of interest (noting that $s_{P(real)} + s_{P(image)} = 0$; see Figure 7.18). Therefore,

$$s = s_{real} + s_{image} = \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P}{P - 100}\right) - \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P}{P + 100}\right)$$

$$s = 0.333 = \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P + 100}{P - 100}\right);$$

$$\ln\left(\frac{P + 100}{P - 100}\right) = 0.0356$$

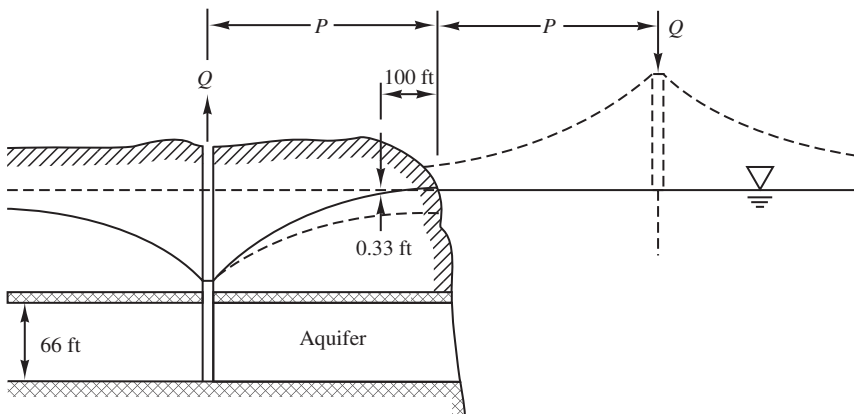


Figure 7.20

or

$$P + 100 = 1.036(P - 100); \text{ which yields } P = 5,660 \text{ ft}$$

Alternate Solution

To determine the drawdown created by two wells, we will apply Equation 7.12a:

$$s = s_{ob} + \frac{Q_A}{2\pi T} \ln\left(\frac{r_{Ao}}{r_A}\right) + \frac{Q_B}{2\pi T} \ln\left(\frac{r_{Bo}}{r_B}\right)$$

The observation well in this case is at the boundary where $s_{ob} = 0.0$ ft. In addition, the drawdown at the point of interest is $s = 0.333$ ft, based on a local mandate. The other variables are as follows: Q_A is the discharge well pumping rate, and Q_B is the recharge (image) well pumping rate. Making appropriate substitutions yields the minimum distance P :

$$0.333 = 0.0 + \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P}{P - 100}\right) + \frac{(-1.55)}{2\pi(0.0264)} \ln\left(\frac{P}{P + 100}\right)$$

$$0.333 = \frac{1.55}{2\pi(0.0264)} \ln\left(\frac{P + 100}{P - 100}\right)$$

or

$$P + 100 = 1.036(P - 100); \text{ which yields } P = 5,660 \text{ ft}$$

Example 7.16

An otherwise infinitely large confined aquifer is bounded on one side by an impermeable barrier. The aquifer transmissivity is 5,000 ft²/day, and the storage coefficient is 0.0002. A well placed 200 ft from the boundary is pumped at a constant rate of 20,000 ft³/day. Determine the drawdown at half the distance between the well and the boundary three days after pumping has begun.

Solution

An image well, having the same characteristics as the real well, is placed on the other side of the boundary. The distance between the image well and the boundary is 200 ft. The distance from the image well to the point where the drawdown is to be calculated is 300 ft. We will calculate the drawdowns for the real well and the image well separately using Equations 7.20 and 7.21 and add them using the principle of superposition.

For the real well,

$$u = \frac{r^2 S}{4 T t} = \frac{(100)^2(0.0002)}{4(5,000)(3)} = 3.33 \times 10^{-5}$$

From Table 7.3, $W(u) = 9.741$. Then

$$s = \frac{Q_w}{4\pi T} W(u) = \frac{20,000}{4\pi(5,000)}(9.741) = 3.10 \text{ ft}$$

For the image well,

$$u = \frac{r^2 S}{4 T t} = \frac{(300)^2(0.0002)}{4(5,000)(3)} = 3.00 \times 10^{-4}$$

From Table 7.3, $W(u) = 7.535$. Then

$$s = \frac{Q_w}{4\pi T} W(u) = \frac{20,000}{4\pi(5,000)}(7.535) = 2.40 \text{ ft}$$

The total drawdown is $3.10 + 2.40 = 5.50$ ft.

Example 7.17

Suppose the boundary in Example 7.16 is a river rather than a barrier. Calculate the drawdown and explain why it is different than the drawdown obtained in Example 7.16.

Solution

In this case, the image well is a recharge well. The drawdowns caused by the real well and the image well are calculated in the same manner as in Example 7.16. However, in this case, the drawdown caused by the image well is negative (-2.40 ft.). So the resulting drawdown is $3.10 - 2.40 = 0.70$ ft. This is smaller than the drawdown obtained in Example 7.16. The reason is that the river produces a constant head boundary that prevents the piezometric surface from dropping as much between the river and the well. Alternatively, if you look at the discharge well and the image well in the equivalent hydraulic system (Figure 7.18), the recharge well is building up the piezometric surface so it will not drop as much.

7.6 Surface Investigations of Groundwater

Locating subsurface groundwater using information obtained on the earth's surface is an ancient art known as *divination*. Some people still practice the "art" by using Y-shaped sticks or metal rods known as *divining rods*. (The senior author's grandmother was a practicing "water witch.") At the turn of the twentieth century, geophysical methods had been developed for petroleum and mineral explorations. A few of these methods have proven useful for locating and analyzing groundwater. Information obtained by surface methods can only provide indirect indications of groundwater. Correct interpretation of the data usually requires supplemental information that can only be obtained by subsurface investigations. Two of the most commonly used geophysical methods are described below.

7.6.1 The Electrical Resistivity Method

Electrical resistivity of rock formations varies over a wide range. The measured resistivity of a particular formation depends on a variety of physical and chemical factors such as the material and the structure of the formation; the size, shape, and distribution of pores; and the water content. The distinguishable difference between a dry rock formation and the same formation with large amounts of water filling the interstitial spaces is the key to detecting groundwater.

The procedure involves measuring the electrical potential difference between two electrodes placed in the ground surface. When an electric current is applied through two other electrodes outside but along the same line with the potential electrodes, an electrical field penetrates the ground and forms a current flow network, as shown in Figure 7.21.

A deeper penetration of the electrical field will occur by increasing the spacing between the electrodes. The variation in apparent resistivity is plotted against the electrode spacing from which a smooth curve can be drawn.*

The interpretation of such a resistivity-spacing curve in terms of subsurface formations is frequently complex and often difficult. Nevertheless, with certain supplemental data from subsurface investigations to substantiate the surface measurement, correct predictions of the existence and depth of groundwater aquifers can often be made.

* H. M. Mooney and W. W. Wetzel, *The Potentials About a Point Electrode and Apparent Resistivity Curves for a Two-, Three-, and Four-Layered Earth* (Minneapolis: University of Minnesota Press, 1956).

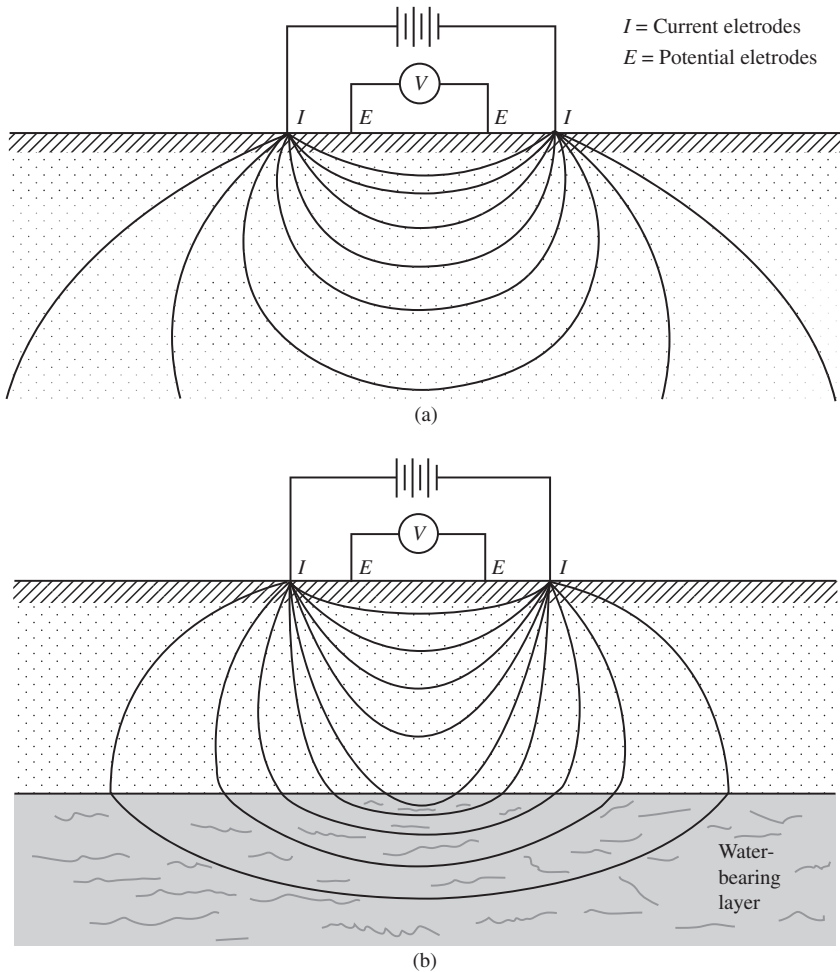


Figure 7.21 Arrangement of electrodes for resistivity determination: (a) current lines in a homogeneous medium and (b) current lines distorted by the presence of a water-bearing layer

7.6.2 Seismic Wave Propagation Methods

By shocking the earth's surface with a small explosion or the impact of a heavy weight, the time required for the sound or shock wave to reach a certain point at a known distance away can be measured. Seismic waves propagate through a transfer medium the same way that light waves do. They may be refracted or reflected at the interface of any two materials of different elastic properties. A change in propagation velocity takes place at the interface. The wave speed is greatest in solid igneous rock and the least in unconsolidated formations. The water content in a particular formation will significantly alter the wave speed in the formation. Because the seismic wave is traveling several hundred meters deep into the ground, subsurface information may be obtained by placing several seismometers at various distances from the shock point along the same line. The wave travel time is plotted against the distance, as shown in Figure 7.22. A sudden change in the slope of the time-spacing curve can be interpreted to determine the depth of a groundwater table.

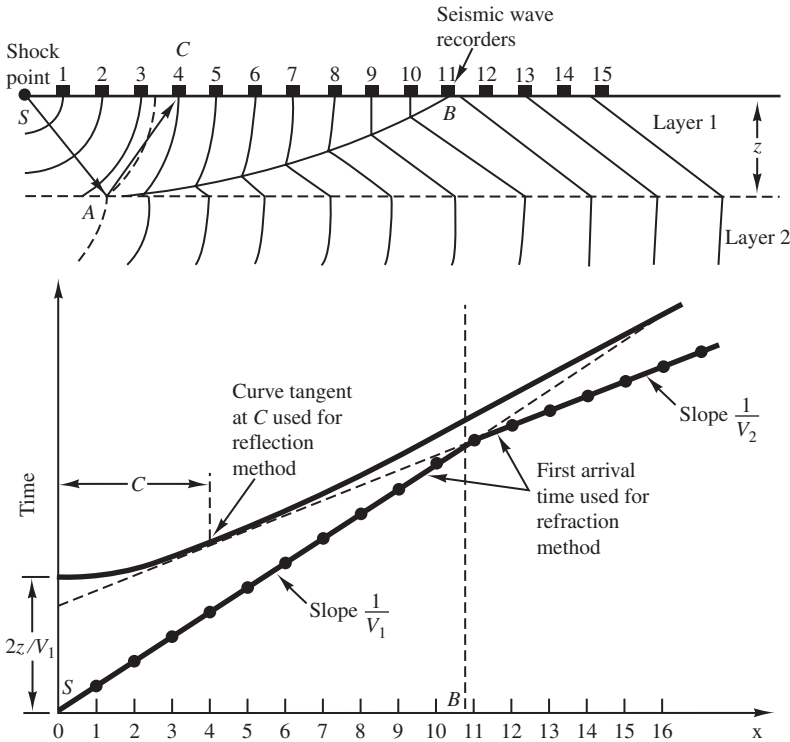


Figure 7.22 Propagation of seismic waves in a two-layer medium. Waves propagate at speed V_1 in the upper (dry) layer and at a higher speed (V_2) in the lower (water-bearing) layer. For points to the lower right of line AB , the wave refracted at the point A through the lower layer (2) and reflected back to the surface arrives sooner than the wave propagating directly through the upper layer (1).

7.7 Seawater Intrusion in Coastal Areas

Along the coastline, freshwater coastal aquifers are in contact with seawater. Under natural conditions, fresh groundwater is discharged into the sea under the water table, as shown in Figure 7.23. However, with an increased demand for groundwater in certain coastal regions, the seaward flow of fresh groundwater has been reduced or even reversed, causing saltwater from the sea to enter and penetrate freshwater aquifers. This phenomenon is commonly known as *seawater intrusion*. If the seawater travels far enough inland and enters water supply wells, the groundwater supply becomes useless. Furthermore, once a coastal aquifer is contaminated by salt, it is very difficult to remove the salt from the formation, and the aquifer may be permanently damaged. Engineering prevention and control of seawater intrusion will be discussed in this section.

Overdrafting of coastal aquifers results in lowering the water table in unconfined aquifers or the piezometric surface in confined aquifers. The natural gradient originally sloping toward the sea is reduced or reversed. Because of the difference in densities of saltwater and freshwater, an interface is formed when the two liquids are in contact. The shapes and movements of the interface are governed by the pressure balance of the saltwater on one side of the interface and freshwater on the other side.

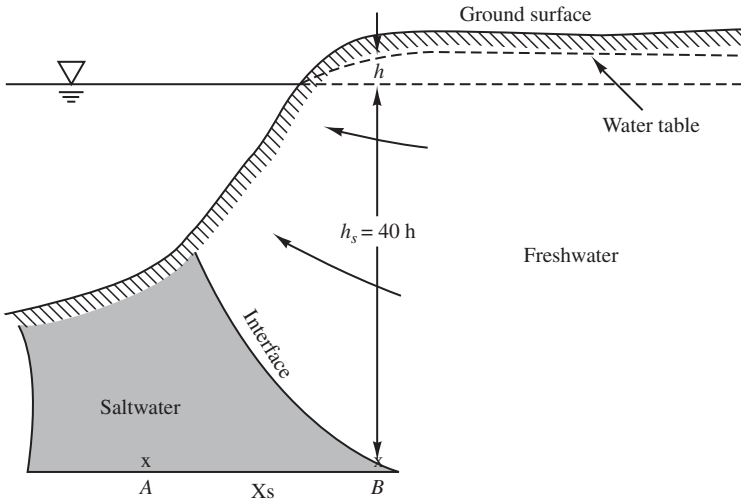


Figure 7.23 Schematic representation of freshwater and saltwater distribution in unconfined coastal aquifers

It has been found that the interface that occurs underground does not take place at sea level but at a depth below sea level that is approximately 40 times the height of the freshwater table above sea level, as shown in Figure 7.23. This distribution is caused by the equilibrium of hydrostatic pressure that exists between these two liquids, which have different densities.

Figure 7.23 displays a cross section of a coastal aquifer. The total hydrostatic pressure at point A and at depth h_s , below sea level is

$$P_A = \rho_s g h_s$$

where ρ_s is the density of the saltwater and g is the gravitational acceleration. Similarly, the hydrostatic pressure at point B inland, at the same depth as A, and on the interface is

$$P_B = \rho g h + \rho g h_s$$

where ρ is the density of freshwater. For a stationary interface, the pressure at A and B must be the same, and we may write

$$\rho_s g h_s = \rho g h + \rho g h_s \tag{7.48}$$

Solving (Equation 7.48) for h_s yields

$$h_s = \left[\frac{\rho}{\rho_s - \rho} \right] h \tag{7.49}$$

By taking $\rho_s = 1.025 \text{ g/cm}^3$ and $\rho = 1.000 \text{ g/cm}^3$, the above relationship yields

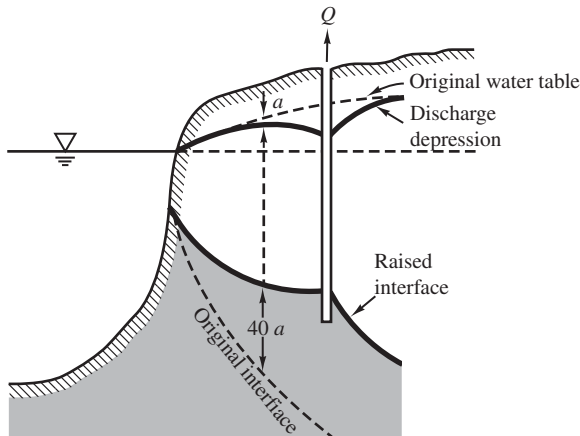
$$h_s = \left[\frac{1.000}{1.025 - 1.000} \right] h = 40h \tag{7.50}$$

This is commonly known as the *Ghyben-Herzberg relation*.

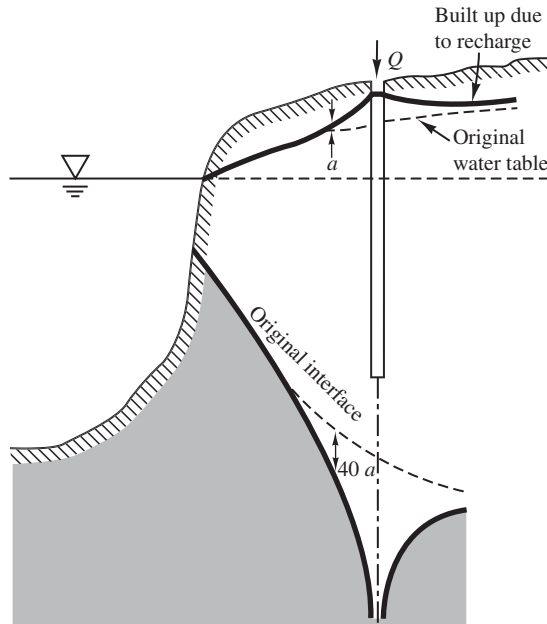
This relation shows that a small depression in the water table near the coastline caused by well pumping could cause a big rise in the interface. Similarly, a buildup of the water table

near the coastline caused by artificial recharge could drive the saltwater wedge deep into the ground, thus forcing it to move seaward. These phenomena are schematically demonstrated in Figure 7.24.

Obviously, artificial recharge of an overdrafted coastal aquifer is an effective method of controlling seawater intrusion. By proper management, artificial recharge of the aquifer can eliminate the overdraft and maintain the proper water table level and gradient.



(a)



(b)

Figure 7.24 Seawater intrusion under the influence of (a) a discharge well and (b) a recharge well

In addition to the artificial recharge, several other methods have been effectively applied for control of seawater intrusion. The most common methods are described as follows:

1. *Pumping Trough*: A pumping trough is a line of discharge wells situated along the coastline. By pumping the wells, a depression (trough line) is formed, as shown in Figure 7.25. Although saltwater is taken into the wells, a certain amount of freshwater in the aquifer is also removed. The freshwater motion is in the seaward direction toward the wells. This movement of fresh groundwater can stabilize the saltwater and freshwater interface.
2. *Pressure Ridge*: A pressure ridge is a series of recharge wells installed parallel to the coastline. Freshwater is pumped into the coastal aquifer to maintain a freshwater pressure ridge along the coastline to control the saltwater intrusion. The pressure ridge must be large enough to repel the seawater and must be located far enough inland; otherwise, the saltwater inland of the ridge will be driven farther inland, as demonstrated in Figure 7.26. Inevitably, a small amount of freshwater will be wasted to the sea; the

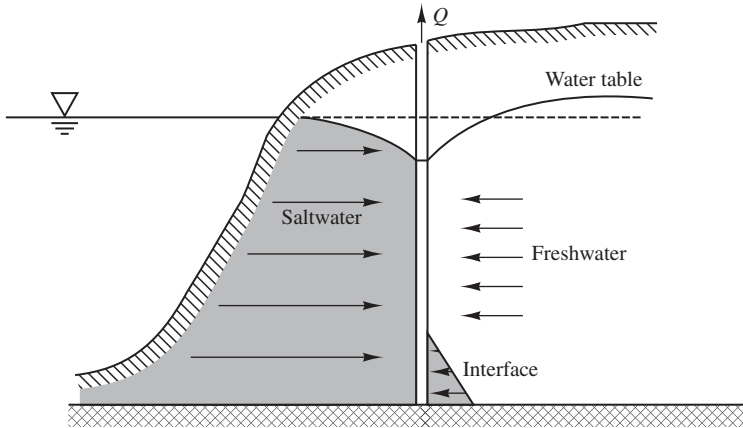


Figure 7.25 Control of seawater by a pumping trough

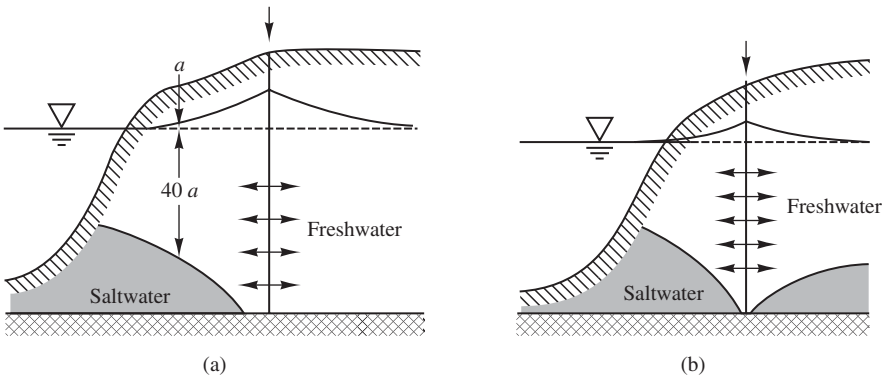


Figure 7.26 Control of seawater by a pressure ridge: (a) proper recharge arrangement and (b) improper recharge arrangement

remainder that moves landward can be used to supply part of the pumping draft. Reclaimed wastewater may be used to meet part of the recharge need. This method's advantage is that it does not deplete the usable groundwater capacity, but the disadvantages of high initial and operating costs and the need for supplemental freshwater often make small-scale operations impractical.

3. **Subsurface Barriers:** Subsurface barriers may be built along the coastline to reduce the coastal aquifer permeability. In relatively shallow-layer aquifers, subsurface dikes may be constructed with sheet piling, bentonite, or even concrete materials. An impermeable subsurface barrier may be formed by injecting flowable materials such as slurry, silicone gel, or cement grout into the aquifers through a line of holes. Subsurface barriers are best suited for certain locations such as narrow, alluvial canyons connected to large inland aquifers. Although the initial cost of installing the barriers may be very high, there is almost no operation or maintenance expense.

7.8 Seepage Through Dam Foundations

In the generic sense, *seepage* is defined as the movement of water through soil. With regards to engineering, seepage is often undesirable and needs to be analyzed and controlled. For example, dams constructed to store water in a reservoir may continuously lose some of their water through seepage. Impermeable concrete dams constructed on an alluvial foundation may lose water through foundation seepage, whereas earth dams may lose water through the dam's embankment. Water movement caused by seepage is governed by Darcy's law in the same manner as groundwater flow. Seepage flow can be analyzed fairly quickly and accurately by applying the *flow net* technique.

A flow net is a graphical representation of flow patterns expressed by a family of *streamlines* and their corresponding *equipotential lines*. Streamlines are always drawn in the direction of the flow, and they can be used to divide the flow field into a certain number of flow channels, each carrying the same discharge. Equipotential lines connect all points in the flow field that have equal velocity potential (or equal head). In a properly constructed flow net, the drop in head (Δh) between adjacent equipotential lines typically remains constant. The two sets of lines always meet at right angles and form an orthogonal net in the flow field. Figure 7.27 represents a portion of a flow net formed by a set of streamlines and equipotential lines.

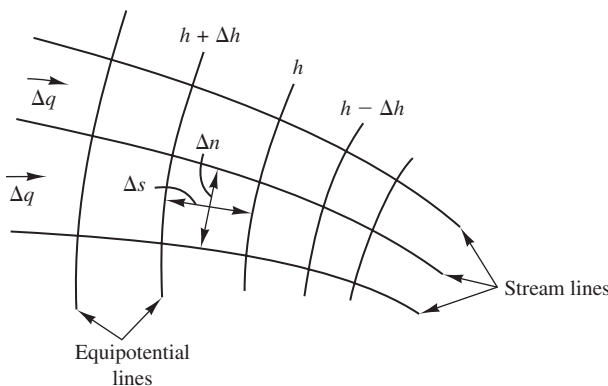


Figure 7.27 A flow net segment

Flow nets are usually constructed so that the distance between a pair of adjacent streamlines (Δn) and the distance between a pair of equipotential lines (Δs) are equal in every cell. The square-net concept, as depicted in Figure 7.27, results in the equation

$$\Delta n = \Delta s$$

for each cell in the flow net. Because distances will be important in the following equations, flow nets are constructed on scale drawings.

The velocity through the cell with the dimensions Δn and Δs noted in Figure 7.27 can be found using Darcy's law for steady flow through a porous media. Thus,

$$V = K \frac{dh}{ds} = K \frac{\Delta h}{\Delta s}$$

The volumetric flow rate through the corresponding flow channel per unit width of the dam is

$$\Delta q = AV = KA \frac{\Delta h}{\Delta s} \quad (7.51)$$

where A is the flow area through the flow channel. Because the flow is per unit width of the dam, the flow area in the cell of interest (Figure 7.27) is

$$A = \Delta n \quad (7.52)$$

Substituting this area into (Equation 7.51) and noting $\Delta n = \Delta s$, we have

$$\Delta q = K(\Delta n) \frac{\Delta h}{\Delta s} = K\Delta h \quad (7.53)$$

Because Δh is a constant value of head drop (loss) between any two adjacent equipotential lines in Figure 7.27, we may write

$$\Delta h = \frac{H}{n} \quad (7.54)$$

H is the difference between the upstream reservoir water level and the tailwater level (as depicted in Figure 7.28), and n is the number of cells, or equipotential drops, in each flow channel of the flow net. Now Equation 7.53 may be rewritten as

$$\Delta q = K \frac{H}{n} \quad (7.55)$$

If there are m different flow channels in the flow net, then the total seepage flow rate per unit width of the dam is

$$q = m\Delta q = K \left(\frac{m}{n} \right) H \quad (7.56)$$

Therefore, the total seepage beneath the dam can be calculated by merely determining the m/n ratio from a graphically constructed flow net and determining the coefficient of permeability of the underlying soil.

Flow nets that depict seepage under a concrete dam, with and without a *cutoff wall*, are provided in Figure 7.28. A cutoff wall is a thin layer of impermeable material or sheet piling partially penetrating the aquifer under the dam. From the two depictions shown in Figure 7.28, it is obvious that the cutoff wall alters the seepage pattern by lengthening flow paths, thus increasing the resistance to flow. Hence, cutoff walls effectively decrease the amount of seepage and can significantly reduce the total uplift force on the base of the dam if they are strategically placed. (See Chapter 8.)

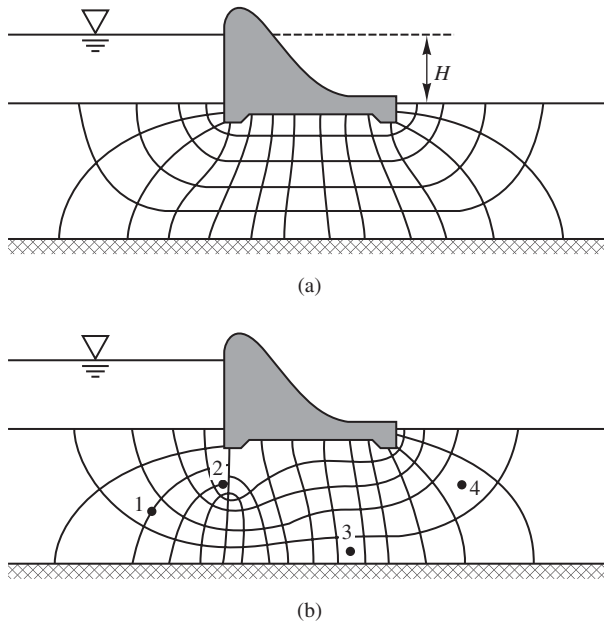


Figure 7.28 Seepage through a previous dam foundation: (a) without a cutoff wall and (b) with a cutoff wall

The construction of flow nets by hand sketching is as much an art as a science. Sophisticated techniques exist for drafting complex flow nets. However, for our purposes, simple sketches will be relied on to help gain an understanding of the fundamental principles. Some useful guidelines in the construction of simple flow nets include:

- Construct a scale drawing denoting all impervious boundaries (i.e., impermeable or low-permeability natural strata or artificial boundaries such as sheet piling).
- Sketch two to four streamlines entering and leaving the pervious boundaries at right angles and flowing essentially parallel to the impervious boundaries.
- Equipotential lines are drawn perpendicular to the flow lines, forming a flow net that is made up of cells that are essentially square (equal median lines).
- In regions of uniform flow, the cells are of equal size. In diverging flow, the cells increase in size; in converging flow, they decrease in size.

It may be helpful to read these rules again while referring to the flow nets depicted in Figure 7.28.

Example 7.18

A concrete gravity dam built on an alluvial channel bed, as shown in Figure 7.28, stores water at a depth of 50 m. If the coefficient of permeability is $K = 2.14$ m/day, estimate the seepage per meter width of dam (a) without a cutoff wall and (b) with a cutoff wall.

Solution

From Equation 7.56,

$$q = K \left(\frac{m}{n} \right) H$$

- (a) For the dam without the cutoff wall [Figure 7.28 (a)], we count the number of flow channels ($m = 5$) and the number of cells along a flow channel (i.e., the number of equipotential drops) ($n = 13$). Applying Equation 7.56 yields

$$q = (2.14) \left(\frac{5}{13} \right) (50) = 41.2 \text{ m}^3/\text{day per meter of dam width}$$

- (b) For the dam with the cutoff wall [Figure 7.28 (b)], the number of flow channels ($m = 5$) and the number of equipotential drops ($n = 16$) results in

$$q = (2.14) \left(\frac{5}{16} \right) (50) = 33.4 \text{ m}^3/\text{day per meter dam width}$$

Flow nets also allow you to determine energy heads, position heads, pressure heads, and seepage velocities at any location under the dam. Refer to Problem 7.8.2 for additional information about the valuable information that can be gleaned from flow nets.

7.9 Seepage Through Earth Dams

Because an earth dam is built with pervious materials, it is of particular engineering concern. Excessive seepage through an earth dam may produce *sloughing* (slippage) of the downstream embankment and *pipng* (removal of soil by exiting seepage water). Either scenario may lead to a complete failure of the dam. Therefore, seepage analysis should be performed for every earth dam by applying the method of flow nets.

Seepage through an earth dam can be treated as flow through unconfined porous media. The upper surface of the flow, known as the *surface of saturation* or *phreatic surface*, is at atmospheric pressure. The typical shape of a *phreatic line* in a homogeneous earth dam is shown in Figure 7.29. The phreatic line is a streamline whose intersection with the equipotential lines is equally spaced vertically by the amount of $\Delta h = H/n$, where H is the total head available and n is the number of equipotential drops counted in a graphic flow net. This line, which provides the upper boundary of the flow net, must be initially located by trial. An empirical rule for locating the phreatic line was suggested by Casagrande* and is shown in Figure 7.29.

Section *DF* on the lower part of the downstream dam face must be protected against soil pipng, which may eventually lead to dam failure. The seepage water may be removed permanently

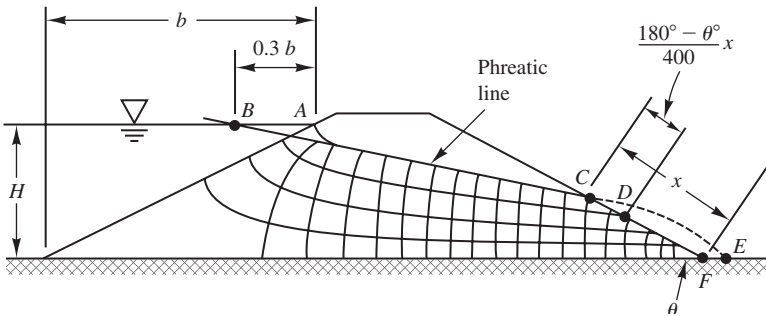


Figure 7.29 Seepage flow net through a homogeneous earth dam. (A large portion of the phreatic line *AD* can be approximated by the parabola *BCE* with *F* as the focus and passing through the point *B*. Point *A* on the upstream face of the dam is the intersection of the water surface with the dam, Point *D* is the downstream transition where the seepage is exposed to the atmosphere.)

* A. Casagrande, "Seepage through dams," *J. New Eng. Water Works Assoc.*, 51 (1937): 139.

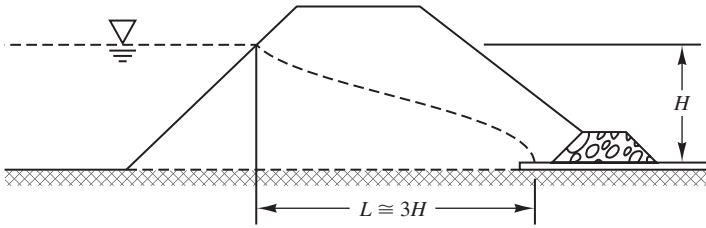


Figure 7.30 Drainage blanket in an earth dam

from the downstream surface by a properly designed drainage system. For a nonstratified, homogeneous earth dam, a narrow, longitudinal drain will effectively intercept all the water seeping through the embankment. Figure 7.30 schematically shows the dimensions of a typical earth dam drainage blanket.

The total discharge through an earth dam can be determined by using a graphically constructed flow net, as discussed in Section 7.8. Equation 7.56 gives the seepage discharge per unit width of the dam:

$$q = K \left(\frac{m}{n} \right) H$$

where m is the number of flow channels and n is the number of equipotential drops in the flow net.

PROBLEMS (SECTION 7.1)

- 7.1.1. A cylindrical sand sample is removed from a 6-inch-diameter well (borehole). When the sand from the 12-inch long sample is dried and poured into a graduated cylinder filled with water, it displaces 3,450 ml of water. Estimate the porosity of the sand sample.
- 7.1.2. Equation 7.1 is the fundamental equation for determining porosity of an aquifer material. However, porosity can also be determined from $\alpha = 1 - (\rho_b/\rho_s)$ where ρ_b is the bulk density of the sample and ρ_s is the density of the solids in the sample. Using the definition of density, derive this expression from Equation 7.1.
- 7.1.3. A limestone sample is oven dried and weighed (157 N). It is then saturated with kerosene and weighed again (179 N). Finally, it is submerged in kerosene. The displaced kerosene is collected and weighed (65.0 N). Determine the porosity of the limestone.
- 7.1.4. Referring to Example 7.1, answer the following questions.
 - (a) Why is the permeability measurement from the unconsolidated sample not likely to match the field (in situ) permeability?
 - (b) If an undisturbed (in situ) sand sample was obtained (instead of the unconsolidated sample) from a test well using a thin tube sampler, would there still be reasons to doubt the accuracy of the laboratory determined permeability? Why or why not?
 - (c) Estimate the time it takes a molecule of water to move from one end of the sample to the other end.
 - (d) Estimate the actual velocity of a water molecule as it travels through the permeameter.
- 7.1.5. A permeameter experiment is being set up to determine the permeability of some very fine sands. The cylindrical samples to be tested will be 30 cm long and have a diameter of 10 cm. To improve the accuracy of the experiment, it is proposed that about 50 ml of water be collected before the

permeability calculations are performed. If a head difference of 40 cm is used, how much time will each experiment take in minutes? If tracer tests are to be performed later, how much time (on average in hours) will it take a conservative tracer to pass through the sample?

- 7.1.6. The experiment in Example 7.1 was done at room temperature (20°C). Apply Equation 7.3 and determine the discharge in 5 min if the same experiment had been performed at a temperature of 5°C. What is the permeability coefficient of the sample at this temperature?
- 7.1.7. A chemical spill occurs at a manufacturing plant, and groundwater sampling reveals that the conservative pollutant is now in the groundwater directly under the location of the spill. If the aquifer is composed of sand and gravel, how many hours will it take the pollutant to travel 82 feet to the property boundary, assuming the aquifer is homogeneous and the water table has a slope of 2 percent.
- 7.1.8. A high-water table exists behind a retaining wall (Figure P7.1.8). Weep holes are drilled at the bottom of the retaining wall every 3.0 m to alleviate the hydrostatic pressure. If the permeability is 10^{-5} m/s, determine the seepage (in cm^3/sec) through each weep hole.

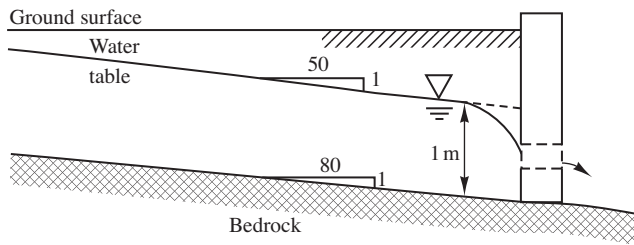


Figure P7.1.8

- 7.1.9. At the Rocky Mountain Arsenal near Denver, Colorado, many impermeable rock outcroppings exist. Groundwater elevations in the vicinity of one of these confined areas is shown in the water table map in Figure P7.1.9. Given a typical cross section, determine the flow rate through the confined opening. Assume a hydraulic conductivity of 0.00058 ft/sec.

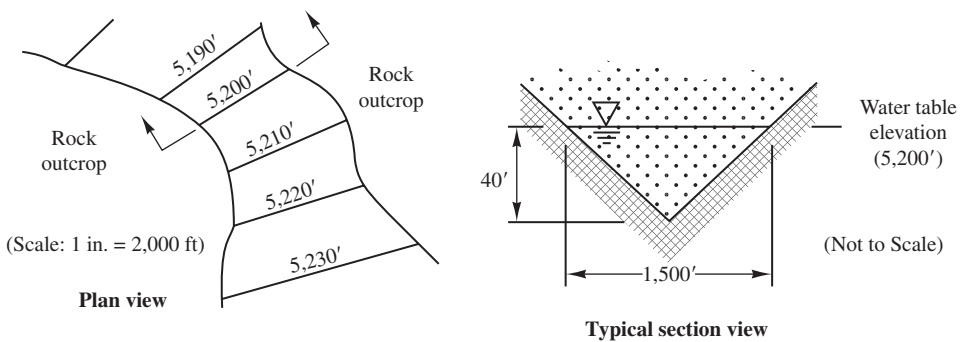


Figure P7.1.9

- 7.1.10. An unconfined aquifer ($K = 12.2$ m/day) is separated from an underlying confined aquifer ($K = 15.2$ m/day) by a semipervious layer ($K = 0.305$ m/day) that is 1.5 meters thick as shown in Figure P7.1.10. Explain why there is flow from the unconfined aquifer to the confined aquifer. Also, determine the flow rate per square meter moving through the semipervious layer.

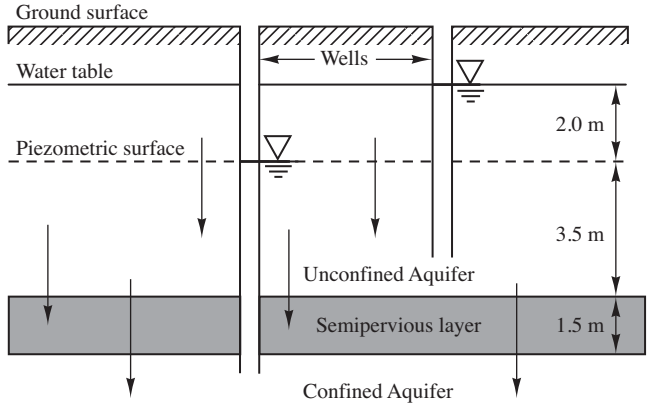


Figure P7.1.10

(SECTION 7.2)

- 7.2.1. Sketch the area described in Equation 7.5 through which the radial flow passes. Also verify Equations 7.6 and 7.14 by integrating Equations 7.5 and 7.13 using the boundary conditions given.
- 7.2.2. A comment is made in the paragraph that follows Equation 7.14 suggesting the selection of the radius of influence (r_0) is somewhat arbitrary (i.e., the discharge is not overly sensitive to this variable). Suppose the radius of influence for a 20-cm-diameter well is approximately 400 m (plus or minus 50 meters). Determine the percent error in discharge that results from this 12.5-percent range in radius of influence. Assume the 50-m-thick, unconfined aquifer (course sand) and the drawdown at the well is 10 m.
- 7.2.3. A 16-inch radius well draws water from a confined aquifer at the rate of 1,570 gpm. The confined aquifer is 100 feet thick with a piezometric surface (before pumping) that is, 350 feet above the bottom of the confined aquifer. The drawdown at the well is 100 feet. Determine the radius of influence if the aquifer's hydraulic conductivity is 4.01×10^{-4} ft/sec.
- 7.2.4. A discharge well is located near the center of a circular island, as depicted in Figure P7.2.4. The island is approximately 800 m in diameter, and the 30-cm-diameter well is pumped at the rate of

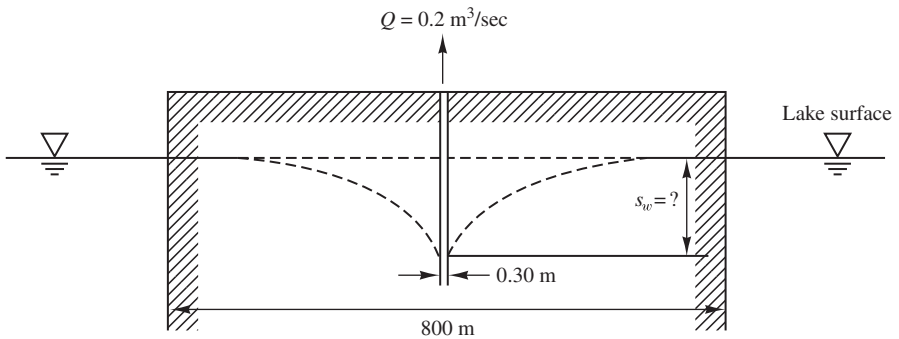


Figure P7.2.4

0.200 m³/sec. The underlying unconfined aquifer is made up of coarse sand and is 40 m deep. Estimate the drawdown at the well.

- 7.2.5. A confined aquifer (Figure P7.2.5) has a thickness of 10.0 m and a permeability of 1.30×10^{-4} m/sec. When a 30-cm-diameter well is pumped at the rate of 30 m³/hr, the drawdown in the piezometric surface is 15 m at the well. Determine the drawdown in the piezometric surface at a distance of 30 m from the well.

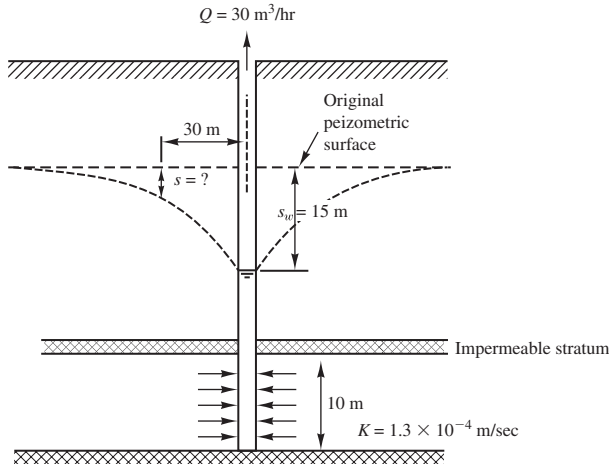


Figure P7.2.5

- 7.2.6. A pharmaceutical industry owns a discharge well that completely penetrates a confined aquifer and is pumped at a constant rate of 2,150 m³/day. The aquifer transmissivity is known to be 880 m²/day. The steady-state drawdown measured at a distance 80 m from the pumped well is 2.72 m. What is the drawdown impact of the industrial well on a domestic well that is 100 m away? If a second industrial well is installed with the same pumping rate and at a distance of 140 m from the domestic well, what is the drawdown impact from both wells in operation. (*Hint:* For confined aquifer flow, the drawdown produced by multiple wells at a particular location is sum of the drawdowns produced by the individual wells.)
- 7.2.7. Two discharge wells (1 and 2) penetrating a confined aquifer are pumped at constant rates of 2,950 m³/day and 852 m³/day, respectively. The steady-state drawdown measured at observation well 1 is 1.02 m (50 m from well 1 and 90 m from well 2). The steady-state drawdown measured at observation well 2 is 0.242 m (180 m from well 1 and 440 m from well 2). Determine the transmissivity of the aquifer.
- 7.2.8. An industrial manufacturer owns a 12-in-diameter well that completely penetrates a 130-ft-thick, unconfined aquifer with a coefficient of permeability of 0.00055 ft/sec. The well-pumping rate is 3.5 cfs, and the radius of influence is 500 feet. Another industry plans to move into an adjacent property and install a well with the same characteristics. If the new well is 250 feet away, what is the impact in additional drawdown at the existing well? (*Hint:* For unconfined aquifers, the drawdown equations are not linear. However, an estimate of the impact of the two identical pumped wells can be made by using the radius of influence as an observation well with zero drawdown when both wells are in operation.)
- 7.2.9. A fully penetrating well in a 33-m-thick confined aquifer pumps at a constant rate of 2,000 m³/day for a long time. In two observation wells located 20 m and 160 m from the well, the difference in

hydraulic head is 2.0 m. If the undisturbed head is 250 m and the head in the furthest observation well is 249 m, calculate the radius of influence of the well.

- 7.2.10.** After pumping a 12-in.-diameter well for a long time, equilibrium conditions have been reached in a confined aquifer. The coefficient of permeability (based on lab tests) is 7.55×10^{-4} ft/sec. At a location of about 90 ft from the well, two closely spaced observation wells indicate the slope of the piezometric surface to be 0.0222. If the aquifer thickness is 50 ft, what is the well discharge in gallons per minute?
- 7.2.11.** A well is to be installed in an unconfined sandy aquifer to lower the water table for a large construction project. The steady-state drawdown must be at least 1.5 m within a distance of 30 m from the well and 3.0 m within a distance of 3.0 m from the well. The hydraulic conductivity of the sand is 1.00×10^{-4} m/sec. A relatively impermeable clay with a hydraulic conductivity 1.00×10^{-10} m/sec forms the base of the aquifer. The depth of the water in the aquifer above the clay layer before pumping is 8.2 m. Calculate the required discharge from the well to meet the design conditions. Assume a radius of influence of 150 m.
- 7.2.12.** A field pump test is performed to determine the transmissivity of a high-capacity aquifer. The confined aquifer thickness is 20 feet and has a porosity of 0.26. Unfortunately, the flow data for the well test was misplaced by the field crew. Can you still estimate the transmissivity given the following field data?
- Two observation wells are located 500 ft and 1,000 ft from the pumped well.
 - The piezometric surface of the two wells at equilibrium differs by 42.8 feet.
 - The time it takes a conservative tracer to move from the outer observation well to the inner observation well is 49.5 hours.

(SECTION 7.3)

- 7.3.1.** A discharge well that fully penetrates a confined aquifer is pumped at a constant rate of $300 \text{ m}^3/\text{hr}$. The aquifer transmissivity is $25.0 \text{ m}^2/\text{hr}$, and the storage coefficient is 0.00025. Determine the drawdown at a distance 100 m from the well at 10, 50, and 100 hours after pumping has begun.
- 7.3.2.** A discharge well penetrating a confined aquifer will be pumped at a constant rate of $50,000 \text{ ft}^3/\text{day}$. How long can the well be pumped at this rate if the drawdown at a point 300 ft from the well cannot exceed 3.66 ft? The aquifer transmissivity is $12,000 \text{ ft}^2/\text{day}$, and the storage coefficient is 0.0003.
- 7.3.3.** An industrial discharge well that fully penetrates a confined aquifer is pumped at a constant rate of $300 \text{ m}^3/\text{hr}$, but only on rare occasions when needed. The industry would like to install a second well on their property to increase their withdrawal capacity. However, the drawdown of the aquifer at a nearby residential well (a distance of 100 m from the first well) cannot exceed 10.5 m. If both wells are turned on at the same time and run for four days, and the second well is pumped at the rate of $200 \text{ m}^3/\text{hr}$, how close can the second well be placed to the residential well before the drawdown limitation is violated? The aquifer transmissivity is $25.0 \text{ m}^2/\text{hr}$, and the storage coefficient is 0.00025.
- 7.3.4.** Two wells penetrating a confined aquifer are 600 ft apart. Each well will be pumped at $40,000 \text{ ft}^3/\text{day}$. However, the pumping from the second well will start a day and a half after the first one. Determine the drawdown at a point halfway between the two wells three days after the pumping has begun in the first well. The aquifer transmissivity is $10,000 \text{ ft}^2/\text{day}$, and the storage coefficient is 0.0005.
- 7.3.5.** A discharge well penetrating a confined aquifer will be pumped at a rate of $800 \text{ m}^3/\text{hr}$ for two days, and then the pumping rate will be reduced to $500 \text{ m}^3/\text{hr}$. The aquifer transmissivity is $40 \text{ m}^2/\text{hr}$, and the storage coefficient is 0.00025. Determine the drawdown at a location 50 m from the pumped well three days after pumping has started.

- 7.3.6. An unconfined aquifer has an early time storage coefficient of 0.0005, a storage coefficient of 0.10, and a permeability of 5 ft/day. The aquifer thickness is 500 ft. A well penetrating this aquifer will be pumped at a rate of 10,000 ft³/day. Determine the drawdown 50 ft from the well two days after pumping has begun.

(SECTION 7.4)

- 7.4.1. A confined aquifer, 30 m thick, has a piezometric surface 75 m above the top of the confined aquifer. A 40-cm-diameter well draws water from the aquifer at the rate of 0.1 m³/sec. If the drawdown at the well is 30 m and the drawdown in an observation well 50 m away is 10 m, what is the transmissivity and the radius of influence?
- 7.4.2. Sketch the free surface of the water table produced by a well located in the middle of a circular island as depicted in Figure P7.4.2. If the well yields 7.5 gpm (gal/min), what is the coefficient of permeability? Also determine the drawdown at a distance of 150 ft from the well.

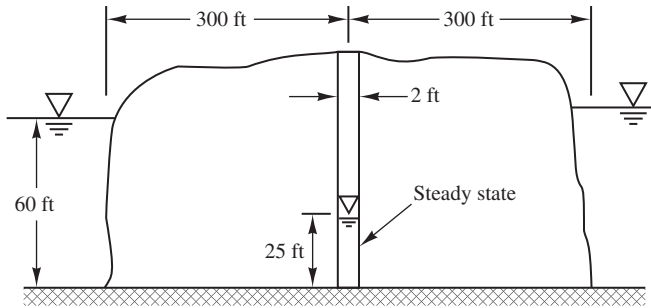


Figure P7.4.2

- 7.4.3. A field test is conducted in a confined aquifer by pumping a constant discharge of 14.9 m³/hr from a 20-cm-diameter well. After an approximate steady state is reached, a drawdown of 0.98 m is measured in the pumped well. Also measured are drawdowns of 0.78 m, 0.63 m, 0.59 m, and 0.56 m, respectively, at 3.0 m, 45.0 m, 90.0 m, and 150 m from the pumped well. Determine the transmissivity of this aquifer. Also determine how far away from the well you must be before the drawdown becomes less than 0.5 m.
- 7.4.4. A field test is conducted in an unconfined aquifer by pumping 1,300 ft³/hr from an 8-inch-diameter well penetrating the aquifer. The undisturbed aquifer thickness is 46 ft. The drawdowns, measured at steady state at various locations, are tabulated below. Determine the coefficient of permeability and how far away from the well you must be before the drawdown becomes less than 2.5 ft.

r (ft)	0.33	40	125	350
s (ft)	6.05	4.05	3.57	3.05

- 7.4.5. Explain why a nonequilibrium test rather than an equilibrium test is needed to determine the storage coefficient of an aquifer.
- 7.4.6. Derive Equations 7.43 and 7.47.
- 7.4.7. A pumping test was conducted in a confined aquifer using a constant pump discharge of 6.00 m³/hr. The drawdowns measured at an observation well located 22.0 m from the pumped well are shown in the following table. Determine the aquifer transmissivity, the storage coefficient, and the drawdown after 50 hours.

Time (hr)	0.05	0.10	0.20	0.50	1.0	2.0	5.0	10.0	20.0
<i>s</i> (m)	0.47	0.60	0.74	0.93	1.09	1.25	1.46	1.61	1.77

7.4.8. The following drawdown information was collected from an observation well 120 ft away from a 16-in.-diameter well that is pumped at a uniform rate of 1.25 cfs. Determine the permeability and the storage coefficient of the 90-ft-thick unconfined aquifer. (*Hint:* Disregard small time drawdowns if they do not form a straight line on the semilog plot.)

Time (hr)	1	2	3	4	5	6	8	10	12	18	24
<i>s</i> (ft)	0.4	1.0	1.7	2.3	2.9	3.3	4.7	5.7	6.6	8.5	9.6

(SECTION 7.5)

7.5.1. Locate imaginary wells on Figure P7.5.1 that will replace the actual boundaries with an equivalent hydraulic system.

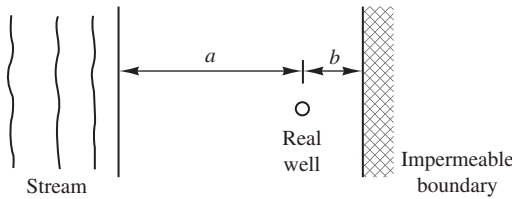


Figure P7.5.1

- 7.5.2. A fully penetrating, 12-in. diameter well pumps groundwater from a 25-ft thick confined aquifer. An impermeable rock stratum is located in the aquifer 105 ft away. Will the impermeable boundary impact the drawdown curve of the well when equilibrium conditions are achieved if the pump rate is 20,000 gallons per day (gpd)? The aquifer permeability is 20 gpd/ft², and the drawdown at the well is 30 feet.
- 7.5.3. A confined aquifer has a thickness of 10.0 m and a transmissivity of 1.30×10^{-3} m²/sec. When a 30-cm-diameter well is pumped at the rate of 30 m³/hr, the drawdown of the piezometric surface at an observation well 30 m away is 9.60 m when it is not impacted by an aquifer boundary. If an identical well is installed in the same aquifer but located 30 m from a fully penetrating impermeable boundary, determine the drawdown at the boundary and at the well.
- 7.5.4. A confined aquifer has a thickness of 10.0 m and a transmissivity of 1.30×10^{-3} m²/sec. When a 30-cm-diameter well is pumped at the rate of 30 m³/hr, the drawdown in the piezometric surface at the well is 15.0 m when it is not impacted by an aquifer boundary. If an identical well is installed in the same aquifer, but located 60 m from a fully penetrating river, what is the drawdown at the well, the boundary, and halfway between the two points?
- 7.5.5. An industrial well taps into an 80-ft-thick confined aquifer that has a transmissivity of 0.0455 ft²/sec. The well is located 600 feet from a completely penetrating stream. A farmer's irrigation well is located halfway between the stream and the industrial well. What is the maximum flow rate that can be pumped from the industrial well to limit the drawdown impact on the irrigation well to 5 feet.
- 7.5.6. A discharge well that fully penetrates a confined aquifer is pumped at a constant rate of 300 m³/hr. The aquifer transmissivity is 25.0 m²/hr, and the storage coefficient is 0.00025. A fully penetrating stream is located 500 m away. Determine the drawdown at a location between the well and the stream (100 m from the well and 400 m from the stream) 50 hours after pumping has begun.

- 7.5.7. A confined aquifer has a thickness of 40.0 ft, a transmissivity of 250 ft²/hr, and a storage coefficient of 0.00023. A 16-in.-diameter well is extracting a flow rate of 10,600 ft³/hr from the aquifer. The well is located 330 ft away from a fully penetrating impermeable boundary. Determine the drawdown at the boundary after 50 hours of pumping. Also determine how long it would take for the drawdown at the boundary to reach 59.0 feet.

(SECTION 7.8)

- 7.8.1. As previously mentioned, constructing a good flow net is more art than science. Because different people would undoubtedly draw different flow nets, we may wonder about the accuracy of using it for seepage calculations. If carefully drawn, there is surprising consistency in seepage predictions from engineer to engineer. To lend greater credibility to this premise, perform the following exercise:
- With a pencil, sketch five more streamlines in Figure 7.28 (a) by bisecting the existing flow channels. Why is the result not a proper flow net? What additional step must be taken? After performing this additional step, recompute the seepage rate of Example 7.18. How do the answers compare?
 - Trace the outline of the dam and boundaries in Figure 7.28 (a). Draw your own flow net using two streamlines instead of four. Estimate the seepage of Example 7.18 using your new flow net. How do the answers compare?
- 7.8.2. Flow nets yield more information than just seepage rates. For example, the total energy head at any location in the flow net can be estimated using equipotential lines. Recall that equal head drops occur from the reservoir water level to the tailwater level in sequential equipotential lines. Given the total energy head at any point, the pressure head can be obtained by subtracting the position head. In addition, the magnitude and direction of the seepage velocity can be estimated at any flow net location using total head differences from adjacent equipotential lines and measured distances from the scale drawing. With this background, estimate the total energy head and the seepage velocity (magnitude and direction) at locations 1, 2, and 4 in Figure 7.28 (b). Assume the upstream water depth is 160 ft (which also acts as a scale), $K = 7.02$ ft/day, and the porosity is 0.40. The bottom of the reservoir is the datum from which total energy heads will be measured.
- 7.8.3. Sheetpiles are used to keep water out of a bridge pier construction site, as depicted in Figure P7.8.3 (drawn to scale). Determine the quantity of seepage (in m³/hr per unit length of sheetpile) that can be expected in order to design an appropriate pump to dewater the construction site. The appropriate seepage dimensions are $d = 3.5$ m, $b = 4.7$ m, and $z = 1.3$ m. The permeability is 0.195 m/day, and the porosity is 0.35. Also determine the approximate exit velocity of the water (in m/sec) next to the sheetpile. (*Note:* If this velocity is high enough, it may transport the soil with it and eventually undermine the sheetpile).

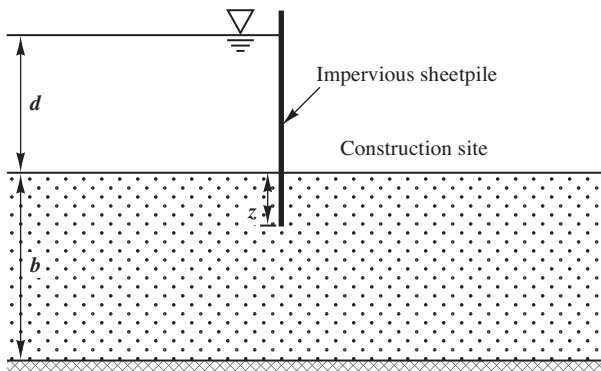


Figure P7.8.3

7.8.4. A concrete dam rests on an alluvial solid foundation as shown in the scale drawing of Figure P7.8.4. Estimate the seepage rate (in m^3/day) for the 100-m-long dam if the permeability is 4.45×10^{-7} m/sec and the upstream water depth is 20 m. Also estimate the energy head (using the dam bottom as the datum) and the seepage velocity immediately below the middle of the dam. The soil porosity is 0.45.

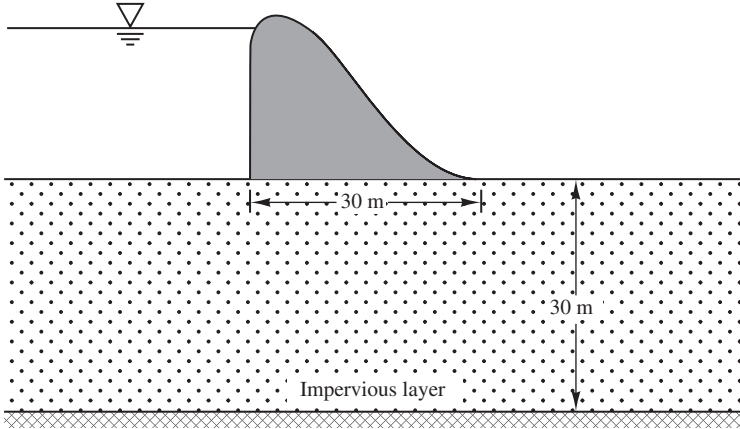


Figure P7.8.4

7.8.5. Place a cutoff wall at the heel of the dam depicted in Figure P7.8.4 much like Example 7.18. The cutoff wall should extend downward one-third of the distance to the impervious layer. Estimate the seepage per unit meter width of dam. Assume the permeability is 4.45×10^{-7} m/sec and the upstream water depth is 20 m. If the seepage per unit meter without the cutoff wall is 4.45×10^{-6} m^3/sec per meter, what is the percent reduction in seepage using the cutoff wall?

(SECTION 7.9)

- 7.9.1.** Determine the seepage in m^3/day through the homogeneous earth dam depicted in Figure 7.29. Also determine the seepage velocity when the water surfaces on the downstream embankment at point *D*. The upstream water depth is 7.0 m, the dam is 80 m long, and silt is the soil used to construct the dam. Assume Figure 7.29 is a scale drawing with the upstream depth of 7.0 m being the scale.
- 7.9.2.** Make a copy of Figure 7.30 and construct a flow net. Use the phreatic line given as the upper streamline with all other streamlines below it terminating in the drain. If the seepage rate is $0.005 \text{ m}^3/\text{day}$ per meter when the upstream water level is 4.24 m, what is the most likely soil type used to construct the dam?
- 7.9.3.** An earth dam, as schematically shown to scale in Figure P7.9.3, is constructed with a uniform material having a coefficient of permeability of 2.00×10^{-6} m/sec on a relatively impervious

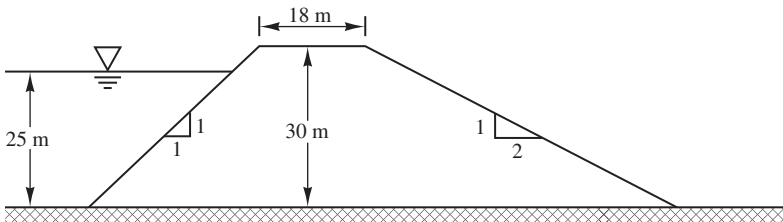


Figure P7.9.3

foundation. The dam is 30 m high, which can be used as the drawing scale. Compute the seepage rate in units of cubic meters per day per unit width of the dam. Assume the phreatic surface emerges on the downstream slope at a distance of $x = 30$ m, as defined in Figure 7.29.

- 7.9.4. Determine the seepage rate per unit width of the dam in Problem 7.9.3 if a drainage blanket (as depicted in Figure 7.30) extends 30 m back from the toe of the dam. Begin the flow net by sketching a phreatic line (uppermost flow line, as depicted in Figure 7.30) entering the drain 5 m from its upstream end; all other flow lines enter the drain between that point and the upstream end.
- 7.9.5. The earth dam depicted in Figure P7.9.5 backs up 30 ft of water. Using the flow net shown, determine the seepage rate (in ft^3/day) for the dam if it is 90 ft long. The permeability coefficient of the soil in the homogeneous dam 3.28×10^{-6} ft/sec with a porosity of 0.40. Also determine the seepage velocity when the water begins to surface on the downstream embankment at point D , as defined in Figure 7.29. The embankment angle on the downstream side is $\theta = 16^\circ$. Assume Figure P7.9.5 is a scale drawing with the upstream depth of 30 ft being the scale.

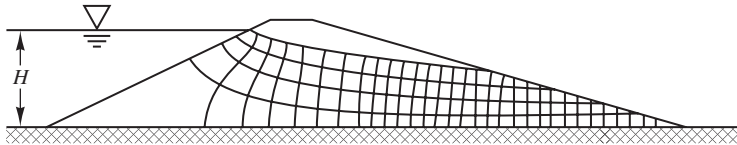


Figure P7.9.5

8



Hydraulic Structures

Water is more useful to people when it is properly controlled, conveyed, and contained. Hydraulic structures are designed and built to serve these purposes. Some of the most common hydraulic structures are pipes, pumps, open channels, wells, water-measuring devices, and stormwater-collection and stormwater-transport systems. These structures are covered in other chapters of this book. Dams, weirs, spillways, culverts, and stilling basins are also common, and they will be addressed in this chapter.

8.1 Functions of Hydraulic Structures

Any classification of hydraulic structures will inevitably be discretionary because many of them can be built to serve more than one purpose. In addition, a general classification of hydraulic structures based on use is not satisfactory because many identical structures may be used to serve completely different purposes. For example, a low head dam could be built across a channel as a device to measure discharge, or it could be built to raise the water level at the entrance to an irrigation canal to permit diversion of water into the canal. Instead of classifying various hydraulic structures into arbitrary categories, we have listed the common functions of hydraulic structures and the basic design criteria.

1. Storage structures are designed to hold water under hydrostatic conditions. A storage structure usually has a large capacity for a relatively small change in hydrostatic head (water elevation).

2. Conveyance structures are designed to transport water from one place to another. The design normally emphasizes delivery of a given discharge with a minimum consumption of energy.
3. Waterway and navigation structures are designed to support water transportation. Maintenance of a minimum water depth under various conditions is critical.
4. Coastline structures are constructed to protect beaches, inlets, harbors, and buildings. Wave action is a key consideration in the design of these structures.
5. Measurement or control structures are used to quantify the discharge in a particular conduit. Stable performance and a one-to-one relationship between the discharge and some indicator (usually elevation) are necessary.
6. Energy-conversion structures are designed to transform hydraulic energy into mechanical or electric energy (e.g., hydraulic turbine systems) or electrical or mechanical energy into hydraulic energy (e.g., hydraulic pumps). The design emphasis is on the system efficiency and the power consumed or produced.
7. Sediment- and fish-control structures are designed to direct or regulate the movement of the nonhydraulic elements in water. An understanding of the basic mechanisms and behaviors of the elements involved is an essential requirement for the design.
8. Energy-dissipation structures are used to control and disperse excess hydraulic energy to prevent channel erosion.
9. Collection structures are designed to gather and admit water to hydraulic systems. A typical example is a surface drainage inlet used to collect surface runoff and direct it into a stormwater conveyance system.

Obviously, detailed consideration of all these functions and their design criteria are beyond the scope of this book. Only the most commonly encountered hydraulic structures are discussed here to demonstrate how fundamental considerations are used in their design.

8.2 Dams: Functions and Classifications

A dam is a barrier structure placed across a watercourse to store water and modify normal stream flow. Dams vary in size from a few meters in height (farm pond dams) to massive structures more than 100 meters in height (large hydroelectric dams). Two of the largest dams in the United States are Hoover Dam (on the Colorado River and located on the border of Arizona and Nevada) and Grand Coulee Dam (on the Columbia River in Washington). Completed in 1936, Hoover Dam is 222 m high and 380 m long, and it can store $3.52 \times 10^{10} \text{ m}^3$ of water. Grand Coulee Dam, completed in 1942, is 168 m high and 1,592 m long, and it can store $1.17 \times 10^{10} \text{ m}^3$ of water. These two dams are dwarfed by the world's largest dams. The highest dam in the world is Rogun in Tajikistan; at 335 m high, it was completed in 1985. The largest dam in the world by reservoir size will arguably be the Three Gorges Dam in the Peoples Republic of China, scheduled for completion in 2011, which will store $3.93 \times 10^{10} \text{ m}^3$ of water.

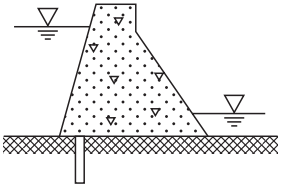
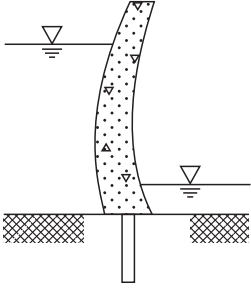
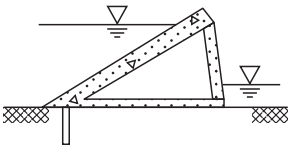
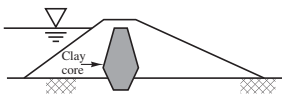
Dams fulfill many functions. Hoover and Grand Coulee dams provide an enormous amount of electricity to the western United States. However, like most large dams, they have multiple purposes. They also provide downstream flood control (see storage routing in Chapter 11), irrigation water to vast amounts of farm land, and recreational opportunities. Dams are also built to provide industrial water, cooling water (for power plants), and municipal water.

Locks and dams are built to support navigation on many large rivers. In the past, dams were built in conjunction with water wheels to supply the power for gristmills.

Dams can be classified in a several ways. It may prove useful to classify them according to how they achieve stability and what materials are used in their construction. One such classification scheme can be seen in Table 8.1. Four types of dams are noted: gravity, arch, buttress, and earth. A typical gravity dam is a massive structure (Figure 8.1). The enormous weight of the dam body provides the necessary stability against overturning (about the toe of the dam) or shear failure (along the bottom). Arch dams are normally built into solid rock foundations that provide resistance to the hydrostatic force by arch actions (Figure 8.1). Combining gravity action and arch action is a common practice. A typical buttress dam supports a slanted concrete slab (upstream face) at intervals with buttress supports. Because much of the cross section is empty space between buttresses, the stability comes from the water weight acting on the face slab. Because of their importance and the basic principles involved, the stability of gravity and arch dams is discussed in the next section.

The most common type of dam encountered is the earth dam. It achieves stability from its mass and from the water on its upstream face. Because these dams are made of porous material, they will continually seep water. Controlling the amount and location of the seepage is an

TABLE 8.1 Classification of Dams

Type	Stability	Material	Cross Section
Gravity	Large mass	Concrete, rock, or masonry	
Arch	Arch action on rock canyon	Concrete	
Buttress	Mass of dam and water on upstream face	Concrete, steel, or timber	
Earth	Mass of dam and water on upstream face	Earth or rock	

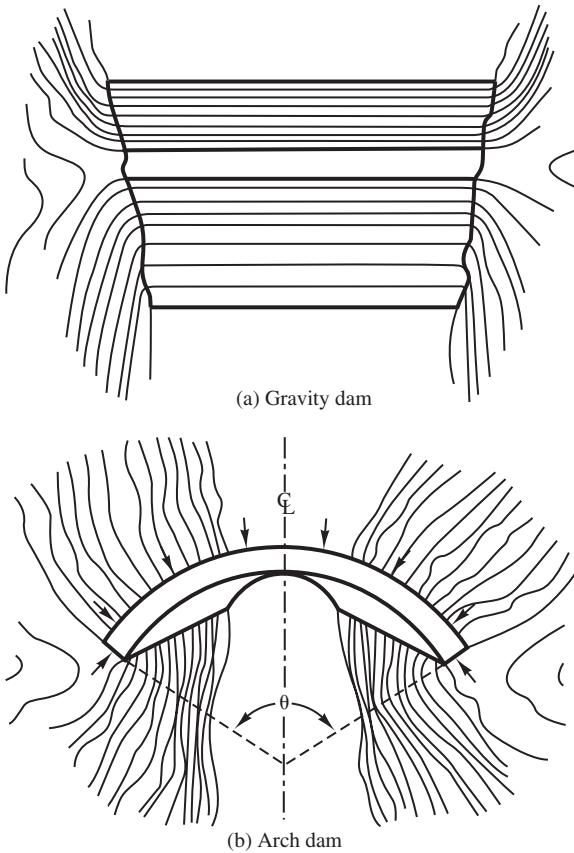


Figure 8.1 Top-view representation of (a) gravity dam and (b) arch dam

important design concern (Chapter 7). Few major dams are likely to be built in the near future, but many small dams are designed and built every year. Most are earth dams built for the purpose of urban stormwater management (Chapter 11). Because of their importance and prevalence, Section 8.4 describes some of the key design considerations and a typical construction scenario.

8.3 Stability of Gravity and Arch Dams

8.3.1 Gravity Dams

The major forces acting on a gravity dam are represented in Figure 8.2. They are

1. the hydrostatic force (F_{HS}),
2. the weight of the dam (W),
3. the uplifting force on the base of the dam (F_u),
4. the sedimentation (silt deposit) pressure force (F_s),
5. the earthquake force on the dam (F_{EQ}), and
6. the earthquake force caused by the water mass behind the dam (F_{EW}).

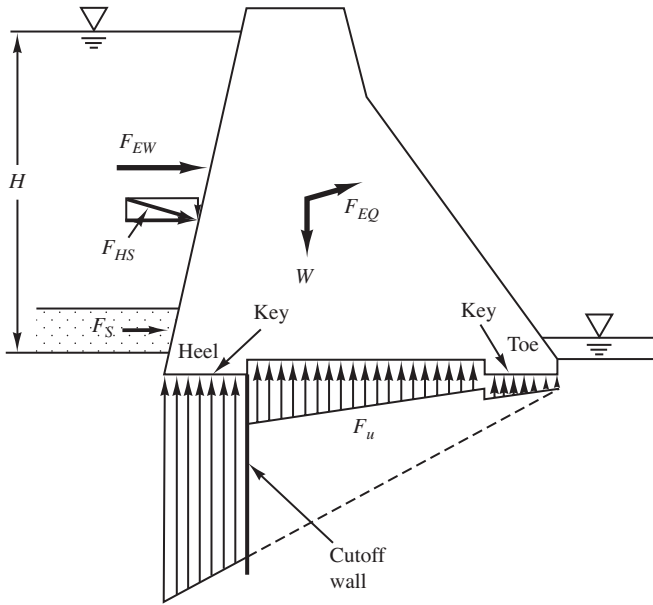


Figure 8.2 Cross-section of a gravity dam

Many gravity dams have a uniform cross section throughout their width that permits a force analysis per unit width of the dam. The binding forces between each unit width segment are neglected in the analysis because they can only add to the stability of the dam.

The hydrostatic force acting on the upstream face of the dam may be resolved into a horizontal component and a vertical component. The horizontal component of the hydrostatic force acts along a horizontal line $H/3$ above the base of the dam. This horizontal force creates a clockwise moment about the *toe of the dam* (labeled in Figure 8.2), and it may cause the dam to fail by overturning. It may also cause dam failure by shearing along a horizontal plane at the base of the dam. The vertical component of the hydrostatic force is equal to the weight of the water mass directly above the upstream face of the dam. It acts along a vertical line that passes through the centroid of that mass. The vertical component of hydrostatic force always forms a counterclockwise moment about the toe. It is a stabilizing factor in gravity dams.

The largest stability force is the weight of the dam, which depends not only on the dimensions but also on the material used. The unit weight of most masonry or solid earth materials is approximately 2.4 to 2.6 times that of water. The importance of this stability force explains the name: *gravity dam*.

The uplifting force on the base of a dam can be determined by foundation seepage analysis (Chapter 7). Acting in a direction opposite to the weight, this force should be minimized in every dam design if possible. It weakens the foundation and tends to overturn the dam. If the foundation soil is porous and homogeneous, then the uplifting pressure on the base varies linearly from full hydrostatic pressure at the *heel of the dam* (i.e., $P = \gamma H$) to the full hydrostatic pressure at the toe. The total resulting uplifting force can be determined by integrating the resulting trapezoidal pressure distribution. The magnitude of the uplifting force as well as the overturning (clockwise) moment can be greatly reduced by installing an impermeable cutoff wall, as

depicted in Figure 8.2. The cutoff wall alters the seepage course by lengthening the pathway, thus reducing the seepage and uplifting force downstream for the cutoff wall.

The water velocity immediately behind the dam is very slow or nearly zero. Consequently, it loses its ability to carry sediments or other suspended matter. These heavier materials are deposited on the bottom of the reservoir, some being near the base of the dam. The silt–water mixture is approximately 50 percent heavier than water (sp. gr. = 1.5) and forms excess pressure force near the heel. Normally, the thickness of the silt layer will increase slowly with time. This force may contribute to dam failure by shear along the base.

In earthquake zones, the forces generated by earthquake motion must be incorporated into dam design. Earthquake forces on dams result from acceleration associated with the earthquake motions. The magnitude of the earthquake force in the dam body (F_{EQ}) is proportional to the acceleration and the mass of the dam body. The force may act in any direction through the centroid of the dam body.

The earthquake force resulting from acceleration of the water body behind the dam is approximately equal to

$$\frac{5}{9} \left(\frac{a\gamma}{g} \right) H^2$$

where a is the earthquake acceleration, γ is the specific weight of water, and H is the hydrostatic head, or depth of water, immediately behind the dam. The earthquake force of the water body acts in a horizontal direction at a distance $(4/3\pi)H$ above the base of the dam.*

To ensure stability, the safety factors against sliding or overturning failures must be greater than 1.0 and are generally much higher. In addition, the maximum pressure exerted on the foundation must not exceed the bearing strength of the foundation.

The *force ratio against sliding* (FR_{slide}) is defined by the ratio of the total horizontal resistance force that the foundation can develop to the sum of all forces acting on the dam that tend to cause sliding. The ratio may be expressed as

$$FR_{slide} = \frac{\mu(\sum F_V) + A_s\tau_s}{\sum F_H} \quad (8.1)$$

where μ is the coefficient of friction between the dam base and the foundation (ordinarily $0.4 < \mu < 0.75$), $\sum F_V$ is the summation of all vertical force components acting on the dam, τ_s is the shear stress strength of keys, and A_s is the total shear area provided by the keys. $\sum F_H$ is the summation of all horizontal force components acting on the dam.

The *keys* (or *keyways*, depicted in Figure 8.2) are dam components built into the foundation to add resistance against dam sliding. Horizontal forces are transmitted to the foundation via the shear force in the keys. The total shear force provided by the keys, $\tau_s A_s$, must be larger than the difference between the total horizontal force acting on the dam, $\sum F_H$, and the friction force provided by the base, $\mu(\sum F_V)$.

$$\tau_s A_s > [\sum F_H - \mu(\sum F_V)]$$

* J. I. Bustamante, "Water pressure on dams subjected to earthquakes," *J. Engr. Mech., Div., ASCE*, 92 (Oct. 1966): 116–127.

The *force ratio against overturning* (FR_{over}) is defined by the ratio of the resisting moments (counterclockwise moments about the toe) to the overturning moments (clockwise moments about the toe):

$$FR_{over} = \frac{Wl_w + (F_{HS})_v l_v}{\sum F_H Y_H + F_u l_u} \tag{8.2}$$

where $(F_{HS})_v$ is the vertical component of the hydrostatic force and $l_w, l_v,$ and l_u are the horizontal distances from the toe to the lines of action of the weight (W), the vertical component of hydrostatic force, and the uplifting force (F_u), respectively. Y_H is the vertical distance measured from the toe to the lines of action of each respective horizontal force component (F_H) acting on the upstream face of the dam.

Oftentimes, we may assume that the vertical pressure on the foundation is a linear distribution between the toe and heel as shown in Figure 8.3. If we let R_V represent the resultant of all vertical forces acting on the base of the dam and P_T and P_H represent the resulting foundation pressure at the toe and heel, respectively, we may write

$$R_V = \frac{(P_T + P_H)}{2} (B)$$

and by equating moments (vertical forces only) about the center line

$$R_V(e) = \left[\frac{(P_T - P_H)}{2} (B) \right] \left(\frac{B}{6} \right)$$

Solving these two equations simultaneously, we have

$$P_T = \left(\frac{R_V}{B} \right) \left(1 + \frac{6e}{B} \right) \tag{8.3}$$

$$P_H = \left(\frac{R_V}{B} \right) \left(1 - \frac{6e}{B} \right) \tag{8.4}$$

The vertical resultant force normally acts through a point on the downstream side of the base centerline. Therefore, P_T is usually the critical pressure in design. The value of P_T must be kept less than the bearing strength of the foundation. The pressure at the heel (P_H) is less

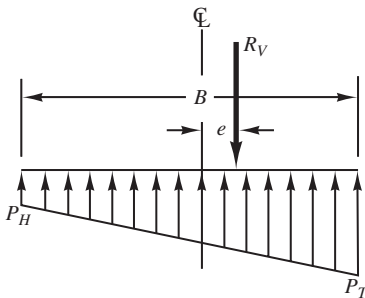


Figure 8.3 Pressure distribution on dam foundation

important. Nevertheless, it is desirable to keep P_H a positive value at all times to prevent tension cracks from developing in the heel region. Negative pressure indicates tension, and masonry materials have very low resistance to tension stress. A positive P_H value can be ensured if the vertical resultant force (R_V) is kept within the middle third of the base, or

$$e < \frac{B}{6} \quad (8.5)$$

The value of e can be found by using the principle of moments; that is, the moment produced by the individual vertical force components about the center line is equal to the moment produced by R_V .

8.3.2 Arch Dams

The force load on an arch dam is essentially the same as on a gravity dam. To resist these forces, the dam foundation must provide a horizontal arch reaction. The large horizontal reactions can only be provided by strong, solid rock abutments at the two ends of the arch (Figure 8.1). Arch dams are usually high dams built in relatively narrow rock canyon sections. The efficiency of using material strength rather than bulk results in a very slender cross section compared to a gravity dam, making arch dams the best choice in many situations. Because arch dams combine the resistance of arch action with gravity, there is high stress in each segment of the dam, and detailed stress analysis is required.

The stability analysis on an arch dam is usually carried out on each horizontal rib. Take a rib situated h meters below the designed reservoir water level. The forces acting in the direction of the dam centerline may be summed up as follows:

$$2R \sin \frac{\theta}{2} = 2r(\gamma h) \sin \frac{\theta}{2}$$

where R is the reaction from the abutment, θ is the central angle of the rib, r is the outer radius (*extrados*) of the arch, and γh is the hydrostatic pressure acting on the rib (Figure 8.4). The previous equation can be simplified for the abutment reaction

$$R = r\gamma h \quad (8.6)$$

This value is determined by considering only the arch reaction for resisting the hydrostatic load on the dam. In practice, however, several other resistive forces, as discussed with gravity dams, should also be included. The analysis should consider the combination resistance from both the arch and gravity actions.

The volume of an arch dam is directly related to the thickness of each rib (t), the width (height) of the rib (B), and the center angle (θ). For the minimum volume of the dam, it can be shown that $\theta = 133^\circ 34'$ (Figure 8.4). Other factors, such as topographic conditions, often prevent the use of this optimal value. Values within the range of $110^\circ < \theta < 140^\circ$ are commonly used in arch dam design.

A simple approach to arch dam design is to keep the central angle constant while the radii vary from rib to rib as shown in Figure 8.4 (a). Another approach frequently used is to keep the radii of the ribs at a constant value and allow variations of the central angle, as shown in Figure 8.4 (b).

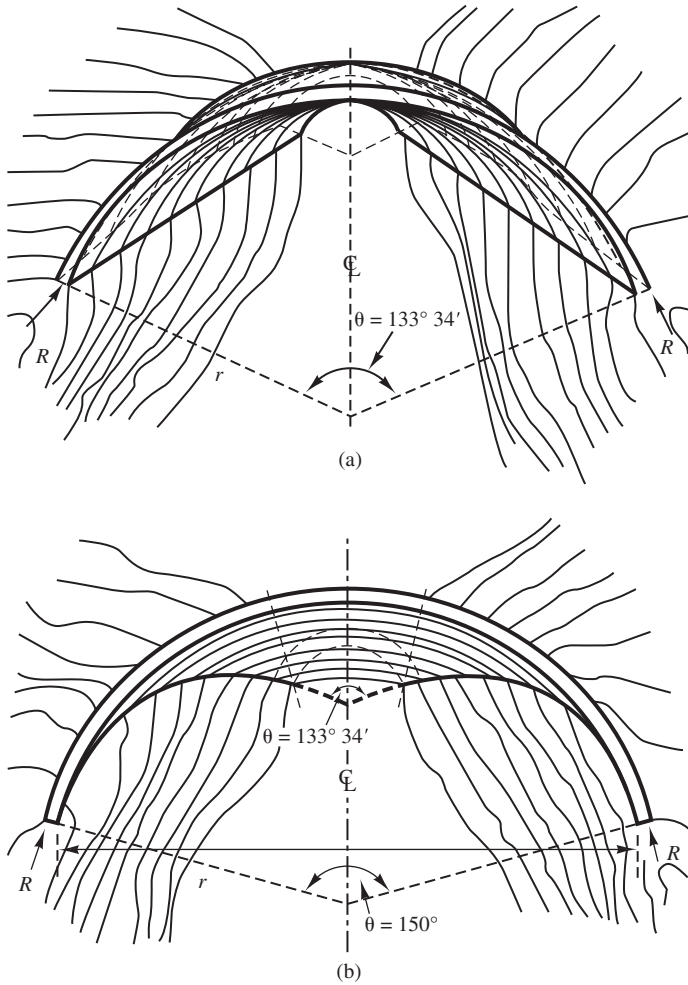


Figure 8.4 Arch dams of (a) constant angle and (b) constant radius

8.4 Small Earth Dams

Small earth dams (or *embankments*) are designed and built for a variety of reasons. For example, small earth dams are frequently used in stormwater-management ponds, mining applications (tailings ponds), farm ponds (irrigation or stock water), constructed wetlands, and flood protection (levees). Because these structures are so common, a basic understanding of critical design features and construction considerations is important. The following paragraphs provide a rudimentary discussion of some of these items. A detailed examination can be found in the classic reference *Design of Small Dams*.*

* U.S. Department of the Interior (Bureau of Reclamation), *Design of Small Dams* (Washington, DC: U.S. Government Printing Office, 1977).

Small earth dams must be designed and constructed properly to fulfill their intended purpose. Special attention must be given to the following features.

- **Foundation:** Placing an embankment on native material without any site preparation will only suffice for very small earth dams. Normally, site clearing, grading or scraping, and compaction are required to provide stability and minimize settling and seepage. Often a trench (also called a *key* or *keyway*) is excavated along the center line of the cross section (Figure 8.5). The trench is then backfilled and compacted, usually with clay, to minimize seepage.
- **Embankment:** The height, slopes, top width, and materials all have to be specified. The height is normally dictated by storage requirements or site limitations such as elevation restrictions or land holdings. The slopes depend somewhat on the material used but are often between 2:1 and 3:1. The top width may have to accommodate maintenance equipment, otherwise 1 to 3 meters is common for small dams. Finally, the embankment

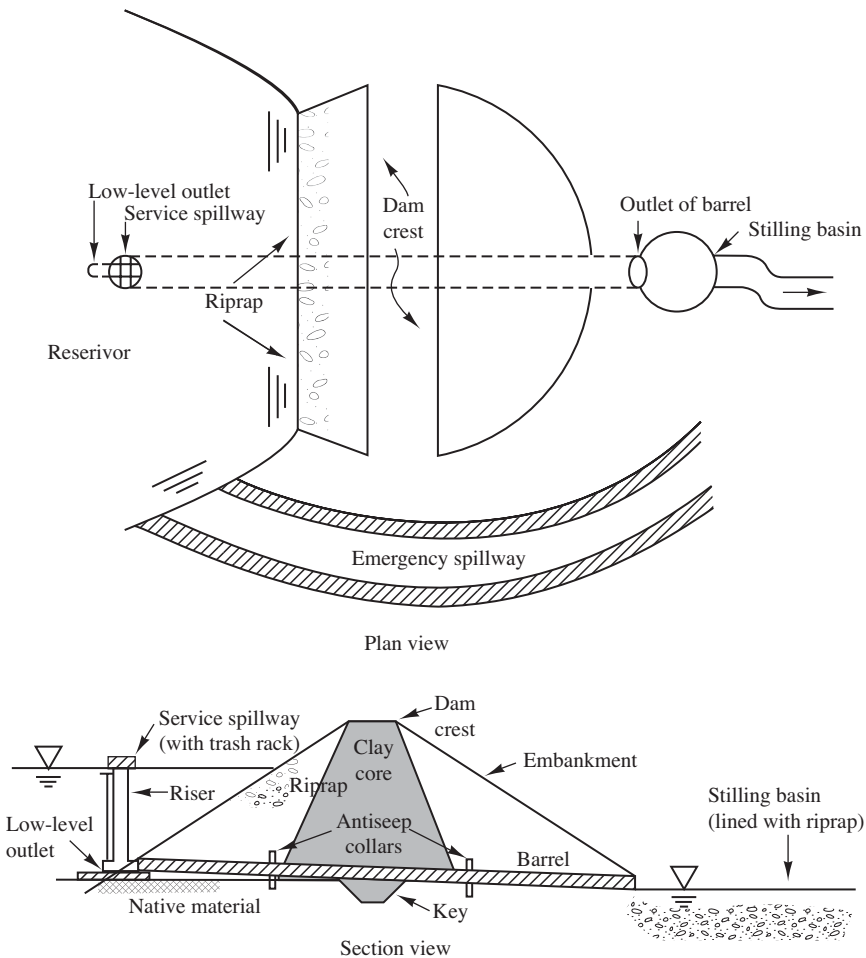


Figure 8.5 Typical small earth dam

materials need to be specified. Small dams and levees are often constructed by using a homogeneous material throughout (*simple embankment*). More commonly, a clay core is placed in the middle of the embankment with more pervious material on either side (*zoned embankment*; see Figure 8.5). The clay core reduces seepage, and the outer materials (silt or silt and sand) provide stability. Compaction testing (with a nuclear density gauge) is often required because the embankment is constructed and compacted in measured lifts.

- *Service Spillway*: An outlet device is necessary to pass the normal stream flow coming into the reservoir downstream. A common outlet device for small earth dams is a *riser and barrel* assembly, often called a *drop inlet* (Figure 8.5). Water flows over the top of the riser pipe, drops into the barrel or pipe, and is passed through the earth embankment. The riser needs to be sized based on expected flows and some acceptable increase in water level. The riser acts as a weir; the design equations will be covered in the next section. The barrel likewise needs to be sized to accommodate the flow without allowing water to back up the riser and choke the flow. It acts much like a culvert; the design equations are covered in Section 8.9. Often the barrel will have a gated outlet on the bottom (*low-level outlet*) to drain the reservoir if necessary. *Antiseep collars* (Figure 8.5) minimize seepage along the barrel. In lieu of antiseep collars, the pipe may be encased in concrete for two-thirds of its length with a filter collar for the final third.
- *Emergency Spillway*: Any earth dam whose failure will cause economic distress downstream or potential loss of life requires an emergency spillway. An emergency spillway is designed to accommodate the flows associated with rare storm events without allowing the dam to be overtopped. (About one-third of all earth dam failures are caused by overtopping and subsequent embankment erosion.) Normally, emergency spillways are constructed by excavating a channel through the native (virgin) material around one end of the dam (Figure 8.5). The control elevation (which permits water to flow through the emergency spillway) is higher than the service spillway elevation, so it is rarely called into use. Design considerations include hydrologic analysis to determine the peak flow rate, channel sizing to pass the peak flow, and lining design to resist the erosive forces associated with the peak flow. Grass-lined emergency spillways are common and hold up well under infrequent use if the slopes are not too steep.

At the end of the construction process, a survey is necessary to confirm final grades, elevations, and distances. Of paramount importance are the elevations for the top of the dam (crest), the service spillway, and the emergency spillway. The appropriate sizes of the service spillway pipes and the emergency spillway channel are also critical.

8.5 Weirs

A *weir* is a flow obstruction that causes water to rise to pass over it. Because weirs are usually installed in streams and channels that have free surfaces, the flow behavior over a weir is governed by gravity forces. One unique application of a weir is to help keep bridges from flooding, as schematically shown in Figure 8.6. By placing an obstruction of adequate dimensions in an otherwise subcritical stream flow, the water level is raised upstream from the weir. With the increased available head, the flow accelerates as it passes over the weir crest. This acceleration causes the depth of water to decrease and attain supercritical flow after passing through critical depth. Some distance downstream from the weir, the flow returns to a normal subcritical depth

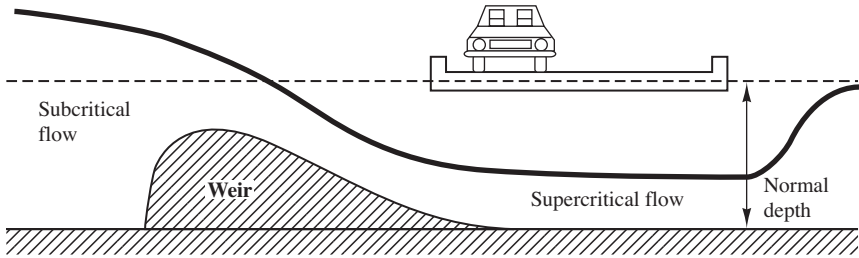


Figure 8.6 Acceleration of flow over a weir

through a hydraulic jump. This arrangement protects the bridge structure from being overtopped. This concept is used in Australia to design “minimum energy” bridges. The cost of these bridges is reduced by minimizing the waterway opening.

Flow acceleration over a weir provides a unique one-to-one relationship between the approaching water height (depth) and the discharge for each type of weir. Thus, weirs are commonly built to measure the discharge in open channels. Weirs are also used to raise stream-flow levels in order to divert water for irrigation and other purposes.

The use of a weir as a flow-measuring device will be discussed in detail in Chapter 9. In this section the hydraulic characteristics of weirs will be presented.

As previously mentioned, a weir increases the flow depth immediately upstream of the weir and reduces the cross-sectional flow area at the crest. The increase of water depth reduces the flow velocity upstream, but the sudden reduction of cross-sectional area causes the flow to speed up quickly as it passes over the crest. The occurrence of critical flow on weirs is the essential feature of weir structures.

The hydraulics of an overflow weir may be examined by using an ideal frictionless weir (Figure 8.7). At the location where critical depth occurs, the discharge per unit width of the weir can be determined by using the critical flow equations (from Chapter 6).

$$\frac{V}{\sqrt{gD}} = 1 \tag{6.11}$$

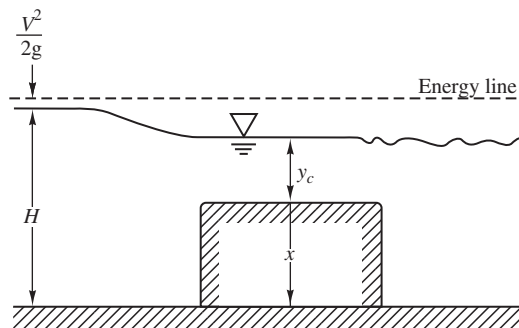


Figure 8.7 Flow over a frictionless weir

and

$$y_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{q^2}{g}} \quad (6.14)$$

Rearranging Equation 6.11 and substituting y_c for D for rectangular channels, we may write

$$\frac{V_c^2}{2g} = \frac{y_c}{2}$$

Therefore, the specific energy at the critical section is

$$E = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2} = \frac{3}{2}y_c \quad (6.8)$$

If the approaching velocity head can be neglected, then the energy of the approaching flow is approximately equal to the water depth upstream of the weir, H . Therefore, for a frictionless weir, we may write an energy balance as

$$E + x = \frac{3}{2}y_c + x = H \quad (8.7)$$

where x is the height of the weir, as depicted in Figure 8.7. Combining Equation 8.7 with Equation 6.14 and defining $H_s = H - x$, we have

$$q = \sqrt{gy_c^3} = \sqrt{g\left(\frac{2H_s}{3}\right)^3} \quad (8.8a)$$

where q is the discharge per unit width of the weir. This is the basic form of the weir equation. In British units, the weir equation is

$$q = 3.09 H_s^{3/2} \quad (8.8b)$$

In the SI system, the equation becomes

$$q = 1.70 H_s^{3/2} \quad (8.8c)$$

The stated discharge coefficients ($3.09 \text{ ft}^{0.5}/\text{sec}$ and $1.70 \text{ m}^{0.5}/\text{sec}$) are higher than the coefficients obtained in experiments because friction loss is neglected in the above analysis. Also note that H_s is defined as the vertical distance from the top of the weir to the upstream water level.

A *sharp-crested weir* is shown in Figure 8.8 (a). Upstream a short distance from the weir, all the velocity vectors are nearly uniform and parallel. However, as flow approaches the weir, the water near the bottom of the channel rises in order to pass over the crest. The vertical component of the flow near the upstream face of the weir causes the lower surface of the stream to separate from the weir and form a *nappe* after the flow passes over the weir [see the left panel in Figure 8.8 (a)]. The nappe usually traps a certain amount of air between its lower surface and the

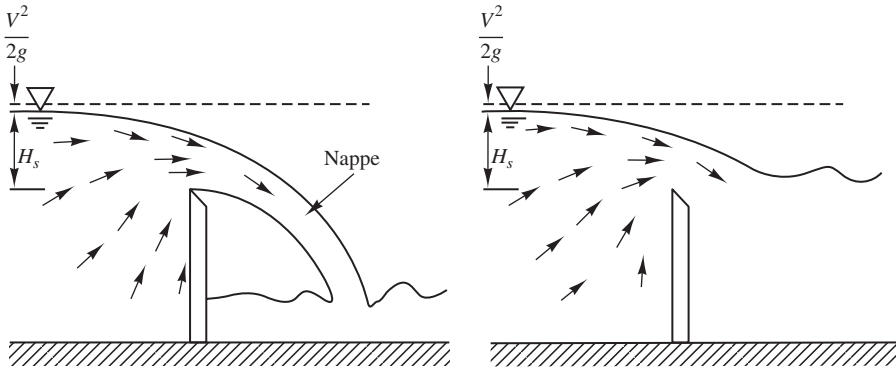


Figure 8.8 (a) Flow over sharp-crested weir (free-falling nappe and submerged flow)

downstream side of the weir. If no means of restoring air is provided, a void will appear that represents a negative pressure on the structure. The nappe will also cling intermittently to the side of the weir and cause the flow to be unstable. The dynamic effect of this unstable flow may result in added negative pressure that may eventually damage the structure.

When the downstream water level rises over the weir crest, the weir is said to be *submerged* [see the right panel in Figure 8.8 (a)]. In this case, the negative pressure no longer exists, and a new set of flow parameters may be considered in the determination of the discharge coefficient.

A *low head dam* is a specific type of weir designed to span a stream or river, raising the upstream water level slightly as the flow passes over its entire length. This allows for a relatively constant diversion of water upstream for open-channel irrigation or power-plant cooling water, two common purposes of these hydraulic structures. Most low head dams are less than 3 m high. Depending on the downstream depth, different hydraulic conditions develop and are depicted in Figure 8.8 (b).

Low head dams must be designed hydraulically to fulfill their purpose, but an additional concern is human safety. Water enthusiasts often underestimate the power of moving water, and these dams can pose formidable dangers. The downstream condition that should be avoided is Case 3 in Figure 8.8 (b). Although this condition may look harmless to the observer, it is responsible for many drowning deaths. This hydraulic condition, which includes a reverse roller, presents at least three dangers. The first danger is the reverse current that ensnares anyone who ventures too close to the backside of the dam. The second danger is the reduced “buoyancy” resulting from the large amounts of entrained air created from the plunging water. The third danger is the force of the water falling over the dam and striking someone who is not able to resist the reverse current. Hydraulic engineers should be aware of the dangers of these structures and avoid designs that are hazardous to the public. Appropriate designs are beyond the scope of this book but can be found in the engineering literature. The most common retrofits for existing structures include strategically placed riprap and altered downstream depths.

Example 8.1

Uniform flow at a depth of 2 meters occurs in a long rectangular channel that is 4 meters wide. The channel is laid on a slope of 0.001, and the Manning coefficient is 0.025. Determine the minimum height of a low weir that can be built on the bottom of this channel to produce critical depth.

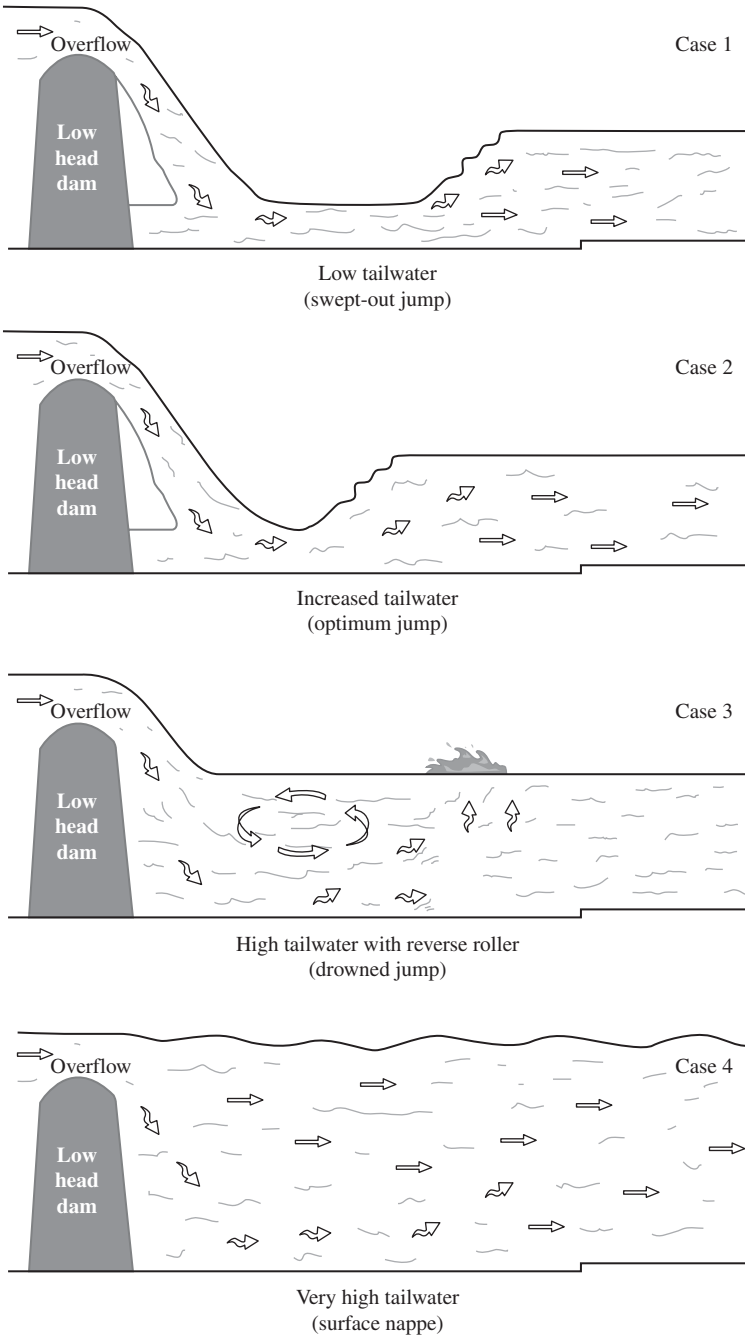


Figure 8.8 (b) Weir flow over low head dams (four hydraulic conditions).
Source: M. A. Robinson et al., "Dangerous dams," *CE News* (Feb. 2007): 24–29.

Solution

For a uniform flow condition, the Manning equation (6.5a) may be used to determine the channel discharge (Q):

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

In this case, $A = (2 \text{ m})(4 \text{ m}) = 8 \text{ m}^2$, $P = 2(2 \text{ m}) + 4 \text{ m} = 8 \text{ m}$, and $R_h = A/P = 1.0 \text{ m}$. Hence,

$$Q = \frac{1}{0.025} (8)(1.0)^{2/3} (0.001)^{1/2} = 10.1 \text{ m}^3/\text{sec}$$

and

$$V = \frac{Q}{A} = \frac{10.1}{8} = 1.26 \text{ m/sec}$$

The specific energy is

$$E = y + \frac{V^2}{2g} = 2 + \frac{(1.26)^2}{2(9.81)} = 2.08 \text{ m}$$

Flow over the weir passes through critical depth. Using Equation 6.14 yields

$$y_c = \sqrt[3]{\frac{Q^2}{gb^2}} = \sqrt[3]{\frac{(10.1)^2}{(9.81)(4)^2}} = 0.87 \text{ m/sec}$$

The corresponding velocity is

$$V_c = \frac{Q}{4y_c} = \frac{10.1}{4(0.87)} = 2.90 \text{ m/sec}$$

and the critical velocity head is

$$\frac{V_c^2}{2g} = 0.43 \text{ m}$$

Now an energy balance is possible between two locations, at the weir and just upstream of the weir, as depicted in Figure 8.9. Assuming no energy loss at the weir, the minimum weir height (x) that can be built to produce critical flow is

$$E = y_c + \frac{V_c^2}{2g} + x$$

$$2.08 = 0.87 + 0.43 + x; \quad x = 0.78 \text{ m}$$

8.6 Overflow Spillways

An *overflow spillway* acts as a safety valve on a dam. Overflow spillways are designed to pass large amounts of water safely over the crest of a dam to maintain target water levels. They often act as emergency spillways or, in conjunction with emergency spillways, to keep the dam from

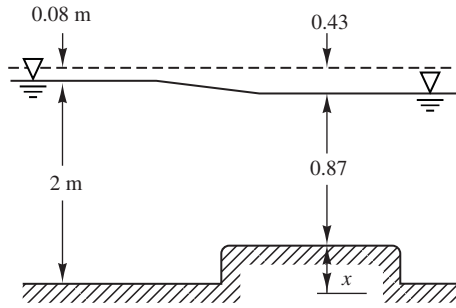


Figure 8.9

being overtopped during storm events. Overflow spillways are common on arch, gravity, and buttress dams. Many earth dams have a concrete section to accommodate an overflow spillway. For small dams, their design shape is not that critical. On large dams, however, their effectiveness is highly dependent on their shape.

In essence, an overflow spillway is a hydraulically efficient weir followed by a steep open channel that allows excess water to flow over a dam at supercritical velocities. The ideal longitudinal profile or shape of an overflow spillway should closely match the underside of the free-falling water nappe of a sharp-crested weir as depicted in Figure 8.10. This will minimize the pressure on the spillway surface. However, caution must be exercised to avoid any negative pressure on the surface. Negative pressure is caused by separation of the high-speed flow from the spillway surface, resulting in a pounding action that can cause significant damage to the spillway structure (e.g., pitting).

The U.S. Waterways Experimental Station suggests a set of simple crest profiles that have been found to agree with actual prototype measurements. The geometry of the U.S. Waterways Experimental Station spillway crest profiles is shown in Figure 8.11.

The discharge of a spillway may be calculated by an equation similar to that derived for flow over a weir (Equation 8.8),

$$Q = CLH_a^{3/2} \tag{8.9}$$

where C is the coefficient of discharge, L is the width of the spillway crest, H_a is the sum of the static head (H_s) and the approaching velocity head ($V_a^2/2g$), at the crest (Figure 8.10). Therefore,

$$H_a = H_s + \frac{V_a^2}{2g} \tag{8.10}$$

The coefficient of discharge of a particular spillway crest is often determined by scaled model tests (Chapter 10) and accounts for the energy losses and the magnitude of the approaching velocity head. The value of the coefficient normally ranges between 1.66 to 2.26 $m^{1/2}/sec$ (or 3 to 4.1 $ft^{1/2}/sec$ in British units). A detailed discussion of weirs is presented in Chapter 9.

Example 8.2

An overflow spillway 80 m wide carries a maximum (design) discharge of 400 m^3/sec . Compute the static (design) head and define the crest profile for the spillway. Consider a 3:1 upstream slope and a 2:1 downstream slope for the crest profile. Assume a discharge coefficient of 2.22 based on model studies and a negligible approach velocity based on the dam height.

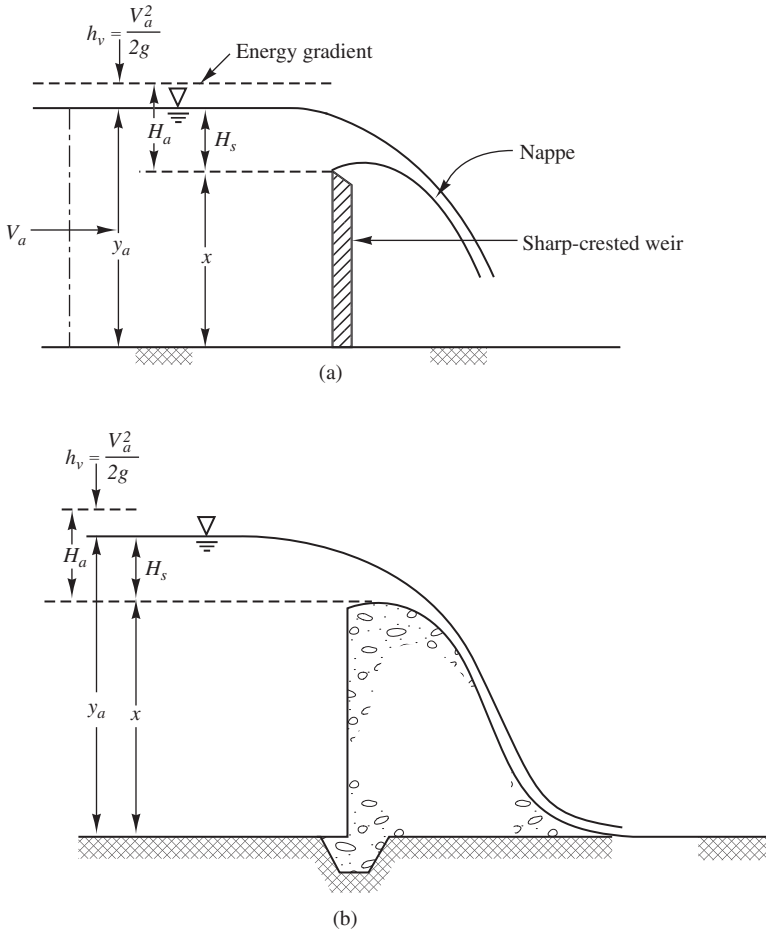


Figure 8.10 Ideal longitudinal profile or shape of an overflow spillway: (a) water nappe over a sharp-crested weir; (b) flow profile of an overflow spillway

Solution

Applying Equation 8.9 and assuming a minimal approach velocity, we obtain

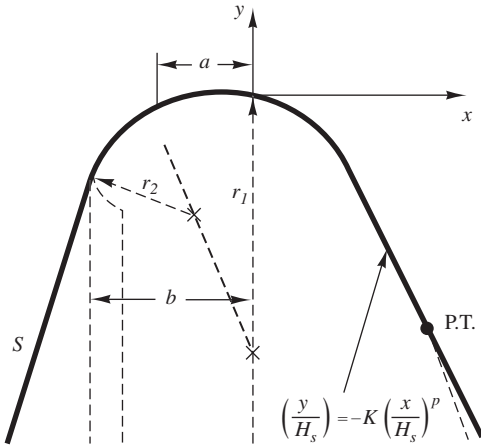
$$Q = 2.22LH_s^{3/2}$$

and

$$H_s = \left(\frac{Q}{2.22L} \right)^{2/3} = \left(\frac{400}{2.22(80)} \right)^{2/3} = 1.72 \text{ m}$$

From the table in Figure 8.11, we have

$a = 0.139 H_s = 0.239 \text{ m};$	$r_1 = 0.68 H_s = 1.170$
$b = 0.237 H_s = 0.408 \text{ m};$	$r_2 = 0.21 H_s = 0.361 \text{ m}$
$K = 0.516;$	$P = 1.836$



Upstream slope (vertical /horizontal)

	3/0	3/1	3/2	3/3
a/H_s	0.175	0.139	0.115	0
b/H_s	0.282	0.237	0.214	0.199
r_1/H_s	0.50	0.68	0.48	0.45
r_2/H_s	0.20	0.21	0.22	-
K	0.500	0.516	0.515	0.534
p	1.850	1.836	1.810	1.776

Figure 8.11 Overflow spillway profile

and from Figure 8.11,

$$\left(\frac{y}{H_s}\right) = -K\left(\frac{x}{H_s}\right)^P = -0.516\left(\frac{x}{H_s}\right)^{1.836}$$

The downstream end of the profile curve will be matched to a straight line with slope 2:1. The position of the point of tangency is determined by

$$\frac{d\left(\frac{y}{H_s}\right)}{d\left(\frac{x}{H_s}\right)} = -KP\left(\frac{x}{H_s}\right)^{P-1} = -0.947\left(\frac{x}{H_s}\right)^{0.836} = -2$$

Hence,

$$\begin{aligned} \frac{X}{H_s} &= 2.45 & X_{P.T.} &= 4.21 \text{ m} \\ \frac{Y}{H_s} &= -2.67 & Y_{P.T.} &= -4.59 \text{ m} \end{aligned}$$

The crest profile curve of the spillway is shown in Figure 8.12.

8.7 Side-Channel Spillways

A side-channel spillway carries water away from an overflow spillway in a channel parallel to the spillway crest (Figure 8.13).

The discharge over the entire width (L) of an overflow spillway can be determined by Equation 8.9, and the discharge through any section of the side channel at a distance x from the upstream end of the channel is

$$Q_x = xCH_a^{3/2} \tag{8.11}$$

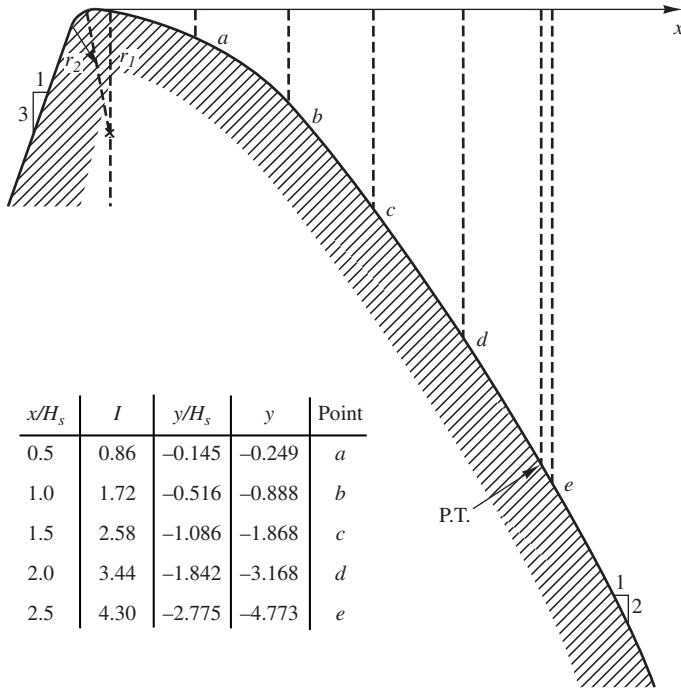


Figure 8.12

The side-channel spillway must provide a slope steep enough to carry away the accumulating flow in the channel. However, a minimum slope and depth at each point along the channel is desired in order to minimize construction costs. For this reason, an accurate

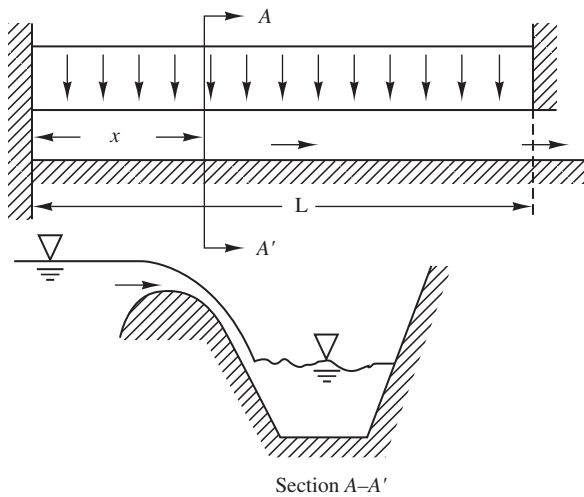


Figure 8.13 Side-channel spillway

water surface profile for the maximum design discharge is important in the side-channel spillway design.

The flow profile in the side channel cannot be analyzed by the energy principle (i.e., Chapter 6, gradually-varied flow profile) because of the highly turbulent flow conditions that cause excess energy loss in the channel. However, an analysis based on the momentum principle has been validated by both model and prototype measurements.

The momentum principle considers the forces and change of momentum between two adjacent sections, a distance of Δx apart, in the side channel,

$$\Sigma F = \rho(Q + \Delta Q)(V + \Delta V) - \rho QV \tag{8.12}$$

where ρ is the density of water, V is the average velocity, and Q is the discharge at the upstream section. The symbol Δ signifies the incremental change at the adjacent downstream section.

The forces represented on the left-hand side of Equation 8.12 usually include the weight component of the water body between the two sections in the direction of the flow [$(\rho g A \Delta x) \sin \theta$], the unbalanced hydrostatic forces

$$\rho g A \bar{y} \cos \theta - \rho g (A + \Delta A) (\bar{y} + \Delta \bar{y}) \cos \theta$$

and a friction force, F_f , on the channel bottom. Here A is the water cross-sectional area, \bar{y} is the distance between the centroid of the area and the water surface, and θ is the angle of the channel slope.

The momentum equation may thus be written as

$$\begin{aligned} \rho g A \Delta x \sin \theta + \left[\rho g A \bar{y} - \rho g (A + \Delta A) (\bar{y} + \Delta \bar{y}) \right] \cos \theta - F_f \\ = \rho (Q + \Delta Q)(V + \Delta V) - \rho QV \end{aligned} \tag{8.13}$$

Let $S_0 = \sin \theta$ for a reasonably small angle, and $Q = Q_1, V + \Delta V = V_2, A = (Q_1 + Q_2)/(V_1 + V_2)$, and $F_f = \gamma A S_f \Delta x$; the above equation may be simplified to

$$\Delta y = -\frac{Q_1(V_1 + V_2)}{g(Q_1 + Q_2)} \left(\Delta V + V_2 \frac{\Delta Q}{Q_1} \right) + S_0 \Delta x - S_f \Delta x \tag{8.14}$$

where Δy is the change in water surface elevation between the two sections. This equation is used to compute the water surface profile in the side channel. The first term on the right-hand side represents the change in water surface elevation between the two sections resulting from the impact loss caused by the water falling into the channel. The middle term represents the change from the bottom slope, and the last term represents the change caused by friction in the channel. Relating the water surface profile to a horizontal datum, we may write

$$\Delta z = \Delta y - S_0 \Delta x = -\frac{Q_1(V_1 + V_2)}{g(Q_1 + Q_2)} \left(\Delta V + V_2 \frac{\Delta Q}{Q_1} \right) - S_f \Delta x \tag{8.15}$$

Note that when $Q_1 = Q_2$ or when $\Delta Q = 0$, Equation 8.15 reduces to

$$\Delta z = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) - S_f \Delta x \quad (8.16)$$

which is the energy equation for constant discharge in an open channel as derived in Chapter 6.

Example 8.3

A 20-ft overflow spillway discharges water into a side-channel spillway with a horizontal bottom slope. If the overflow spillway ($C = 3.7 \text{ ft}^{1/2}/\text{sec}$) is under a head of 4.2 feet, determine the depth change from the end of the side channel (after it has collected all of the water from the overflow spillway) to a point 5 feet upstream. The concrete ($n = 0.013$) side channel is rectangular with a 10-ft bottom. The water passes through critical depth at the end of the side channel.

Solution

The flow at the end of the side channel (Equation 8.9) is

$$Q = CL(H_a)^{3/2} = (3.7)(20)(4.2)^{3/2} = 637 \text{ ft}^3/\text{sec}$$

The flow at a point 5 feet upstream (Equation 8.11) is

$$Q = xC(H_a)^{3/2} = (15)(3.7)(4.2)^{3/2} = 478 \text{ ft}^3/\text{sec}$$

Solving for critical depth (Equation 6.14) at the end of the channel, we have

$$y_c = [Q^2/gb^2]^{1/3} = [(637)^2/\{(32.2)(10)^2\}]^{1/3} = 5.01 \text{ ft}$$

The solution method employs a finite-difference solution scheme (Equation 8.14) and an iterative process can be employed to compute the upstream depth. The upstream depth (or depth change, Δy) is estimated, and Equation 8.14 is solved for a depth change. The two depth changes are compared, and a new estimate is made if they are not nearly equal. Table 8.2 displays the solution. Because side-channel spillway profile computations involve implicit equations, computer algebra software (e.g., Mathcad, Maple, and Mathematic) or spreadsheet programs will prove very helpful.

8.8 Siphon Spillways

Water that is passing through a closed conduit will experience negative pressure when the conduit is elevated above the hydraulic grade line (pressure line) as described in Section 4.2. A spillway designed to discharge water in a closed conduit under negative pressure is known as a *siphon spillway*. A siphon spillway begins to discharge water under negative pressure when the reservoir level reaches a threshold elevation that primes the conduit. Before this, the water overflows the spillway crest in the same manner as that of the overflow spillway described in Section 8.6 [Figure 8.14 (a)]. However, if the water that flows into the reservoir exceeds the capacity of the spillway, the water level at the crest will rise until it reaches and passes the level of the crown (C). At this point, the conduit is primed and siphon action begins changing free surface flow into pressure flow. Theoretically, the discharge head is increased by the amount

TABLE 8.2 Side-Channel Profile Computations (Example 8.3)

(1) Δx	(2) Δy	(3) y	(4) A	(5) Q	(6) V	(7) $Q_1 + Q_2$	(8) $V_1 + V_2$	(9) ΔQ	(10) ΔV	(11) R_h	(12) S_f	(13) Δy
-	—	5.01	50.1	637	12.7	—	—	—	—	—	—	—
5	-1.00	6.01	60.1	478	7.95	1,115	20.7	159	4.75	2.73	0.0013	-2.48
	-2.48	7.49	74.9	478	6.38	1,115	19.1	159	6.32	3.00	0.0007	-2.68
	-2.72	7.73	77.3	478	6.18	1,115	18.9	159	6.52	3.04	0.0007	-2.71

Column (1) Distances upstream (ft) from the end of the channel
 Column (2) The assumed change in depth (ft) between sections
 Column (3) Channel depth (ft) obtained from the assumed Δy
 Column (4) Channel cross-sectional area (ft²) corresponding to the depth
 Column (5) Channel discharge (ft³/sec) based on location along the spillway
 Column (6) Average channel velocity, $V = Q/A$ (ft/sec)
 Column (7–10) Variables needed for Equation 8.14. Recall that subscript 1 refers to the upstream section and subscript 2 refers to the downstream section.
 Column (11) Hydraulic radius (ft) found by dividing area by wetted perimeter.
 Column (12) Friction slope found from Manning’s equation [$n^2 V^2 / (2.22 R_h^{4/3})$]. Average values of V and R_h should be used. However, because the friction loss is small, upstream values are used for convenience.
 Column (13) Change in channel depth between sections found using Equation 8.14. This value is compared to the assumed value in column (2), and another estimate is made if they do not correspond. Balance occurs when the depth 5 ft upstream is 7.73 ft.

equal to $H - H_a$ [Figure 8.14 (b)], and the discharge rate can thus be substantially increased. The large head allows rapid discharge of the excess water until it drops to the spillway entrance elevation.

The portion of the spillway conduit rising above the hydraulic grade line (HGL) is under negative pressure. Because the hydraulic grade line represents zero atmospheric pressure, the vertical distance measured between the HGL and the conduit (immediately above the HGL) indicates the negative pressure head ($-P/\gamma$) at that location. The crown of a siphon is the

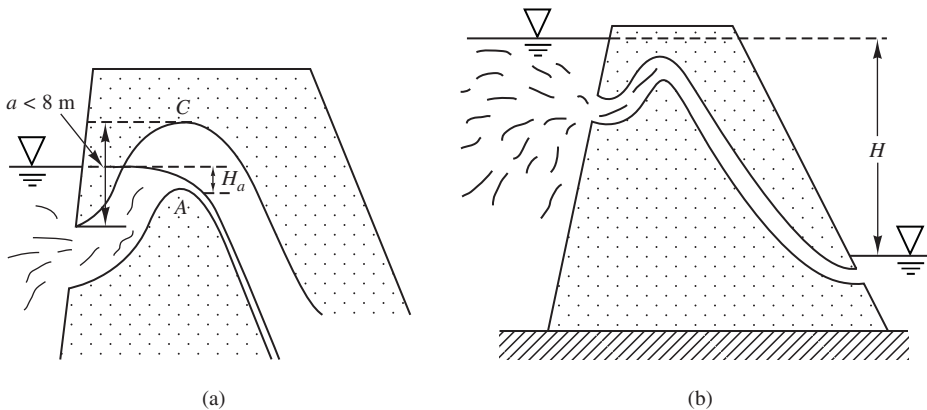


Figure 8.14 Schematic representation of a siphon spillway: (a) beginning stage; (b) siphoning stage

highest point in the conduit, so it is subjected to the maximum negative pressure. The maximum negative pressure at a spillway crown must not be allowed to decrease below the vapor pressure of water at the temperature measured.

If the negative pressure at any section in the conduit drops below the water vapor pressure, the liquid vaporizes, and vast numbers of small vapor cavities form. These vapor bubbles are carried down the conduit with the flow. When a bubble reaches the region of higher pressure, the vapor condenses into liquid form and sudden collapse takes place. As the bubble collapses, water surrounding the bubble rushes in at great speed to fill the cavity. All this water collides in the cavity with a great deal of momentum, creating potentially damaging pressure. The process is referred to as *cavitation* and was addressed in reference to pipelines in Section 4.2.

Under normal conditions, atmospheric pressure is equivalent to a 10.3-m (33.8-ft) water column height. Therefore, the maximum distance between the crown (highest point in the siphon) and the water surface elevation in the reservoir is limited to approximately 8 m [Figure 8.14 (a)]. The difference (10.3 m – 8.0 m = 2.3 m) accounts for the vapor pressure head, the velocity head, and the head losses between the reservoir and crown.

Example 8.4

The siphon spillway depicted in Figure 8.15 was primed during a flood event and has lowered the reservoir level considerably. However, it is still operating under pressure flow. The 40-m-long siphon has a constant cross section of 1 m by 1 m. The distance between the entrance and the crown is 10 m, the friction factor (*f*) is 0.025, the inlet loss coefficient is 0.1, the bend loss coefficient (at the crown) is 0.8, and the exit loss coefficient is 1.0. Determine the discharge and the pressure head at the crown section.

Solution

The energy relationship between point 1 (the upstream reservoir) and point 2 (the outlet) may be written as

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + 0.1 \frac{V^2}{2g} + 0.8 \frac{V^2}{2g} + 0.025 \left(\frac{L}{D}\right) \frac{V^2}{2g} + 1.0 \frac{V^2}{2g}$$

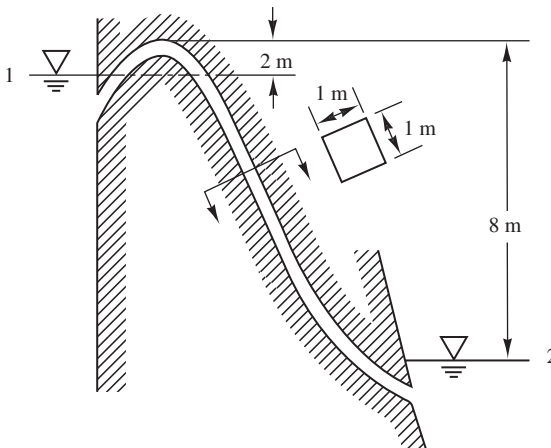


Figure 8.15 Siphon spillway

The last four terms on the right side of the equation represent the energy losses: entrance, bend, friction, and exit, respectively. In this case, $V_1 = V_2 = 0$, $P_1/\gamma = P_2/\gamma = 0$, $z_1 = 6$ m, and $z_2 = 0$, and V is the siphon velocity. Thus, the above equation may be simplified to

$$6 = \left[1 + 0.1 + 0.8 + 0.025 \left(\frac{40}{1} \right) \right] \frac{V^2}{2g}$$

$$V = 6.37 \text{ m/sec}$$

Hence, the discharge Q is

$$Q = AV = (1)^2(6.37) = 6.37 \text{ m}^3/\text{sec}$$

The energy relationship between point 1 (the reservoir) and point C (the crown) may be expressed as

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + 6 = \frac{V_c^2}{2g} + \frac{P_c}{\gamma} + 8 + 0.1 \frac{V_c^2}{2g} + 0.025 \left(\frac{10}{1} \right) \frac{V_c^2}{2g} + 0.8 \frac{V_c^2}{2g}$$

This equation may be simplified because $V_1 = 0$, $P_1/\gamma = 0$, and $V_c = V = 6.37$ m/sec.

$$6 = \frac{(6.37)^2}{2(9.81)}(1 + 0.1 + 0.8 + 0.25) + \frac{P_c}{\gamma} + 8$$

Therefore, the pressure head at the crown section is

$$\frac{P_c}{\gamma} = -6.45 \text{ m}$$

8.9 Culverts

Culverts are hydraulic structures that provide passage of stream flow from one side of a road, highway, or railroad embankment to the other. They come in a variety of sizes, shapes (e.g., circular, box, arch), and materials; concrete and corrugated metal are the most common materials. Typically, the primary design objective is to determine the most economical culvert that will carry the design discharge without exceeding an allowable upstream elevation.

The major components of a culvert include the inlet, the pipe barrel, the outlet, and an outlet energy dissipater, if necessary. Inlet structures protect embankments from erosion and improve the hydraulic performance of culverts. Outlet structures are designed to protect culvert outlets from scouring.

Although culverts appear to be simple structures, the hydraulics can be complex and involve the principles of pressure pipe flow, orifice flow, and open-channel flow. The hydraulic operation of culverts may be grouped into four flow classifications (Figure 8.16) that represent the most common design conditions.

- (a) submerged inlet and submerged outlet producing (pressure) pipe flow,
- (b) submerged inlet with full pipe flow but unsubmerged (free-discharge) outlet,
- (c) submerged inlet with partially full (open-channel) pipe flow, and
- (d) unsubmerged inlet and outlet producing open-channel flow throughout.

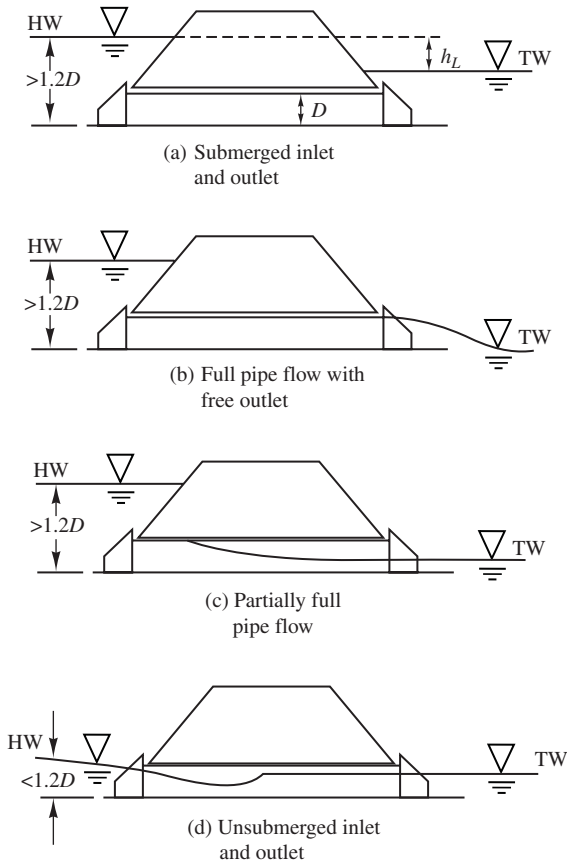


Figure 8.16 Common culvert flow classifications

The hydraulic principles used to analyze these four classifications of culvert flow are described in the following paragraphs.

- (a) Submergence of culvert outlets [Figure 8.16 (a)] may be the result of inadequate drainage downstream or large flood flows in the downstream channel. In this case, the culvert discharge is primarily affected by the tailwater (TW) elevation and the head loss of the flow through the culvert, regardless of the culvert slope. The culvert flow can be treated as a pressure pipe, and the head loss (h_L) is the sum of the entrance loss (h_e), friction loss (h_f), and exit loss (h_d):

$$h_L = h_e + h_f + h_d \tag{8.17a}$$

Substituting Equations 3.34, 3.28 (where $S = h_f/L$), and 3.37, we have in SI units,

$$h_L = k_e \left(\frac{V^2}{2g} \right) + \frac{n^2 V^2 L}{R_h^{4/3}} + \frac{V^2}{2g} \tag{8.17b}$$

and in BG units,

$$h_L = k_e \left(\frac{V^2}{2g} \right) + \frac{n^2 V^2 L}{2.22 R_h^{4/3}} + \frac{V^2}{2g} \tag{8.17c}$$

Approximate values for entrance coefficients are $k_e = 0.5$ for a square-edged entrance and $k_e = 0.2$ for a well-rounded entrance. Common values for the Manning roughness coefficient are $n = 0.013$ for concrete pipes and $n = 0.024$ for corrugated metal pipes. Since energy principles govern, the head loss can be added to the TW elevation to obtain the headwater (HW) elevation as shown in Figure 8.16 (a), or

$$\text{HW} = \text{TW} + h_L$$

In a true design situation, a culvert must be sized to convey a certain discharge (design flow) without exceeding a specified headwater elevation. In this case, Equation 8.17b is rearranged to express a direct relationship between the discharge and the dimensions of the culvert for a given elevation difference (h_L) between the tailwater and headwater. For a circular culvert (in SI units),

$$h_L = \left[k_e + \left(\frac{n^2 L}{R_h^{4/3}} \right) (2g) + 1 \right] \frac{8Q^2}{\pi^2 g D^4} \tag{8.18}$$

where Q is the discharge, D is the diameter, and R_h is the hydraulic radius of the culvert barrel. The hydraulic radius is $D/4$ for pipes flowing full. For culverts with noncircular cross sections, the head loss may be calculated by Equation 8.17b with the corresponding hydraulic radius calculated by dividing the cross-sectional area (A) by the wetted perimeter (P).

- (b) If the flow rate conveyed by a culvert has a normal depth that is larger than the barrel height, then the culvert will flow full even if the tailwater level drops below the outlet crown [Figure 8.16 (b)]. The discharge is controlled by the head loss and the HW elevation. The hydraulic principles are the same as discussed above for condition (a)—that is, the energy equation is appropriate and the head loss is found using the same expressions. However, in condition (a), the head loss is added to the tailwater elevation to obtain the headwater elevation. In this case, the head loss is added to the outlet crown elevation. Based on model and full scale studies done by the Federal Highway Administration (FHWA), the flow actually exits the pipe barrel somewhere between the crown and critical depth. For our purposes, the outlet crown will be used and represents a conservative estimate.
- (c) If the normal depth is less than the barrel height, with the inlet submerged and free discharge at the outlet, then a partially full pipe flow condition will normally result, as illustrated in Figure 8.16 (c). The culvert discharge is controlled by the entrance conditions (headwater, barrel area, and edge conditions). The discharge can be calculated by the orifice equation

$$Q = C_d A \sqrt{2gh} \tag{8.19}$$

where h is the hydrostatic head above the center of the pipe opening (orifice) and A is the cross-sectional area. C_d is the coefficient of discharge; common values used in practice are $C_d = 0.60$ for a square-edged entrance and $C_d = 0.95$ for a well-rounded entrance.

- (d) When the hydrostatic head at the entrance is less than $1.2D$, air will break into the barrel and the culvert will no longer flow under pressure. In this case, the culvert slope and the barrel wall friction dictate the flow depth as they do in the open-channel flow regime. Although several flow situations can occur, two situations are the most common. If the culvert slope is steep, flow passes through critical depth at the entrance and quickly attains normal (supercritical) depth in the culvert barrel. If the culvert slope is mild, then the flow depth will approach normal (subcritical) depth in the culvert barrel and pass through critical depth at the end of the barrel if the tailwater is low. If the tailwater is higher than critical depth, then flow depths can be computed by applying the water surface profile procedures developed for open channels in Chapter 6.

The FHWA* classifies the various culvert flow regimes into two types of flow control: *inlet control* and *outlet control*. Basically stated, if the culvert barrel can pass more flow than the entrance allows into the culvert, it is considered inlet control. If the culvert entrance allows more flow into the barrel than it can convey, it is considered outlet control. Flow classification (a) and (b) above are outlet control, and flow classification (c) is inlet control. Note that the culvert capacity equation in flow classification (c) is not affected by barrel length, roughness, or tailwater depth because only the entrance conditions limit capacity. Flow classification (d) can be either inlet control (steep slope) or outlet control (mild slope). FHWA Hydraulic Design Series 5 contains culvert hydraulic principles, equations, nomographs, and computer algorithms (which are included in many proprietary and nonproprietary software packages) to analyze and design roadway culverts. Despite the variety of culvert shapes and materials, and the complexity and variety of flow situations, the fundamental principles of culvert hydraulics have been covered in the preceding discussion.

Example 8.5

The design flow for a proposed corrugated steel culvert is $5.25 \text{ m}^3/\text{sec}$. The maximum available headwater (H) is 3.2 m above the culvert invert (inside bottom) as shown in Figure 8.17. The culvert is 40 m long and has a square-edged entrance and a slope of 0.003. The outlet is not submerged (free discharge). Determine the required diameter.

Solution

Flow classification (a) is not possible because the outlet is not submerged. Flow classification (d) is not possible because the entrance is likely to be submerged. Therefore, the pipe will be sized for flow classifications (b) and (c).

Assuming full pipe flow or flow classification (b), the energy balance for this culvert (Figure 8.17) may be expressed as

* J. M. Normann, R. J. Houghtalen, and W. J. Johnston, *Hydraulic Design of Highway Culverts*, 2nd ed., Hydraulic Design Series No. 5. Washington, DC: U.S. Department of Transportation, Federal Highway Administration (May 2005).

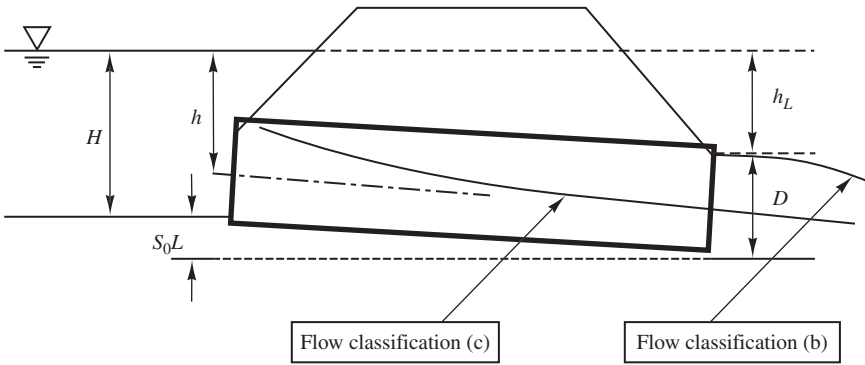


Figure 8.17 Corrugated steel culvert flow profile

$$\begin{aligned}
 H + S_0L &= D + h_L \\
 h_L &= H + S_0L - D \\
 h_L &= 3.2 + 0.003(40) - D \\
 h_L &= 3.32 - D
 \end{aligned}$$

where the tailwater depth is assumed to equal D , the diameter of the culvert. Also, from Equation 8.18, we have

$$\begin{aligned}
 h_L &= \left(K_e + \frac{n^2L}{R_h^{4/3}}(2g) + 1 \right) \frac{8Q^2}{\pi^2gD^4} \\
 h_L &= \left[0.5 + \frac{(0.024)^2(40)}{(D/4)^{4/3}} \{2(9.81)\} + 1 \right] \frac{8(5.25)^2}{\pi^2(9.81)D^4}
 \end{aligned}$$

Equating the two head loss equations from above results in

$$D + \left(1.5 + \frac{2.87}{D^{4/3}} \right) \left(\frac{2.28}{D^4} \right) = 3.32$$

This implicit equation is solved, yielding $D = 1.41$ m.

Assuming partially full pipe flow or flow classification (c), the discharge is controlled by the entrance condition only. In this case, the head (h) is measured above the centerline of the pipe, and we have

$$h + \frac{D}{2} = 3.2$$

or

$$h = 3.2 - \frac{D}{2}$$

Now we can substitute this expression for head into Equation 8.19 for orifice flow:

$$Q = C_d A \sqrt{2gh} = C_d (\pi D^2/4) \sqrt{2gh}$$

$$5.25 = 0.60 (\pi D^2/4) \sqrt{2(9.81)(3.2 - D/2)}$$

This expression yields $D = 1.25$ m.

We have obtained two different pipe diameters, but which one represents the required size? Assuming full pipe flow, we have determined that a pipe of 1.41 m is required to pass the design flow through the barrel (i.e., outlet control). Assuming partially full pipe flow, we have determined that a pipe of 1.25 m is required to get the design flow into the barrel (i.e., inlet control). Therefore, the required diameter is 1.41 m, and the culvert will operate under outlet control according to flow classification (b). Culvert barrels come in standard sizes, so it is likely that a diameter of 1.5 m will be used.

8.10 Stilling Basins

When the water velocity at the outlet of a hydraulic structure is high, the excessive amount of kinetic energy carried by the flow may be damaging to the receiving channel and even undermine the outlet of the hydraulic structure. This situation often occurs at the end of a spillway where water is highly accelerated and direct disposal in the downstream channel could produce enormous erosion. To avoid damage, numerous *energy dissipators* are available. A *stilling basin* is an effective energy dissipater that produces a controlled hydraulic jump. Much of the damaging energy is lost in the transition from supercritical to subcritical flow, as was discussed in Chapter 6. The stilling basin may be either horizontal or inclined to match the slope of the receiving channel. In either case, it should provide obstructions and friction forces sufficient to overcome the gravitational forces so that the flow is decelerated and a hydraulic jump is produced within the confines of the stilling basin.

The relationship between the energy to be dissipated and the depth of flow in the stilling basin is contained in the Froude number (V/\sqrt{gD}), as will be discussed in Section 10.4. Recall that the Froude number was defined in Equation 6.12, where D is the hydraulic depth and $D = y$ for rectangular channels. Generally speaking, no special stilling basin is needed when the flow from the outlet of a hydraulic structure has a Froude number less than 1.7. As the Froude number increases, energy dissipators such as baffles, sills, and blocks may be installed along the floor of the basin to enhance the kinetic energy reduction within the limited basin length. The U.S. Bureau of Reclamation (USBR) has developed a comprehensive set of curves to define the dimensions of the stilling basin and the various types of energy dissipators contained therein. These curves, which are based on extensive experimental data, are shown in Figures 8.18, 8.19, and 8.20. Selection of the appropriate stilling basin from the three different designs (types IV, III, and II) is based on the entry Froude number and velocity, as mentioned in the figure captions. Note that the USBR uses d instead of y to denote flow depth with subscripts 1 (flow entering the basin) and 2 (flow leaving the basin). Also, F is used instead of N_F for the Froude number.

The design of effective stilling basins requires that special attention be given to the downstream depth. Recall from Chapter 6 that tailwater (TW) depth dictates the type and location of the hydraulic jump. In the case of a stilling basin, the actual TW depth will be dictated by the downstream channel characteristics and the spillway design flow. As an approximation, the Manning equation can be used, assuming the flow is uniform in the downstream channel. Once TW depth is determined, the floor of the stilling basin needs to be adjusted so that the TW/d_2 ratio given in Figures 8.18 to 8.20 will be satisfied. Again, recall from Chapter 6 that d_2 is the depth of flow

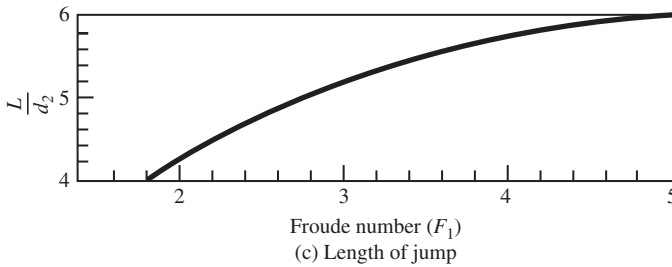
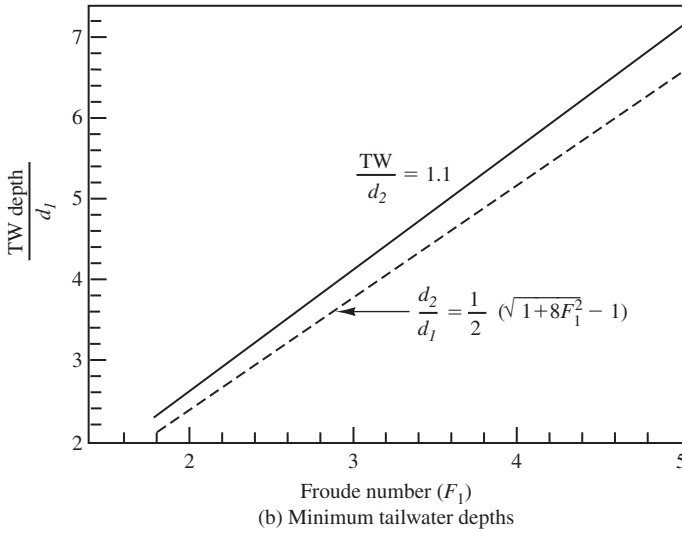
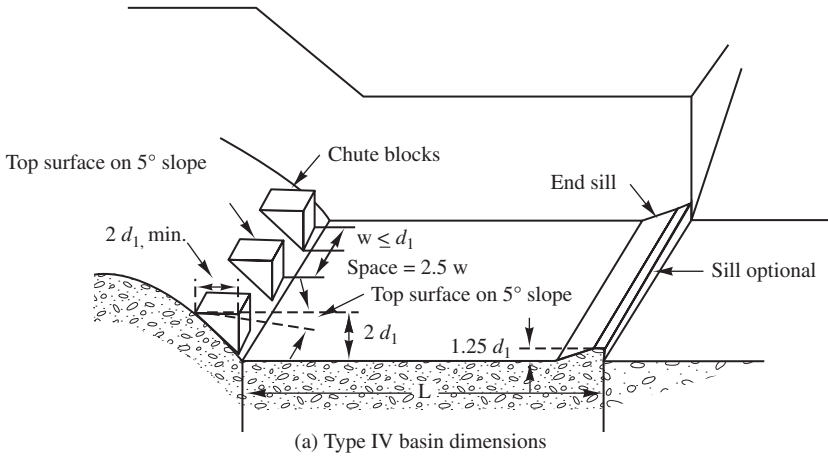
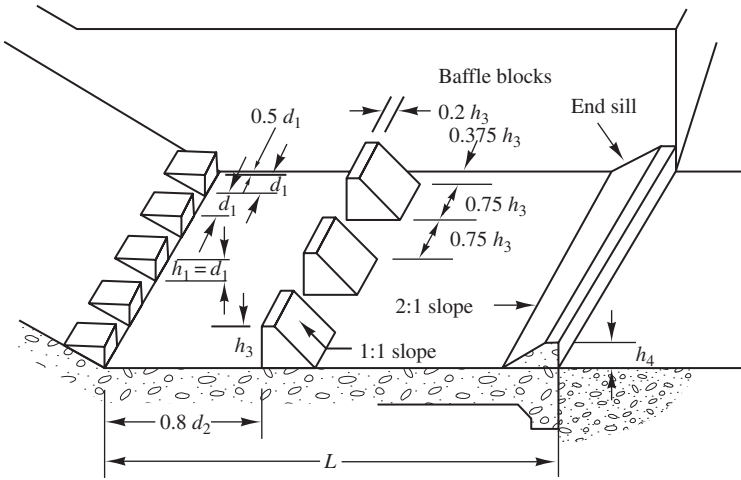
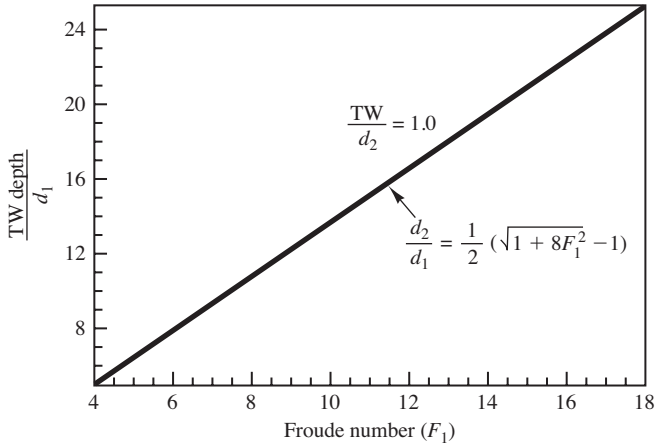


Figure 8.18 U.S. Bureau of Reclamation Type IV energy dissipator (for approaching Froude number between 2.5 and 4.5). *Source:* Courtesy of U.S. Bureau of Reclamation.

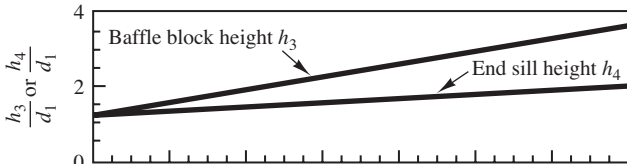
leaving the hydraulic jump. If the TW/d₂ ratio is not satisfied, then the jump may move out of the basin (if the TW is too low) or it may occur on the spillway (if the TW is too high). Ideally, the stilling basin performance should be checked for flows other than the design discharge.



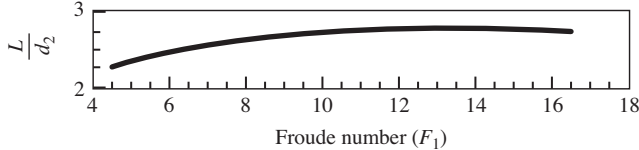
(a) Type III basin dimensions



(b) Minimum tailwater depths



(c) Height of baffle blocks and end sill



(d) Length of jump

Figure 8.19 U.S. Bureau of Reclamation Type III energy dissipator (for approaching Froude number above 4.5 and approaching velocity less than 20 m/sec).

Source: Courtesy of U.S. Bureau of Reclamation.

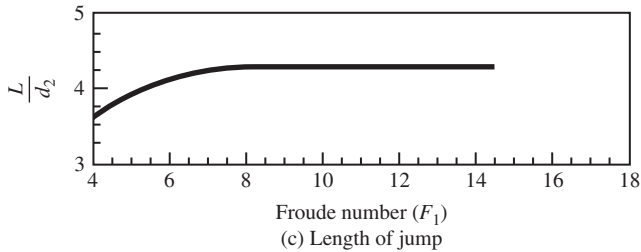
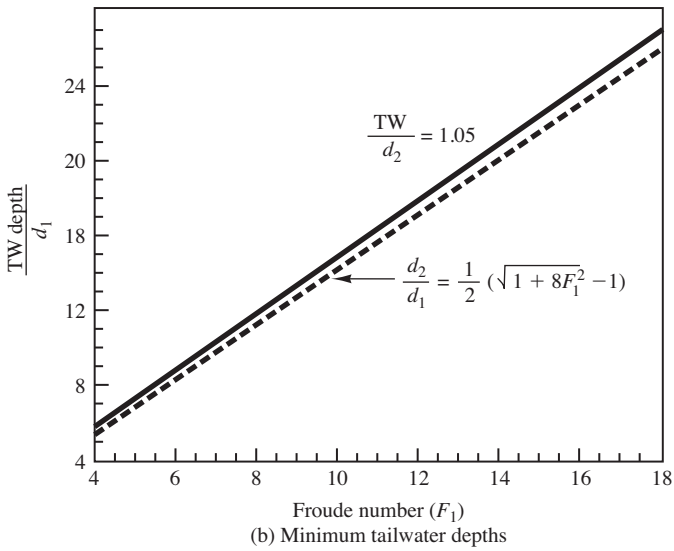
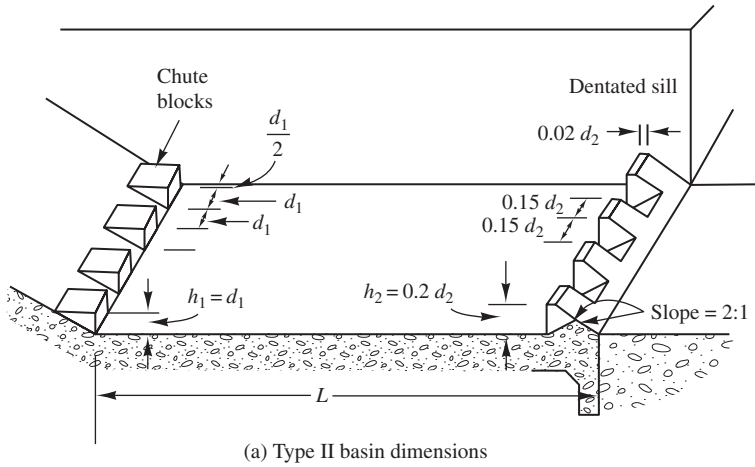


Figure 8.20 U.S. Bureau of Reclamation Type II energy dissipator (for approaching Froude number above 4.5). *Source:* Courtesy of U.S. Bureau of Reclamation.

PROBLEMS
(SECTION 8.3)

8.3.1. A gravity dam is depicted in Figure P8.3.1. If a force ratio against sliding of 1.3 is required, is the 33-m-high dam safe? Assume the coefficient of friction between the dam base and the foundation is 0.6, the specific gravity of concrete is 2.5, and full uplift forces exist on the base of the dam. Neglect earthquake and sedimentation forces.

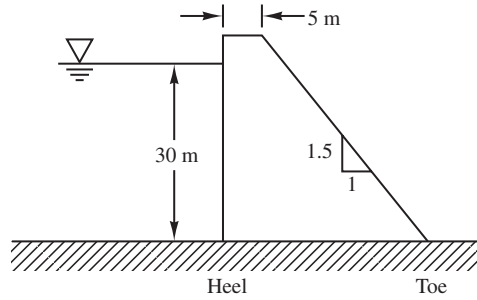


Figure P8.3.1

8.3.2. A gravity dam is depicted in Figure P8.3.1. A force ratio of 2.0 against overturning is required. Determine whether the dam is safe if the water level rises to the top of the 33-m-high dam during a flood event. Assume the specific gravity of concrete is 2.5, and full uplift forces exist on the base of the dam. Neglect earthquake and sedimentation forces.

8.3.3. Typical force ratios for a gravity dam are 2.0 for overturning and 1.2 to 1.5 for sliding. Determine if these requirements are met in the gravity dam depicted in Figure P8.3.3. Assume that the uplift force takes a triangular distribution with maximum magnitude one-third that of the hydrostatic pressure at the heel and zero at the toe. The reservoir is full (design capacity) with 3 ft of freeboard, and the concrete has a specific gravity of 2.65. The coefficient of friction between the dam base and the foundation is 0.65. Neglect earthquake and sedimentation forces.

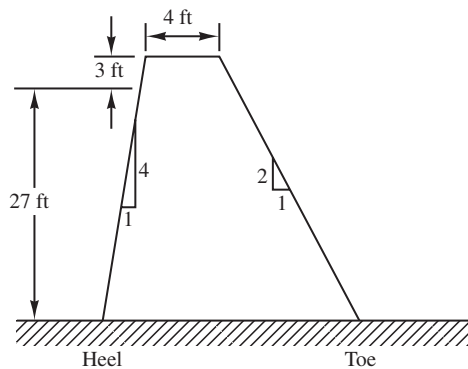


Figure P8.3.3

8.3.4. The specific gravity of the dam shown in Figure P8.3.4 is 2.63, and the coefficient of friction between the dam and the foundation is 0.53. The depth of the water is 27.5 m when the reservoir is filled to design capacity. Assume that the uplift force has a triangular distribution with the maximum magnitude of 60 percent of the full hydrostatic pressure at the heel of the dam. Determine the force ratios against sliding and overturning. Neglect earthquake and sedimentation forces.

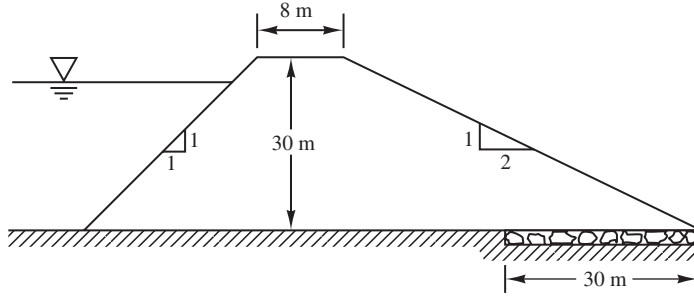


Figure P8.3.4

- 8.3.5. Determine the foundation pressure at the heel and toe of the dam in Problem 8.3.1.
- 8.3.6. For the concrete gravity dam shown in Figure P8.3.3, compute the foundation bearing pressure at the heel and toe. Assume that the uplift force takes a triangular distribution with maximum magnitude one-third that of the hydrostatic pressure at the heel and zero at the toe. The reservoir is full to its designed elevation with 3 ft of freeboard, and the masonry has a specific gravity of 2.65.
- 8.3.7. Prove that if the resultant vertical force (R_v) passes through the middle one-third of the base in a concrete, gravity dam, none of the concrete along the base of the dam will be in tension.
- 8.3.8. A V-canyon supports a 100-ft high, constant-angle (120°) arch dam. If the canyon is 60 feet wide at the top and the design freeboard is 6 ft (i.e., the water level is 6 feet below the top of the dam), determine the abutment reactions at dam heights of 25, 50, and 75 feet above the bottom of the canyon.
- 8.3.9. A constant-angle arch dam ($\theta = 150^\circ$) is designed to span a vertical-walled canyon 150 m wide. The height of the dam is 78 m, including 3 m of freeboard. If the dam has 4 m thickness at the crest and has a symmetrical cross section that increases its thickness to 11.8 m at the base, determine the masonry stress in the dam using the cylinder method (constant radii) for approximate analysis at
 - (a) the crest,
 - (b) the midheight, and
 - (c) the dam base.

(SECTION 8.5)

- 8.5.1. A weir 1.05 m in height is built across the floor of a 4-m-wide rectangular channel. If the water depth just upstream of the weir is 1.52 m, what is the water depth on the crest and the discharge in the channel? Neglect friction loss and the velocity head upstream.
- 8.5.2. It is necessary to obtain the elevation of the crest of a weir in a channel without stopping the flow. The water surface elevation upstream of the weir is 96.1 ft above mean sea level (MSL). Determine the elevation of the crest of the weir above MSL if the weir is 4.90 ft wide and discharges $30.9 \text{ ft}^3/\text{sec}$. Assume the weir friction loss and the velocity head upstream are negligible.
- 8.5.3. A 3.05-m-wide frictionless weir rises 1.10 m above the channel bottom. The water depth upstream of the weir is 1.89 m. Determine the discharge in the channel by two different equations and the velocity of the water going over the weir. Assume the upstream velocity head is negligible.
- 8.5.4. Verify the following:
 - (a) that Equation 6.14 can be obtained from Equation 6.11;
 - (b) that the discharge coefficient for the weir equation using SI units (Equation 8.8c) is indeed 1.70; and
 - (c) that the discharge coefficient would be lower if the weir was not assumed to be frictionless.

- 8.5.5.** A weir 0.78 m in height is built across the floor of a 4-m-wide rectangular channel. If the water depth just upstream of the weir is 2.00 m, what is the flow rate in the channel? Neglect friction loss and the velocity head upstream. Also determine the flow rate if the velocity head is not neglected.
- 8.5.6.** A rectangular channel carries a discharge of $2.00 \text{ m}^3/\text{sec}$ per meter of channel width over a weir crest that is 1.40 m in height. The energy grade line upstream of the weir measures 2.70 m above the channel bottom. Determine the true coefficient of discharge in the weir equation, considering the energy losses. Also determine the amount of the energy loss in meters.
- 8.5.7.** A weir is being considered for flow measurement in a long, concrete irrigation canal with a bottom slope of 0.002. The maximum depth of flow in the 15-ft-wide canal is 6 feet, but the channel contains an additional 4 feet of freeboard. Determine the maximum height of a flow-measurement weir before the backwater exceeds the freeboard, assuming the weir is frictionless.

(SECTION 8.6)

- 8.6.1.** A 31.2-ft-wide overflow spillway is subject to a head of 3.25 feet under design conditions. The coefficient of discharge is 3.42 based on model studies.
- Determine the spillway discharge, assuming a negligible approach velocity.
 - Determine the spillway discharge under flood conditions if the head increases to 4.10 ft and the discharge coefficient increases to 3.47.
 - Approximate the spillway discharge under flood conditions (considering the approach velocity) if the dam is 30 feet high (i.e., the spillway crest). Use the flood flow from part b to determine the approach velocity, which is why the solution is approximate.
- 8.6.2.** Assume the spillway in Example 8.2 is 6 meters high. Recompute the static head on the spillway by accounting for the approach velocity. What percent error was introduced in the static head by ignoring the approach velocity?
- 8.6.3.** Determine the maximum approach velocity (as a function of H_0) that can occur before an error of 2 percent enters into the calculation for spillway discharge. Compute the function in British units (i.e., $g = 32.2 \text{ ft}/\text{sec}^2$).
- 8.6.4.** A 21-m-wide overflow spillway has a discharge coefficient of 1.96 at flood stage. Flood stage occurs at a static head of between 1.7 m and 3.1 m. The spillway is 15 m high. Determine the maximum discharge the spillway is able to pass; (a) ignore the approach velocity and (b) include the approach velocity.
- 8.6.5.** An overflow spillway is designed to discharge $214 \text{ m}^3/\text{sec}$ under a maximum head of 1.86 m. Determine the width and the profile of the spillway crest if the upstream and downstream slopes are 1:1. Assume $C = 2.22$.
- 8.6.6.** Determine the maximum discharge for a 104-ft-wide overflow spillway having an available head of 7.2 ft. Also determine the profile of the spillway crest having a vertical upstream slope and a 1.5:1 downstream slope. Assume $C = 4.02$.

(SECTION 8.7)

- 8.7.1.** In Example 8.3, the side-channel spillway passed through critical depth (5.01 ft) at the end of the channel. Five feet upstream, the depth was determined to be 7.73 ft in the horizontal (0 percent slope) channel. Determine the depth 5 ft upstream if the side-channel slope is 5 percent.
- 8.7.2.** Answer the following:
- What happens to the $\cos \theta$ term in Equation 8.13 (it does not appear in Equation 8.14)?
 - Verify that $F_f = \gamma AS_f \Delta x$. (*Hint:* See Chapter 6.)
 - Determine the largest channel slope that would keep S_0 within 1 percent of $\sin \theta$. (Recall that the assumption $S_0 = \sin \theta$ for a reasonably small angle was used in the derivation of Equation 8.14.)

- 8.7.3. The flow at the end of a 30-m-long side channel spillway is $36.0 \text{ m}^3/\text{sec}$. A 30-m-long overflow spillway, which is under a head of 0.736 m , contributes the flow to the side-channel spillway. If the side channel has a bottom width of 3 m ($n = 0.020$) and a bottom slope of 0.01 , what is the depth 10 m upstream from the end of the channel (where it passes through critical depth)?
- 8.7.4. The design engineer working on the spillway described in Example 8.3 would like to try an alternative design. Determine the depth 5 ft and 10 ft upstream if the overflow spillway is lengthened to 25 ft but still accommodates the design discharge ($637 \text{ ft}^3/\text{sec}$) uniformly along its entire length. In addition, the side-channel width is shortened to 8 ft .
- 8.7.5. A 90-m -long, overflow spillway ($C = 2.00$) operating under a head of 1.22 m contributes flow to a side-channel spillway. The rectangular side-channel spillway ($n = 0.015$) is 4.6 m wide and has a bottom slope of 0.001 . Define the water surface profile (at 30-m intervals) upstream from the location where the overflow spillway stops contributing discharge (where the depth is 9.80 m). Further down the channel, the flow passes through critical depth as shown in Figure P8.7.5.

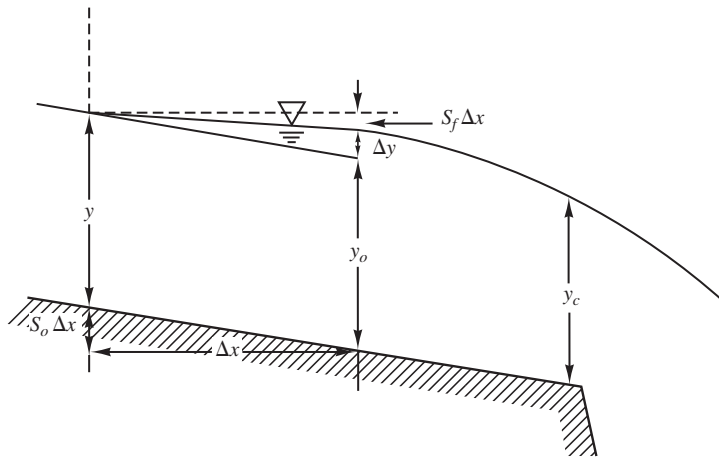


Figure P8.7.5

- 8.7.6. The overflow spillway of Example 8.2 discharges into a side-channel spillway ($n = 0.013$) with a horizontal bottom slope. The wall opposite the overflow spillway crest is vertical, and the depth of water at the exit end of the side channel is critical depth. Determine the depth of water at the beginning (upstream end) of the channel using 20-m (Δx) increments. The width of the channel bottom is 10 m .

(SECTION 8.8)

- 8.8.1. The reservoir level in Example 8.4 will continue to fall as the siphon empties the reservoir. If the water temperature is 20°C , what is the maximum elevation difference between the crown and the falling reservoir level before the pressure at the crown drops below the vapor pressure of water? Assume the flow rate remains constant as the reservoir empties (i.e., the downstream water level is falling at the same rate as the reservoir level).
- 8.8.2. Answer the following questions by referring to Example 8.4.
 - (a) If the water temperature is 20°C , what is the allowable (negative) pressure head before the onset of cavitation?

- (b) Should the bend loss at the crown be included in the computations for determining the pressure head at the crown?
- (c) The maximum difference between the upstream reservoir pool level and the siphon crown was stated to be approximately 8 m [Figure 8.14 (a)] to prevent cavitation. Is this “rule of thumb” applicable here?

- 8.8.3.** A rectangular siphon (3 ft by 6 ft) discharges into a pool with an elevation of 335 ft MSL. Determine the discharge when the siphon primes at an upstream pool elevation of 368 ft MSL if losses are 10.5 ft, excluding the exit loss. Also determine the pressure head at the siphon crown under these conditions if 3.5 ft of losses accrue before the crown. The siphon crown is at an elevation of 366.5 ft.
- 8.8.4.** A 60-m-long siphon spillway discharges water at the rate of $0.32 \text{ m}^3/\text{sec}$. The siphon crown is 1.2 m above the water surface elevation of the reservoir and a length of 10 m away from the siphon entrance. If the siphon diameter is 30 cm, the friction factor is 0.02, and the head loss coefficients are 0.2 (inlet) and 1.0 (exit), what is the elevation difference between the siphon crown and the downstream pool? What is the pressure at the crown in kN/m^2 ?
- 8.8.5.** A siphon spillway (Figure P8.8.5) with a cross-sectional area of 12 ft^2 is used to discharge water to a downstream reservoir 60 ft below the crest of the spillway. If the upstream reservoir level is 7.5 ft above the intake, what is the pressure head at the crest if the siphon has been primed? Assume the frictional head loss is equal to twice the velocity head and is evenly distributed throughout its length. The entrance and exit loss coefficients are 0.5 and 1.0, respectively, and the siphon crest is one-fourth the total length from the entrance. Also determine whether cavitation is a concern if the water temperature is 68°F .

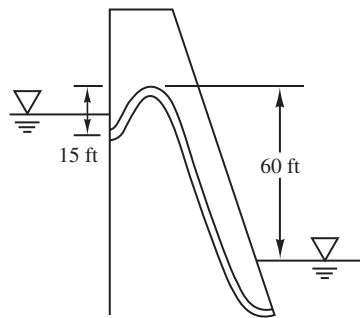


Figure P8.8.5

- 8.8.6.** Determine the required siphon diameter and maximum height of the siphon crest above the entrance given the following design conditions: $Q = 5.16 \text{ m}^3/\text{sec}$, K (entrance) = 0.25, K (exit) = 1.0, K (siphon bend) = 0.7, $f = 0.022$, siphon length = 36.6 m, length to crest = 7.62 m, upstream pool elevation = 163.3 m MSL, downstream pool elevation = 154.4 m MSL, and $(P/\gamma)_{max} = -10.1 \text{ m}$.
- 8.8.7.** A siphon spillway is designed to discharge $20 \text{ m}^3/\text{sec}$ with a head above the crest of h_s , a crest elevation of 30 m, and an outlet elevation of 0 m. The allowable gauge pressure at the crest is -8 m of water column during design flow. The crest section is followed in order by a vertical section, a 90° bend with centerline radius of curvature of 3 m, and a horizontal section at elevation 0 m. The distance from the entrance to the siphon crest is 3.2 m, the vertical section is 30 m long, and the vertical section to outlet distance is 15 m. If the siphon conduit has a Manning’s coefficient $n = 0.025$ and the coefficients of entrance and of the bends (combined) are, respectively, $K_e = 0.5$ and $K_b = 0.3$, what is the area of the siphon required to satisfy the given requirements?

(SECTION 8.9)

- 8.9.1.** In Example 8.5, a culvert diameter of 1.5 m may be required if this is the next largest standard size. As the design engineer, you would like to use a 1.25 m diameter culvert to save your client money. Would this smaller diameter culvert meet the design requirements if:
- (a) a well-rounded entrance was used on the 1.25-m pipe?
 - (b) a well-rounded entrance was used along with increasing the pipe slope to 1.0 percent while maintaining the maximum available headwater at 3.2 m?
- 8.9.2.** Derive Equation 8.19, the orifice equation, by balancing energy between points (1) and (2) in Figure P8.9.2. Initially, assume no energy losses. What does the variable h represent? What does the variable C_d represent?

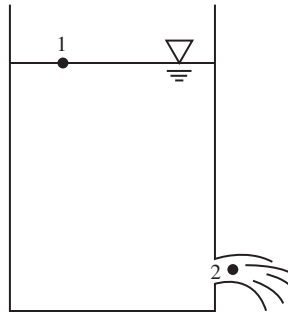


Figure P8.9.2

- 8.9.3.** Stage recorders have registered stream depths upstream (4.05 m) and downstream (3.98 m) of a culvert during a flood event. The 2 m by 2 m concrete culvert (square-edged entrance) is 15 m long and has a slope of 3.0 percent. Based on this information, determine the flood flow rate that passed through the culvert. Assume the stream depths are measured above the culvert inverts on the upstream and downstream ends.
- 8.9.4.** Determine the flow rate in Problem 8.9.3 if the outlet was not submerged. (*Hint:* Assume hydraulic flow classification (c). Once the flow rate is found, verify that it is flow classification (c) by checking normal depth.)
- 8.9.5.** A concrete culvert is required to convey a design flow of 654 cfs over a length of 330 ft. and a slope of 0.015 ft/ft. Because there is a large lake downstream, the tailwater elevation is constant at 526.4 ft MSL, which will submerge the culvert outlet. The headwater elevation cannot exceed 544.4 ft. MSL without overtopping the roadway embankment. Find the required diameter for (a) a single barrel culvert and (b) twin barrels, assuming a square-edged entrance.
- 8.9.6.** A 1.5-m-diameter culvert (concrete barrel, well-rounded entrance) that is 20 m long is installed on a slope of 2 percent. The design discharge is 9.5 m³/sec, and the inlet will be submerged but not the outlet. Determine the depth that will result upstream (above the invert) during the design flood event.
- 8.9.7.** Determine the size of a circular, corrugated metal culvert with these design conditions: a 60.0-m length, a 0.10-m/m slope, and a flow of 2.5 m³/sec. The outlet will be unsubmerged, but the inlet (square-edged) will be submerged with a headwater depth of 2.0 m above the culvert invert.
- 8.9.8.** A rectangular concrete culvert (square-edged entrance) is placed on a slope of 0.09 ft/ft. The culvert is 4.0 ft × 4.0 ft and 140 ft long. The tailwater level is 2.0 ft below the culvert crown at the outlet. Determine the discharge if the headwater level is (a) 1.5 ft above the crown at the inlet, (b) coincident with the crown, and (c) 1.5 ft below the crown.

(SECTION 8.10)

- 8.10.1.** A horizontal rectangular stilling basin (USBR type III) is used at the outlet of a spillway to dissipate energy. The spillway discharges $350 \text{ ft}^3/\text{sec}$ and has a uniform width of 35 ft. At the point where the water enters the basin, the velocity is 30 ft/sec. Compute the following:
- (a) the sequent depth of the hydraulic jump,
 - (b) the length of the jump,
 - (c) the energy loss in the jump, and
 - (d) the efficiency of the jump defined as the ratio of specific energy after the hydraulic jump to the specific energy before the jump.
- 8.10.2.** A spillway carries a discharge of $22.5 \text{ m}^3/\text{sec}$ with the outlet velocity of 15 m/sec at a depth of 0.2 m. Select an adequate USBR stilling basin and determine the sequent depth, the length of the hydraulic jump, and the energy loss.
- 8.10.3.** An increase in discharge through the spillway in Problem 8.10.2 to $45 \text{ m}^3/\text{sec}$ will increase the spillway outlet depth to 0.25 m. Select an adequate USBR stilling basin and determine the sequent depth, the length, the energy loss, and the efficiency of the hydraulic jump (defined as the ratio of specific energy after the hydraulic jump to the specific energy before the jump).

9



Water Pressure, Velocity, and Discharge Measurements

Measurement of water pressure, velocity, and discharge provides fundamental data for analysis, design, and operation of every hydraulic system. A wide variety of measurement devices and methods are available for use in the laboratory and the field. The devices used to determine pressure, velocity, and discharge are based on the fundamental laws of physics and fluid mechanics. In general, each measurement device is designed to perform under certain conditions; hence, there are limitations to each device. Proper selection of a measurement device for a particular application should be based on an understanding of the fundamental principles that are discussed in this chapter. Details about the installation and operation of specific measurement devices can be found in specialized literature such as the fluid meter publications of the American Society of Mechanical Engineers (ASME) and manufacturers' literature.

9.1 Pressure Measurements

Pressure at any point in a liquid is defined as the normal force exerted by the liquid on a unit surface area. A common method of access to measure this force in a vessel is through a hole or opening in the boundary wall. If a vertical tube is connected to the opening, then the height that the contained water rises to in the tube represents the pressure head (P/γ). For typical pressure ranges in hydraulic applications, the height of piezometers becomes impractical and manometers can be used instead. Manometers are U-shaped transparent tubes that can make use of a dense gauge fluid that is immiscible with water. Manometry principles were covered in Chapter 2.

Manometers are capable of detecting pressure in stationary and moving liquids. When the water in a vessel is stationary, the manometer reading reflects the hydrostatic pressure at the opening in the boundary wall. If the water is moving in the vessel, then the pressure at the opening decreases with increasing flow velocity at the opening. The amount of pressure decrease can be calculated by Bernoulli's principle.

It is very important that the openings in the boundary wall satisfy certain characteristics so that the true water pressure can be registered. The openings must be flush with the surface and normal to the boundary. Figures 9.1 (a) and (b) schematically display various correct and incorrect openings for pressure measurement of moving liquids. The plus signs (+) indicate that the opening registers a higher than actual pressure value, and the minus signs (-) indicate that the opening registers a lower than actual pressure value. To eliminate the irregularities and variations that might cause significant errors, multiple pressure openings can be constructed at a given cross section in a closed conduit. For example, the multiple openings can be connected to a single manometer column that would register an average pressure at the cross section. This multiopening system is effective in sections of relatively straight pipe where the velocity profiles are reasonably symmetric and the pressure difference existing between one side of the pipe and the other is very small. If a large pressure difference exists between any of the openings, then measurement error can develop because water may flow out of the higher-pressure opening and into the lower-pressure opening through the manometer. Caution must be taken to ensure that no net flow occurs through any pressure openings.

Other devices are available to measure water pressure. For example, to increase pressure-measurement sensitivity, inclined manometers may be used in which a small change in pressure can drive the indicator fluid a great distance along the sloped manometer tube (Figure 9.2). Differential manometers measure pressure differences between two vessels and were discussed in Chapter 2. Bourdon tube gauges are semimechanical devices. They contain a curved tube that is sealed on one end and connected to the pressurized water through the vessel wall opening on

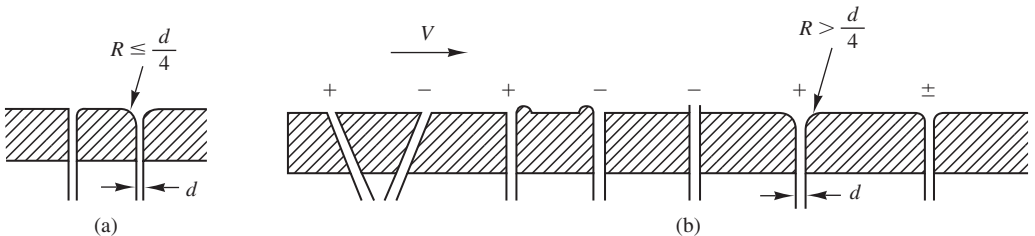


Figure 9.1 Pressure openings: (a) correct connections and (b) incorrect connections

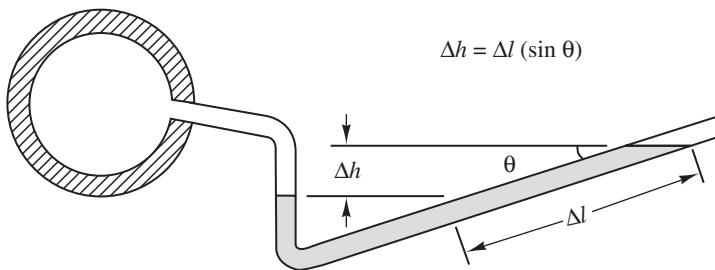


Figure 9.2 Inclined manometer

the other end. An increase in pressure within the tube causes it to straighten slightly, which is reflected on an analog scale or a digital readout.

Manometer systems and analog Bourdon tube gauges may be used to measure pressure in water under relatively steady flow conditions. However, they are not suitable for applications in time-varying flow fields that require high-frequency response both in probes and in recording systems. Electronic pressure cells or transducers (transmitters) are available commercially for these applications. Generally, these devices convert strain on a diaphragm caused by the water into an electrical signal proportional to the pressure. Digital readouts over time can be captured using computer software for operational control or assessment. An abundance of literature from the manufacturers of these devices is available on the Internet.

Example 9.1

In Figure 9.3, water is flowing in the pipe, and mercury (sp. gr. = 13.6) is the manometer fluid. Determine the pressure in the pipe in psi and in inches of mercury.

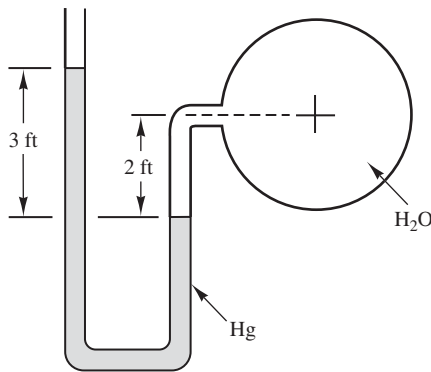


Figure 9.3 Mercury manometer

Solution

A horizontal surface of equal pressure can be drawn from the mercury–water interface across to the mercury side of the manometer. Equal pressures exist at both locations because (1) we have the same liquid (mercury), (2) both locations are at the same elevation, and (3) the mercury is interconnected. (Review Section 2.3: Surfaces of Equal Pressure.) Based on manometry principles,

$$\begin{aligned}
 (3 \text{ ft})(\gamma_{Hg}) &= P + (2 \text{ ft})(\gamma) \\
 (3 \text{ ft})(13.6)(62.3 \text{ lb/ft}^3) &= P + (2 \text{ ft})(62.3 \text{ lb/ft}^3) \\
 P &= 2,420 \text{ lb/ft}^2 = 16.8 \text{ psi}
 \end{aligned}$$

Pressure can be expressed as the height of any fluid. For mercury,

$$\begin{aligned}
 h &= P/\gamma_{Hg} = (2,420 \text{ lb/ft}^2)/[(13.6)(62.3 \text{ lb/ft}^3)] \\
 h &= 2.86 \text{ ft of Hg (34.3 inches)}
 \end{aligned}$$

9.2 Velocity Measurements

Water velocity in every conduit varies from near-zero values close to the stationary boundary to a maximum value near midstream. It is interesting to measure the velocity distribution in pipes and open channels. This is done by making local measurements at several positions in a cross

section. The measurements should only be made with velocity probes of small size so that local flow patterns will not be disturbed by the presence of the probe in the flow field. Instruments commonly used for velocity measurement are *Pitot tubes* and *current meters*.

Pitot tubes* are hollow tubes bent to measure pressure in the flowing stream. The probe usually consists of two tubes that are bent in such a way that the open end of one tube is perpendicular to the velocity vector, and the other is parallel to the flow as depicted in Figure 9.4 (a). To facilitate measurements, the two tubes are usually combined into one concentric construction—that is, a smaller tube is inside a large one [Figure 9.4 (b)].

At the probe tip (0), a stagnation point is produced where the velocity is zero. The pressure sensed at this opening is the stagnation point pressure, or *stagnation pressure*. At the side openings (1), the flow velocity (V) is practically undisturbed. These openings sense the static (or ambient) pressure at the site.

Applying Bernoulli's equation between the two positions, 0 and 1, and neglecting the small vertical distance in between, we may write

$$\frac{P_0}{\gamma} + 0 = \frac{P_1}{\gamma} + \frac{V^2}{2g}$$

It is evident that the stagnation pressure head (P_0/γ) is a combination of the static pressure head (P_1/γ) and the dynamic pressure head ($V^2/2g$) (i.e., the conversion of the velocity head to pressure head at the probe tip). From this expression, the flow velocity can be determined:

$$V^2 = 2g \left(\frac{P_0 - P_1}{\gamma} \right) = 2g \left(\frac{\Delta P}{\gamma} \right) \quad (9.1a)$$

or

$$V = \sqrt{2g(\Delta P/\gamma)} \quad (9.1b)$$

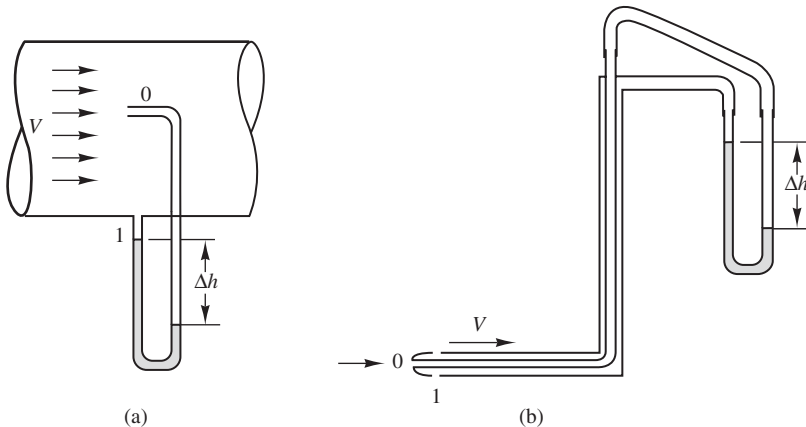


Figure 9.4 Schematic representation of a Pitot tube: (a) separated tubes and (b) combined tubes

* Henri de Pitot (1695–1771) first used an open-ended glass tube with a 90° bend to measure velocity distribution in the Seine River. The rise of water elevation in the vertical part of the tube indicated stagnation pressure. Pitot did not apply Bernoulli's principle to obtain the correct velocity as discussed in this section.

The quantity $\Delta P/\gamma$ indicates the pressure head difference between the two openings in the probe. It is a function of the liquid column displacement (Δh) in the manometer shown in Figure 9.4. The pressure difference (ΔP) is found using manometry principles that were discussed in Chapter 2 and are reviewed next in Example 9.2.

Pitot tubes are widely used to measure the pressure and the velocity of flowing water. They are both reliable and accurate because they involve a simple physical principle and a simple setup. The Pitot tube is very useful for measuring water velocities under conditions in which the exact direction of the stream flow cannot be determined. In these cases, misalignments of the probe in the flow are likely to occur. The *Prandtl Pitot tube* shown in Figure 9.4 (b) has approximately a 1 percent error at an angle of 20° to the direction of the stream flow.

The outer diameter of a Pitot tube is typically small—say, 5 mm. The two pressure tubes inside are much smaller. Because of the small diameters of these tubes, caution must be used to keep air bubbles from becoming trapped inside. Surface tension at the interface can produce a significant effect in the small tubes and yield unreliable readings.

Example 9.2

A Pitot tube is used to measure velocity at a certain location in a water pipe. The manometer indicates a pressure difference (column height) of 14.6 cm. The indicator fluid has a specific gravity of 1.95. Compute the velocity.

Solution

Referring to Figure 9.4 (b), let x be the distance from position 1 to the interface between the water and manometry fluid (left side) and let γ_m be the specific weight of the manometry fluid. Applying manometry principles from Chapter 2 yields

$$P_1 - \gamma x + \gamma_m \Delta h - \gamma \Delta h + \gamma x = P_o$$

or

$$P_o - P_1 = \Delta P = \Delta h(\gamma_m - \gamma)$$

Substituting this into Equation 9.1a,

$$\begin{aligned} V^2 &= 2g\Delta h \left(\frac{\gamma_m - \gamma}{\gamma} \right) = 2g\Delta h [(sp.gr.)_m - 1] \\ V^2 &= 2(9.81) \left(\frac{14.6}{100} \right) [1.95 - 1.0] \\ V &= 1.65 \text{ m/sec} \end{aligned}$$

Current meters are frequently used to measure the speed of water in open channels. There are two different types of mechanical current meters: cup and propeller.

The *cup type* current (velocity) meter usually consists of four to six evenly shaped cups mounted radially about a vertical axis of rotation [Figure 9.5 (a)]. The moving water rotates the cups around the axis at a rate proportional to the water velocity. A mechanical or fiber-optic sensor relays each revolution to an electronic data-collection device. A conversion is made to flow velocity, and the data are stored or presented as a digital readout. Most cup-type current meters do not register velocity below a few centimeters per second because of starting friction.

The propeller-type current meter has a horizontal axis of rotation [Figure 9.5 (b)]. It is more suitable for measuring higher velocity ranges in the midstream regions and is less susceptible to interference by weeds and debris.

Depending on the design and construction of the current meter, the speed of axial rotation may not be linearly proportional to the speed of the water current. For this reason, each current

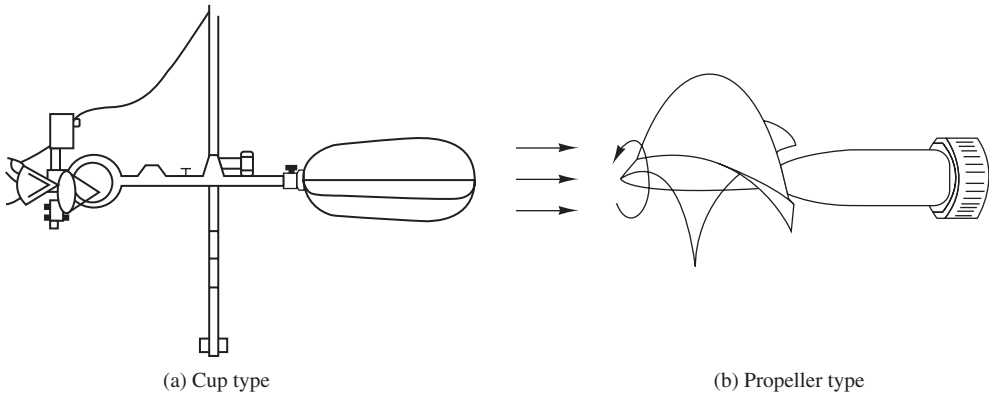


Figure 9.5 Current meters: (a) cup type and (b) propeller type

meter must be individually calibrated before it is used for field measurements. Calibrations may be carried out by towing the probe through still water at constant speeds. However, a calibration curve covering the range of applicable speeds is generally provided by the manufacturer.

Acoustic Doppler (sonar) *velocity* meters can be used to measure the velocity of water in open channels. They rely on the Doppler principle by transmitting acoustic pulses along various paths. Because a sonic pulse moving with the current moves faster than one against the current, time-of-arrival differences can be used to determine flow velocity. Additional information can be obtained from the manufacturers of these devices.

9.3 Discharge Measurements in Pipes

Although pipe flow measurements can be accomplished by several different methods, the simplest and most reliable measurement is the *volumetric* (or *weight*) *method*. This method requires only a stopwatch and an open tank to collect the water flowing from the pipe. The discharge rate can be determined by measuring the water volume (or weight) collected per unit time. Because of its absolute reliability, this method is frequently used for calibration of various types of flow meters. It is impractical for most operational applications because flowing water is totally diverted into a container when a measurement is made. However, in some cases there are water towers or ground level tanks within the pipe network that can be used for measurement.

Flow diversion is not necessary to obtain accurate flow measurements in pressurized pipe flow. Pipe flow rates can be correlated to variations in energy (head) distribution associated with a sudden change in pipe cross-sectional geometry. This principle is utilized in Venturi meters, nozzle meters, and orifice meters.

A *Venturi meter* is a precisely engineered section of pipe with a narrow throat. Two piezometric openings are installed at the entrance and at the throat as shown in Figure 9.6. Applying the Bernoulli equation at sections 1 and 2, and neglecting head loss, we get

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 \quad (9.2)$$

The continuity equation between the two sections is

$$A_1 V_1 = A_2 V_2 \quad (9.3)$$

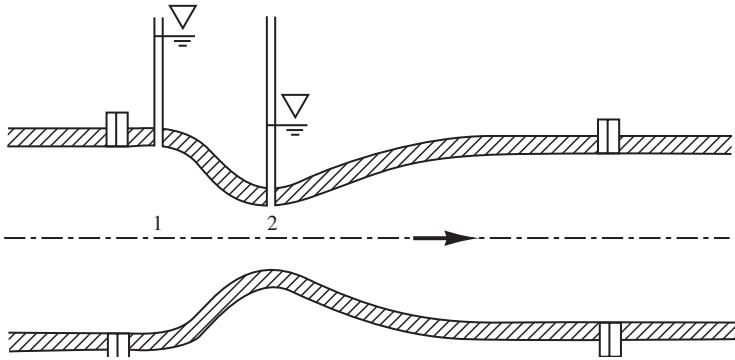


Figure 9.6 Venturi meter

where A_1 and A_2 are the pipe and throat cross-sectional areas, respectively. Substituting Equation 9.3 into Equation 9.2 and rearranging, we obtain

$$Q = \frac{A_1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} \sqrt{2g\left(\frac{P_1 - P_2}{\gamma} + z_1 - z_2\right)} \tag{9.4}$$

The equation may be simplified to

$$Q = C_d A_1 \sqrt{2g \left[\Delta \left(\frac{P}{\gamma} + z \right) \right]} \tag{9.5a}$$

where the dimensionless discharge coefficient C_d is evaluated as

$$C_d = \frac{1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} \tag{9.5b}$$

For Venturi meters installed in a horizontal position,

$$Q = C_d A_1 \sqrt{2g \left(\frac{\Delta P}{\gamma} \right)} \tag{9.5c}$$

The coefficient C_d can be directly computed from the values of A_1 and A_2 . The difference between this theoretically computed value and that obtained from experiments (which accounts for losses) should not exceed a few percentage points for well-manufactured Venturi meters.

For satisfactory operation, the meter should be installed in a section of the pipe where the flow is relatively undisturbed before it enters the meter. To ensure this, a section of straight and uniform pipe that is free from fittings and at least 30 diameters in length must be provided upstream of the meter installation.

Example 9.3

A 6-cm (throat) Venturi meter is installed in a 12-cm-diameter horizontal water pipe. A differential (mercury–water) manometer installed between the throat and the entry section registers a mercury (sp. gr. = 13.6) column reading of 15.2 cm. Calculate the discharge.

Solution

From manometry principles (as reviewed in Example 9.2),

$$\Delta P = \Delta h(\gamma_{Hg} - \gamma)$$

or

$$\begin{aligned} \Delta P/\gamma &= \Delta h\left(\frac{\gamma_{Hg} - \gamma}{\gamma}\right) = \Delta h[(sp. gr.)_{Hg} - 1] \\ &= (15.2 \text{ cm})(13.6 - 1.0) \end{aligned}$$

$$A_1 = (\pi/4)(12)^2 = 113 \text{ cm}^2 \quad A_2 = (\pi/4)(6)^2 = 28.3 \text{ cm}^2$$

The dimensionless coefficient (C_d) can be calculated from the area ratio by using Equation 9.5b:

$$C_d = \frac{1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} = 0.259$$

To determine the discharge, apply Equation 9.5c for a horizontally installed Venturi meter:

$$Q = 0.259\left(\frac{113}{10,000}\right)\sqrt{2(9.81)\left[\left(\frac{15.2}{100}\right)12.6\right]} = 0.0179 \text{ m}^3/\text{sec}$$

Nozzle meters and *orifice meters* (Figure 9.7) are also based on variations in energy (head) distribution associated with a sudden change in pipe cross-sectional geometry. In fact, the discharge equations for the nozzle meter and the orifice meter have the same form as that derived for the Venturi meter (Equation 9.5a). The main difference in application is that the value of the coefficient of discharge for the nozzle meters and orifice meters would be different from the theoretical value, C_d , calculated by using Equation 9.5b. This is primarily because of separation of the stream flow from the pipe wall boundary immediately downstream from the flow constriction (*vena contracta*).

Nozzle meters and orifice meters produce a significant amount of head loss because most of the pressure energy that converts to kinetic energy (to speed the fluid through the narrow

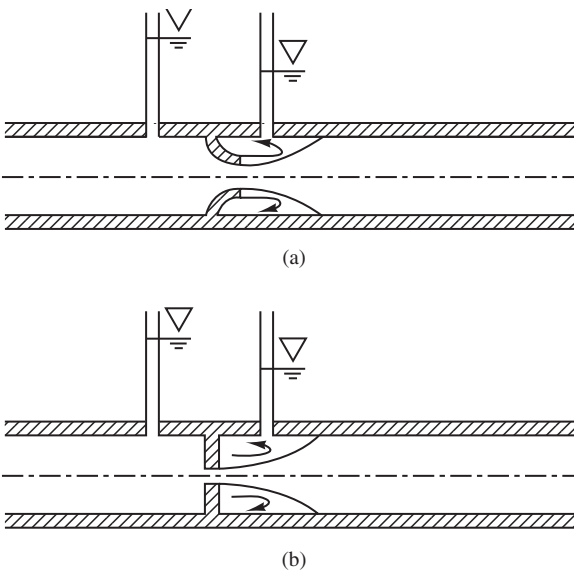


Figure 9.7 (a) Nozzle meter and (b) orifice meter

opening) cannot be recovered. The coefficient of discharge may vary significantly from one meter to another. The value depends not only on the status of flow in the pipe (the pipe's Reynolds number) but also on the area ratio between the nozzle (or orifice) and the pipe, the location of the pressure taps, and the upstream and downstream condition of the pipe flow. For this reason, on-site calibration is recommended for each meter installed. If installation of a pipe orifice meter is made without an on-site calibration, then reference should be made to the manufacturer's data, and the detailed installation requirements should be followed.

As previously stated, the coefficient of discharge for nozzle meters and orifice meters cannot be computed directly from the area ratio, A_1/A_2 . The discharge equations (Equations 9.5a and 9.5c) must be modified by an experimental, dimensionless coefficient, C_v :

$$Q = C_v C_d A_1 \sqrt{2g \left[\Delta \left(\frac{P}{\gamma} + z \right) \right]} \tag{9.6a}$$

where z is the difference in elevations between the two pressure taps. For horizontal installations,

$$Q = C_v C_d A_1 \sqrt{2g \left(\frac{\Delta P}{\gamma} \right)} \tag{9.6b}$$

Extensive research on nozzle meters has been sponsored by ASME and the International Standards Association to standardize the nozzle geometry, installation, specification, and experimental coefficients. One of the typical ASME flow nozzle installations commonly used in the United States with the corresponding experimental coefficients is shown in Figure 9.8.

Compared with Venturi meters and nozzle meters, orifice meters are affected even more by flow conditions. For this reason, detailed instructions of installation and calibration curves must be provided by the manufacturer for each type and size. If a meter is not installed strictly according to the instructions, then it should be calibrated meticulously on site.

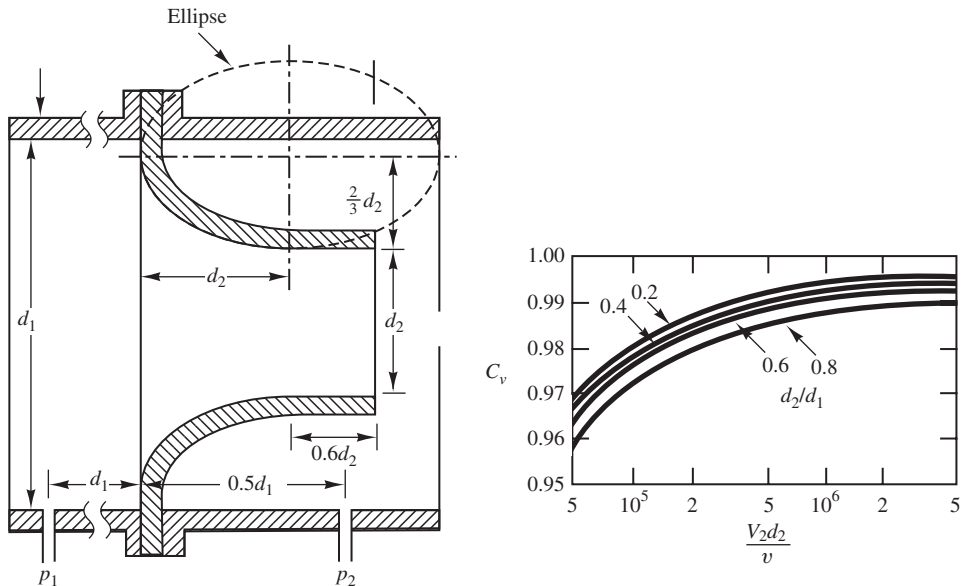


Figure 9.8 ASME nozzle dimensions and coefficients

Example 9.4

The Venturi meter in Example 9.3 is replaced with an ASME flow nozzle meter. During operation, the attached differential (mercury–water) manometer registers a mercury column reading of 15.2-cm. The water in the pipeline is 20°C. Determine the discharge.

Solution

The diameter ratio is $d_2/d_1 = 6/12 = 0.5$, and $C_d = 0.259$ (from Example 9.3). Assume that the experimental meter coefficient $C_v = 0.99$. The corresponding discharge can be calculated from Equation 9.6b as

$$Q = (0.99)(0.259) \left(\frac{113}{10,000} \right) \sqrt{2(9.81) \left[\left(\frac{15.2}{100} \right) (12.6) \right]} = 0.0178 \text{ m}^3/\text{sec}$$

This value must be verified by checking the corresponding Reynolds number of the nozzle. The N_R value calculated based on the discharge is

$$N_R = \frac{V_2 d_2}{\nu} = \frac{\left[\frac{(0.0178)}{(\pi/4)(0.06)^2} \right] (0.06)}{1.00 \times 10^{-6}} = 3.78 \times 10^5$$

With this Reynolds number value, the chart in Figure 9.8 gives a better value of the experimental coefficient, $C_v = 0.986$. Hence, the correct discharge is

$$Q = (0.986/0.99)(0.0178) = 0.0177 \text{ m}^3/\text{sec}$$

A *bend meter* (Figure 9.9) measures the pressure difference between the outer and inner sides of a bend in a pipeline. Centrifugal force developed at a pipe bend forces the main stream to flow closer to the outer wall of the pipe at the bend. A difference in pressure is developed between the inside and outside of the bend. The pressure difference increases as the flow rate increases. The relationship between the measured pressure difference and the discharge in the pipe can be calibrated for flow rate determinations. The discharge equation may be expressed as

$$Q = C_d A \sqrt{2g \left(\frac{P_o}{\gamma} - \frac{P_i}{\gamma} \right)} \quad (9.7)$$

where A is the pipe cross-sectional area and P_i and P_o are the local pressure values registered at the inside and outside of the pipe bend, respectively. C_d is the dimensionless discharge coefficient, which can be determined by calibration on site. Note that Equation 9.7 is appropriate if the elevation difference between taps (Δz) is negligible. Otherwise, the elevation difference must be included with the pressure difference.

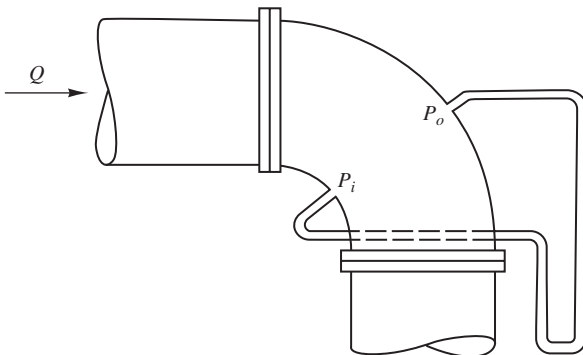


Figure 9.9 Bend meter

If the bend meter cannot be calibrated in place, then the pipe discharge may still be determined within an accuracy of approximately 10 percent if the pipe flow's Reynolds number is sufficiently large and if at least 30 diameters of straight pipe are provided upstream from the bend. In such a case, the discharge coefficient is approximated by

$$C_d = \frac{R}{2D} \tag{9.8}$$

where R is the center line radius at the bend and D is the pipe diameter.

Bend meters are inexpensive and convenient. An elbow already in the pipeline may be used without additional installation cost or added head loss.

9.4 Discharge Measurements in Open Channels

A *weir* is a simple overflow structure extending across a channel and normal to the direction of the flow. Various types of weirs exist, and they are generally classified by shape. Weirs may be either sharp-crested (useful for measuring flow) or broad-crested (incorporated into hydraulic structures with flow measurement as a secondary function).

9.4.1 Sharp-Crested Weirs

Sharp crested weirs (Figure 9.10) include the following four basic types:

1. horizontal weirs without end contractions,
2. horizontal weirs with end contractions,
3. V-notch weirs, and
4. trapezoidal weirs.

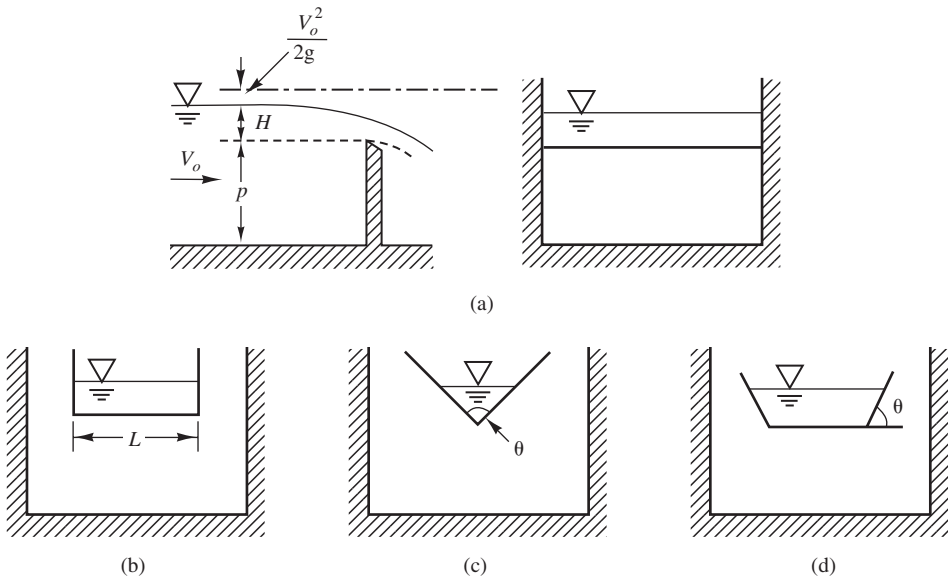


Figure 9.10 Common sharp-crested weirs: (a) uncontracted horizontal weir, (b) contracted horizontal weir, (c) V-notched weir, and (d) trapezoidal weir

An *uncontracted horizontal weir* extends across the entire width of a uniform reach in the channel. A standard uncontracted, horizontal weir should meet the following requirements.

1. The crest of the weir should be horizontal, sharp-edged, and normal to the flow.
2. The weir plate should be vertical and have a smooth upstream surface.
3. The approach channel should be uniform and the water surface should be free from large surface waves.

The basic discharge equation for a standard, uncontracted, horizontal weir (Figure 9.10) is

$$Q = CLH^{3/2} \quad (9.9)$$

where L is the length of the crest, H is the head on the weir, and C is the discharge coefficient having units of length^{0.5}/time. Discharge coefficients are often derived from experimental data by governmental agencies such as the U.S. Bureau of Reclamation (USBR). Using British units (H , L , p in feet, and Q in cubic feet per second), the discharge coefficient in ft^{0.5}/sec may generally be expressed as

$$C = 3.22 + 0.40 \frac{H}{p} \quad (9.10a)$$

where p is the weir height (Figure 9.10). In the SI unit system, the coefficient becomes

$$C = 1.78 + 0.22 \frac{H}{p} \quad (9.10b)$$

A *contracted horizontal weir* has a crest that is shorter than the width of the channel. Thus, water contracts both horizontally and vertically in order to flow over the crest. The weir may be contracted at either end or at both ends. The general discharge equation may be expressed as

$$Q = C \left(L - \frac{nH}{10} \right) H^{3/2} \quad (9.11)$$

where n is the number of contractions at the end [$n = 1$ for a contraction at one end and $n = 2$ for contractions at both ends, as shown in Figure 9.10 (b)]. The coefficient of discharge, C , should be determined by calibration in place. Note that the value will depend on the unit system used because C has units of length^{0.5}/time.

A standard contracted horizontal weir is one whose crest and sides are so far removed from the bottom and sides of the channel that full contraction is developed. The dimensions of a standard contracted horizontal weir are shown in Figure 9.11 (a), where the weir length (L) is denoted as b . The discharge equation of this standard weir is given by the USBR* as

$$Q = 3.33 (L - 0.2H) H^{3/2} \quad (9.12a)$$

This expression was developed for the British measurement system with L and H in feet and Q in cubic feet per second. In the SI unit system, the equation becomes

$$Q = 1.84(L - 0.2H) H^{3/2} \quad (9.12b)$$

* U.S. Bureau of Reclamation, *Water Measurement Manual* (Washington, DC: U.S. Government Printing Office, 1967).

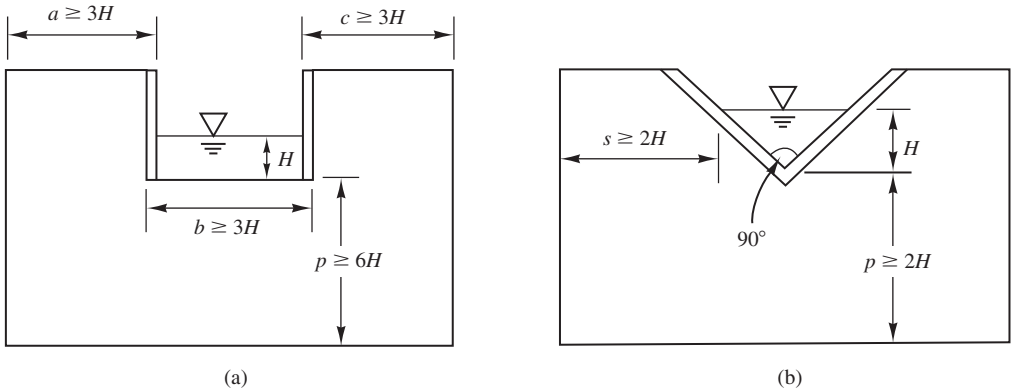


Figure 9.11 USBR standard weirs: (a) contracted horizontal weir and (b) 90° V-notch weir

A *V-notch weir* is especially useful when accuracy in measurement is desired for a large range of water depths. The discharge equation for a V-notch weir takes the general form

$$Q = C \left(\tan \frac{\theta}{2} \right) H^{5/2} \tag{9.13}$$

where θ is the weir angle as shown in Figure 9.10 (c), and the discharge coefficient (C) is determined by calibration in place. Note that the value will depend on the unit system used because C has units of length^{0.5}/time.

A standard USBR 90° V-notch weir consists of a thin plate with each side of the notch inclined 45° from the vertical [Figure 9.11(b)]. The weir operates like a contracted horizontal weir, and all requirements stated for the standard, uncontracted, horizontal weir apply. The minimum distances of the sides of the weir from the channel banks should be at least twice the head on the weir, and the minimum distance from the weir crest to the channel bottom should be at least twice the head on the weir. The discharge equation of the standard 90°-V-notch weir is given by the USBR as

$$Q = 2.49H^{2.48} \tag{9.14}$$

This expression was developed for the British measurement system with H in feet and Q in cubic feet per second.

The *trapezoidal weir* has hydraulic characteristics in between that of the contracted horizontal weir and the V-notch weir. The general discharge equation developed for the contracted horizontal weir may be applied to the trapezoidal weir with an individually calibrated discharge coefficient.

The USBR standard trapezoidal weir (Figure 9.12) is also known as the *Cipolletti weir*. It has a horizontal crest, and the sides incline outwardly at a slope of 1:4 (horizontal to vertical). All requirements stated for the standard, uncontracted, horizontal weir apply. The height of the weir crest should be at least twice the head of the approach flow above the crest (H), and the distances from the sides of the notch to the sides of the channel should also be at least twice the head. The discharge equation for the Cipolletti weir is given by the USBR as

$$Q = 3.367LH^{3/2} \tag{9.15}$$

This expression was developed for the British measurement system with L and H in feet and Q in cubic feet per second.

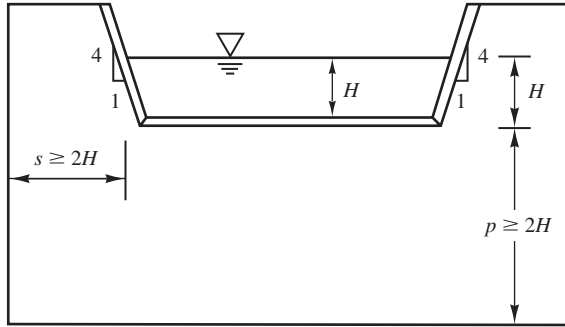


Figure 9.12 USBR standard trapezoidal weir

Example 9.5

Laboratory measurements are made on a contracted (both sides) horizontal weir with a crest length of 1.56 m. The measured discharge is 0.25 m³/sec under a head of $H = 0.2$ m. Determine the discharge coefficient in the given (SI) units.

Solution

Applying Equation 9.11 for the contracted horizontal weir, we get

$$Q = C \left(L - \frac{nH}{10} \right) H^{3/2}$$

Here $L = 1.56$ m, $H = 0.2$ m, and $n = 2$ for contraction at both ends;

$$0.25 = C \left(1.56 - \frac{2(0.2)}{10} \right) (0.2)^{3/2}$$

$$C = 1.84 \text{ m}^{0.5}/\text{sec}$$

9.4.2 Broad-Crested Weirs

A broad-crested weir provides a stretch of elevated channel floor over which critical flow takes place (Figure 9.13). Depending on the height of the weir in relation to the depth of the approaching channel, the discharge equation may be derived from the balance of forces and momentum between the upstream approach section (1) and the section of minimum depth (2) on the crest of the weir. For a unit width of the weir, the following equation may be written as

$$\rho q \left(\frac{q}{y_2} - \frac{q}{y_1} \right) = \frac{1}{2} \gamma \left[y_1^2 - y_2^2 - h(2y_1 - h) \right] \tag{9.16}$$

where q is the discharge per unit width, h is the weir height measured from the channel floor, and y_1 and y_2 are the upstream and downstream depths, respectively.

The conditions provided above are not sufficient to simplify Equation 9.16 into a one-to-one relationship between the approach water depth and the discharge. An additional equation was obtained from experimental measurements* for the average flow

$$y_1 - h = 2y_2 \tag{9.17}$$

* H. A. Doeringsfeld and C. L. Barker, "Pressure-momentum theory applied to the broad-crested weir," *Trans. ASCE*, 106 (1941): 934-946.

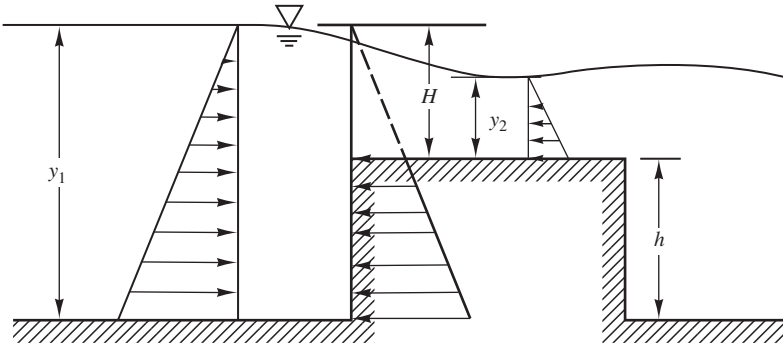


Figure 9.13 Broad-crested weir

Substituting Equation 9.17 into Equation 9.16 and simplifying, we have

$$q = 0.433 \sqrt{2g} \left(\frac{y_1}{y_1 + h} \right)^{1/2} H^{3/2} \tag{9.18}$$

The total discharge over the weir is

$$Q = Lq = 0.433 \sqrt{2g} \left(\frac{y_1}{y_1 + h} \right)^{1/2} LH^{3/2} \tag{9.19}$$

where L is the overflow length of the weir crest (as seen in a plan view) and H is the height of the approach water above the weir crest.

Considering the limit of a weir with zero height ($h = 0$) to infinity ($h \rightarrow \infty$), Equation 9.19 may vary from $Q = 1.92LH^{3/2}$ to $Q = 1.36LH^{3/2}$ with the units in meters and seconds.

9.4.3 Venturi Flumes

The use of a weir is probably the simplest method for measuring discharge in open channels. However, there are disadvantages to using weirs, including the relatively high energy loss and the sedimentation deposited in the pool immediately upstream of the weir. These difficulties can be partially overcome by using a *critical flow flume* called a *Venturi flume*.

A variety of Venturi flumes have been designed for field application. Most of the flumes operate with a submerged outflow condition and create critical depth at a contracted section (throat) followed by a hydraulic jump at the exit. The discharge through the flume can be calculated by reading the water depth from the observation wells located at the critical flow section and at another reference section.

The most extensively used critical flow flume in the United States is the *Parshall flume*, developed by R. L. Parshall* in 1920. The flume was experimentally developed for the British measurement system. It has fixed dimensions as shown in Figure 9.14 and Table 9.1. Empirical discharge equations were developed to correspond to each flume size. These equations are listed in Table 9.2.

* R. L. Parshall and C. Rohwer, *The Venturi flume*. Colorado Agricultural Experimental Station Bulletin No. 265 (1921); R. L. Parshall, "The improved Venturi flume," *Trans. ASCE*, 89 (1926): 841–851.

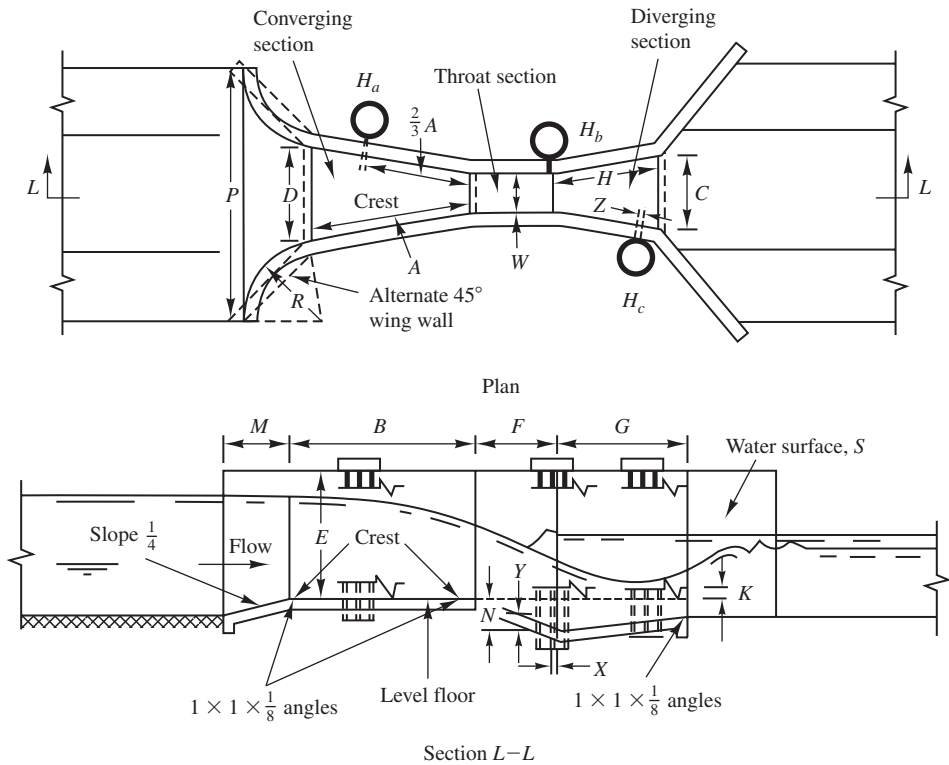


Figure 9.14 Parshall flume dimensions. *Source:* Courtesy of U.S. Bureau of Reclamation.

In Equations 9.20 through 9.24, Q is the discharge in cubic feet per second (cfs), W is the throat width in feet, and H_a is the water level reading from observation well a measured in feet. These equations are derived strictly for the units stated and for the dimensions specified in Figure 9.14 and Table 9.1. They do not have an equivalent version in the metric system.

When the ratio of gauge reading H_b (from observation well b) to gauge reading H_a (from observation well a) exceeds the following values:

- 0.50 for flumes 1, 2, and 3 in. wide
- 0.60 for flumes 6 in. and 9 in. wide
- 0.70 for flumes 1 ft to 8 ft wide
- 0.80 for flumes 10 ft to 50 ft wide

the flow is said to be submerged. The effect of the downstream submergence is to reduce the discharge through the flume. In this case, the discharge computed by the above equations should be corrected by considering both readings H_a and H_b .

Figure 9.15 shows the rate of submerged flow through a 1-ft Parshall flume. The diagram is made applicable to larger flumes (up to 8 ft) by multiplying the corrected discharge for the 1-ft flume by a factor given for the particular size selected.

Figure 9.16 shows the rate of submerged flow through a 10-ft Parshall flume. The diagram is also made applicable to larger size flumes (up to 50 ft) by multiplying the corrected discharge for the 10-ft flume by a factor given for the particular size used.

TABLE 9.1 Parshall Flume Dimensions

Source	W	A	$\frac{2}{3}A$	B	C	D	E	F	G	H	K	M	N	P	R	X	Y	Z	Flow (cfs)	
	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	min	max
1*	0 ft	1 ft	0 ft	1 ft	0 ft	0 ft	0 ft	0 ft	0 ft	0 ft	0 ft	—	0 ft	—	—	0 ft	0 ft	0 ft		
	1-in.	$2\frac{9}{32}$	$9\frac{17}{32}$	2	$3\frac{21}{32}$	$6\frac{19}{32}$	6 to 9	3	8-in.	$8\frac{1}{8}$	$\frac{3}{4}$	—	$1\frac{1}{8}$	—	—	$\frac{5}{16}$	$\frac{1}{2}$ in.	$\frac{1}{8}$ in.	0.01	0.19
	2-in.	$4\frac{5}{16}$	$10\frac{7}{8}$	4	$5\frac{5}{16}$	$8\frac{13}{32}$	6 to 10	$4\frac{1}{2}$	10-in.	$10\frac{1}{8}$	$\frac{7}{8}$	—	$1\frac{11}{16}$	—	—	$\frac{5}{8}$	1-in.	$\frac{1}{4}$ in.	0.02	0.47
	3-in.	$6\frac{3}{8}$	$12\frac{1}{4}$	6	7	$10\frac{3}{16}$	12 to 18	6	12-in.	$12\frac{5}{32}$	1	—	$2\frac{1}{4}$	—	—	1	$1\frac{1}{2}$	$\frac{1}{2}$ in.	0.03	1.13
2	0 ft	2 ft	1 ft	2 ft	1 ft	1 ft	2 ft	1 ft	2 ft		0 ft	1 ft	0 ft	2 ft	1 ft	0 ft	0 ft			
	6-in.	$\frac{7}{16}$	$4\frac{5}{16}$	0	$3\frac{1}{2}$	$3\frac{5}{8}$	0	0	0-in.	—	3	0	$4\frac{1}{2}$	$11\frac{1}{2}$	4	2	3-in.	—	0.05	3.9
	9-in.	$10\frac{5}{8}$	$11\frac{1}{8}$	10	3	$10\frac{5}{8}$	6	0	6-in.	—	3	0	$4\frac{1}{2}$	$18\frac{1}{2}$	4	2	3-in.	—	0.09	8.9
	1 ft	4 ft	3 ft	4 ft	2 ft	2 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	4 ft	1 ft	0 ft	0 ft			
	0-in.	6	0	$4\frac{7}{8}$	0	$9\frac{1}{4}$	0	0	0-in.	—	3	3	9	$10\frac{3}{4}$	8	2	3-in.	—	0.11	16.1
	6-in.	9	2	$7\frac{7}{8}$	6	$16\frac{3}{8}$	0	0	0-in.	—	3	3	9	18	8	2	3-in.	—	0.15	24.6
	2 ft	5 ft	3 ft	4 ft	3 ft	3 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	6 ft	1 ft	0 ft	0 ft			
	0-in.	0	4	$10\frac{7}{8}$	0	$11\frac{1}{2}$	0	0	0-in.	—	3	3	9	1	8	2	3-in.	—	0.42	33.1
	3 ft	5 ft	3 ft	5 ft	4 ft	5 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	7 ft	1 ft	0 ft	0 ft			
	0-in.	6	8	$4\frac{3}{4}$	0	$1\frac{7}{8}$	0	0	0-in.	—	3	3	9	$3\frac{1}{2}$	8	2	3-in.	—	0.61	50.4
	4 ft	6 ft	4 ft	5 ft	5 ft	6 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	8 ft	2 ft	0 ft	0 ft			
	0-in.	0	0	$10\frac{5}{8}$	0	$4\frac{1}{4}$	0	0	0-in.	—	3	6	9	$10\frac{3}{4}$	0	2	3-in.	—	1.3	67.9
	5 ft	6 ft	4 ft	6 ft	6 ft	7 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	10 ft	2 ft	0 ft	0 ft			
	0-in.	6	4	$4\frac{1}{2}$	0	$6\frac{5}{8}$	0	0	0-in.	—	3	6	9	$1\frac{1}{4}$	0	2	3-in.	—	1.6	85.6
6 ft	7 ft	4 ft	6 ft	7 ft	8 ft	3 ft	2 ft	3 ft		0 ft	1 ft	0 ft	11 ft	2 ft	0 ft	0 ft				
0-in.	0	8	$10\frac{3}{8}$	0	9	0	0	0-in.	—	3	6	9	$3\frac{1}{2}$	0	2	3-in.	—	2.6	103.5	

(continued)

TABLE 9.1 Parshall Flume Dimensions (continued)

Source	W	A	$\frac{2}{3}A$	B	C	D	E	F	G	H	K	M	N	P	R	X	Y	Z	Flow (cfs)	
	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	<u>ft.</u> in.	min	max
2	<u>7 ft</u>	<u>7 ft</u>	<u>5 ft</u>	<u>7 ft</u>	<u>8 ft</u>	<u>9 ft</u>	<u>3 ft</u>	<u>2 ft</u>	<u>3 ft</u>		<u>0 ft</u>	<u>1 ft</u>	<u>0 ft</u>	<u>12 ft</u>	<u>2 ft</u>	<u>0 ft</u>	<u>0 ft</u>		3.0	121.4
	0-in.	6	0	4 $\frac{1}{4}$	0	11 $\frac{3}{8}$	0	0	0-in.	—	3	6	9	6	0	2	3-in.	—		
	<u>8 ft</u>	<u>8 ft</u>	<u>5 ft</u>	<u>7 ft</u>	<u>9 ft</u>	<u>11 ft</u>	<u>3 ft</u>	<u>2 ft</u>	<u>3 ft</u>		<u>0 ft</u>	<u>1 ft</u>	<u>0 ft</u>	<u>13 ft</u>	<u>2 ft</u>	<u>0 ft</u>	<u>0 ft</u>		3.5	139.5
3	<u>10 ft</u>		<u>6 ft</u>	<u>14 ft</u>	<u>12 ft</u>	<u>15 ft</u>	<u>4 ft</u>	<u>3 ft</u>	<u>6 ft</u>		<u>0 ft</u>		<u>1 ft</u>			<u>0 ft</u>	<u>1 ft</u>		6.0	200
	0-in.	—	0	0	0	7 $\frac{1}{4}$	0	0	0-in.	—	6	—	1 $\frac{1}{2}$	—	—	9	0-in.	—		
	<u>12 ft</u>		<u>6 ft</u>	<u>16 ft</u>	<u>14 ft</u>	<u>18 ft</u>	<u>5 ft</u>	<u>3 ft</u>	<u>8 ft</u>		<u>0 ft</u>		<u>1 ft</u>			<u>0 ft</u>	<u>1 ft</u>		8.0	350
	0-in.	—	8	0	8	4 $\frac{3}{4}$	0	0	0-in.	—	6	—	1 $\frac{1}{2}$	—	—	9	0-in.	—		
	<u>15 ft</u>		<u>7 ft</u>	<u>25 ft</u>	<u>18 ft</u>	<u>25 ft</u>	<u>6 ft</u>	<u>4 ft</u>	<u>10 ft</u>		<u>0 ft</u>		<u>1 ft</u>			<u>0 ft</u>	<u>1 ft</u>		8.0	600
	0-in.	—	8	0	4	0	0	0	0-in.	—	9	—	6	—	—	9	0-in.	—		
	<u>20 ft</u>		<u>9 ft</u>	<u>25 ft</u>	<u>24 ft</u>	<u>30 ft</u>	<u>7 ft</u>	<u>6 ft</u>	<u>12 ft</u>		<u>1 ft</u>		<u>2 ft</u>			<u>0 ft</u>	<u>1 ft</u>		10	1,000
	0-in.	—	4	0	0	0	0	0	0-in.	—	0	—	3	—	—	9	0-in.	—		
	<u>25 ft</u>		<u>11 ft</u>	<u>25 ft</u>	<u>29 ft</u>	<u>35 ft</u>	<u>7 ft</u>	<u>6 ft</u>	<u>13 ft</u>		<u>1 ft</u>		<u>2 ft</u>			<u>0 ft</u>	<u>1 ft</u>		15	1,200
	0-in.	—	0	0	4	0	0	0	0-in.	—	0	—	3	—	—	9	0-in.	—		
	<u>30 ft</u>		<u>12 ft</u>	<u>26 ft</u>	<u>34 ft</u>	<u>40 ft</u>	<u>7 ft</u>	<u>6 ft</u>	<u>14 ft</u>		<u>1 ft</u>		<u>2 ft</u>			<u>0 ft</u>	<u>1 ft</u>		15	1,500
	0-in.	—	8	0	8	4 $\frac{3}{4}$	0	0	0-in.	—	0	—	3	—	—	9	0-in.	—		
<u>40 ft</u>		<u>16 ft</u>	<u>27 ft</u>	<u>45 ft</u>	<u>50 ft</u>	<u>7 ft</u>	<u>6 ft</u>	<u>16 ft</u>		<u>1 ft</u>		<u>2 ft</u>			<u>0 ft</u>	<u>1 ft</u>		20	2,000	
0-in.	—	0	0	4	9 $\frac{1}{2}$	0	0	0-in.	—	0	—	3	—	—	9	0-in.	—			
<u>50 ft</u>		<u>19 ft</u>	<u>27 ft</u>	<u>56 ft</u>	<u>60 ft</u>	<u>7 ft</u>	<u>6 ft</u>	<u>20 ft</u>		<u>1 ft</u>		<u>2 ft</u>			<u>0 ft</u>	<u>1 ft</u>		25	3,000	
0-in.	—	4	0	8	9 $\frac{1}{2}$	0	0	0-in.	—	0	—	3	—	—	9	0-in.	—			

* Tolerance on the throat width (W) is $\pm\frac{1}{64}$ inch; tolerance on other dimensions is $\pm\frac{1}{32}$ inch. The throat sidewalls must be parallel and vertical.

Source (1) Colorado State University Technical Bulletin No. 61.

Source (2) U.S. Department of Agriculture: Soil Conservation Service Circular No. 843.

Source (3) Colorado State University Technical Bulletin No. 426-A.

TABLE 9.2 Parshall Flume Discharge Equations

Throat Width	Discharge Equation	Free Flow Capacity (cfs)
3 in.	$Q = 0.992H_a^{1.547}$ (9.20)	0.03 to 1.9
6 in.	$Q = 2.06H_a^{1.58}$ (9.21)	0.05 to 3.9
9 in.	$Q = 3.07H_a^{1.53}$ (9.22)	0.09 to 8.9
1 to 8 ft	$Q = 4WH_a^{1.522}W^{0.026}$ (9.23)	Up to 140
10 to 50 ft	$Q = (3.6875W + 2.5)H_a^{1.6}$ (9.24)	Up to 2,000

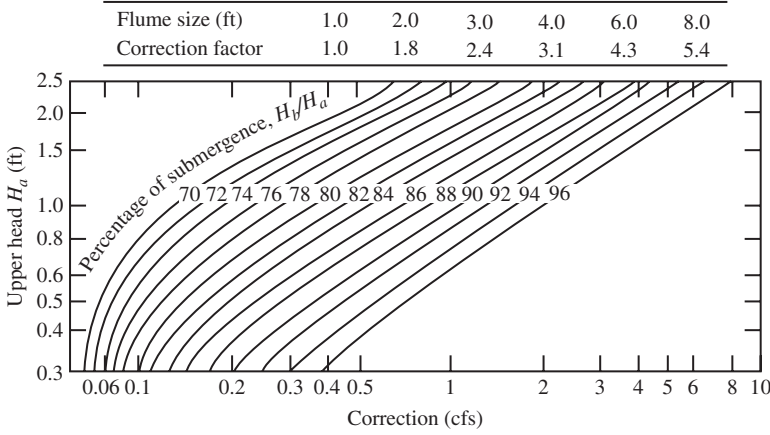


Figure 9.15 Flow-rate correction for a 1-ft submerged Parshall flume. *Source:* After R. L. Parshall, *Measuring Water in Irrigation Channels with Parshall Flumes and Small Weirs*, U.S. Soil Conservation Service, Circular 843 (1950); R. L. Parshall, *Parshall Flumes of Large Size*, Colorado Agricultural Experimental Station Bulletin No. 426A (1953).

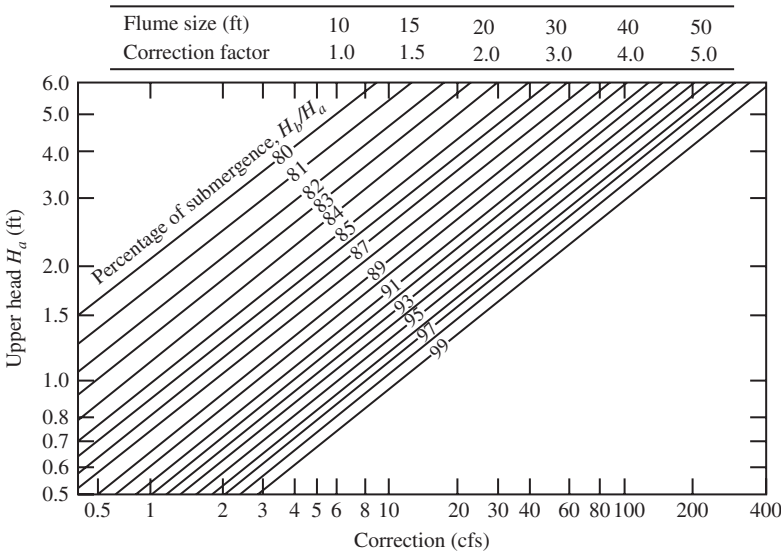


Figure 9.16 Flow-rate correction for a 10-ft submerged Parshall flume. *Source:* After R. L. Parshall, *Measuring Water in Irrigation Channels with Parshall Flumes and Small Weirs*, U.S. Soil Conservation Service, Circular 843 (1950); R. L. Parshall, *Parshall Flumes of Large Size*, Colorado Agricultural Experimental Station Bulletin No. 426A (1953).

Example 9.6

A 4-ft Parshall flume is installed in an irrigation channel to monitor the rate of flow. The readings at gauges H_a and H_b are 2.5 ft and 2.0 ft, respectively. Determine the channel discharge.

Solution

$$H_a = 2.5 \text{ ft} \quad H_b = 2.0 \text{ ft}$$

Submergence, $H_b/H_a = 80\%$.

Equation (9.23) provides the value of the unsubmerged discharge

$$Q_u = 4WH_a^{1.522} \cdot w^{0.026} = 4(4)(2.5)^{1.522}(4)^{0.026} = 67.9 \text{ cfs}$$

Under the given conditions, the flume is operating at 80 percent submergence, and the value should be corrected accordingly.

From Figure 9.15, we find the flow rate correction for a 1-ft Parshall flume to be 1.8 cfs. For the 4-ft flume, the corrected flow rate is

$$Q_c = 3.1(1.8) = 5.6 \text{ cfs}$$

The corrected discharge of the channel is

$$Q = Q_u - Q_c = 67.9 - 5.6 = 62.4 \text{ cfs}$$

PROBLEMS
(SECTION 9.1)

- 9.1.1.** Water is poured into an open-ended U-tube (Figure P9.1.1). Then oil is poured into one leg of the U-tube, causing the water surface in one leg to rise 6 in. above the oil–water interface in the other leg. The oil column measures 8.2 in. What is the specific gravity of the oil?

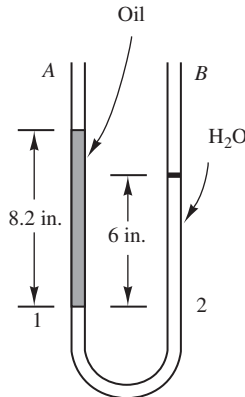


Figure P9.1.1

- 9.1.2.** Determine the pressure in the pipe shown in Figure P9.1.2 if $y = 1.24 \text{ m}$, $h = 1.02 \text{ m}$, and the manometry fluid is mercury (sp. gr. = 13.6). Also determine the height the pressurized water would climb in a piezometer if it was used to measure pressure at the same location.

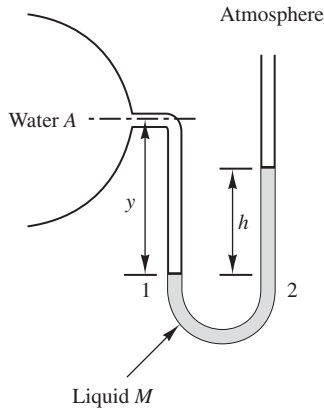


Figure P9.1.2

- 9.1.3. An inclined oil manometer is used to measure pressure in a wind tunnel. Determine the specific gravity of the oil if a pressure change of 323 pascals causes a 15-cm reading change on the 15° incline.

(SECTION 9.2)

- 9.2.1. A plexiglass tube (piezometer) is mounted on a pipe as shown in Figure P9.2.1. Another plexiglass tube with a 90° bend (i.e., Pitot tube) is inserted into the center of the pipe and directed toward the current. For a given discharge, the piezometer registers a water height of 320 cm, and the Pitot tube registers a water height of 330 cm. Determine the water pressure (kPa) and the flow velocity (m/sec). Is the flow velocity the average pipe velocity? Explain.

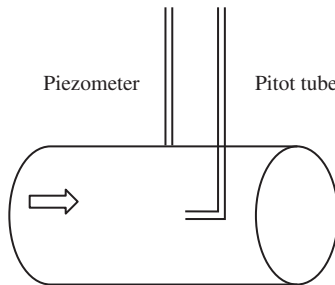


Figure P9.2.1

- 9.2.2. Show that Equation 9.1b can be written in the following form

$$V = \sqrt{2g\Delta h(sp. gr. - 1)}$$

if the pipe fluid is water but the manometry fluid is not (and water from the pipe extends all the way to the manometry fluid). Also, sp. gr. is the specific gravity of the manometry fluid. [Hint: Apply manometry principles to the sketch shown in Figure 9.4(b) and combine this with Bernoulli principles.]

- 9.2.3. Pitot tube measurements in a 5-ft-diameter water tunnel indicate that the stagnation pressure and the static pressure differ by a mercury column height of 5.65 inches. What is the water velocity? Estimate the discharge in the tunnel. Why is it just an estimate?

- 9.2.4.** Refer to Figure 9.4 (b) and answer the following questions:
- If the pipe fluid and the manometry fluid are water (with air in between), determine the pipe velocity if $\Delta h = 34.4$ cm.
 - If the pipe fluid is water, which extends all the way to the manometry fluid (mercury; sp. gr. = 13.6), what is the pipe velocity if $\Delta h = 34.4$ cm?
- 9.2.5.** Refer to Figure 9.4 (b) and answer the following questions:
- If the pipe fluid and the manometry fluid are oil (with air in between), what is the pipe velocity if $\Delta h = 18.8$ in? [sp. gr. (oil) = 0.85.]
 - If the pipe fluid is oil, which extends all the way to the manometry fluid (mercury; sp. gr. = 13.6), what is the pipe velocity if $\Delta h = 18.8$ in?
- 9.2.6.** Referring to Figure 9.4 (a), determine the maximum measurable velocity if the maximum Pitot tube scale length is 25 cm. The pipe fluid is water, which extends all the way to the manometry fluid, which is mercury with a specific gravity of 13.6. Estimate the maximum discharge in the 10-cm-diameter pipe. Why is it just an estimate?

(SECTION 9.3)

- 9.3.1.** Determine the flow rate in 50-cm diameter pipe that contains a 20-cm (throat) Venturi meter if the pressure taps read 290 kN/m^2 and 160 kN/m^2 .
- 9.3.2.** A Venturi meter with a 4-in. throat is installed in an 8-in. diameter (vertical) waterline. The pressure difference reading between the throat and entry section in a mercury–water manometer is 2.01 feet. Determine the flow rate if the flow is moving downward and the distance between the pressure taps is 0.80 feet.
- 9.3.3.** A discharge of $0.0820 \text{ m}^3/\text{sec}$ passes through a horizontal, 20-cm waterline containing a 10-cm ASME flow nozzle. Determine the pressure difference that will register across the flow meter in pascals.
- 9.3.4.** Determine the discharge in the 40-cm diameter waterline shown in Figure P9.3.4. The nozzle meter has a throat diameter of 16 cm and is built and installed according to the ASME standard. The distance between pressure taps is 25 cm.

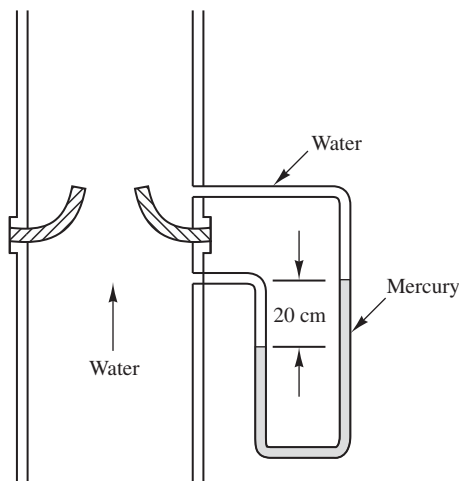


Figure P9.3.4

- 9.3.5.** A 12-in. orifice meter ($C_v = 0.675$) is installed in a vertical segment of a 21-in. -diameter waterline. If the largest flow rate expected is $12.4 \text{ ft}^3/\text{sec}$, what is the necessary length of a vertical U-tube for the differential (mercury–water) manometer? There is a 9-inch difference between manometer taps, and the flow is in the upward direction.
- 9.3.6.** A maintenance crew discovers an orifice meter while working on an old water line. The engineer in charge would like to know the size of the orifice plate opening without dismantling the delicate meter. A specification plate on the side of the meter reads “ $C_v = 0.605$.” For a flow rate of $0.00578 \text{ m}^3/\text{sec}$, the attached mercury–water manometer registers a pressure difference of 9.0 cm. If the horizontal pipeline has an inside diameter of 10 cm, what is the size of the orifice plate?
- 9.3.7.** A bend meter is installed in a 75-cm-diameter water pipe as shown in Figure 9.9. The installation delivered 51 m^3 of water in 1 min. Determine the pressure head difference (in cm) that will register in a mercury–water manometer when the water pipe and bend are in a horizontal position. The bend radius is 80 cm.
- 9.3.8.** A bend meter is installed in a 75-cm-diameter water pipe with an 80-cm bend radius as shown in Figure 9.9. The flow rate in the pipe is $0.850 \text{ m}^3/\text{sec}$, and the pressure difference reading between the outside and inside taps registers 5.26 cm on a mercury–water manometer. Determine the coefficient of discharge for the bend meter if the pipe bend is in a vertical position and flow is downward. Note that in the vertical position the pressure taps are mounted along a 45° line from the horizontal datum.

(SECTION 9.4)

- 9.4.1.** An uncontracted, horizontal weir (sharp-crested) is 4.5 m long and 3.1 m high. Determine the discharge if the upstream water depth is 4.4 m. Determine the discharge for the same upstream depth if a contracted weir (both ends) with a 2.4-m crest length and the same height is being proposed as a replacement for the original weir. Assume the same discharge coefficient.
- 9.4.2.** The crest of a standard, USBR 90° V-notch weir is 3 feet above the bottom of an irrigation channel. It is used to measure 25.6 cfs of flow to a customer. At this flow rate, the depth of water upstream of the weir is at the channel capacity. If the customer needs to increase the flow rate to 33.3 cfs, what is the required crest length of a standard, contracted horizontal weir (with the same crest height) that will accommodate the new demand without flooding the upstream channel?
- 9.4.3.** What is the crest length of the Cipolletti weir (USBR standard trapezoidal weir) required to accommodate a flow up to $0.793 \text{ m}^3/\text{sec}$ if the maximum head is limited to 0.259 m?
- 9.4.4.** A broad-crested rectangular weir is 1 m high and has a crest length of 3 m. The weir is constructed with a well-rounded upstream corner and a smooth surface. What is the discharge if the head is 0.4 m?
- 9.4.5.** Determine the discharge in cubic meters per second measured by a 15-ft Parshall flume if the gauge reading H_a is 1 m and the gauge reading H_b is 0.5 m.
- 9.4.6.** An uncontracted, horizontal weir is 1.5 m high and 4.5 m long. Determine the discharge over this weir when the upstream depth is 2.2 m. Determine the weir height if the same discharge was desired without exceeding an upstream depth of 1.8 m.
- 9.4.7.** Flow occurs over a sharp-crested, uncontracted horizontal weir 3.5 ft high under a head of 1.0 ft. If this weir replaced another weir, which was one-half its height, what change in depth occurred in the upstream channel?
- 9.4.8.** The flow through an 8-ft Parshall flume is 129 cfs when H_a is 2.50 ft. Determine the downstream water level (H_b) that would produce this flow rate.
- 9.4.9.** The discharge in a rectangular channel 4 m wide is constant. A depth of 2.3 m is maintained by a 1.7-m-high contracted (both ends) horizontal weir ($C = 1.86$) with a 1-m-long horizontal crest.

This weir is to be replaced by an uncontracted horizontal weir that will maintain the same upstream depth. Determine the height of the weir if the discharge coefficient is based on Equation 9.10b.

- 9.4.10.** Derive Equation 9.18 from the impulse-momentum equation (9.16), showing all steps.
- 9.4.11.** Equation 9.19 is the general expression for a broad-crested weir. Considering the limit of a weir with zero height ($h = 0$) to infinity ($h \rightarrow \infty$), Equation 9.19 may vary from $Q = 1.92LH^{3/2}$ to $Q = 1.36LH^{3/2}$ with H in meters and Q in meters per seconds. Verify these expressions and find equivalent expressions for the BG system of units.
- 9.4.12.** In Chapter 8, it was stated that an essential feature of weirs is that flow achieves critical depth passing over the weir. Because that is the case, derive Equation 9.9 using Equation 6.14, which relates critical depth to the flow rate. [*Hint:* Critical depth will also have to be related to the head on the weir by an energy balance (ignoring losses). What does the C term represent?]
- 9.4.13.** Laboratory tests on a 60° V-notch weir gave the following results: for $H = 0.3$ m, $Q = 0.22$ m³/sec; and for $H = 0.6$ m, $Q = 0.132$ m³/sec. Determine the discharge equation for this V-notch weir in both SI and BG forms.



Hydraulic Similitude and Model Studies

The use of small models for predicting the behavior of hydraulic structures dates back at least to Leonardo da Vinci.* But the methods developed for using the results of experiments conducted on a *scaled model* to predict quantitatively the performance of a full-size hydraulic structure (i.e., the *prototype*) were not realized fully until the beginning of the twentieth century. The principles on which the model studies are based constitute the theory of *hydraulic similitude*. In other words, are the hydraulic relationships in the prototype sufficiently similar to those in the model? Assessment of the appropriate physical quantities and fundamental hydraulic relationships (both static and dynamic) involved in the actual performance of the structure is known as *dimensional analysis*.

All significant hydraulic structures are now designed and built after certain preliminary model studies have been completed. Such studies may be conducted for any one or more of the following purposes:

1. to determine the discharge coefficient of a large measurement structure, such as an overflow spillway or a weir;
2. to develop an effective method for energy dissipation at the outlet of a hydraulic structure;
3. to reduce energy loss at an intake structure or at a transition section;

* Leonardo da Vinci (1452–1519), a genius, Renaissance scientist, engineer, architect, painter, sculptor, and musician.

4. to develop an efficient, economic spillway or other type of flood-releasing structure for a reservoir;
5. to determine an average time of travel in a temperature control structure, for example, in a cooling pond at a power plant;
6. to establish the best cross section, location, and dimensions of various structural components such as the breakwater, the docks, and the locks in harbor and waterway design; and
7. to determine the dynamic behaviors of the floating, semi-immersible, and bottom structures in transportation or offshore facilities.

River models have also been extensively used in hydraulic engineering to determine

1. the pattern a flood wave travels through a river channel;
2. the effect of artificial structures such as bends, levees, dikes, jetties, and training walls on the sedimentation movements in the channel reach, as well as the impacts to upstream and downstream channel reaches; and
3. the direction and force of natural and anthropogenic currents in channels or harbors and their effect on navigation and marine life.

10.1 Dimensional Homogeneity

When a physical phenomenon is described by an equation or a set of equations, all terms in each of the equations must be kept dimensionally homogeneous.* In other words, all terms in an equation must be expressed in the same units.

In fact, to derive a relationship among several parameters involved in a physical phenomenon, one should always check the equation for homogeneity of units. If all terms in the relationship do not result in the same units on both sides of the equation, then one can be sure that pertinent parameters are missing or misplaced, or extraneous terms have been included.

Based on the conceptual and physical understanding of the phenomenon and the principle of dimensional homogeneity, the solution of many hydraulic problems may be formulated. For example, we understand that the speed of surface wave propagation on water (C) is related to the gravitational acceleration (g) and the water depth (y). Generally, we may write

$$C = f(g, y) \quad (10.1)$$

where f is used to express a function. The units of the physical quantities involved—length (L) and time (T)—are indicated in the brackets:

$$\begin{aligned} C &= [LT^{-1}] \\ g &= [LT^{-2}] \\ y &= [L] \end{aligned}$$

* There are a few exceptions such as empirical equations (e.g., Section 3.6).

TABLE 10.1 Dimensions of Physical Quantities Commonly Used in Hydraulic Engineering

Quantity	Dimension	Quantity	Dimension
Length	L	Force	MLT^{-2}
Area	L^2	Pressure	$ML^{-1}T^{-2}$
Volume	L^3	Shear stress	$ML^{-1}T^{-2}$
Angle (radians)	None	Specific weight	$ML^{-2}T^{-2}$
Time	T	Modulus of elasticity	$ML^{-1}T^{-2}$
Discharge	L^3T^{-1}	Coefficient of compressibility	$M^{-1}L^2T^2$
Linear velocity	LT^{-1}	Surface tension	MT^{-2}
Angular velocity	T^{-1}	Momentum	MLT^{-1}
Acceleration	LT^{-2}	Angular momentum	ML^2T^{-1}
Mass	M	Torque	ML^2T^{-2}
Moment of inertia	ML^2	Energy	ML^2T^{-2}
Density	ML^{-3}	Power	ML^2T^{-3}
Viscosity	$ML^{-1}T^{-1}$	Kinematic viscosity	L^2T^{-1}

Because the left-hand side of Equation 10.1 has the units of $[LT^{-1}]$, those units must appear explicitly on the right-hand side as well. Thus, d and g must combine as a product and the function (f) must be the square root. Therefore,

$$C = \sqrt{gy}$$

as discussed in Chapter 6 (Equation 6.11).

The dimensions of the physical quantities most commonly used in hydraulic engineering are listed in Table 10.1.

10.2 Principles of Hydraulic Similitude

Similarity between hydraulic models and prototypes may be achieved in three basic forms:

1. geometric similarity,
2. kinematic similarity, and
3. dynamic similarity.

Geometric similarity implies similarity of form. The model is a geometric reduction of the prototype and is accomplished by maintaining a fixed ratio for all homologous lengths between the model and the prototype.

The physical quantities involved in geometric similarity are length (L), area (A), and volume (Vol). To keep the homologous lengths in the prototype (L_p) and the model (L_m), a constant ratio (L_r) requires adherence to the following expression:

$$\frac{L_p}{L_m} = L_r \tag{10.2}$$

An area (A) is the product of two homologous lengths; hence, the ratio of the homologous area is also a constant and can be expressed as

$$\frac{A_p}{A_m} = \frac{L_p^2}{L_m^2} = L_r^2 \tag{10.3}$$

A volume (Vol) is the product of three homologous lengths. The ratio of the homologous volume can be expressed as

$$\frac{Vol_p}{Vol_m} = \frac{L_p^3}{L_m^3} = L_r^3 \quad (10.4)$$

Example 10.1

A geometrically similar open-channel model is constructed with a 5:1 scale. If the model measures a discharge 7.07 cfs (ft^3/sec), what is the corresponding discharge in the prototype?

Solution

The discharge ratio can be found using the expression $Q = AV$, which requires the area and velocity ratios. The area ratio between prototype and model using Equation 10.3 is

$$\frac{A_p}{A_m} = \frac{L_p^2}{L_m^2} = L_r^2 = 25$$

The velocity ratio between the prototype and the model is

$$\frac{V_p}{V_m} = \frac{\frac{L_p}{T}}{\frac{L_m}{T}} = \frac{L_p}{L_m} = L_r = 5$$

Note that, for geometric similarity, the time ratio from the prototype to the model remains unscaled. Accordingly, the discharge ratio is

$$\frac{Q_p}{Q_m} = \frac{A_p V_p}{A_m V_m} = (25)(5) = 125$$

Thus, the corresponding discharge in the prototype is

$$Q_p = 125 Q_m = 125(7.07) = 884 \text{ cfs}$$

Kinematic similarity implies similarity in motion. Kinematic similarity between a model and the prototype is attained if the homologous moving particles have the same velocity ratio along geometrically similar paths. Thus, kinematic similarity involves the scale of time as well as length. The ratio of times required for homologous particles to travel homologous distances in a model and its prototype is

$$\frac{T_p}{T_m} = T_r \quad (10.5)$$

Velocity (V) is defined in terms of distance per unit time; thus, the ratio of velocities can be expressed as

$$\frac{V_p}{V_m} = \frac{\frac{L_p}{T_p}}{\frac{L_m}{T_m}} = \frac{L_p}{L_m} \frac{T_m}{T_p} = \frac{L_r}{T_r} \quad (10.6)$$

Acceleration (a) is defined in terms of length per unit time squared; thus, the ratio of homologous acceleration is

$$\frac{a_p}{a_m} = \frac{\frac{L_p}{T_p^2}}{\frac{L_m}{T_m^2}} = \frac{\frac{L_p}{L_m}}{\frac{T_p^2}{T_m^2}} = \frac{L_r}{T_r^2} \tag{10.7}$$

Discharge (Q) is expressed in terms of volume per unit time; thus,

$$\frac{Q_p}{Q_m} = \frac{\frac{L_p^3}{T_p}}{\frac{L_m^3}{T_m}} = \frac{\frac{L_p^3}{L_m^3}}{\frac{T_p}{T_m}} = \frac{L_r^3}{T_r} \tag{10.8}$$

Kinematic models constructed for hydraulic machinery may frequently involve *angular displacement* (θ) expressed in radians, which is equal to the tangential displacement (L) divided by the length of the radius (R) of the curve at the point of tangency. The ratio of angular displacements may be expressed as

$$\frac{\theta_p}{\theta_m} = \frac{\frac{L_p}{R_p}}{\frac{L_m}{R_m}} = \frac{\frac{L_p}{L_m}}{\frac{R_p}{R_m}} = \frac{\frac{L_p}{L_m}}{\frac{L_p}{L_m}} = 1 \tag{10.9}$$

Angular velocity (N) in revolutions per minute is defined as angular displacement per unit time; thus, the ratio

$$\frac{N_p}{N_m} = \frac{\frac{\theta_p}{T_p}}{\frac{\theta_m}{T_m}} = \frac{\frac{\theta_p}{\theta_m}}{\frac{T_p}{T_m}} = \frac{1}{T_r} \tag{10.10}$$

Angular acceleration (α) is defined as angular displacement per unit time squared; thus,

$$\frac{\alpha_p}{\alpha_m} = \frac{\frac{\theta_p}{T_p^2}}{\frac{\theta_m}{T_m^2}} = \frac{\frac{\theta_p}{\theta_m}}{\frac{T_p^2}{T_m^2}} = \frac{1}{T_r^2} \tag{10.11}$$

Example 10.2

A 10:1 scale model is constructed to study the flow movement in a cooling pond. The designed discharge from the power plant is 200 m³/sec, and the model can accommodate a maximum flow rate of 0.1 m³/sec. What is the appropriate time ratio?

Solution

The length ratio between the prototype and the model is

$$L_r = \frac{L_p}{L_m} = 10$$

The discharge ratio is $Q_r = \frac{200}{0.1} = 2,000$, and

$$Q_r = \frac{Q_p}{Q_m} = \frac{\frac{L_p^3}{T_p}}{\frac{L_m^3}{T_m}} = \left(\frac{L_p}{L_m}\right)^3 \left(\frac{T_m}{T_p}\right) = L_r^3 T_r^{-1}$$

Substituting the length ratio into the discharge ratio gives the time ratio

$$T_r = \frac{T_p}{T_m} = \frac{L_r^3}{Q_r} = \frac{(10)^3}{2,000} = 0.5$$

or

$$T_m = 2T_p$$

Therefore, a unit time period measured in the model is equivalent to two time periods in the prototype pond.

Dynamic similarity implies similarity in the forces involved in motion. Dynamic similarity between a model and its prototype is attained if the ratio of homologous forces (prototype to model) is kept at a constant value, or

$$\frac{F_p}{F_m} = F_r \quad (10.12)$$

Many hydrodynamic phenomena may involve several different kinds of forces in action. Typically, models are built to simulate the prototype on a reduced scale and may not be capable of simulating all the forces simultaneously. In practice, a model is designed to study the effects of only a few *dominant forces*. Dynamic similarity requires that the ratios of these forces be kept the same between the model and the prototype. Hydraulic phenomena governed by various types of force ratios are discussed in Sections 10.3, 10.4, 10.5, and 10.6. Because force is equal to mass (M) multiplied by acceleration (a) and because mass is equal to density (ρ) multiplied by volume (Vol), the force ratio is expressed as

$$\begin{aligned} \frac{F_p}{F_m} &= \frac{M_p a_p}{M_m a_m} = \frac{\rho_p Vol_p a_p}{\rho_m Vol_m a_m} \\ &= \frac{\rho_p}{\rho_m} \frac{L_p^3}{L_m^3} \frac{T_p^2}{T_m^2} = \frac{\rho_p}{\rho_m} \frac{L_p^4}{L_m^4} \frac{1}{\frac{T_p^2}{T_m^2}} = \rho_r L_r^4 T_r^{-2} \end{aligned} \quad (10.13)$$

and the mass (force divided by acceleration) ratio may be expressed as

$$\frac{M_p}{M_m} = \frac{\frac{F_p}{a_p}}{\frac{F_m}{a_m}} = \frac{F_p}{F_m} \frac{a_m}{a_p} = F_r T_r^2 L_r^{-1} \tag{10.14}$$

Work is equal to force multiplied by distance, so the ratio of homologous work in dynamic similarity is

$$\frac{\overline{W}_p}{\overline{W}_m} = \frac{F_p L_p}{F_m L_m} = F_r L_r \tag{10.15}$$

Power is the time rate of doing work; thus, the power ratio may be expressed as

$$\frac{P_p}{P_m} = \frac{\frac{\overline{W}_p}{T_p}}{\frac{\overline{W}_m}{T_m}} = \frac{\overline{W}_p}{\overline{W}_m} \frac{1}{\frac{T_p}{T_m}} = \frac{F_r L_r}{T_r} \tag{10.16}$$

Example 10.3

A 59,700-w (80-hp) pump is used to power a water-supply system. The model constructed to study the system has an 8:1 scale. If the velocity ratio is 2:1, what is the power needed for the model pump?

Solution

By substituting the length ratio into the velocity ratio, the time ratio is obtained:

$$V_r = \frac{L_r}{T_r} = 2; \quad L_r = 8$$

$$T_r = \frac{L_r}{2} = \frac{8}{2} = 4$$

Assume that the same fluid is used in the model and the prototype, because an alternative was not specified. Thus, $\rho_r = 1$, and the force ratio is calculated from Equation 10.13 as

$$F_r = \rho_r L_r^4 T_r^{-2} = \frac{(1)(8)^4}{(4)^2} = 256$$

From Equation 10.16, the power ratio is

$$P_r = \frac{F_r L_r}{T_r} = \frac{(256)(8)}{(4)} = 512$$

and the power required for the model pump is

$$P_m = \frac{P_p}{P_r} = \frac{59,700}{512} = 117 \text{ W} = 0.157 \text{ hp}$$

Example 10.4

The model designed to study the prototype of a hydraulic machine must

1. be geometrically similar,
2. have the same discharge coefficient defined as $Q/(A\sqrt{2gH})$, and
3. have the same ratio of peripheral speed to the water-discharge velocity $[\omega D/(Q/A)]$.

Determine the scale ratios in terms of discharge (Q), head (H), diameter (D), and rotational angular velocity (ω).

Solution

It is important to recognize that although the energy head (H) is expressed in units of length, it is not necessarily modeled as a linear dimension. To have the same ratio of peripheral speed to the water discharge velocity, we have

$$\frac{\omega_p D_p}{Q_p/A_p} = \frac{\omega_m D_m}{Q_m/A_m}$$

or

$$\frac{\omega_r D_r A_r}{Q_r} = \frac{T_r^{-1} L_r L_r^2}{L_r^3 T_r^{-1}} = 1$$

To have the same discharge coefficient, we have

$$\frac{Q_p/(A_p \sqrt{2gH_p})}{Q_m/(A_m \sqrt{2gH_m})} = \frac{Q_r}{A_r \sqrt{(gH)_r}} = 1$$

or

$$\frac{L_r^3 T_r^{-1}}{L_r^2 (gH)_r^{1/2}} = 1$$

from which we get

$$(gH)_r = \frac{L_r^2}{T_r^2}$$

Because the gravitational acceleration (g) is the same for the model and prototype, we may write,

$$H_r = L_r^2 T_r^{-2}$$

The other ratios asked for are

Discharge ratio Q_r : $L_r^3 T_r^{-1}$

Diameter ratio: $D_r = L_r$, and

Angular speed ratio: $v = T_r^{-1}$

10.3 Phenomena Governed by Viscous Forces: Reynolds Number Law

Water in motion always involves inertial forces. When the inertial forces and the viscous forces can be considered to be the only forces that govern the motion, then the ratio of these forces acting on homologous particles in a model and its prototype is defined by the Reynolds number law:

$$N_R = \frac{\text{inertial force}}{\text{viscous force}} \quad (10.17)$$

The inertial forces defined by Newton’s second law of motion, $F = ma$, can be expressed by the ratio in Equation 10.13:

$$F_r = M_r \frac{L_r}{T_r^2} = \rho_r L_r^4 T_r^{-2} \tag{10.13}$$

The viscous force defined by Newton’s law of viscosity,

$$F = \mu \left(\frac{dV}{dL} \right) A$$

may be expressed by

$$F_r = \frac{\mu_p \left(\frac{dV}{dL} \right)_p A_p}{\mu_m \left(\frac{dV}{dL} \right)_m A_m} = \mu_r L_r^2 T_r^{-1} \tag{10.18}$$

where μ is the viscosity and V denotes the velocity.

Equating values of F_r from Equations 10.13 and 10.18, we get

$$\rho_r L_r^4 T_r^{-2} = \mu_r L_r^2 T_r^{-1}$$

from which

$$\frac{\rho_r L_r^4 T_r^{-2}}{\mu_r L_r^2 T_r^{-1}} = \frac{\rho_r L_r^2}{\mu_r T_r} = \frac{\rho_r L_r V_r}{\mu_r} = 1 \tag{10.19}$$

Reformulating the above equation, we may write

$$\frac{\left(\frac{\rho_p L_p V_p}{\mu_p} \right)}{\left(\frac{\rho_m L_m V_m}{\mu_m} \right)} = (N_R)_r = 1$$

or

$$\frac{\rho_p L_p V_p}{\mu_p} = \frac{\rho_m L_m V_m}{\mu_m} = N_R \tag{10.20}$$

Equation 10.20 states that when the inertial force and the viscous force are considered to be the only forces governing the motion of the water, the Reynolds number of the model and the prototype must be kept at the same value.

If the same fluid is used in both the model and the prototype, the scale ratios for many physical quantities can be derived based on the Reynolds number law. These quantities are listed in Table 10.2.

TABLE 10.2 Scale Ratios for the Reynolds Number Law (water used in both model and prototype, $\rho_r = 1, \mu_r = 1$)

Geometric Similarity		Kinematic Similarity		Dynamic Similarity	
Length	L_r	Time	L_r^2	Force	1
Area	L_r^2	Velocity	L_r^{-1}	Mass	L_r^3
Volume	L_r^3	Acceleration	L_r^{-3}	Work	L_r
		Discharge	L_r	Power	L_r^{-1}
		Angular velocity	L_r^{-2}		
		Angular acceleration	L_r^{-4}		

Example 10.5

To study a transient process, a model is constructed at a 10:1 scale. Water is used in the prototype, and it is known that viscous forces are the dominant ones. Compare the time, velocity, and force ratios if the model uses

- (a) water or
 (b) oil that is five times more viscous than water, with $\rho_{oil} = 0.8\rho_{water}$.

Solution

- (a) From Table 10.2,

$$T_r = L_r^2 = (10)^2 = 100$$

$$V_r = L_r^{-1} = (10)^{-1} = 0.1$$

$$F_r = 1$$

- (b) From the Reynolds number law,

$$\frac{\rho_p L_p V_p}{\mu_p} = \frac{\rho_m L_m V_m}{\mu_m}$$

we have

$$\frac{\rho_r L_r V_r}{\mu_r} = 1$$

Because the ratios of viscosity and density are, respectively,

$$\mu_r = \frac{\mu_p}{\mu_m} = \frac{\mu_{water}}{\mu_{oil}} = \frac{\mu_{water}}{5\mu_{water}} = 0.2$$

$$\rho_r = \frac{\rho_p}{\rho_m} = \frac{\rho_{water}}{\rho_{oil}} = \frac{\rho_{water}}{0.8\rho_{water}} = 1.25$$

From the Reynolds number law,

$$V_r = \frac{\mu_r}{\rho_r L_r} = \frac{(0.2)}{(1.25)(10)} = 0.016$$

The time ratio is

$$T_r = \frac{L_r}{V_r} = \frac{(10)}{(0.016)} = 625 \quad \text{or} \quad T_r = \frac{L_r}{V_r} = \frac{\rho_r L_r^2}{\mu_r} = \frac{(1.25)(10)^2}{(0.2)} = 625$$

The force ratio, then, is

$$F_r = \frac{\rho_r L_r^4}{T_r^2} = \frac{(1.25)(10)^4}{(625)^2} = 0.032 \quad \text{or}$$

$$F_r = \frac{\rho_r L_r^4}{T_r^2} = \frac{(\rho_r L_r^4)}{\left(\frac{\rho_r^2 L_r^4}{\mu_r^2}\right)} = \frac{\mu_r^2}{\rho_r} = \frac{(0.2)^2}{1.25} = 0.032$$

The T_r and F_r equations that were solved first simplify the computations. However, the reformulated equations that rely on the Reynolds number law (given in terms of ρ and μ) demonstrate the importance of selecting the model fluid. The properties of the liquid used in the model, especially the viscosity, greatly affect the performance in the Reynolds number models.

10.4 Phenomena Governed by Gravity Forces: Froude Number Law

In some flow situations, inertial forces and gravity forces are considered to be the only dominant forces. The ratio of the inertial forces acting on the homologous elements of the fluid in the model and prototype can be defined by Equation 10.13, restated as

$$\frac{F_p}{F_m} = \rho_r L_r^4 T_r^{-2} \tag{10.13}$$

The ratio of gravity forces, represented by the weight of the homologous fluid elements involved, may be expressed as

$$\frac{F_p}{F_m} = \frac{M_p g_p}{M_m g_m} = \frac{\rho_p L_p^3 g_p}{\rho_m L_m^3 g_m} = \rho_r L_r^3 g_r \tag{10.21}$$

Equating the values from Equations 10.13 and 10.21, we obtain

$$\rho_r L_r^4 T_r^{-2} = \rho_r L_r^3 g_r$$

which can be rearranged as

$$g_r L_r = \frac{L_r^2}{T_r^2} = V_r^2$$

or

$$\frac{V_r}{g_r^{1/2} L_r^{1/2}} = 1 \tag{10.22}$$

TABLE 10.3 Scale Ratios for the Froude Number Law ($g_r = 1, \rho_r = 1$)

Geometric Similarity		Kinematic Similarity		Dynamic Similarity	
Length	L_r	Time	$L_r^{1/2}$	Force	L_r^3
Area	L_r^2	Velocity	$L_r^{1/2}$	Mass	L_r^3
Volume	L_r^3	Acceleration	1	Work	L_r^4
		Discharge	$L_r^{5/2}$	Power	$L_r^{7/2}$
		Angular velocity	$L_r^{-1/2}$		
		Angular acceleration	L_r^{-1}		

Equation 10.22 may be expressed as

$$\frac{\left(\frac{V_p}{g_p^{1/2} L_p^{1/2}} \right)}{\left(\frac{V_m}{g_m^{1/2} L_m^{1/2}} \right)} = (N_F)_r = 1$$

and hence,

$$\frac{V_p}{g_p^{1/2} L_p^{1/2}} = \frac{V_m}{g_m^{1/2} L_m^{1/2}} = N_F \text{ (Froude number)} \tag{10.23}$$

In other words, when the inertial force and the gravity force are considered to be the only forces that dominate the fluid motion, the Froude number of the model and the Froude number of the prototype should be kept equal.

If the same fluid is used for both the model and the prototype, and they are both subjected to the same gravitational force field, then many physical quantities can be derived based on the Froude number law. These quantities are listed in Table 10.3.

Example 10.6

An open-channel model 30 m long is built to satisfy the Froude number law. What is the flow in the model for a prototype flood of 700 m³/sec if the scale used is 20:1? Also determine the force ratio.

Solution

From Table 10.3, the discharge ratio is

$$Q_r = L_r^{5/2} = (20)^{2.5} = 1,790$$

Thus, the model flow should be

$$Q_m = \frac{Q_p}{Q_r} = \frac{700 \text{ m}^3/\text{sec}}{1,790} = 0.391 \text{ m}^3/\text{sec} = 391 \text{ L}/\text{sec}$$

The force ratio is

$$F_r = \frac{F_p}{F_m} = L_r^3 = (20)^3 = 8,000$$

10.5 Phenomena Governed by Surface Tension: Weber Number Law

Surface tension is a measure of molecular energy on the surface of a liquid body. The resulting force can be significant in the motion of small surface waves or the control of evaporation from a large body of water such as a water storage tank or reservoir.

Surface tension, denoted by σ , is measured in terms of force per unit length. Hence, the resulting force is $F = \sigma L$. The ratio of analogous surface tension forces in prototype and model is

$$F_r = \frac{F_p}{F_m} = \frac{\sigma_p L_p}{\sigma_m L_m} = \sigma_r L_r \quad (10.24)$$

Equating the surface tension force ratio to the inertial force ratio (Equation 10.13) gives

$$\sigma_r L_r = \rho_r \frac{L_r^4}{T_r^2}$$

Rearranging gives

$$T_r = \left(\frac{\rho_r}{\sigma_r} \right)^{1/2} L_r^{3/2} \quad (10.25)$$

By substituting for T_r from the basic relationship of $V_r = L_r/T_r$, Equation 10.25 may be rearranged to give

$$V_r = \frac{L_r}{\left(\frac{\rho_r}{\sigma_r} \right)^{1/2} L_r^{3/2}} = \left(\frac{\sigma_r}{\rho_r L_r} \right)^{1/2}$$

or

$$\frac{\rho_r V_r^2 L_r}{\sigma_r} = 1 \quad (10.26)$$

Hence,

$$\frac{\rho_p V_p^2 L_p}{\sigma_p} = \frac{\rho_m V_m^2 L_m}{\sigma_m} = N_W \text{ (Weber number)} \quad (10.27)$$

In other words, the Weber number must be kept at the same value in the model and in the prototype for studying phenomena governed by inertial and surface tension forces. If the same liquid is used in the model and the prototype, then $\rho_r = 1.0$ and $\sigma_r = 1.0$, and Equation 10.26 can be simplified to

$$V_r^2 L_r = 1$$

or

$$V_r = \frac{1}{L_r^{1/2}} \quad (10.28)$$

Because $V_r = L_r/T_r$, we may also write

$$\frac{L_r}{T_r} = \frac{1}{L_r^{1/2}}$$

Thus,

$$T_r = L_r^{3/2} \quad (10.29)$$

10.6 Phenomena Governed by Both Gravity and Viscous Forces

Both gravity and viscous forces may be important in the study of surface vessels moving through water or shallow water waves propagated in open channels. These phenomena require that both the Froude number law and the Reynolds number law be satisfied simultaneously; that is, $(N_R)_r = (N_F)_r = 1$, or

$$\frac{\rho_r L_r V_r}{\mu_r} = \frac{V_r}{(g_r L_r)^{1/2}}$$

Assuming that both the model and the prototype are affected by the earth's gravitational field ($g_r = 1$) and because $\nu = \mu/\rho$, the above relationship may be simplified to

$$\nu_r = L_r^{3/2} \quad (10.30)$$

This requirement can only be met by choosing a special model fluid with a kinematic viscosity ratio to water equal to the three-halves power of the scale ratio. In general, this requirement is difficult to meet. For example, a 1:10 scale model would require that the model fluid have a kinematic viscosity of 31.6 times less than that of water, which is obviously impossible.

However, two courses of action may be available, depending on the relative importance of the two forces in the particular phenomenon. In the case of ship resistance, the ship model may be built according to the Reynolds model law and may operate in a towing tank in accordance with the Froude number law. In the case of shallow water waves in open channels, empirical relationships such as Manning's formula (Equation 6.4) may be used as an auxiliary condition for the wave measurements, according to the Froude number law.

10.7 Models for Floating and Submerged Bodies

Model studies for floating and submerged bodies are performed to obtain information on

1. friction drag along the boundary of the moving vessel,
2. form drag resulting from boundary flow separation caused by the body shape,
3. forces expended in the generation of gravity waves, and
4. stability of the body in withstanding waves and wave forces on the body.

The first two phenomena are strictly governed by viscous forces, and therefore the models should be designed according to the Reynolds number law. The third phenomenon is governed by the gravity force and must be analyzed by applying the Froude number law. All three measurements may be performed simultaneously in a towing tank filled with water. In analyzing

the data, however, the friction forces and the form drag forces are first computed from the measurements by using known formulas and drag coefficients. The remaining force measured in towing the vessel through the water surface is the force expended in generating the gravity waves (wave resistance), and it is scaled up to the prototype values by the Froude number law.

The analysis procedure is demonstrated in Example 10.7. For subsurface vessels such as submarines, the effect of surface waves on the vessels may be neglected. Hence, the Froude number model is not needed. To study stability and wave forces on stationary offshore structures, the effect of inertial force must be taken into consideration. The inertial force, defined as $F_i = M'a$, can be calculated directly from the prototype dimensions. Here, M' is the mass of water displaced by the portion of the structure immersed below the waterline (also known as the *virtual mass*), and a is the acceleration of the water mass.

Example 10.7

A ship model, with a maximum cross-sectional area of 0.780 m^2 immersed below the waterline has a characteristic length of 0.9 m . The model is towed in a wave tank at the speed of 0.5 m/sec . For the particular shape of the vessel, it is found that the drag coefficient can be approximated by $C_D = (0.06/N_R^{0.25})$ for $10^4 \leq N_R \leq 10^6$, and $C_D = 0.0018$ for $N_R > 10^6$. The Froude number law is applied for the 1:50 model. During the experiment, a total force of 0.400 N is measured. Determine the total resistance force on the prototype vessel.

Solution

Based on the Froude number law (Table 10.3), we may determine the velocity ratio as

$$V_r = L_r^{1/2} = (50)^{1/2} = 7.07$$

Hence, the corresponding velocity of the vessel is

$$V_p = V_m V_r = 0.5(7.07) = 3.54 \text{ m/sec}$$

The Reynolds number of the model is

$$N_R = \frac{V_m L_m}{\nu} = \frac{0.5(0.9)}{1.00 \times 10^{-6}} = 4.50 \times 10^5$$

and the drag coefficient for the model is

$$C_{D_m} = \frac{0.06}{(4.50 \times 10^5)^{1/4}} = 0.00232$$

The drag force on a vessel is defined as $D = C_D \left(\frac{1}{2} \rho A V^2 \right)$, where ρ is the water density and A is the projected area of the immersed portion of the vessel on a plane normal to the direction of the motion. Thus, the model drag force can be calculated as

$$D_m = C_{D_m} \left(\frac{1}{2} \rho_m A_m V_m^2 \right) = \frac{1}{2} (0.00232)(998)(0.78)[(0.5)^2] = 0.226 \text{ N}$$

The model wave resistance is the difference between the measured towing force and the drag force:

$$F_w = 0.400 - 0.226 = 0.174 \text{ N}$$

For the prototype, the Reynolds number is

$$N_R = \frac{V_p L_p}{\nu_p} = \frac{V_p L_r L_m}{\nu_p} = \frac{3.54(50)(0.9)}{1.00 \times 10^{-6}} = 1.59 \times 10^8$$

Thus, the drag coefficient of the prototype vessel is $C_{D_p} = 0.0018$, and the drag force is

$$D_p = C_{D_p} \left(\frac{1}{2} \rho_p A_p V_p^2 \right) = C_{D_p} \left(\frac{1}{2} \rho_p A_m L_r^2 V_p^2 \right)$$

$$D_p = 0.0018 \left(\frac{1}{2} (998) (0.780) (50)^2 (3.54)^2 \right) = 21,900 \text{ N}$$

The wave resistance on the prototype vessel is calculated by applying the Froude number law (Table 10.3):

$$F_{w_p} = F_{w_r} F_{w_m} = L_r^3 F_{w_m} = (50)^3 (0.174) = 21,800 \text{ N}$$

Hence, the total resistance force on the prototype is

$$F = D_p + F_{w_p} = 21,900 + 21,800 = 43,700 \text{ N}$$

10.8 Open-Channel Models

Open-channel models may be used to study velocity–slope relationships and the effects of flow patterns on the changes in channel bed configuration. For the former applications, relatively long reaches of the river channel can be modeled. A special example is the U.S. Army Corps of Engineers—Waterways Experiment Station in Vicksburg, Mississippi, where the Mississippi River was once modeled on one site. In these applications, where changes in bed configuration are only of secondary concern, a *fixed-bed model* may be used. Basically, this model is used in studying the velocity–slope relationship in a particular channel; therefore, the effect of bed roughness is important.

An empirical relationship, such as the Manning equation (Equation 6.4), may be used to assume the similarity between the prototype and the model:

$$V_r = \frac{V_p}{V_m} = \frac{\frac{1}{n_p} R_{h_p}^{2/3} S_p^{1/2}}{\frac{1}{n_m} R_{h_m}^{2/3} S_m^{1/2}} = \frac{1}{n_r} R_{h_r}^{2/3} S_r^{1/2} \quad (10.31)$$

If the model is built with the same scale ratio for the horizontal dimensions (\bar{X}) and vertical dimensions (\bar{Y}), known as the *undistorted model*, then

$$R_{h_r} = \bar{X}_r = \bar{Y}_r = L_r \quad \text{and} \quad S_r = 1$$

and

$$V_r = \frac{1}{n_r} L_r^{2/3} \quad (10.32)$$

The Manning's roughness coefficient $n \propto R_h^{1/6}$ (Equation 6.3), so we may write

$$n_r = L_r^{1/6} \quad (10.33)$$

This often results in a model velocity so small (or, conversely, the model roughness will be so large) that realistic measurements cannot be made. In addition, the model water depth may be so shallow that the physical characteristics of the flow may be altered. These issues may be

resolved by using a distorted model in which the vertical scale and the horizontal scale do not have the same value, usually, a smaller vertical scale ratio, $\bar{X}_r > \bar{Y}_r$. This means that

$$S_r = \frac{S_p}{S_m} = \frac{\bar{Y}_r}{\bar{X}_r} < 1$$

Hence, $S_m > S_p$, and the result is a larger slope for the model. The use of the Manning equation requires that the flow be fully turbulent in both model and prototype.

Open-channel models involving problems of sediment transport, erosion, or deposit require *movable bed models*. A movable channel bed consists of sand or other loose material that can be moved in response to the forces of the current at the channel bed. Normally, it is impractical to scale the bed material down to the model scale. A vertical scale distortion is usually employed on a movable bed model in order to provide a sufficient tractive force to induce bed material movement. Quantitative similarity is difficult to attain in movable bed models. For any sedimentation studies performed, it is important that the movable bed model be quantitatively verified by a number of field measurements.

Example 10.8

An open-channel model is built to study the effects of tidal waves on sedimentation movement in a 10-km river reach (the reach meanders in an area 7 km long). The mean depth and width of the reach are 4 m and 50 m, respectively, and the discharge is 850 m³/sec. Manning’s roughness value is $n_p = 0.035$. If the model is to be constructed in a laboratory room 18 m long, determine a convenient scale, the model discharge, and the model roughness coefficient. Verify that turbulent flow prevails in the model.

Solution

In surface wave phenomena the gravitational forces are dominant. The Froude number law will be used for the modeling. The laboratory length will limit the horizontal scale:

$$\bar{X}_r = \frac{L_p}{L_m} = \frac{7,000}{18} = 389$$

We will use $L_r = 400$ for convenience.

It is judged reasonable to use a vertical scale of $\bar{Y}_r = 80$ (enough to measure surface gradients). Recall that the hydraulic radius is the characteristic dimension in open-channel flow and that for a large width-to-depth ratio the hydraulic radius is roughly equal to the water depth. Thus, we can make the following approximation:

$$R_{h_r} = \bar{Y}_r = 80$$

Because

$$N_F = \frac{V_r}{g_r^{1/2} R_{h_r}^{1/2}} = 1 \tag{10.22}$$

then

$$V_r = R_{h_r}^{1/2} = \bar{Y}_r^{1/2} = (80)^{1/2}$$

Using Manning’s formula (or Equation 10.31),

$$V_r = \frac{V_p}{V_m} = \frac{1}{n_r} R_{h_r}^{2/3} S_r^{1/2} \quad S_r = \frac{\bar{Y}_r}{\bar{X}_r}$$

we have

$$n_r = \frac{R_{h_r}^{2/3} S_r^{1/2}}{V_r} = \frac{\bar{Y}_r^{2/3} \left(\frac{\bar{Y}_r}{\bar{X}_r} \right)^{1/2}}{\bar{Y}_r^{1/2}} = \frac{\bar{Y}_r^{2/3}}{\bar{X}_r^{1/2}} = \frac{(80)^{2/3}}{(400)^{1/2}} = 0.928$$

Hence,

$$n_m = \frac{n_p}{n_r} = \frac{0.035}{0.928} = 0.038$$

The discharge ratio is

$$Q_r = A_r V_r = \bar{X}_r \bar{Y}_r V_r = \bar{X}_r \bar{Y}_r^{3/2} = (400)(80)^{3/2} = 2.86 \times 10^5$$

Thus, the model discharge required is

$$Q_m = \frac{Q_p}{Q_r} = \frac{850}{2.86 \times 10^5} = 0.00297 \text{ m}^3/\text{sec} = 2.97 \text{ L/sec}$$

To use the Manning formula, turbulent flow must be ensured in the model. To verify the turbulent flow condition in the model, it is necessary to calculate the value of the model Reynolds number.

The horizontal prototype velocity is

$$V_p = \frac{850 \text{ m}^3/\text{sec}}{(4 \text{ m})(50 \text{ m})} = 4.25 \text{ m/sec}$$

Hence,

$$V_m = \frac{V_p}{V_r} = \frac{4.25}{(80)^{1/2}} = 0.475 \text{ m/sec}$$

The model depth is

$$Y_m = \frac{Y_p}{Y_r} = \frac{4}{80} = 0.05 \text{ m}$$

The model Reynolds number is

$$N_R = \frac{V_m Y_m}{\nu} = \frac{(0.475)(0.05)}{1.00 \times 10^{-6}} = 23,800$$

which is much greater than the critical Reynolds number (2,000). Hence, the flow is turbulent in the model.

10.9 The Pi Theorem

Complex hydraulic engineering problems often involve many variables. Each variable usually contains one or more dimensions. In this section, the Pi theorem is introduced to reduce the complexity of these problems when coupled with experimental model studies. The Pi theorem relies on dimensional analysis to group several independent variables into dimensionless combinations, thereby reducing the number of control variables required in the experiment. In addition to variable reduction, dimensional analysis indicates those factors that significantly influence the phenomenon; thus it guides the direction in which the experimental (model) work should be conducted.

Physical quantities in hydraulic engineering may be expressed either in the force–length–time (FLT) system or in the mass–length–time (MLT) system. These two systems are connected by Newton’s second law, which states that force equals mass times acceleration, or $F = ma$. Through this relationship, conversion can be made from one system to the other.

The steps in dimensional analysis can be demonstrated by examining a simple flow phenomenon such as the drag force exerted on a sphere as it moves through a viscous fluid. Our general understanding of the phenomenon is that the drag force is related to the size (diameter) of the sphere (D), the velocity of the sphere (V), the viscosity of the fluid (μ), and the fluid density (ρ). Thus, we may express the drag force as a function of D , V , μ , and ρ , or we may write:

$$F_d = f(D, V, \rho, \mu)$$

A generalized approach to dimensional analysis of the phenomenon may be made through the *Buckingham Pi theorem*. This theorem states that if a physical phenomenon involves n dimensional variables in a dimensionally homogenous equation described by m fundamental dimensions, then the variables may be combined into $(n - m)$ dimensionless groups for analysis. For drag force on a moving sphere, a total of five dimensional variables are involved. The previous equation may thus be expressed as

$$f'(F_d, D, V, \rho, \mu) = 0$$

These five variables ($n = 5$) are described by the fundamental dimensions, M , L , and T ($m = 3$). Because $n - m = 2$, we can express the function using two Π groups:

$$\mathcal{O}(\Pi_1, \Pi_2) = 0$$

The next step is to arrange the five dimensional parameters into two dimensionless Π groups. This is accomplished by selecting *m-repeating variables* (for this problem, we need three repeating variables) that will appear in each of the dimensionless Π groups. The repeating variables must contain all m -dimensions, must be independent (i.e., cannot be combined to form a dimensionless variable of their own), and should be the most dimensionally simple of all the variables in the experiment. In this case, we will select the following three repeating variables: *sphere diameter* (simple dimensionally and contains the length dimension), *velocity* (simple dimensionally and contains the time dimension), and *density* (the simplest variable left that contains the mass dimension). At this point, the three repeating variables are combined with the two nonrepeating variables to form the two Π groups.

$$\begin{aligned} \Pi_1 &= D^a V^b \rho^c \mu^d \\ \Pi_2 &= D^a V^b \rho^c F_D^d \end{aligned}$$

The values of the exponents are determined by noting the Π groups are dimensionless, and they can be replaced by $M^0 L^0 T^0$.

Because most hydraulic studies involve certain common dimensionless groups such as the Reynolds number, the Froude number, or the Weber number, as discussed in the previous sections in this chapter, one should always look for them in performing the dimensional analysis. To determine the Π_1 group, we write the dimensional expression:

$$M^0 L^0 T^0 = (L)^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d$$

where the dimensions are found in Table 10.1. Based on this algebraic relationship:

$$\text{for } M: 0 = c + d$$

$$\text{for } L: 0 = a + b - 3c - d$$

$$\text{for } T: 0 = -b - d$$

There are four unknowns in the three conditions given. We can always solve for three of them in terms of the fourth—say, d . From the equations above, we have:

$$c = -d, b = -d, \text{ and, by substitution, } a = -d$$

Thus,

$$\Pi_1 = D^{-d}V^{-d}\rho^{-d}\mu^d = \left(\frac{\mu}{DV\rho}\right)^d = \left(\frac{DV\rho}{\mu}\right)^{-d}$$

where the dimensionless variable combination results in the Reynolds number (N_R).

Working in similar fashion with the Π_2 group, we get

$$\Pi_2 = \frac{F_D}{\rho D^2 V^2}$$

Note that the dimensions for the drag force in the MLT system are (ML/T^2) based on Newton's second law, $F = ma$ as displayed in Table 10.1.

Finally, returning to the original condition that $\mathcal{O}(\Pi_1, \Pi_2) = 0$, we may write $\Pi_1 = \mathcal{O}'(\Pi_2)$ or $\Pi_2 = \mathcal{O}''(\Pi_1)$. Thus,

$$\frac{F_D}{\rho D^2 V^2} = \mathcal{O}''(N_R)$$

where \mathcal{O}'' is the undefined function we are seeking. In other words, the dimensionless grouping of variables on the left side of the equation, which includes the drag force, is a function of the Reynolds number. We have now reduced the original problem from five variables to two dimensionless variables that contain the original five variables.

Now the full benefit of the Pi theorem should be evident to the reader. Based on the original problem formulation, a five-variable experiment would have been difficult to set up and tedious to analyze. However, the new two variable experiment is much less complex. To find the appropriate relationship or function, \mathcal{O}'' , an experiment can be devised whereby the drag force is measured as the Reynolds number is changed. The resulting data can be graphed [$F_D/(\rho D^2 V^2)$ vs. N_R] and analyzed by statistical software to define the functional relationship. Incidentally, the Reynolds number can be changed easily by altering the experimental velocity [because $N_R = (\rho DV)/\mu$] without changing the experimental fluid (required to change ρ and μ), a needlessly tedious endeavor.

It should be emphasized that dimensional analysis does not provide solutions to a problem; rather, it provides a guide to point out the relationships among the parameters that are applicable to the problem. If one omits an important parameter, the results are incomplete and thus can lead to incorrect conclusions. However, if one includes parameters that are unrelated to the problem, additional dimensionless groups that are irrelevant to the problem will result. Thus, the successful application of dimensional analysis depends, to a certain degree, on the engineer's basic understanding of the hydraulic phenomenon involved. These considerations can be demonstrated by the following example problem.

Example 10.9

A broad crested weir model is designed to study the discharge per foot of the prototype. Derive an expression for this discharge using the Buckingham Pi theorem. Assume that the overflowing water sheet is relatively thick so that the surface tension and thus viscosity of the fluid can be neglected.

Solution

Based on our general understanding of the phenomenon, we may assume that the discharge (q) would be affected by the spillway head (H), the gravitational acceleration (g), and the spillway height (h). Thus, $q = f(H, g, h)$ or $f'(q, H, g, h) = 0$.

In this case, $n = 4$, $m = 2$ (not 3, because none of the terms involve mass). According to the Pi theorem, there are $n - m = 2$ dimensionless groups, and

$$\emptyset(\Pi_1, \Pi_2) = 0$$

Based on the rules used to guide the selection of repeating variables, we will use spillway head (which is simple dimensionally and contains the length dimension) and spillway discharge (which is simple dimensionally and contains the time dimension). Note that we could not use spillway height as a repeating variable once spillway head was selected because they can be combined to form a dimensionless parameter (i.e., h/H). However, gravitational acceleration (g) could have been selected as the second repeating variable without affecting the problem solution. (The reader is encouraged to complete Problem 10.9.1 to verify that this statement is accurate.)

Using q and H as the basic repeating variables, we have

$$\Pi_1 = q^{a_1} H^{b_1} g^{c_1}$$

$$\Pi_2 = q^{a_2} H^{b_2} h^{c_2}$$

From the Π_1 group, we have

$$L^0 T^0 = \left(\frac{L^3}{TL}\right)^{a_1} L^{b_1} \left(\frac{L}{T^2}\right)^{c_1}$$

Thus,

$$L: \quad 0 = 2a_1 + b_1 + c_1$$

$$T: \quad 0 = -a_1 - 2c_1$$

Hence,

$$c_1 = -\frac{1}{2} a_1 \quad b_1 = -\frac{3}{2} a_1$$

$$\Pi_1 = q^{a_1} H^{-\frac{3}{2} a_1} g^{-\frac{1}{2} a_1} = \left(\frac{q}{g^{1/2} H^{3/2}}\right)^{a_1}$$

From the Π_2 group we have

$$L^0 T^0 = \left(\frac{L^3}{TL}\right)^{a_2} L^{b_2} L^{c_2}$$

Thus,

$$L: \quad 0 = 2a_2 + b_2 + c_2$$

$$T: \quad 0 = -a_2$$

Hence,

$$a_2 = 0 \quad b_2 = -c_2$$

$$\Pi_2 = q^0 H^{-c_2} h^{c_2} = \left(\frac{h}{H}\right)^{c_2}$$

Note that this dimensionless variable would not have resulted if both h and H were used as repeating variables. Now that the two dimensionless groups are identified to be

$$\left(\frac{q}{g^{1/2} H^{3/2}}\right) \quad \text{and} \quad \left(\frac{h}{H}\right)$$

we may return to

$$\mathcal{O}(\Pi_1, \Pi_2) = \mathcal{O}\left(\frac{q}{g^{1/2} H^{3/2}}, \frac{h}{H}\right) = 0$$

or

$$\frac{q}{g^{1/2} H^{3/2}} = \mathcal{O}'\left(\frac{h}{H}\right)$$

$$q = g^{1/2} H^{3/2} \mathcal{O}'\left(\frac{h}{H}\right)$$

The result indicates that the discharge per unit length of the spillway is proportional to \sqrt{g} and $H^{3/2}$. The flow rate is also influenced by the ratio (h/H) , as shown in Chapter 9 (Equation 9.19).

PROBLEMS (SECTION 10.2)

- 10.2.1.** Uniform flow at a depth of 2 meters occurs in a rectangular channel that is 4 meters wide. The channel is laid on a slope of 0.001, and the Manning coefficient is 0.025. Determine the maximum depth and width of a geometrically similar channel if the flow in the model must be limited to 0.081 m³/sec.
- 10.2.2.** A 1:15 scale model will be used to study the circulation patterns in a rectangular detention pond. The pond has a bottom length of 40 m and a bottom width of 10 m. All sides of the pond have 3:1 (H:V) slopes. If the design depth is 5 m, what is the depth, surface area, and storage volume of the model? (*Note:* The storage volume expression is $Vol = LWd + (L + W)zd^2 + (4/3)z^2d^3$, where L and W are the length and width of the bottom, d is the depth, and z is the side slope.)
- 10.2.3.** A shipping channel with a design flow rate of 2,650 cfs is experiencing a large accumulation of sediment. A model study is proposed for the 3.20-mile-long channel, but the available model length is only 70 feet. A time ratio of 10 will be used. Select a suitable length scale and determine the model discharge.
- 10.2.4.** A proposed reservoir will be drained using a gated opening that is governed by the orifice equation: $Q = C_d A (2gh)^{1/2}$ (Equation 8.19), where C_d is the coefficient of discharge. A 1:150 scale model is used, and the model reservoir is drained in 18.3 minutes. How many hours should it take to drain the prototype? Assume the discharge coefficients of the prototype and the model are identical.
- 10.2.5.** A 1:5 scale model at 1,200 rpm is used to study the prototype of a centrifugal pump that produces 1.00 m³/sec at 30-m head when rotating at 400 rpm. Determine the model discharge and

head. (*Hint:* In this case, the head is not a linear dimension. See Example 10.4 for a more complete explanation.)

- 10.2.6.** An overflow spillway with a 100-m-long crest will convey a design discharge of $1,150 \text{ m}^3/\text{sec}$ under a permitted maximum head of 3.00 m. The operation of the prototype spillway is studied on a 1:50 scale model in a hydraulic laboratory. The time ratio on the model is $L_r^{1/2}$. The model velocity measured at the end (toe) of the spillway is 3.00 m/sec. Determine the flow rate in the model and the Froude numbers [$N_F = V/(gd)^{1/2}$, where d is the flow depth] at the toe of the spillway in the model and the prototype.
- 10.2.7.** A 1:20 scale model of a prototype energy dissipation structure is constructed to study force distribution and water depths. A velocity ratio of 7.75 is used. Determine the force ratio and prototype discharge if the model discharge is $10.6 \text{ ft}^3/\text{sec}$.
- 10.2.8.** A 1:50 scale model is used to study the power requirements of a prototype submarine. The model will be towed at a speed 50 times greater than the speed of the prototype in a tank filled with seawater. Determine the conversion ratios from the prototype to the model for the following quantities: (a) time, (b) force, (c) power, and (d) energy.
- 10.2.9.** A seawall has been proposed to dissipate wave forces on a beach front. A 3-ft-long, 1:30 model is used to study the effects on the prototype. If the total force measured on the model is 0.510 lbs and the velocity scale is 1:10, what is the force per unit length of the prototype?
- 10.2.10.** The moment exerted on a gate structure is studied in a laboratory water tank with a 1:125 scale model. The moment measured on the model is $1.5 \text{ N}\cdot\text{m}$ on the 1-m long gate arm. Determine the moment exerted on the prototype.

(SECTION 10.3)

- 10.3.1.** The motion of a submarine is being studied in a laboratory. The prototype speed of interest is 5 m/sec in the ocean. Inertial forces and viscous forces govern the motion. At what theoretical speed must a 1:10 model be towed to establish similarity between the model and the prototype? Assume that the seawater and towing tank water are the same.
- 10.3.2.** Verify the scale ratios given in Table 10.2 (Reynolds Number Law) for (a) velocity, (b) time, (c) acceleration, (d) discharge, (e) force, and (f) power.
- 10.3.3.** A 4-ft diameter oil pipeline is being proposed for a remote location. The oil has a specific gravity of 0.8 and a dynamic viscosity of $9.93 \times 10^{-5} \text{ lb}\cdot\text{sec}/\text{ft}^2$. A model will be used to study the pipeline flow conditions using a 0.5-ft diameter pipe and water at normal conditions (68.4°F). If the design flow rate in the prototype is 125 cfs (ft^3/sec), what is the required flow in the model?
- 10.3.4.** The moment exerted on a ship's rudder is studied with a 1:20 scale model in a water tunnel using the same temperature water as the river water. Inertial and viscous forces govern the fluid motion. The torque measured on the model is $10 \text{ N}\cdot\text{m}$ for a water tunnel velocity of 20 m/sec. Determine the corresponding water velocity and torque for the prototype.
- 10.3.5.** A 1:10 scale model of a water-supply piping system is to be tested at 20°C to determine the total head loss in the prototype that carries water at 85°C . The prototype is designed to carry $5.0 \text{ m}^3/\text{sec}$. Determine the model discharge.
- 10.3.6.** A structure is built underwater on the ocean floor where a strong current of 5 m/sec is measured. Inertial forces and viscous forces are dominant. The structure is to be studied by a 1:25 model in a water tunnel using seawater that is the same density ($\rho = 1,030 \text{ kg}/\text{m}^3$) and temperature (4°C) as that measured in the ocean. What speed must the water tunnel provide in order to study the force load on the structure from the current? If the required water tunnel velocity is judged to be impractical, can the study be performed in a wind tunnel using air at 20°C ? Determine the air speed in the wind tunnel.

(SECTION 10.4)

- 10.4.1.** If a 1:1,000 scale tidal basin model is used to study the operation of a prototype satisfying the Froude number law, what length of time in the model represents a period of one day in the prototype?
- 10.4.2.** Verify the scale ratios given in Table 10.3 (Froude number law) for (a) velocity, (b) time, (c) acceleration, (d) discharge, (e) force, and (f) power.
- 10.4.3.** An ogee spillway has a design flow rate of 14,100 cfs (ft^3/sec). An energy dissipater is being designed to force a hydraulic jump at the end of a spillway channel. The initial depth of flow in the 100-ft wide prototype is expected to be 2.60 ft. Assuming inertial and gravity forces are dominant, determine the discharge of the 1:10 scale model and the velocity in the prototype and model.
- 10.4.4.** An overflow spillway with a 300-m crest is designed to discharge $3,600 \text{ m}^3/\text{sec}$. A 1:20 model of a portion of the cross section of the dam is built in a laboratory flume 1 m wide. Calculate the required laboratory flow rate assuming viscosity and surface tension effects are negligible.
- 10.4.5.** A 1:25 model is built to study a stilling basin at the outlet of a steep spillway chute assuming inertial and gravity forces are dominant. The stilling basin consists of a horizontal floor (apron) with U.S. Bureau of Reclamation (USBR) type II baffles installed to stabilize the location of the hydraulic jump. The prototype has a rectangular cross section 82.0 ft wide designed to carry a 2,650 cfs (ft^3/sec) discharge. The velocity immediately before the jump is 32.8 ft/sec. Determine the following:
- (a) the model discharge,
 - (b) the velocity in the model immediately before the jump,
 - (c) the Froude number of the prototype and the model at this location, and
 - (d) the prototype depth downstream of the jump (see Section 8.10, Figure 8.21).
- 10.4.6.** The 120-m-long crest of an overflow spillway will discharge $1,200 \text{ m}^3/\text{sec}$ of floodwater from a reservoir with a permitted maximum head of 2.75 m. The operation of the prototype spillway is studied on a 1:50 scale model in a hydraulic laboratory assuming inertial and gravity forces are dominant.
- (a) Determine the velocity at the bottom (toe) of the prototype spillway if the velocity in the model spillway at this location measures 3.54 m/sec.
 - (b) Determine the Froude number at the bottom of the prototype spillway.
 - (c) If a USBR type II stilling basin (Section 8.10) that is 50 m wide is used to dissipate the energy at the toe of the spillway, what is the prototype depth downstream of the jump?
 - (d) What is the energy dissipation (in kilowatts) in the prototype?
 - (e) What is the energy removal efficiency of the dissipater in the prototype?

(SECTION 10.5)

- 10.5.1.** A measuring device includes certain small glass tubes of a given geometry. To study the surface tension effect, a 5:1 scale model (larger than prototype) is built. Determine the discharge and force ratios assuming the same liquid is used in the model and the prototype.
- 10.5.2.** Determine the surface tension of a liquid in the prototype if a time ratio of 2 is established with a 1:10 scale model. The surface tension of the liquid in the model is 150 dyn/cm. Also determine the force ratio. Assume the densities of the fluids in the prototype and the model are approximately the same.
- 10.5.3.** A model is built to study the surface tension phenomena in a reservoir. Determine the conversion ratios between the model and the prototype for the following quantities if the model is built with a 1:100 scale: (a) rate of flow, (b) energy, (c) pressure, and (d) power. The same fluid is used in the model and prototype.

(SECTION 10.7)

- 10.7.1.** A ship 100 m long moves at 1.5 m/sec in freshwater at 20°C. A 1:100 scale model of the prototype ship is to be tested in a towing tank containing a liquid of specific gravity 0.90. What viscosity must this liquid have for both Reynolds and Froude number laws to be satisfied?
- 10.7.2.** A 1:250 ship model is towed in a wave tank and wave resistance of 10.7 N is measured. Determine the corresponding prototype wave resistance on the prototype.
- 10.7.3.** A concrete caisson 60 m wide, 120 m long, and 12 m high is to be towed in seawater in the longitudinal direction to an offshore construction site where it will be sunk. The calculated floating depth of the caisson is 8 m, with 4 m remaining above the water surface. A 1:100 model is built to study the operation of the prototype. If the model is towed in a wave tank using seawater, what is the model speed that corresponds to the prototype speed of 1.5 m/sec? The model study considers both the skin drag and form drag (Reynolds number) and the resistance resulting from the generation of gravity waves in motion (Froude number).
- 10.7.4.** A barge model 1 m long is tested in a towing tank at a speed of 1 m/sec. Determine the prototype velocity if the prototype is 150 m in length. The model has a 2-cm draft and is 10 cm wide. The drag coefficient is $C_D = 0.25$ for $N_R > 5 \times 10^4$, and the towing force required to tow the model is 0.3 N. What force is required to tow the barge in waterways?

(SECTION 10.8)

- 10.8.1.** A model is built to study flow in a stream segment. The stream has an average depth of 1.2 feet and is roughly 20 feet wide with a flow rate of 94.6 cfs. An undistorted model with a 1:100 scale is constructed to study the velocity–slope relationship. If the reach has a Manning’s coefficient of 0.045, what is the model values of roughness and velocity?
- 10.8.2.** A new laboratory site is available for modeling the channel of Example 10.8 so that the length is no longer a restriction, but the roughness coefficient of the material to be used in the movable bed is $n_m = 0.018$. Determine the appropriate horizontal scale (using the same vertical scale) and the corresponding model velocity.
- 10.8.3.** Determine the model roughness coefficient, the model velocity, and the model flow rate for Example 10.8 if the vertical scale used was 400, just like the horizontal scale. Are the values obtained for the model reasonable? Will the model flow remain fully turbulent?
- 10.8.4.** A 1:300 scale model is constructed to study the discharge–depth relationship in a river reach with Manning’s coefficient $n = 0.031$. If the model discharges 52 L/sec and has Manning’s coefficient $n = 0.033$, what is an adequate vertical scale ratio and the corresponding flow rate for the prototype? (Assume the channel has a large width-to-depth ratio, and the hydraulic radius is roughly equal to the water depth.)
- 10.8.5.** A model study is proposed for a barge channel ($n_p = 0.03$) that has a sedimentation problem. The channel conveys a flow rate of 10,600 ft³/sec. The following model parameters have been established: a vertical scale ratio of 1:65 and a roughness coefficient of $n_m = 0.02$. Determine the horizontal scale ratio, the time ratio, and the model flow. (Assume the channel has a large width-to-depth ratio, and the hydraulic radius is roughly equal to the water depth.)

(SECTION 10.9)

- 10.9.1.** A model of a broad crested weir has been designed and constructed to study the discharge per foot (q) of the prototype. Because the overflowing water sheet is relatively thick, the surface tension and viscosity of the fluid are not critical to the analysis. However, the weir (prototype) discharge is affected by the weir head (H), the gravitational acceleration (g), and the weir height (h). Use the

Buckingham Pi theorem (with weir head and gravitational acceleration as repeating variables) to derive an expression for the discharge per foot of weir.

- 10.9.2.** Use the Buckingham Pi theorem to derive an expression for the power developed by a motor in terms of the torque and rotational speed (i.e., Equation 5.3). (*Hint:* Use the FLT system of units instead of the MLT system.)
- 10.9.3.** Steady flow of an incompressible, Newtonian fluid occurs through a long, smooth-walled horizontal pipe. Use the Buckingham Pi theorem to derive an expression for the pressure drop that occurs per unit length of pipe (ΔP_l) using the pipe diameter (D), the pipe velocity (V), and the fluid density (ρ) as repeating variables. The fluid viscosity (μ) is the only other pertinent variable. (*Hint:* Use Newton's second law to determine the units for (ΔP_l) and (μ) in the MLT system.)
- 10.9.4.** If the viscosity (μ) and density (ρ) of a liquid are included as variables in the dimensional analysis of Example 10.9, use the Buckingham Pi theorem to develop all of the dimensionless groups if spillway height (h), the gravitational acceleration (g), and the viscosity are selected as the repeating variables. Use the FLT system of units instead of the MLT system.
- 10.9.5.** Determine an expression for the velocity (V) of an air bubble rising through a stationary liquid. The pertinent variables are bubble diameter (D), gravitational acceleration (g), viscosity (μ), density (ρ), and surface tension (σ). Use bubble diameter, density, and viscosity as repeating variables and the MLT unit system.
- 10.9.6.** A liquid of density ρ and viscosity μ flows down an open channel of width W on a channel slope $\sin \theta$. The mean speed V is believed to depend on, among other things, the depth (d), the gravitational acceleration (g), and the roughness height (ϵ). Find the dimensionless parameters that may affect the coefficient k in the formula $V = k \sqrt{dg \sin \theta}$.



Hydrology For Hydraulic Design

Despite their apparent similarity, the terms *hydrology* and *hydraulics* should not be confused. As previously stated in Chapter 1, hydraulics is a branch of engineering that applies fluid mechanics principles to problems dealing with the collection, storage, control, transport, regulation, measurement, and use of water. In contrast, hydrology is a science dealing with the properties, distribution, and circulation of the Earth's water. Thus, hydrology generally refers to natural processes whereas hydraulics generally refers to processes designed, constructed, and controlled by humans.

Even though hydrology and hydraulics represent distinct disciplines, they are inexorably linked in engineering practice. Many hydraulic projects require a hydrologic study to establish the *design flow rate* (Q). Indeed, the design flow rate is critical in establishing the appropriate size and design of many hydraulic structures. For example, rainfall events result in water flowing over the land surface to natural or human-made channels. The design of stormwater pipes, channels, ponds, and low-impact development devices is predicated on establishing appropriate design flow rates for these structures. Where runoff data are available, the statistical methods discussed in Chapter 12 can be used to determine a design flow rate. However, at most project sites, only rainfall data are available. In that event, we would have to use hydrologic methods to determine the design runoff rate using the available rainfall information.

This chapter is not meant to be a comprehensive coverage of hydrology. Entire books are devoted to that purpose. However, to better understand earlier chapters on open-channel flow and hydraulic structures, an introduction to hydrologic concepts and design methods is appropriate.

This will give the reader an appreciation for the level of effort required to arrive at the flow rate on which many hydraulic analyses and designs depend. In addition, some hydrologic methods are presented to equip the reader with design tools that are popular and effective in establishing design flow rates. The hydrologic methods presented are primarily appropriate for small urban watersheds. A majority of the hydraulic design activity done in the United States is for hydraulic infrastructure in the urban environment.

11.1 The Hydrologic Cycle

Water is found everywhere on Earth, even in the driest desert. It is present in one of three phases and resides on our planet's surface, beneath its crust, and in the overlying atmosphere. Most of Earth's water is found in the oceans. However, it is constantly being cycled between the ocean, the air, and the land. This phenomenon is called the *hydrologic cycle*.

The hydrologic cycle is a complex process with many subcycles, so a brief overview may be helpful. Water from the oceans evaporates by absorbing energy from the sun. This increases the water vapor in the overlying air mass. *Condensation* and *precipitation* occur when this vapor-laden air mass cools, usually because it rises in the atmosphere. If the precipitation occurs over land, the water can take a number of paths. Some is caught by buildings, trees, or other vegetation (*interception*). Most of this water eventually evaporates back into the atmosphere. Rainfall that makes it to the ground is either stored in depressions, infiltrates into the ground, or runs off the land driven by gravitational forces. The *depression storage* water either infiltrates or evaporates. *Infiltration* water is either held in the soil pores or moves downward to the water table. Water held in soil pores can be used by plants and released back to the atmosphere through the process of *transpiration*. Water that drains down to the groundwater *aquifer* often ends up in rivers and eventually the ocean. This is also the ultimate destination of *surface runoff* water. Figure 11.1 is a simple depiction of the process.

A *hydrologic budget* represents a quantitative accounting of the water within a system. A *closed system* does not allow mass transfer across its boundaries. Because the amount of water on our planet is relatively fixed, Earth's hydrologic cycle represents a closed system. Unfortunately, most situations require an accounting of boundary transfer and are called *open systems*. For example, accounting for the water available for use in a water-supply reservoir is dramatically affected by boundary transfers. Accounting for the stream flow entering and leaving the reservoir is necessary. It may also be necessary to account for infiltration of water at the lake bottom and evaporation from and precipitation on the lake surface. Figure 11.2 identifies the parameters involved in the budget. Note that the water in the reservoir represents the control volume and is bounded by a control surface.

In equation form, the hydrologic budget is defined as

$$P + Q_i - Q_o - I - E - T = \Delta S \quad (11.1)$$

where most of the variables are defined in Figure 11.2. The term ΔS represents the change in storage over the time period of interest. Transpiration (T) appears in Equation 11.1 but does not play a part in the hydrologic budget of the reservoir system. If the values of evaporation and infiltration are insignificant, the hydrologic budget of the reservoir system is represented by

$$P + Q_i - Q_o = \Delta S \quad (11.2)$$

Hydrologists often confront the task of developing a hydrologic budget for a *watershed*, a tract of land that contributes (drains) surface water to a stream at a particular point of interest

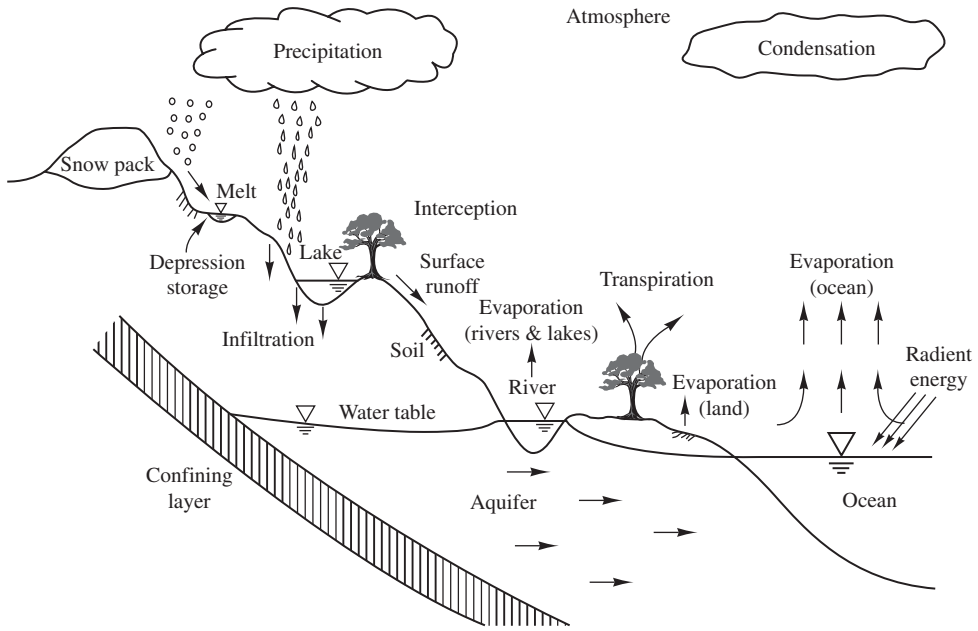


Figure 11.1 The hydrologic cycle

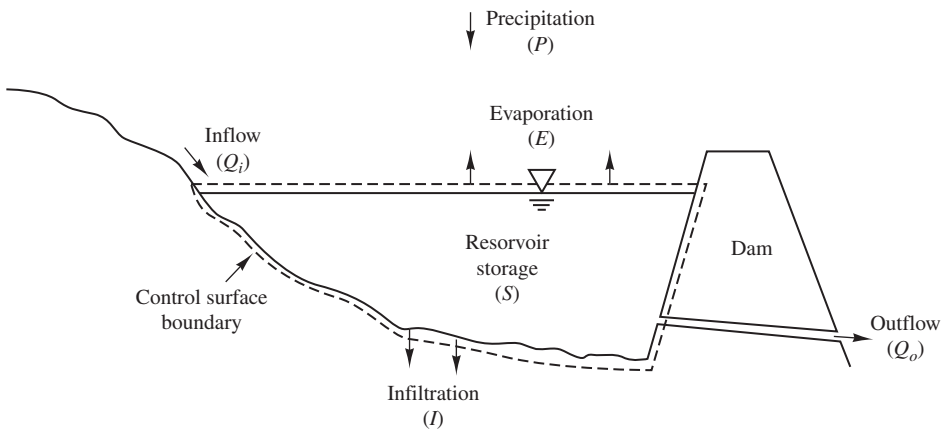


Figure 11.2 Hydrologic budget for a reservoir

(i.e., design point). An accurate topographic map is necessary to properly delineate a watershed boundary. Figure 11.3 depicts the delineation procedure for a stream named Nelson Brook. The watershed is delineated in two steps:

1. identify the design point with a circle on the topographic map and designate all peak elevations with X's close to the design point and upstream to the headwaters of the watershed [Figure 11.3 (a)], and
2. circumscribe the stream with a watershed boundary that passes through the peak points and the design point [Figure 11.3 (b)].

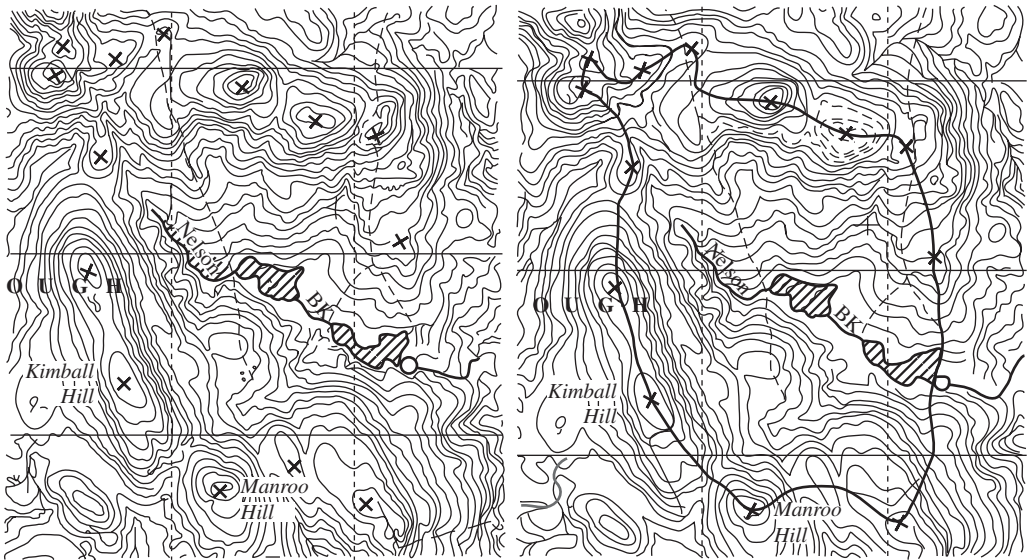


Figure 11.3 Watershed delineation: (a) peak point identification; (b) circumscribing boundary. *Source:* Adapted from the Natural Resource Conservation Service (www.nh.nrcs.usda.gov/technical/WS_delineation.html).

The watershed boundary should intersect contours in a perpendicular fashion. Note that any significant precipitation that occurs within the watershed boundary will produce surface runoff that eventually flows into Nelson Brook and through the design point. Any significant precipitation outside the watershed boundary will produce surface runoff that does not end up passing through the design point. It is often helpful to draw arrows on the topographic map that represents the flow direction of surface runoff, passing through the contour lines in a perpendicular fashion from higher elevations to lower elevations. From these flow arrows, it is easy to see what area contributes flow to the design point.

Watershed delineation is somewhat intuitive. It helps to visualize the topography from the elevation contours, and your ability to do this improves with practice. Modern geographic information systems will often perform the watershed delineation task automatically once the design point is designated, but visual checking and field verification help to avoid program malfunctions in flat or disturbed terrain. Once a watershed is delineated, a hydrologic budget can be performed. Other terms that are often used interchangeably for watershed include catchment, drainage area, and drainage basin, although the last term encompasses an area that produces sub-surface as well as surface flow.

The following example problem demonstrates the application of the hydrologic budget to a watershed. The units used for the various phases of the hydrologic cycle are typical, although they make the computations more tedious because of the many conversions that are necessary.

Example 11.1

A rainfall event occurs over the Nelson Brook watershed [Figure 11.3 (b)]. The average rainfall (R/F) intensities and average stream flows at the design point for 1-hr time increments were measured during each hour of the 7-hr storm and are shown in the following table.

Parameter	Hour						
	1	2	3	4	5	6	7
Stream Flow (cfs)	30	90	200	120	80	40	20
R/F Intensity (in./hr)	0.5	2.5	1.0	0.5	0.0	0.0	0.0

Determine the quantity of water in acre-feet that was added to the groundwater table during the event if the watershed is 350 acres. (An acre-foot is the quantity of water that would cover one acre of land to a depth of 1 foot.) Assume the small pond and wetlands in the watershed (cross-hatched in Figure 11.3) have negligible storage capacity. (Note: Stream flows are given in cubic feet per second, or cfs.)

Solution

The Nelson Brook watershed is an open system. Therefore, we need to define the boundaries of this system—that is, the control volume. It is obvious that the watershed boundary will represent the areal limits of our system. Then we need to put a roof on the top and a floor underneath. Water passing through the roof (mass transfer) is quantified using the precipitation data. Water passing through the floor is represented as infiltration. Equation 11.1 can now be applied to this system over the 7-hr storm duration using the 1-hr average values:

$$P + Q_i - Q_o - I - E - T = \Delta S$$

where

$$P = [(0.5 + 2.5 + 1.0 + 0.5)\text{in/hr}] (1 \text{ hr}) (1 \text{ ft}/12 \text{ in.}) (350 \text{ acres}) = 131 \text{ ac-ft};$$

$$Q_i = 0 \text{ (because no stream flow enters our control volume);}$$

$$Q_o = [(30 + 90 + 200 + 120 + 80 + 40 + 20) \text{ cfs}] (1 \text{ hr}) (3,600 \text{ sec}/1 \text{ hr}) (1 \text{ ac}/43,560 \text{ ft}^2) = 47.9 \text{ ac-ft};$$

$$E = 0 \text{ (assume that evaporation during the 7-hr storm is negligible);}$$

$$T = 0 \text{ (assume that transpiration is also negligible); and}$$

$$\Delta S = 0 \text{ (no change in storage in the control volume during the storm).}$$

Therefore, $I = 131 - 48 = 83.1$ ac-ft is added to the groundwater table.

Actually, this represents the upper limit of what may be added to the groundwater table. The assumptions of no evaporation and transpiration are reasonable when compared to the large volumes of water passing through the system. However, all of the infiltration may not end up in the groundwater table. Depending on how saturated the soil is when the storm begins, a certain amount of the 83 ac-ft will replenish the soil moisture in the soil mantle above the water table.

A hydrologic budget is an important tool for engineers and hydrologists. It provides a means of quantifying water as it passes through various phases of and positions in the hydrologic cycle. To capitalize on it, the hydrologist needs to quantify the budget components. Reliable estimates of all the components are often difficult to obtain. Precipitation and runoff are the two components of the hydrologic cycle that are the easiest to quantify and of greatest interest in hydraulic design, and they usually represent a large transport of mass. The next two sections describe methods of quantifying precipitation and surface runoff.

Several governmental agencies collect and distribute hydrologic information. These agencies establish standard units of measurement as well. For example, the National Weather Service has an extensive system of gauges to measure rainfall depths (in inches) and rainfall intensities (in inches per hour). The U.S. Geological Survey (USGS) has an extensive network of stream gauges to measure stream flow (in cubic feet per second, cfs). The U.S. Army Corps of Engineers collects information on lake evaporation (in inches). Lake volumes and other large quantities of water are often expressed in acre-feet.

11.2 Precipitation

Precipitation is one of the most important phases in the hydrologic cycle. It represents the process by which water vapor is removed from the air and is distributed over Earth's surface in solid or liquid form. Fortunately, precipitation is relatively easy to measure compared to other phases of the hydrologic cycle. However, it does exhibit tremendous variability in time and space; quantifying this variability for design purposes is a challenge.

Three prerequisites are necessary for precipitation to occur: (1) significant atmospheric moisture must be available in (2) the presence of cool temperatures and (3) condensation nuclei. The primary source of atmospheric moisture is oceanic evaporation. However, transpiration by plants and evaporation from land and freshwater surfaces provides additional moisture. The moisture present in an air mass is often measured in terms of its *absolute humidity* (mass of water vapor per unit volume of air). When an air mass holds all of the water vapor it is capable of at a certain temperature, it is said to be *saturated*. Because the capacity of air to hold water decreases with decreasing temperature, water vapor condenses to the liquid state when the air is cooled below a certain temperature (*dew point*). For this to occur, the water must attach to *condensation nuclei*, usually particles of ocean salt or combustion by-products. When the water droplets are of sufficient mass to overcome air resistance, precipitation occurs in one of many forms (e.g., hail, snow, sleet, rain).

The most common way of producing precipitation is through the lifting and thus cooling of air masses laden with water vapor. Lifting takes place by mechanical means or through a thermodynamic process. *Orographic precipitation* occurs through mechanical lifting. The process begins when winds carry moist air from a water surface (usually the ocean) to a land surface. If a mountain range blocks the path of the wind, the moist air must rise to pass over the range. The increase in altitude causes the air to expand and its pressure to decrease, resulting in a lower temperature. The lowering of the air temperature results in an increase in relative humidity. When the air temperature is lowered to a point where the saturation humidity is reached, water vapor condenses and precipitation takes place.

Thermodynamic lifting produces the other distinct categories of precipitation. *Convective precipitation* occurs in the tropics and in big cities during the summer. The process begins with the rapid heat gain that occurs on Earth's surface during the day. Subsequently, the overlying air mass is heated and absorbs a lot of water vapor as evaporation accelerates. The heated air expands and rises. As the rising air mass cools, it condenses and precipitation occurs. Light showers or intense thunderstorms may result.

Cyclonic precipitation is typified by frontal storms. Air masses from high-pressure regions move toward low-pressure regions. Unequal heating of Earth's surface creates the pressure differences. When a warm air mass laden with moisture meets a cold air mass, the warm air rises. Condensation and precipitation occur along the front.

Figure 11.4 depicts the average annual precipitation throughout the continental United States. The variation of precipitation is caused by many factors. For example, atmospheric moisture decreases with increasing latitude because of the reduced evaporation potential and the decreased capacity of cold air to hold water vapor. Also, orographic effects are in evidence along the West Coast, particularly in the Pacific Northwest. Heavier precipitation is produced on the windward (ocean) side of the Cascade Mountains and lighter amounts on the leeward side. Perhaps the biggest factor affecting rainfall is distance from a moisture source. The coastal areas in general receive more rainfall than inland areas. To a lesser extent, this effect is apparent on the windward (eastern) side of the Great Lakes.

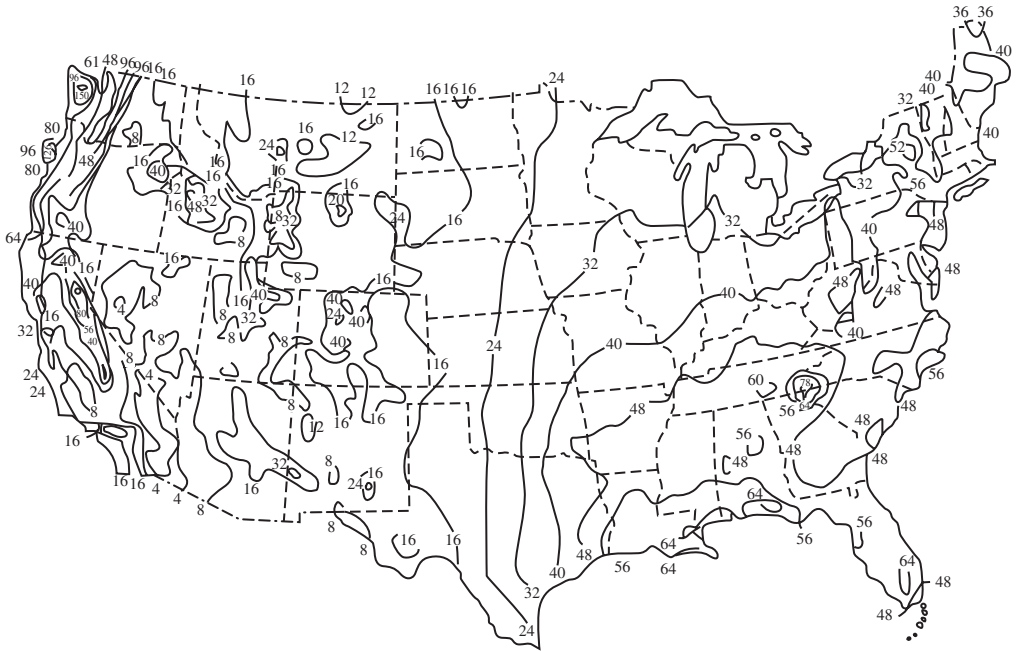


Figure 11.4 Mean annual precipitation (in inches) in the United States. *Source:* U.S. National Weather Service.

Hydraulic engineers often need to estimate rainfall magnitudes for design purposes. The maximum amount of rainfall anticipated for a particular duration is required for the design of flood-control and stormwater-management structures. The frequency and duration of moderate rainfall events is critical for the design of *non-point source pollution* (i.e., natural and anthropogenic pollutants that build up on the land surface and are washed off during frequent but small rainfall events). The minimum amount of rainfall anticipated for a drought period is required for the design of large-scale water storage and water-supply systems.

Precipitation is measured by distributing rain gauges throughout a watershed. The three major types of rain gauges are (1) weighing, (2) float and siphon, and (3) tipping bucket. If the gauges are evenly distributed in the watershed (Figure 11.5), a simple arithmetic mean may suffice for an average rainfall depth determination. If the areal distribution of gauges is uneven or the precipitation is quite variable, then a weighted average is necessary. The *Thiessen method* and the *isohyetal method* are two such methods that were developed before the advent of hydrologic computer models.

The Thiessen method provides a weighting factor for each gauge based on its areal impact. The gauge stations are located on a watershed map, and straight lines are drawn connecting each gauge to nearby gauges (Figure 11.6). Perpendicular bisectors of the connection lines form polygons around each gauge. This identifies an effective area for each gauge. The polygon areas are determined and expressed as percentages of the watershed area. A weighted average precipitation is computed by multiplying the precipitation at each gauge by its associated area percentage and summing the products. This can provide more accurate results than those obtained by simple arithmetic averages. However, the Thiessen method assumes a linear variation of precipitation between stations that may misrepresent localized orographic influences.

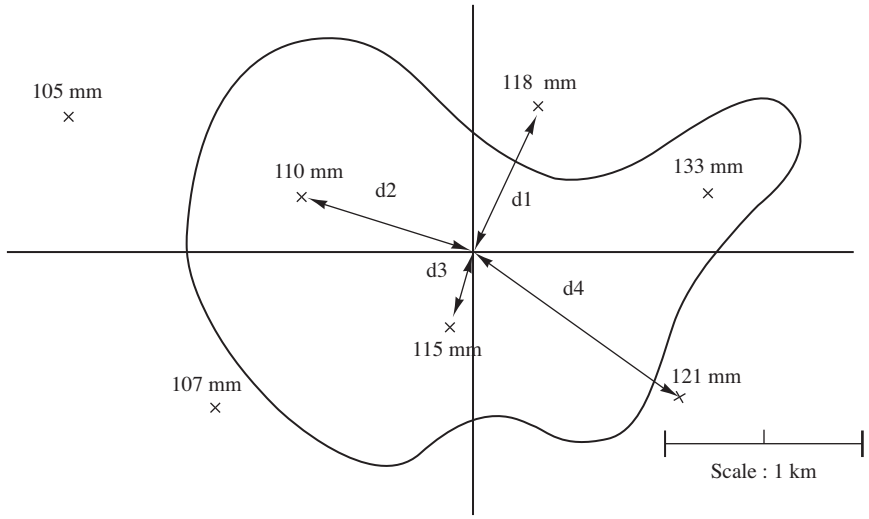


Figure 11.5 Watershed gauge locations and precipitation amounts. (Distances from the watershed centroid to the nearest gauge in each quadrant are depicted.)

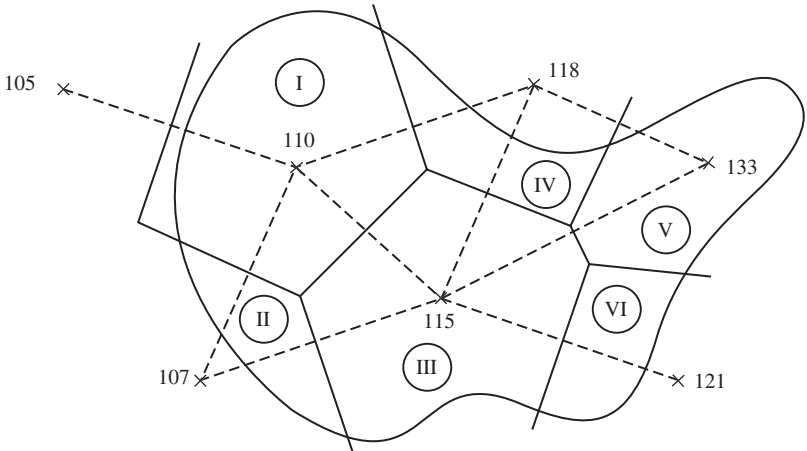


Figure 11.6 Thiessen method for determining average watershed precipitation

A more accurate method for averaging precipitation when accounting for nonlinear variation is the isohyetal method. The precipitation depth measured at each gauge is placed on the watershed map at the gauge location. Contours of equal precipitation (isohyets) are drawn, and areas between isohyets are determined (Figure 11.7). Then average precipitations for all areas are estimated (usually the average value of the two boundary isohyets). These precipitation values are multiplied by the area percentage and summed to obtain the weighted average precipitation.

Modern hydrologic models generally use other methods to estimate average precipitation. These methods include inverse-distance weighting, grid-based, and Kriging (or Kriging).

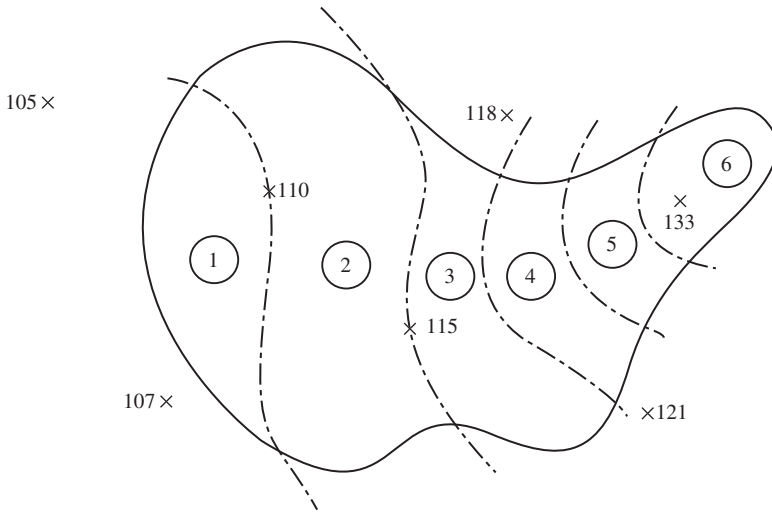


Figure 11.7 Isohyetal method for determining average watershed precipitation

The centroid of the watershed is required for the inverse-distance weighting method, which is used to break the watershed up into quadrants (Figure 11.5). The distance to the nearest gauge in each quadrant is used to establish gauge weights; the closer a gauge is to the centroid, the greater the weighting factor. The weighting equation and associated procedure are provided in Example 11.2.

Grid-based and Kriging methods are more advanced procedures for rainfall estimation. Grid-based methods rely on radar information, which has become more accurate and reliable. The radar information, generally in 4 km by 4 km grid format for preselected time intervals, can be obtained from the National Weather Service. In modern hydrologic models, the watershed is divided into matching grids, and precipitation determinations over the watershed are computed within the model. Kriging is a geostatistical method for data interpolation. Rainfall predictions are made at unobserved locations using observed values at nearby gauge sites. The method relies on linear least-squares algorithms. This method is not used in many hydrologic models, but is likely to become more common as existing models become more sophisticated. The development of this algorithm is beyond the scope of this book.

Example 11.2

Compute the average rainfall depth over the watershed shown in Figures 11.5, 11.6, and 11.7 by the four methods discussed.

Solution

(a) Arithmetic method:

Average the rainfall depths from the gauges in the watershed:

$$(133 + 115 + 110)/3 = 119.3 \text{ mm}$$

Discussion: Would you ever include a gauge outside the watershed?

(b) Thiessen method:

Area	Gauge Precipitation (mm)	Area Factor	Weighted Precipitation (mm)
I	110	0.30	33.0
II	107	0.07	7.5
III	115	0.30	34.5
IV	118	0.09	10.6
V	133	0.16	21.3
VI	121	0.08	9.7
Total	—	1.00	116.6 mm

(c) Isohyetal method:

Area	Average Precipitation (mm)	Area Factor	Weighted Precipitation (mm)
1	109.0	0.18	19.6
2	112.5	0.36	40.5
3	117.5	0.18	21.2
4	122.5	0.11	13.5
5	127.5	0.08	10.2
6	133.0	0.09	12.0
Total	—	1.00	117.0 mm

(d) Inverse-distance weighting method (*Note: AutoCAD will determine centroids of areas.*)

Quadrant	Distance (<i>d</i>) from Gauge to Centroid (km)	Gauge Precipitation (mm)	Weighting Factor ^a	Weighted Precipitation (mm)
1	0.78	118	0.15	17.7
2	0.82	110	0.14	15.4
3	0.38	115	0.65	74.8
4	1.31	121	0.06	7.3
Total	—	—	1.00	115.2 mm

^aWeighting factor in quadrant 1: $w_1 = (1/d_1^2) / [1/d_1^2 + 1/d_2^2 + 1/d_3^2 + 1/d_4^2]$.

11.3 Design Storm

A *design event* is used as a basis for designing a hydraulic structure. It is presumed that the structure will function properly if it can accommodate the design event at full capacity. However, the structure would fail to function as intended if the magnitude of the design event is exceeded. For economic reasons, some risk of failure is allowed in selecting the design event. As discussed in detail in Chapter 12, this risk is usually tied to the *return period* of the design event. The return period is defined as the average number of years between occurrences of a hydrologic event with a certain magnitude or greater. The inverse of the return period represents the probability that this magnitude will be exceeded in any given year. For example, if a 25-year event (or an event

that has a return period of 25 years) is selected as the design event, then there is a $1/25 = 0.04 = 4$ percent probability that the design event will be exceeded in any given year.

Statistical methods are used to analyze historical records to determine the return periods of hydrologic events with different magnitudes (discussed in detail in Chapter 12). If historical runoff data are available at the project site, then we can determine the *design discharge* directly once the return period is selected. However, if only historical rainfall data are available, then we should select the *design storm* and calculate the corresponding runoff by using a hydrologic rainfall-runoff model. A design storm is characterized in terms of a return period, average rainfall intensity or rainfall depth, rain duration, time distribution of rainfall, and the spatial distribution of rainfall. The *rainfall intensity* refers to the time rate of rainfall. Local curves that relate the average rainfall intensity, duration, and the return period (or frequency of occurrence) (known as *IDF curves*) can be used in selecting a design rainfall. The procedure to determine these curves is discussed in Chapter 12. Alternatively, one can use the regional IDF relationships such as those shown in Figure 11.8.

The Soil Conservation Service* (SCS) presented several dimensionless rainfall distributions for different regions of the United States as shown in Figure 11.9. The four distributions are displayed in Figure 11.10 and tabulated in Table 11.1, where t = time, P_T = total depth of

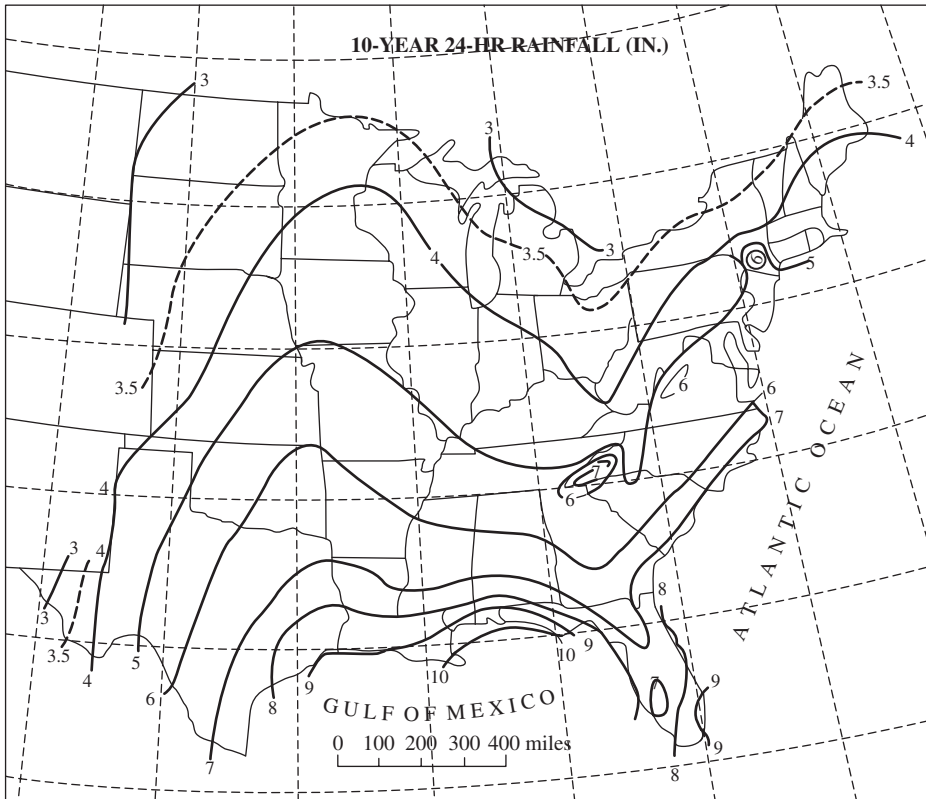


Figure 11.8 Typical rainfall frequency map (10-yr, 24-hr storm). *Source:* National Weather Service.

* Soil Conservation Service, *Urban Hydrology for Small Watersheds* (Technical Release 55). Washington, DC: U.S. Department of Agriculture, Engineering Division, 1986. [Note: The Soil Conservation Service is now called the Natural Resources Conservation Service (NRCS), although the methodologies presented here still use the old name.]

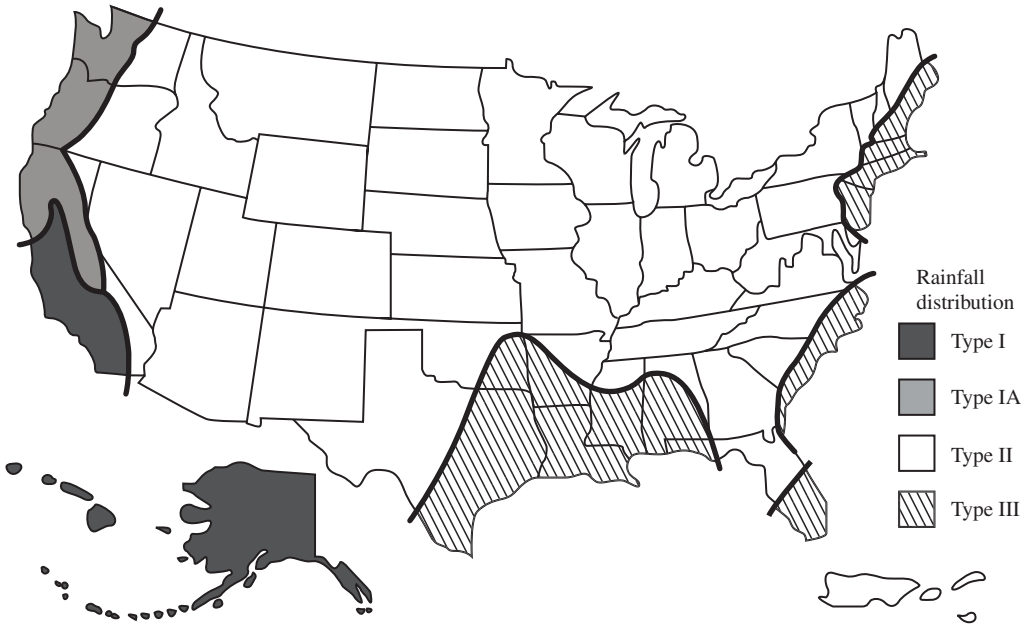


Figure 11.9 Location of the four SCS rainfall distributions. Source: Soil Conservation Service.

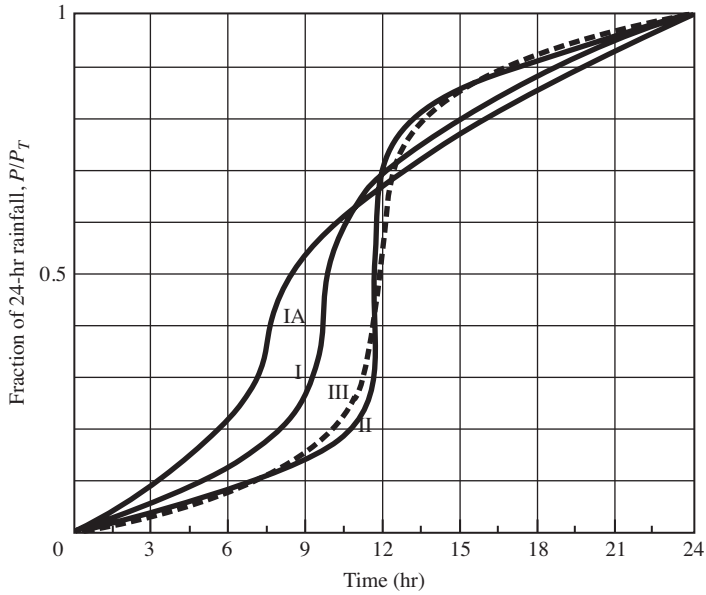


Figure 11.10 The four SCS 24-hour rainfall distributions. Source: Soil Conservation Service.

TABLE 11.1 SCS 24-Hour Rainfall Distributions

t (hrs)	Type I	Type IA	Type II	Type III	t (hrs)	Type I	Type IA	Type II	Type III
	P/P_T	P/P_T	P/P_T	P/P_T		P/P_T	P/P_T	P/P_T	P/P_T
0.0	0.000	0.000	0.000	0.000	12.5	0.706	0.683	0.735	0.702
0.5	0.008	0.010	0.005	0.005	13.0	0.728	0.701	0.776	0.751
1.0	0.017	0.022	0.011	0.010	13.5	0.748	0.719	0.804	0.785
1.5	0.026	0.036	0.017	0.015	14.0	0.766	0.736	0.825	0.811
2.0	0.035	0.051	0.023	0.020	14.5	0.783	0.753	0.842	0.830
2.5	0.045	0.067	0.029	0.026	15.0	0.799	0.769	0.856	0.848
3.0	0.055	0.083	0.035	0.032	15.5	0.815	0.785	0.869	0.867
3.5	0.065	0.099	0.041	0.037	16.0	0.830	0.800	0.881	0.886
4.0	0.076	0.116	0.048	0.043	16.5	0.844	0.815	0.893	0.895
4.5	0.087	0.135	0.056	0.050	17.0	0.857	0.830	0.903	0.904
5.0	0.099	0.156	0.064	0.057	17.5	0.870	0.844	0.913	0.913
5.5	0.112	0.179	0.072	0.065	18.0	0.882	0.858	0.922	0.922
6.0	0.125	0.204	0.080	0.072	18.5	0.893	0.871	0.930	0.930
6.5	0.140	0.233	0.090	0.081	19.0	0.905	0.884	0.938	0.939
7.0	0.156	0.268	0.100	0.089	19.5	0.916	0.896	0.946	0.948
7.5	0.174	0.310	0.110	0.102	20.0	0.926	0.908	0.953	0.957
8.0	0.194	0.425	0.120	0.115	20.5	0.936	0.920	0.959	0.962
8.5	0.219	0.480	0.133	0.130	21.0	0.946	0.932	0.965	0.968
9.0	0.254	0.520	0.147	0.148	21.5	0.956	0.944	0.971	0.973
9.5	0.303	0.550	0.163	0.167	22.0	0.965	0.956	0.977	0.979
10.0	0.515	0.577	0.181	0.189	22.5	0.974	0.967	0.983	0.984
10.5	0.583	0.601	0.203	0.216	23.0	0.983	0.978	0.989	0.989
11.0	0.624	0.623	0.236	0.250	23.5	0.992	0.989	0.995	0.995
11.5	0.654	0.644	0.283	0.298	24.0	1.000	1.000	1.000	1.000
12.0	0.682	0.664	0.663	0.600					

rainfall, and P = rainfall depth accumulated up to time t . Note that the design storm lasts for 24 hours. However, the shorter and more intense rainfalls are nested within the 24-hr duration, and therefore these distributions are appropriate for both small and large watersheds. Applying the SCS procedure results in a *design storm hyetograph*, a relationship between rainfall intensity and time for a given location and return period. Design storm hyetographs are the primary input into rainfall-runoff models.

Example 11.3

Determine a 10-year, 24-hour storm hyetograph for Virginia Beach, Virginia.

Solution

From Figure 11.8 we determine that the 10-year, 24-hour rainfall for Virginia Beach, Virginia is 6 inches. Likewise Figure 11.9 indicates that SCS Type II hyetographs can be used for Virginia Beach. (Virginia Beach is on the border of Type II and III hyetographs, but the review agency has dictated a Type II.) The calculations are performed in tabular form as shown in Table 11.2.

TABLE 11.2 Example Design Storm Hyetograph

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t_1 (hr)	t_2 (hr)	P_1/P_T	P_2/P_T	P_1 (in.)	P_2 (in.)	$\Delta P = P_2 - P_1$ (in.)	$i = \Delta P/\Delta t$ (in./hr)
0.0	0.5	0.000	0.005	0.000	0.030	0.030	0.060
0.5	1.0	0.005	0.011	0.030	0.066	0.036	0.072
1.0	1.5	0.011	0.017	0.066	0.102	0.036	0.072
1.5	2.0	0.017	0.023	0.102	0.138	0.036	0.072
2.0	2.5	0.023	0.029	0.138	0.174	0.036	0.072
2.5	3.0	0.029	0.035	0.174	0.210	0.036	0.072
3.0	3.5	0.035	0.041	0.210	0.246	0.036	0.072
3.5	4.0	0.041	0.048	0.246	0.288	0.042	0.084
4.0	4.5	0.048	0.056	0.288	0.336	0.048	0.096
4.5	5.0	0.056	0.064	0.336	0.384	0.048	0.096
5.0	5.5	0.064	0.072	0.384	0.432	0.048	0.096
5.5	6.0	0.072	0.080	0.432	0.480	0.048	0.096
6.0	6.5	0.080	0.090	0.480	0.540	0.060	0.120
6.5	7.0	0.090	0.100	0.540	0.600	0.060	0.120
7.0	7.5	0.100	0.110	0.600	0.660	0.060	0.120
7.5	8.0	0.110	0.120	0.660	0.720	0.060	0.120
8.0	8.5	0.120	0.133	0.720	0.798	0.078	0.156
8.5	9.0	0.133	0.147	0.798	0.882	0.084	0.168
9.0	9.5	0.147	0.163	0.882	0.978	0.096	0.192
9.5	10.0	0.163	0.181	0.978	1.086	0.108	0.216
10.0	10.5	0.181	0.203	1.086	1.218	0.132	0.264
10.5	11.0	0.203	0.236	1.218	1.416	0.198	0.396
11.0	11.5	0.236	0.283	1.416	1.698	0.282	0.564
11.5	12.0	0.283	0.663	1.698	3.978	2.280	4.560
12.0	12.5	0.663	0.735	3.978	4.410	0.432	0.864
12.5	13.0	0.735	0.776	4.410	4.656	0.246	0.492
13.0	13.5	0.776	0.804	4.656	4.824	0.168	0.336
13.5	14.0	0.804	0.825	4.824	4.950	0.126	0.252
14.0	14.5	0.825	0.842	4.950	5.052	0.102	0.204
14.5	15.0	0.842	0.856	5.052	5.136	0.084	0.168
15.0	15.5	0.856	0.869	5.136	5.214	0.078	0.156
15.5	16.0	0.869	0.881	5.214	5.286	0.072	0.144
16.0	16.5	0.881	0.893	5.286	5.358	0.072	0.144
16.5	17.0	0.893	0.903	5.358	5.418	0.060	0.120
17.0	17.5	0.903	0.913	5.418	5.478	0.060	0.120
17.5	18.0	0.913	0.922	5.478	5.532	0.054	0.108
18.0	18.5	0.922	0.930	5.532	5.580	0.048	0.096
18.5	19.0	0.930	0.938	5.580	5.628	0.048	0.096
19.0	19.5	0.938	0.946	5.628	5.676	0.048	0.096
19.5	20.0	0.946	0.953	5.676	5.718	0.042	0.084
20.0	20.5	0.953	0.959	5.718	5.754	0.036	0.072
20.5	21.0	0.959	0.965	5.754	5.790	0.036	0.072
21.0	21.5	0.965	0.971	5.790	5.826	0.036	0.072
21.5	22.0	0.971	0.977	5.826	5.862	0.036	0.072
22.0	22.5	0.977	0.983	5.862	5.898	0.036	0.072
22.5	23.0	0.983	0.989	5.898	5.934	0.036	0.072
23.0	23.5	0.989	0.995	5.934	5.970	0.036	0.072
23.5	24.0	0.995	1.000	5.970	6.000	0.030	0.060

Table 11.2 is set up to determine the rainfall intensity for the time increments between times t_1 and t_2 tabulated in columns 1 and 2. The corresponding cumulative rain to total rain ratios are tabulated in columns 3 and 4, respectively. These values are obtained from Table 11.1 for the Type II distribution. In this example, $P_T = 6$ in. To find the rainfall intensity between, say, $t = 9.5$ hours and $t = 10$ hours, we first note the precipitation ratios are 0.163 and 0.181, respectively, for 9.5 and 10 hours. Thus, the rainfall depth at $t = 9.5$ hours is $P = (0.163)(6) = 0.978$ in., and that at $t = 10$ hours is $P = (0.181)(6) = 1.086$ in. A depth of $1.086 - 0.978 = 0.108$ in. is produced between $t = 9.5$ and 10 hours, and, therefore, the intensity is

$0.11 \text{ in.}/0.5 \text{ hr} = 0.216 \text{ in.}/\text{hr}$ Similarly, for the period between 10 and 10.5 hr, the intensity is equal to $[(6)(0.203) - 6(0.181)]/0.5 = 0.264 \text{ in.}/\text{hr}$ The intensity distribution of the whole design-storm hyetograph can be determined by repeating the calculations for all the time intervals.

11.4 Surface Runoff and Stream Flow

As precipitation falls to Earth, a portion of it must satisfy the various demands of *interception*, *depression storage*, and *soil-moisture replenishment*. The rainfall intercepted is caught by the leaves and stems of vegetation or by human-made structures such as rooftops. Depression storage includes water retained on the ground as puddles or small isolated wetlands. Soil moisture is held as capillary water in the smaller pore spaces of the soil or as hygroscopic water adsorbed to the surface of soil particles. After satisfying these three demands, the water that remains on the ground surface is often called *excess precipitation*.

Excess precipitation may follow two major paths to a stream. The portion that travels as overland flow across the ground surface to the nearest channel is commonly known as *surface runoff* (Figure 11.1). The rest infiltrates into the soil and may end up in the stream via *groundwater* flow. A third path that accounts for some minor contribution to stream flow is *interflow*. Interflow is water that makes its way to the stream through the soil above the groundwater table.

Overland flow is classified as *direct runoff*. This water reaches the stream channel shortly after it falls to Earth's surface and, for small drainage basins, is completely discharged within a day or two. The portion of water that percolates downward through the soil to the groundwater table and eventually arrives in a nearby stream is called *groundwater runoff*. Usually, it takes a much longer period of time for the groundwater runoff to reach the stream. The characteristics of direct runoff and groundwater runoff differ so greatly that they are treated separately in analyzing a particular runoff event. A technique that employs a separation of the two will be discussed in the next section. It should be noted that once groundwater runoff and direct runoff combine in a stream, there is no practical means of differentiating between them.

In the stream flow phase of the hydrologic cycle, water is usually concentrated in a single channel. This makes it ideal for measurement as water departs from the watershed. Stream flow analysis requires a record of discharge over time. Usually, this is obtained by recording water levels or stages above some datum over time and converting the stage to discharge. A stage-discharge relationship at a given stream location is known as a *rating curve*. A stream flow measuring station is called a *gauge* (or *gauging station*).

The simplest way to measure a stream or river stage is with a staff gauge, a vertical scale fixed to the bottom of the channel so that a height (stage) reading may be obtained by visual inspection. The USGS operates an extensive system of continuous-recording stream gauges. These gauge houses (Figure 11.11) are built adjacent to the stream and contain a stilling well that is connected to the stream with intake pipes. Changes in stream level are obtained using a float connected to a data recorder. If blockage of the pipe is a problem, pressure transducers can be correlated to stage readings stored in data recorder. In remote locations, satellite telemetry has proven cost-effective in retrieving data.

Several methods of discharge measurement in open channels are discussed in Chapter 9. They all involve the construction of a measuring device in the streambed such as a weir or a Venturi flume. However, it is not always practical or economical to build one of these devices

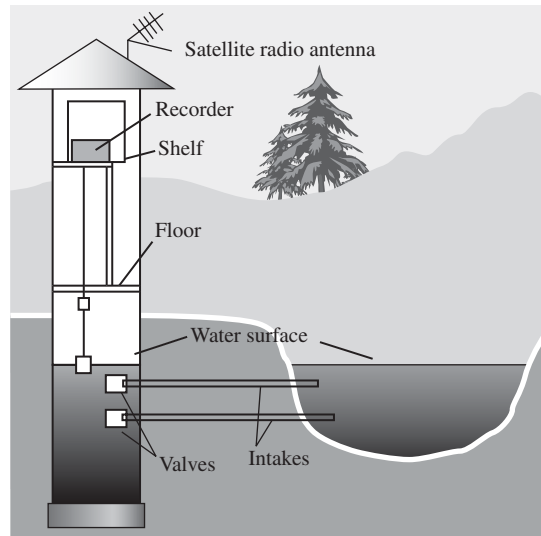


Figure 11.11 Schematic of a stream gauging station. *Source:* USGS, 2008.

where rating curves (stage vs. discharge) are needed (e.g., at USGS stream gauge locations). If this is the case, then the stream discharge can be measured in either a natural or human-made channel using the following procedure.

The stream cross section is divided into several vertical sections as shown in Figure 11.12. In each section, flow velocities vary from zero at the channel bottom to a maximum value near the surface. Many field tests have shown that the average velocity for each section can be represented well by taking the average of the velocities measured by a current meter located at 20 percent and 80 percent of the depth below the water surface. A current meter (discussed in Section 9.2) measures velocity using a rotating-cup wheel, much like an anemometer measures wind velocity. The average velocity in each vertical section is multiplied by the cross-sectional area to give the discharge in each section; these are summed to yield the total discharge in the channel at that stage.

The accuracy of the measurement increases with the number of vertical sections. However, the accuracy must be limited to those measurements that can be made in a reasonable amount of time. This is especially true if the stage is changing rapidly because the discharge is often highly dependent on the stage.

Repeating the above procedure at various stages provides the information needed to develop a rating curve (i.e., flow vs. stage) at a specific location. Once the rating curve is

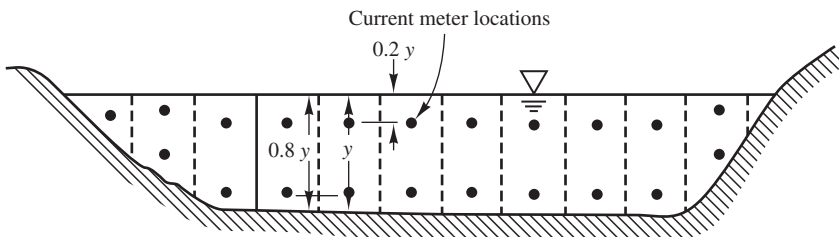


Figure 11.12 Procedure for discharge measurement with current meter. *Source:* USGS, 2008.

constructed, continuous recording of the river stage is sufficient to define stream flow as a function of time. This information is critical in establishing reliable estimates of design flow rates. The next few sections will verify the importance of reliable stream flow records.

11.5 Rainfall-Runoff Relationships: The Unit Hydrograph

Rainfall-runoff models are often used in engineering practice to determine the runoff from a watershed given a rainfall hyetograph. The unit hydrograph theory presented by Sherman* forms the basis for some of the rainfall-runoff models currently used.

Measurement of rainfall and stream flow is a prerequisite to understanding the complex rainfall-runoff process. Stream flow measurements during and after a storm produce the data necessary to plot a *hydrograph*, a graph of discharge over time. Rainfall often is measured and plotted along the same time scale (i.e., a *hyetograph*). A typical hydrograph resulting from a single storm consists of a rising limb, a peak, and a recession limb [Figure 11.13 (a)].

In analyzing hydrographs, engineers frequently separate the direct surface runoff from the groundwater contribution. One of the simplest methods, and one that is as easily justified as any other, calls for a dividing line from the point at which the rising limb begins to the point of greatest curvature on the recession side. The area between the hydrograph (curve) and the groundwater contribution (straight line) represents the volume of *direct surface runoff*. The area below the straight line represents the groundwater contribution, also known as the *base flow*. Figure 11.13 (a) depicts this hydrograph-separation technique.

Two other simple base-flow separation techniques are depicted in Figure 11.13 (b). In the first method, a horizontal straight line is drawn connecting the point where the rising limb begins (*A*) to a point on the recession curve (*D*). The area below the line *A–D* is attributed to the base flow. In the second method, the base flow before the rising limb begins (*A*) is projected forward to the peak flow time (*B*) in the form of a straight line. Then, this point (*B*) is connected to a point (*C*) on the recession limb at time *N* after the peak. An empirical formula is used to determine *N* such as $N = A^{0.2}$ for large watersheds where *N* is in days and *A* is watershed area in square kilometers. The area below the line *A–B–C* is assumed to represent the base flow. Although empirical and very approximate, this method reflects the fact that the groundwater contribution should initially decrease as the stream flow rate and water surface elevation increase. As the storm progresses, rainfall infiltrates into the ground, eventually making it to the groundwater table. As the groundwater table rises and the water surface elevation in the stream recedes, the base flow in the stream increases.

Hydrographs can be considered mathematical functions. They represent a watershed's response to a given stimulus (rainfall). In other words, if two storms occur over the same watershed at different times with the same intensity, pattern, and antecedent moisture conditions (i.e., soil-moisture conditions), we would expect identical hydrographs to be produced. Effectively using a hydrograph for design purposes requires the determination of a *unit hydrograph*, a hydrograph that results from a rainfall event of nearly uniform intensity and aerial distribution over the watershed that produces 1 inch (or 1 centimeter—i.e., a unit of depth) of runoff. Figure 11.14 depicts graphically the stimulus (rainfall) and response (unit hydrograph). The effective duration of the storm (t_e) is the length of the runoff-producing rainfall and should be nearly uniform in intensity over the effective storm duration.

*L. K. Sherman, "Streamflow from rainfall by unit-graph method," *Engineering News Record*, 108: 501–505, 1932.

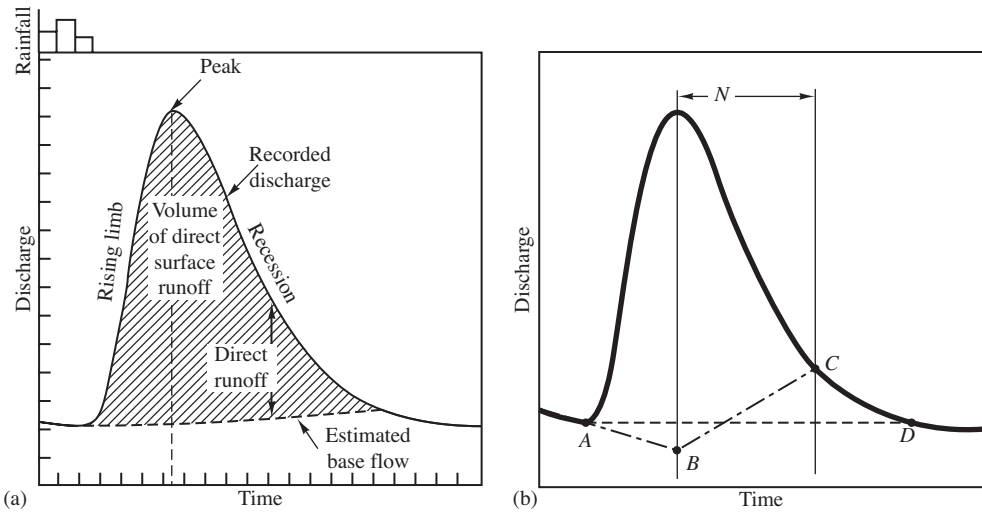


Figure 11.13 (a) Typical hydrograph (separation of base flow); (b) alternate base-flow separation techniques

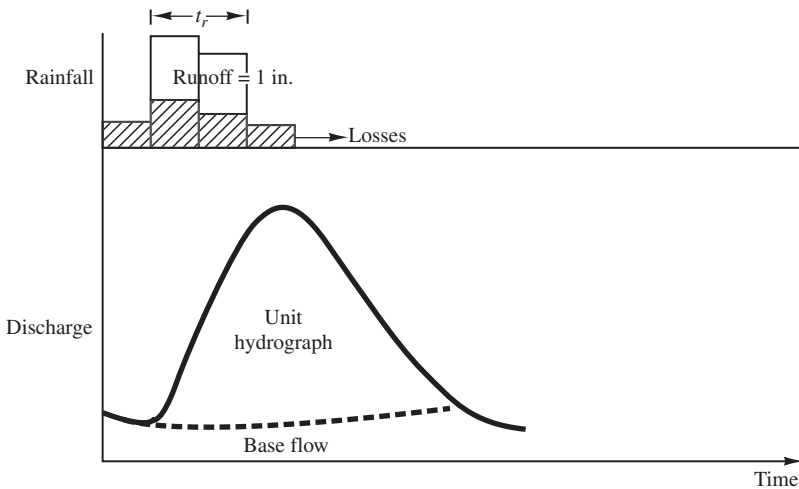


Figure 11.14 Unit hydrograph produced by 1 inch of runoff

The derivation of a unit hydrograph involves six steps:

1. A reasonably uniform storm (intensity and areal coverage) is selected for which reliable rainfall and stream flow data are available. A plot of the data is helpful (Figure 11.14).
2. The amount of stream flow from groundwater contribution (base flow) is estimated.
3. Direct runoff (i.e., flow rates resulting from surface runoff from the rainfall event) is computed by subtracting the base-flow estimates from the measured stream flow.

4. The average depth of surface runoff is computed. Generally, this is performed using finite-difference methods to sum the direct runoff values and transform the sum into a volume (i.e., multiply the sum by the time interval used). This resulting volume is divided by the drainage area to determine the average depth of surface runoff.
5. A unit hydrograph is obtained by dividing the direct runoff flows (step 3) by the average depth of surface runoff (step 4).
6. Finally, the unit hydrograph's storm duration is established by examining the rainfall data and determining the duration of effective precipitation or runoff-producing precipitation. For example, if the majority of the precipitation occurs in a 2-hr time period, then it is a 2-hr unit hydrograph.

The following example problem highlights the process.

Example 11.4

A storm occurs over the Pierre Creek watershed. Measurements of rainfall and stream flow are listed in the following table. The drainage area that contributes stream flow to the gauge site is 1,150 km². Determine the unit hydrograph from this rainfall event for this watershed.

Day	Hour	Rainfall (cm)	Stream Flow (m ³ /sec)	Base Flow (m ³ /sec)	Direct Runoff (m ³ /sec)	Unit Hydrograph (m ³ /sec)	Hour After Start
22	0000		170	170	0		
	0600	1.7	150	150	0	0	0
	1200	5.1	400	157	243	38	6
	1800	4.7	750	165	585	91	12
23	0000	0.8	980	172	808	125	18
	0600		890	180	710	110	24
	1200		690	187	503	78	30
	1800		480	195	285	44	36
24	0000		370	202	168	26	42
	0600		300	210	90	14	48
	1200		260	217	43	7	54
25	1800		225	225	0	0	60
	0000		200	200	—	—	—
	0600		180	180	—	—	—
	1200		170	170	—	—	—
		Σ = 12.3			Σ = 3,435		—

Solution

With the data provided, the six-step computation procedure is employed to obtain the coordinates of the unit hydrograph. The values in each column of the solution table just presented are explained as follows:

- Day and Hour: time of rainfall and discharge measurements
- Rainfall: depth of rainfall that occurred within a six-hour increment (for example, 1.7 cm. of rain fell between hour 0000 and 0600 on day 22)

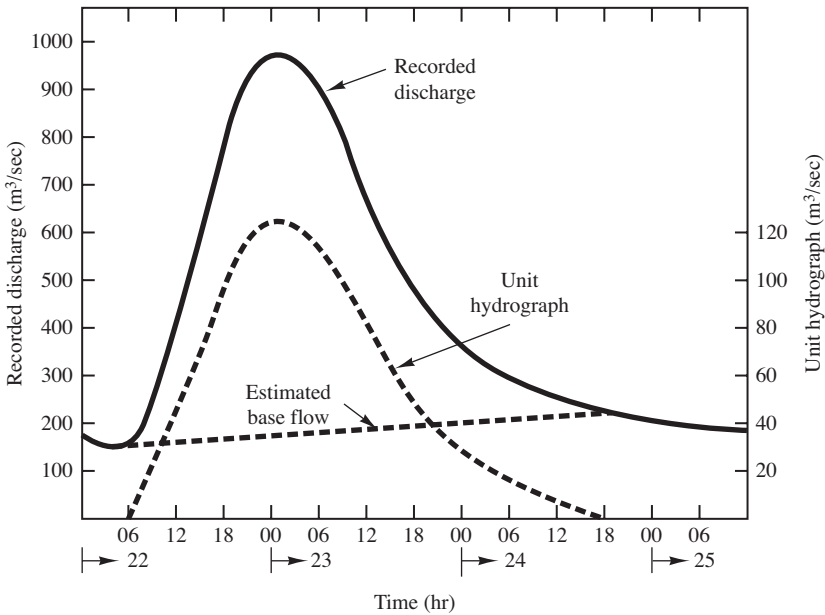


Figure 11.15

- Stream Flow: instantaneous discharge measurements on Pierre Creek (plotted in Figure 11.15)
- Base Flow: base-flow estimates taken from the depicted hydrograph (Figure 11.15)
- Direct Runoff: portion of the rainfall that runs off the land surface and contributes to stream flow (found by subtracting base-flow column from the stream flow column)
- Unit Hydrograph: ordinates found by dividing the direct runoff column by the volume of direct runoff expressed in centimeters over the drainage area (i.e., the depth of runoff, which is found by summing the direct runoff column, multiplying by the time increment—6 hr—and dividing by the drainage area):

$$\text{Depth of Runoff} = \frac{\left(3,435 \frac{\text{m}^3}{\text{sec}}\right)(6 \text{ hrs})\left(\frac{3,600 \text{ sec}}{\text{hr}}\right)\left(\frac{100 \text{ cm}}{\text{m}}\right)^3}{(1,150 \text{ km}^2)\left(\frac{10^5 \text{ cm}}{\text{km}}\right)^2} = 6.45 \text{ cm}$$

- Hrs After Start: hours after start of effective (runoff-producing) rainfall (establishing this time frame is important for using the unit hydrograph as a design tool)

Note that of the 12.3 cm of rainfall that was measured, only 6.45 cm produced runoff. The rest of the rainfall was lost as interception, depression storage, and infiltration. In fact, the first 1.7 cm that fell (time 0000 to 0600) and the last 0.8 cm (time 1800 to 2400) were probably all lost. This leaves the 12 hours from 0600 to 1800 as being effective in producing runoff. Furthermore, note the fairly uniform intensity of rainfall over this time span. Because the effective rainfall occurred over this 12-hr period, it is considered

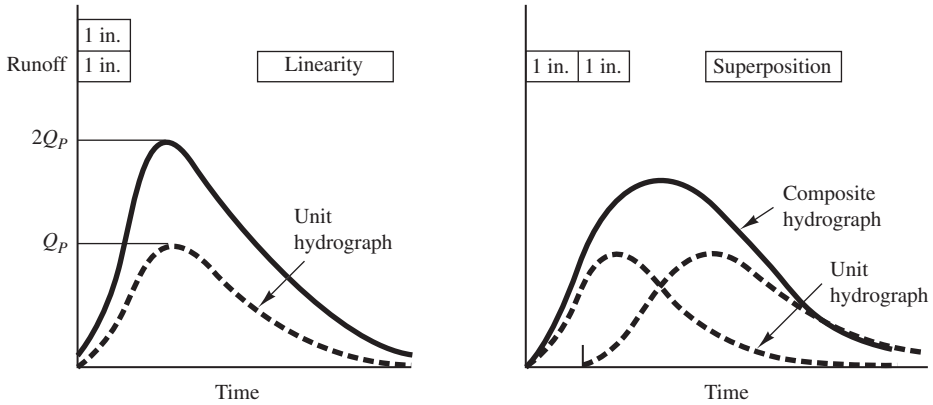


Figure 11.16 Linearity and superposition principles

a 12-hr unit hydrograph. A 6-hr unit hydrograph would look different. (See Problem 11.5.1 for further discussion.) Finally, the resulting unit hydrograph is plotted under the original measured hydrograph in Figure 11.15. How the unit hydrograph is used as a design tool will be described in the next example problem.

A unit hydrograph is unique to a particular watershed. Even on the same watershed, different unit hydrographs result from different duration storms (i.e., different effective rainfall durations). For example, a 6-hr unit hydrograph will likely have a greater peak than a 12-hr unit hydrograph even though both represent the same volume of runoff: 1 inch (or 1 cm) of rainfall depth over the watershed.

A unit hydrograph can be used to predict a watershed’s response to any storm if the principles of linearity and superposition are assumed to be valid. *Linearity* suggests that the response from a watershed to runoff is linear in nature. In other words, if 1 inch of runoff over a given duration (t_r) produces a unit hydrograph, then 2 inches over the same duration would produce a flow rate twice as high at each point in time. *Superposition* suggests that the effects of rainfall on the watershed can be separately accounted for and accumulated; that is, if two unit hydrograph storms occur back to back, their separate responses can be added to produce the composite response of the watershed. Figure 11.16 depicts these concepts. Extensive research has shown that the rainfall-runoff process is exceedingly complex and not completely subject to these principles. Nonetheless, unit hydrograph theory has proven to be a useful design tool when judiciously applied. The following example problem demonstrates how a unit hydrograph can be used to predict a watershed’s response to a future (design) storm.

Example 11.5

A hypothetical 24-hr design storm for the Pierre Creek watershed described in Example 11.4 has led to a prediction of 5 cm of runoff in the first 12 hours and 3 cm of runoff in the subsequent 12 hours. Using the 12-hr unit hydrograph (UH) derived in that example problem, calculate the stream flow expected from this design storm.

Solution

The solution to the problem is provided in the following table.

Hour	Unit Hydrograph (m ³ /sec)	5 × (UH) (m ³ /sec)	3 × (UH) (m ³ /sec)	Base Flow (m ³ /sec)	Stream Flow (Q) (m ³ /sec)
0	0	0	—	190	190
6	38	190	—	198	388
12	91	455	0	206	661
18	125	625	114	214	953
24	110	550	273	222	1,045
30	78	390	375	230	995
36	44	220	330	238	788
42	26	130	234	246	610
48	14	70	132	254	456
54	7	35	78	262	375
60	0	0	42	270	312
66	—	—	21	278	299
72	—	—	0	286	286

Each column is computed as follows.

- Hour: the time since the storm began
- 5 × (UH): stream flow resulting from the 5-cm runoff depth that occurs in the first 12 hours of the storm (the UH developed in Example 11.4 is multiplied by 5 according to the principle of linearity)
- 3 × (UH): stream flow resulting from the 3-cm runoff depth that occurs in the second 12 hours of the storm (the UH from Example 11.4 is multiplied by 3 according to the principle of linearity). Note that the effects of this runoff are delayed by 12 hours based on when the 3 cm of runoff occurs.
- Base Flow: estimated base flow from conditions existing before the storm began
- Stream Flow (Q): total stream flow obtained by adding the previous three columns [i.e., (5 × UH) + (3 × UH) + (base flow)] according to the principles of superposition (the values in this column are plotted in Figure 11.17 to give the total expected stream flow)

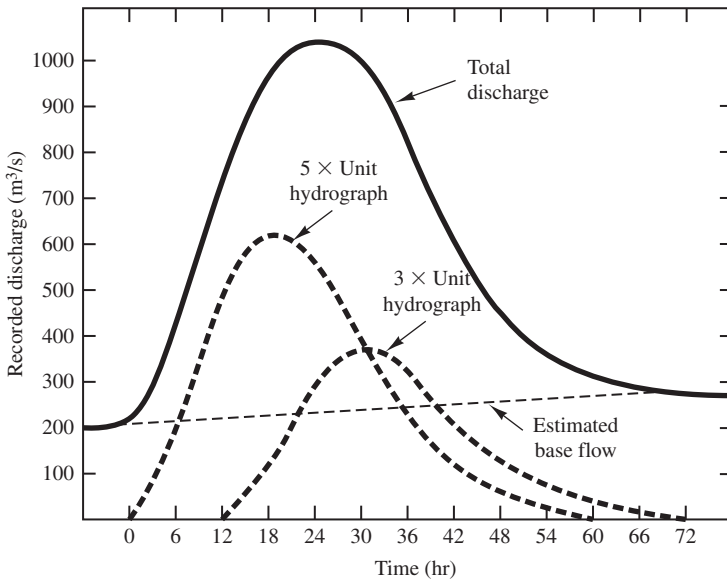


Figure 11.17 Unit hydrograph application example problem

11.6 Rainfall-Runoff Relationships: SCS Procedures

When confronted with the task of designing hydraulic structures on streams, engineers rarely find stream flow gauges at the proposed sites. Without stream flow information, a unit hydrograph and subsequent design hydrographs cannot be developed. *Synthetic unit hydrographs* have been developed to overcome this problem: They draw on data from existing stream gauges and transpose the information to hydrologically similar watersheds that lack gauge information. Because the United States has so many hydrologically dissimilar areas, many synthetic hydrographs have been proposed over the last 50 years. One of the most popular and broadly applicable is the Soil Conservation Service’s synthetic unit hydrograph. (The SCS is now known as the Natural Resources Conservation Service, or NRCS.) The SCS rainfall-runoff models are based on synthetic unit hydrographs. The SCS has also developed procedures to estimate the losses from rainfall and to determine flow times related to watershed runoff.

11.6.1 Losses from Rainfall and Rainfall Excess

Losses from rainfall is a collective reference to that part of the rainfall that does not show up as runoff. In general, the abstractions include *interception*, *depression storage*, *evaporation*, *transpiration*, and *infiltration*. Under design storm conditions, the evaporation and transpiration are usually negligible. *Rainfall excess* is that part of the rainfall that becomes runoff. We sometimes refer to rainfall excess as *direct runoff* or just *runoff*. As discussed in the preceding section (Example 11.5), we need to know the rainfall excess produced over various time intervals to calculate a runoff hydrograph (or stream flow hydrograph) by using a unit hydrograph.

The SCS developed a procedure for determining rainfall excess based on runoff curve numbers and cumulative rainfall depths. The runoff *curve number (CN)* is a basin parameter with a range of 0 to 100. The value of *CN* depends upon the antecedent moisture condition of the soil (Table 11.3), the hydrologic soil group (Table 11.3), the soil cover type (i.e., land use) and condition, and the percent of impervious areas in the watershed. The recommended *CN* values for various urban land use types and some agricultural land uses are given in Table 11.4. These *CNs* are for average moisture conditions preceding a storm. If a watershed is composed of several subareas with different *CNs*, then a weighted average (based on area) or composite *CN* can be obtained for the whole watershed.

TABLE 11.3 SCS Antecedent Moisture Conditions and Hydrologic Soil Groups

Antecedent Moisture Condition (AMC)	Soil Conditions	Hydrologic Soil Group	Descriptions of the Hydrologic Soil Groups
AMC I	Low moisture; soil is dry	A	Soils with high infiltration rate, even when wet; mostly sand and gravel
AMC II	Average moisture; common for design	B	Soils with moderate rate of infiltration when wet; coarse to fine texture
AMC III	High moisture; heavy rain in last few days	C	Soils with slow infiltration rate when wet; moderately fine to fine texture
		D	Soils with low infiltration rate when wet; clay or soil with high water table

TABLE 11.4 SCS Runoff Curve Numbers for AMC II

Cover Description Cover Type and Hydrologic Condition	% Impervious	Hydrologic Soil Groups			
		A	B	C	D
Open space (parks, cemeteries, etc.):					
Poor condition (grass cover < 50%)		68	79	86	89
Fair condition (grass cover 50%–75%)		49	69	79	84
Good condition (grass cover > 75%)		39	61	74	80
Impervious areas (parking lots, etc.)	100	98	98	98	98
Urban districts:					
Commercial and business	85	89	92	94	95
Industrial	72	81	88	91	93
Residential areas (by average lot size):					
1/8 acre or less (town houses)	65	77	85	90	92
1/4 acre	38	61	75	83	87
1/3 acre	30	57	72	81	86
1/2 acre	25	54	70	80	85
1 acre	20	51	68	79	84
2 acre	12	46	65	77	82
Newly graded areas (no vegetation)		77	86	91	94
Agricultural land or open land (good condition)					
Fallow land (crop residue)		76	85	90	93
Row crops (contoured)		65	75	82	86
Small grain crops (contoured)		61	73	81	84
Pasture, grassland, or range		39	61	74	80
Meadow (mowed for hay)		30	58	71	78
Woods–grass combination (orchards)		32	58	72	79
Woods		30	55	70	77

Once the curve number is selected, the cumulative runoff (R) corresponding to cumulative rainfall (P) is calculated by using

$$R = \frac{(P - 0.2S)^2}{(P + 0.8S)} \tag{11.3}$$

where

$$S = \frac{1,000 - 10(CN)}{(CN)} \tag{11.4}$$

where R = cumulative runoff (or rainfall excess) in inches, P = cumulative rainfall in inches, and S = soil-moisture storage deficit in inches at the time runoff (as opposed to rainfall) begins. These equations are valid if $P > 0.2S$, otherwise $R = 0$. Figure 11.18 graphically presents these equations. The runoff produced over a time increment is the difference between the cumulative runoff at the end and at the beginning of the time increment.

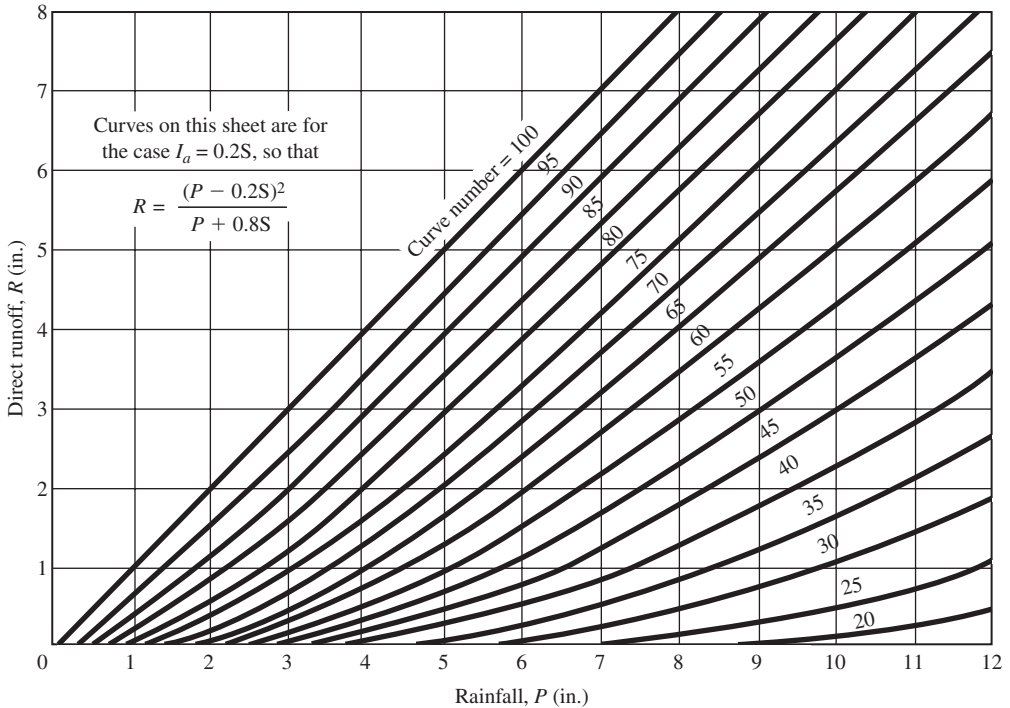


Figure 11.18 The SCS rainfall-runoff relationship. Source: Soil Conservation Service.

Example 11.6

An urban residential district is made of 1/3-acre lots. It is 30-percent impervious, and the hydrologic soil group for the district is identified as group B. Determine the rainfall excess (i.e., runoff) resulting from a 10-hr storm that produces the rainfall intensities tabulated in column 3 of Table 11.5.

TABLE 11.5 SCS Curve Number Method Example

(1) t_1 (hr)	(2) t_2 (hr)	(3) i (in./hr)	(4) ΔP (in.)	(5) P_1 (in.)	(6) P_2 (in.)	(7) R_1 (in.)	(8) R_2 (in.)	(9) ΔR (in.)
0	2	0.05	0.10	0.00	0.10	0.00	0.00	0.00
2	4	0.20	0.40	0.10	0.50	0.00	0.00	0.00
4	8	1.00	2.00	0.50	2.50	0.00	0.53	0.53
6	8	0.50	1.00	2.50	3.50	0.53	1.12	0.59
8	10	0.25	0.50	3.50	4.00	1.12	1.46	0.34

Solution

From Table 11.4, we obtain $CN = 72$ for this urban watershed. The calculations are presented in Table 11.5. The rainfall intensities (i) are tabulated in $\Delta t = 2$ hour intervals in column 3, where t_1 and t_2 listed in columns 1 and 2 mark the beginning and end of these intervals. In column 4, $\Delta P = i \Delta t = i (t_2 - t_1)$ is the incremental rainfall depth accumulating over a time interval. In column 5, P_1 is the cumulative rainfall at t_1 . Obviously, $P_1 = 0$ when the rainfall first begins at $t_1 = 0$. In column 6, P_2 is the cumulative rainfall at t_2 , and it is obtained as $P_2 = P_1 + \Delta P$. The R_1 values listed in column 7 represent the cumulative runoff (or rainfall excess) at time t_1 , and they are obtained by using Equations 11.3 and 11.4 (or Figure 11.18) given CN and P_1 . In column 8, R_2 is the cumulative runoff at t_2 , and it is obtained in the same manner but using P_2 in

Equations 11.3 and 11.4. In column 9, $\Delta R = R_2 - R_1$ is the incremental depth of runoff accumulating over the time interval Δt .

11.6.2 Time of Concentration

Time of concentration is defined as the time it takes runoff to reach the design point from the hydrologically most remote point in the watershed. Although it is difficult to compute accurately, time of concentration is a key parameter in many hydrologic analysis and design procedures. Many design tools are available for determining the time of concentration. Most techniques distinguish between the overland flow phase and the channel flow phase. One popular procedure promoted by the NRCS breaks down the flow time into three components: (1) sheet (overland) flow, (2) shallow concentrated flow, and (3) open-channel flow.

Sheet flow is defined as flow over plane surfaces at very shallow depths (~0.1 ft). Sheet flow occurs throughout a watershed before concentrating in swales and gullies. Resistance to sheet flow (Manning’s *n* values; tabulated in Table 11.6) incorporates raindrop impact, surface drag, resistance from obstacles (e.g., litter, grass, stones), erosion, and sediment transport. Based on a Manning’s kinematic solution, Overton and Meadows* suggest sheet flow travel time (T_{t_1} in hours) is found using

$$T_{t_1} = [0.007(nL)^{0.8}]/(P_2^{0.5}s^{0.4}) \tag{11.5}$$

where *n* is Manning’s sheet flow roughness, *L* is flow length in feet, *P*₂ is the 2-year, 24-hr rainfall in inches, and *s* is the land slope (ft/ft). The National Weather Service and the National Oceanic and Atmospheric Administration evaluate and publish 2-year (return interval), 24-hr precipitation depths for the United States. The sheet flow length was originally limited to 300 feet or less by the NRCS, but currently in practice it is often limited to 100 ft.

TABLE 11.6 Manning’s *n* Values for Sheet Flow^{a,b}

Surface Description	Range of <i>n</i> Values
Concrete, bare soil	0.011
Grass	
short grass	0.15
dense grass	0.24
Bermuda grass	0.41
Range (natural)	0.13
Fallow, no residue	0.05
Cultivated soils	
residue cover <20%	0.06
residue cover >20%	0.17
Woods	
light underbrush	0.4
dense underbrush	0.8

^aE. T. Engman, “Roughness coefficients for routing surface runoff,” *Journal of Irrigation and Drainage Engineering*, 112 (1986).

^bSoil Conservation Service, *Urban Hydrology for Small Watersheds* (Technical Release 55). Washington, DC: Soil Conservation Service, 1986.

* D. E. Overton and M. E. Meadows, *Storm Water Modeling* (New York: Academic Press, 1976): 58–88.

Shallow concentrated flow occurs when two separate planes of sheet flow converge to consolidate flow but fail to form a defined stream or channel. This type of flow often occurs in street gutters. Shallow concentrated flow velocity is estimated from the SCS equations:

$$\text{Unpaved: } V = 16.1345 s^{0.5} \tag{11.6}$$

$$\text{Paved: } V = 20.3282 s^{0.5} \tag{11.7}$$

where V is the average velocity (ft/sec) and s is the watercourse slope (ft/ft). The shallow concentrated flow travel time (T_{t2}) is found by dividing the flow length by the average velocity.

Open-channel flow begins where shallow concentrated flow ends. This transition can be subjective, but it is often accompanied by well-defined stream banks. Field reconnaissance and contour maps are helpful. U.S. Geological Survey quadrangle maps depict channels (or streams) with blue lines. The average velocity for open-channel flow is defined by Manning’s equation as

$$V = (1.49/n)(R_h)^{2/3}(S_e)^{1/2} \tag{11.8}$$

where V is the average velocity (ft/sec), n is Manning’s channel roughness coefficient (Table 11.7), R_h is the hydraulic radius (ft) described in Chapter 6 (Section 2), and S_e is the slope of the energy grade line (ft/ft) or channel bottom slope (S_o) if uniform flow is assumed. The channel flow time (T_{t3}) is found by dividing the channel length by the average velocity.

An alternative procedure for determining the time of concentration for small watersheds (less than 2,000 acres) is based on the runoff curve numbers discussed in the preceding section. The empirical equation is

$$T_c = [L^{0.8}(S + 1)^{0.7}]/(1,140 Y^{0.5}) \tag{11.9}$$

where T_c is the time of concentration (hrs), L is the length of the longest flow path in the watershed (from the design point to the hydrologically most remote point in the watershed located on the drainage divide in feet and often called the *hydraulic length*), Y is the average watershed slope expressed as a percentage, and S is calculated using Equation 11.4.

TABLE 11.7 Typical Manning’s n Values for Channel Flow

Channel Surface	n
Glass, polyvinyl chloride, high-density polyethylene	0.010
Smooth steel, metals	0.012
Concrete	0.013
Asphalt	0.015
Corrugated metal	0.024
Earth excavation, clean	0.022–0.026
Earth excavation, gravel or cobbles	0.025–0.035
Earth excavation, some weeds	0.025–0.035
Natural channels, clean and straight	0.025–0.035
Natural channels, stones or weeds	0.030–0.040
Riprap-lined channel	0.035–0.045
Natural channels, clean and winding	0.035–0.045
Natural channels, winding with pools or shoals	0.045–0.055
Natural channels, weeds with debris or deep pools	0.050–0.080
Mountain streams, gravel or cobbles	0.030–0.050
Mountain streams, cobbles or boulders	0.050–0.070

Example 11.7

Determine the time of concentration (T_c) for a watershed with the following characteristics: sheet flow segment with $n = 0.20$ and $L = 120$ ft (and slope of 0.005); 2-year, 24-hr rainfall is 3.6 inches; a shallow concentrated flow (unpaved) segment where $L = 850$ ft with a slope of 0.0125; and a channel flow segment with $n = 0.025$, $A = 27$ ft², $P = 13$ ft, $S_o = 0.005$, and $L = 6,800$ ft.

Solution

For the sheet flow segment, using Equation 11.5:

$$T_{t_1} = [0.007\{(0.20)(120)\}^{0.8}]/[(3.6)^{0.5} (0.005)^{0.4}] = 0.39 \text{ hrs}$$

For the shallow concentrated flow, from Equation 11.6:

$$V = 16.1345(0.0125)^{0.5} = 1.80 \text{ fps}$$

and

$$T_{t_2} = 850/[(180 \text{ ft/sec})(3,600 \text{ sec/hr})] = 0.13 \text{ hrs}$$

For the channel flow segment, $R_h = A/P = 27/13 = 2.08$ ft. Then, from Equation 11.8:

$$V = (1.49/0.025)(2.08)^{2/3}(0.005)^{1/2} = 6.87 \text{ ft/sec}$$

and

$$T_{t_3} = 6,800/[(6.87 \text{ ft/sec})(3,600 \text{ sec/hr})] = 0.27 \text{ hr}$$

Therefore,

$$T_c = 0.39 + 0.13 + 0.27 = 0.79 \text{ hrs}$$

It is interesting to note that the sheet flow length (120 ft) makes up only 1.5 percent of the total flow length ($120 + 850 + 6,800 = 7,700$ ft). However, 0.39 hours of the time of concentration of 0.79 hr—that is, 49 percent—is attributed to the sheet flow segment of the flow path.

11.6.3 SCS Synthetic Unit Hydrograph

The SCS synthetic unit hydrograph procedure only requires two parameters: (1) the time to peak and (2) the peak discharge. With these parameters, a unit hydrograph can be developed at any ungauged stream location. A design hydrograph evolves from the unit hydrograph as described in Section 11.5. The following procedure is used to develop the unit hydrograph.

The *time to peak* is the elapsed time from the beginning of the effective (runoff-producing) rainfall to the peak discharge as shown in Figure 11.19. The SCS computes the time to peak (T_p , in hours) using the equation

$$T_p = \Delta D/2 + T_L \quad (11.10)$$

where ΔD is the duration of effective rainfall (hrs) and T_L is the lag time (hrs). The *lag time* is the elapsed time from the centroid of effective rainfall to the peak discharge (Figure 11.19). Lag time and storm duration are interrelated parameters and change with each storm.

Time of concentration (T_c), a watershed characteristic, remains relatively constant and is readily determined (Section 11.6.2). SCS has related T_c to the effective rainfall duration (ΔD) and lag time (T_L) empirically in order to compute an optimal time to peak for the synthetic unit hydrograph. SCS recommends that the storm duration be set to

$$\Delta D = 0.133 T_c \quad (11.11)$$

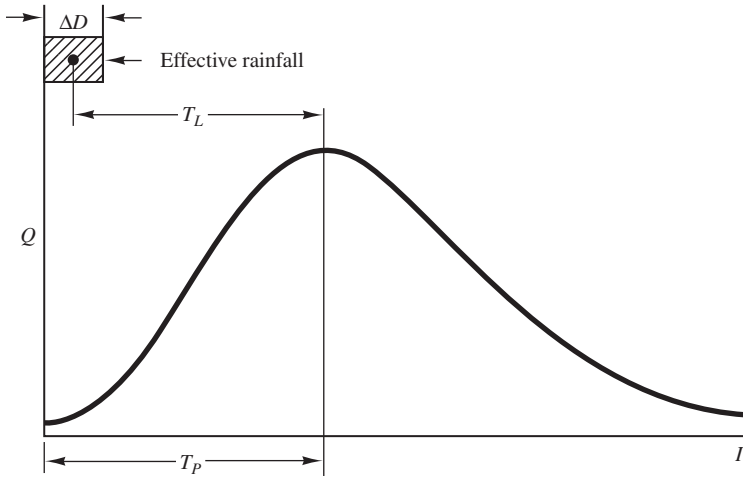


Figure 11.19 SCS hydrograph parameters

based on the characteristics of the SCS curvilinear unit hydrograph. Also, the lag time is related to time of concentration by

$$T_L = 0.6 T_c \tag{11.12}$$

Therefore, the time to peak is computed as

$$T_p = 0.67 T_c \tag{11.13}$$

The time of concentration is determined using Equations 11.5 through 11.8, depending on the flow regime (sheet, shallow concentrated, or channel). Alternatively, Equation 11.9 is used.

The peak discharge (q_p , in cfs) for the SCS synthetic unit hydrograph is determined from

$$q_p = (K_p A)/T_p \tag{11.14}$$

where A is the drainage area (square miles), T_p is the time to peak (hrs), and K_p is an empirical constant. K_p ranges from 300 in flat swampy areas to 600 in steep terrain, but it is often assigned a value of 484. The local NRCS office will provide guidance in evaluating K_p .

The peak discharge and time to peak combine with the dimensionless unit hydrograph coordinates in Table 11.8 ($K_p = 484$ only) to produce the SCS unit hydrograph. The example problem that follows demonstrates the procedure. The dimensionless unit hydrograph is plotted in Figure 11.20, which also displays mass accumulation. Recall that the area under a hydrograph represents runoff volume (or mass). A little more than one-third (37.5 percent) of the total runoff volume accumulates before the peak.

It is important to remember that the shape and peak of a unit hydrograph depends on the effective (runoff-producing) storm duration. Therefore, there are an infinite number of unit hydrographs for a particular watershed. However, for the SCS synthetic unit hydrograph method, the effective storm duration (ΔD) is found using Equation 11.11. To use the resulting unit hydrograph to produce design storms, runoff depths must be developed in units of time that are ΔD in length (or multiples thereof).

TABLE 11.8 Dimensionless Unit Hydrograph and Mass Curve Coordinates for the SCS Synthetic Unit Hydrograph

Time Ratios (t/t_p)	Flow Ratios (q/q_p)	Mass Ratios (Q_d/Q)	Time Ratios (t/t_p)	Flow Ratios (q/q_p)	Mass Ratios (Q_d/Q)
0.0	0.00	0.000	1.7	0.460	0.790
0.1	0.03	0.001	1.8	0.390	0.822
0.2	0.10	0.006	1.9	0.330	0.849
0.3	0.19	0.012	2.0	0.280	0.871
0.4	0.31	0.035	2.2	0.207	0.908
0.5	0.47	0.065	2.4	0.147	0.934
0.6	0.66	0.107	2.6	0.107	0.953
0.7	0.82	0.163	2.8	0.077	0.967
0.8	0.93	0.228	3.0	0.055	0.977
0.9	0.99	0.300	3.2	0.040	0.984
1.0	1.00	0.375	3.4	0.029	0.989
1.1	0.99	0.450	3.6	0.021	0.993
1.2	0.93	0.522	3.8	0.015	0.995
1.3	0.86	0.589	4.0	0.011	0.997
1.4	0.78	0.650	4.5	0.005	0.999
1.5	0.68	0.700	5.0	0.000	1.000
1.6	0.56	0.751			

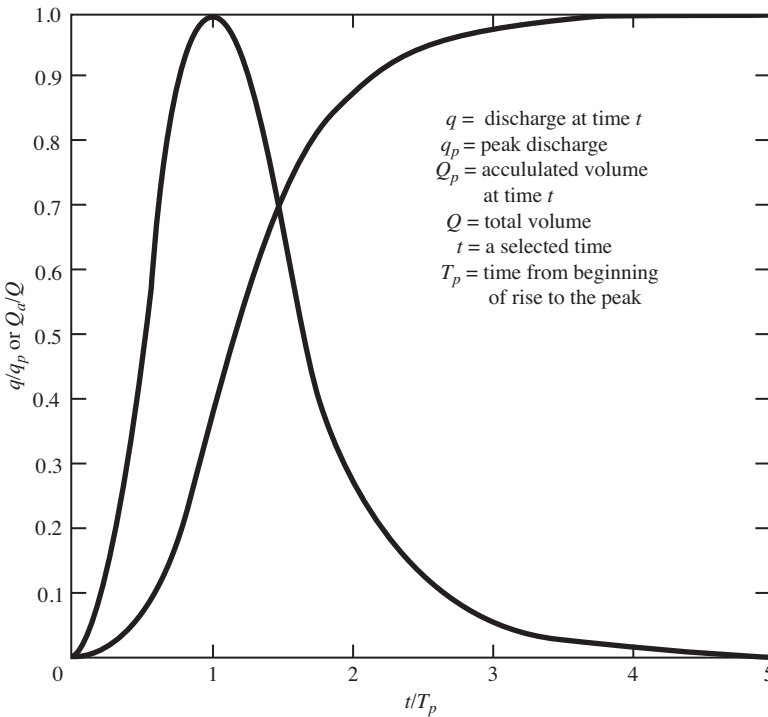


Figure 11.20 The SCS dimensionless unit hydrograph and mass curve. *Source:* Soil Conservation Service.

Example 11.8

A development is being planned for a large tract of land in Albemarle County, Virginia. The development will encompass a 250-acre watershed with a hydraulic length of 4,500 feet. The predominant land use is horse pasture (clay loam) with an average land slope of 8 percent. The land use in the watershed is scheduled

to change over the next ten years to single-family residential (1/2-acre lots). Determine the SCS synthetic unit hydrograph before development.

Solution

Time to peak: The time to peak requires the time of concentration. Applying Equation 11.9,

$$T_c = [L^{0.8}(S + 1)^{0.7}]/(1,140 Y^{0.5})$$

requires a determination of the maximum potential retention (*S*). Thus, from Equation 11.4,

$$S = (1,000/CN) - 10 = (1,000/74) - 10 = 3.51 \text{ in.}$$

where the curve number is found in Table 11.4 (pasture, good condition, and C soils based on clay loam texture and Table 11.3). Substituting the hydraulic length and watershed slope into Equation 11.9 yields

$$T_c = [(4,500)^{0.8}(3.51 + 1)^{0.7}]/[1,140 (8)^{0.5}] = 0.75 \text{ hours (45 minutes)}$$

By applying Equation 11.13, an estimate of the time to peak is

$$T_p = 0.67 T_c = 0.67(0.75) = 0.50 \text{ hours (30 minutes)}$$

Peak discharge: The peak discharge for the watershed is calculated from Equation 11.14 as

$$q_p = (K_p A)/T_p = 484 [250 \text{ acres (1 sq.mi/640 acres)}]/0.5 \text{ hrs} = 378 \text{ cfs}$$

SCS synthetic unit hydrograph: The coordinates of the unit hydrograph displayed in Table 11.9 are extracted from Table 11.8. The peak flow is 378 cfs and occurs 30 minutes into the storm. The storm duration is found using Equation 11.11 and equals

$$\Delta D = 0.133 T_c = (0.133)(0.75) = 0.10 \text{ hrs (6 minutes)}$$

Thus, the unit hydrograph developed is a 6-minute unit hydrograph and can be used to develop design flow rates in the same manner as discussed in Section 11.5. (See Problem 11.6.3.)

TABLE 11.9 SCS Synthetic Unit Hydrograph Coordinates for Example 11.8

Time Ratios (<i>t/t_p</i>)	Flow Ratios (<i>q/q_p</i>)	Time (min)	Flow (cfs)	Time Ratios (<i>t/t_p</i>)	Flow Ratios (<i>q/q_p</i>)	Time (min)	Flow (cfs)
0.0	0.00	0	0	1.7	0.460	51	174
0.1	0.03	3	11	1.8	0.390	54	147
0.2	0.10	6	38	1.9	0.330	57	125
0.3	0.19	9	72	2.0	0.280	60	106
0.4	0.31	12	117	2.2	0.207	66	78
0.5	0.47	15	178	2.4	0.147	72	56
0.6	0.66	18	249	2.6	0.107	78	40
0.7	0.82	21	310	2.8	0.077	84	29
0.8	0.93	24	352	3.0	0.055	90	21
0.9	0.99	27	374	3.2	0.040	96	15
1.0	1.00	30	378	3.4	0.029	102	11
1.1	0.99	33	374	3.6	0.021	108	8
1.2	0.93	36	352	3.8	0.015	114	6
1.3	0.86	39	325	4.0	0.011	120	4
1.4	0.78	42	295	4.5	0.005	135	2
1.5	0.68	45	257	5.0	0.000	150	0
1.6	0.56	48	212				

11.6.4 SCS Design Hydrograph

Suppose we are required to generate a design hydrograph resulting from a 10-yr, 24-hr storm. We can summarize the procedure to obtain this design hydrograph as follows:

1. Determine the 10-yr, 24-hr rainfall amount using Figure 11.8.
2. Select the SCS storm type for the 24-hr hyetograph using Figure 11.9.

3. Determine the runoff resulting from this design rainfall event using the curve number method discussed in Section 11.6.1.
4. Generate the SCS synthetic unit hydrograph as described in Section 11.6.3.
5. Use unit hydrograph theory (linearity and superposition) to construct the 10-year design hydrograph (as performed in Example 11.5).

The design procedure described above is very tedious and is not something one would want to perform more than once. Fortunately, computer programs are available that perform the task quickly and accurately.

11.7 Storage Routing

The design hydrographs developed in the last few sections can be viewed as flood waves. As these waves move downstream, their shape changes. If no additional inflow occurs between an upstream and downstream observation point, then storage (in the channel and floodplain) reduces the flood peak and broadens the flood wave. To reduce the peak even more dramatically requires more storage, and flood-control reservoirs are constructed to take advantage of this benefit. Even though few major reservoirs are being planned or built in the United States, stormwater-management ponds are abundant in developing suburban areas.

Storage routing is the process of evaluating the changes in a storm hydrograph as it passes through a pond or reservoir. In other words, an outflow hydrograph is developed using the inflow hydrograph, storage characteristics of the site, and the hydraulic properties of the outlet device. A hypothetical experiment will help us understand the concept. In Figure 11.21, an empty barrel with a hole in its bottom is positioned under a spigot. The spigot is turned on (at $t = 0$) and held at a steady flow rate (Q_{in}) until it is shut off (at $t = t_0$). Initially, the inflow rate exceeds the outflow rate of the barrel's hole and water begins to build up in the barrel (i.e., storage). The outflow rate increases over time as the depth (head) increases. It reaches a maximum when the spigot is turned off, because no additional inflow is available to increase the depth further. After the spigot is turned off, the barrel takes a while to empty.

Figure 11.21 also depicts the inflow and outflow hydrographs for the experiment. Note the steady inflow and time-varying outflow. Also recall that the area under a hydrograph represents

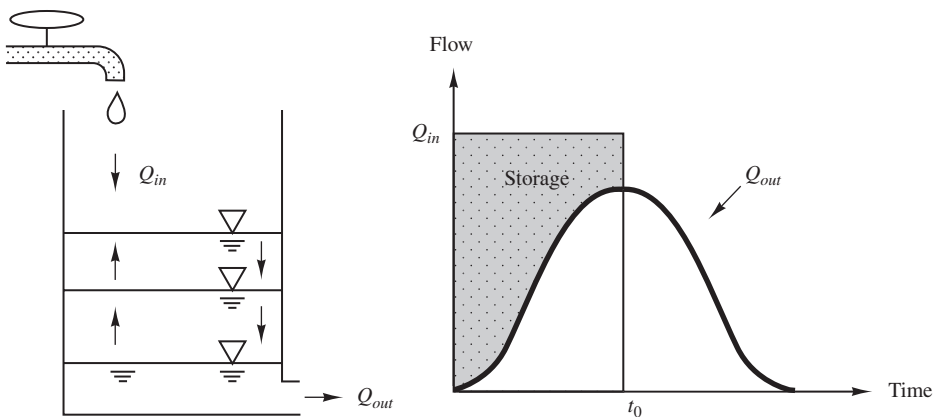


Figure 11.21 The hole-in-the-barrel experiment

a volume of water. Thus, the area under the inflow hydrograph represents the volume of water that entered the barrel, and the area under the outflow hydrograph represents the volume of water that drained from the barrel. The area in between the inflow and outflow hydrographs represents the storage of water in the barrel. This storage accumulates over time until it reaches a maximum when the spigot is shut off (represented by the shaded area in Figure 11.21). From this time forward, the area under the outflow hydrograph represents the volume of water that drains from the barrel after time t_0 . This volume (area) must match the maximum storage volume previously defined. Also, the total area under the inflow hydrograph and the total area under the outflow hydrograph should be equal based on mass balance.

Application of the conservation of mass (with constant density) is necessary to solve the storage routing problem mathematically. Simply stated, the change in storage is equal to inflow minus outflow. In differential form, the equation may be expressed as

$$dS/dt = I - O \tag{11.15}$$

where dS/dt is the rate of change of storage with respect to time, I is the instantaneous inflow, and O is instantaneous outflow. If average rates of inflow and outflow are used, then an acceptable solution can be obtained over a discrete time step (Δt) using

$$\Delta S/\Delta t = \bar{I} - \bar{O} \tag{11.16}$$

where ΔS is the storage change over the time step. Finally, by assuming linearity of flow across the time step, the mass balance equation may be expressed as

$$\Delta S = [(I_i + I_j)/2 - (O_i + O_j)/2] \Delta t \tag{11.17}$$

where subscripts i and j designate whether the inflow is at the beginning or end of the time step. Figure 11.22 depicts the variables in the equation. The linearity assumption improves by decreasing Δt .

The mass–balance relationship in Equation 11.17 contains two unknowns. Because the inflow hydrograph must be defined before the routing calculations are done, the inflow values (I_i and I_j) are known. Likewise, the time increment (Δt) is chosen and the outflow value at the beginning of the time step (O_i) was solved in the previous time-step calculations. That leaves the storage increment (ΔS) and the outflow at the end of the time step (O_j). In fact, these two unknowns are related. As can be seen in Figure 11.22, as O_j increases, ΔS decreases. To solve the mass–balance equation requires another relationship between storage and outflow. Because both storage and outflow (for uncontrolled outlet devices) are related to the depth of water in the reservoir, they are related to one another. This relationship is employed to complete the solution.

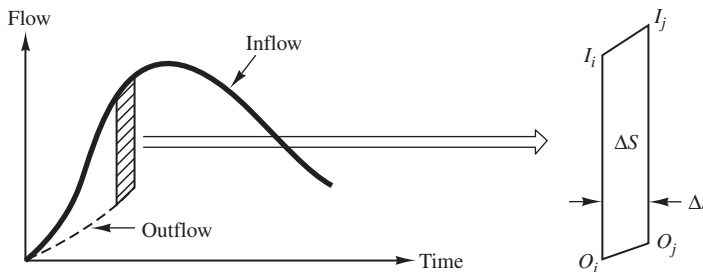


Figure 11.22 Graphical representation of the storage equations

The data requirements to perform storage routing include the following:

- inflow hydrograph (using SCS or other appropriate procedures),
- elevation versus storage relationship for the reservoir, and
- elevation versus discharge (outflow) relationship for the outlet device.

Figure 11.23 displays these data requirements graphically. The procedure for obtaining the stage (elevation) versus storage curve is described in the figure. Also, the two major types of outlet devices are noted with typical stage-versus-discharge relationships.

The modified Puls (or level pool) routing method reformulates Equation 11.17 into

$$(I_i + I_j) + [(2S_i/\Delta t) - O_i] = [(2S_j/\Delta t) + O_j] \tag{11.18}$$

where $(S_j - S_i)$ equals the change in storage (ΔS). The advantage of this expression is that all of the known variables are on the left side of the equation and all of the unknowns are grouped on the right. The solution procedure for the modified Puls routing method is as follows.

1. Determine the appropriate inflow hydrograph for the reservoir. Reservoir routing is performed for a number of reasons (outlet device sizing, storage determination, and evaluation of downstream flooding) but always requires a complete inflow hydrograph.

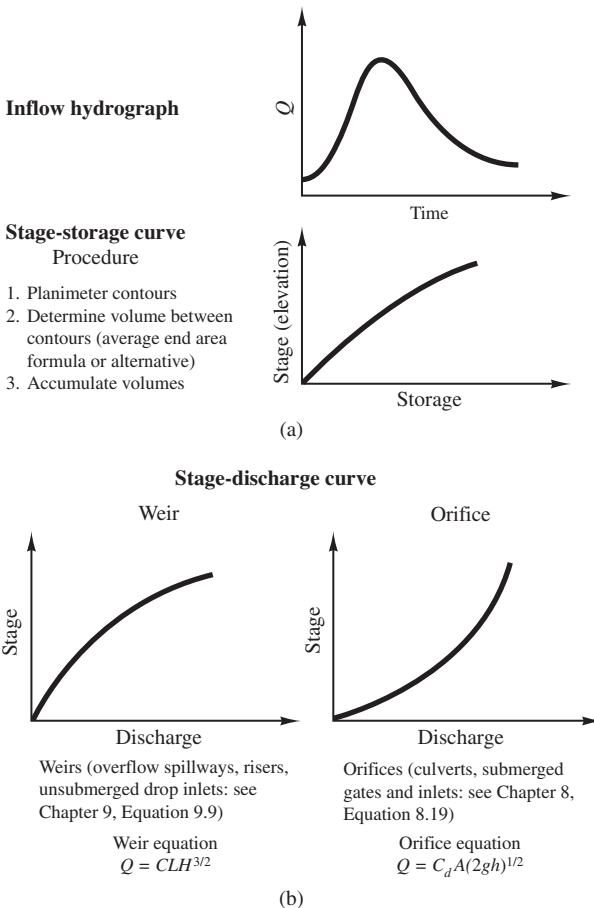


Figure 11.23 Data requirements for storage routing

2. Select a routing interval (Δt). Remember that linearity of inflows and outflows is assumed, so Δt must be chosen accordingly. A generally good estimate is $\Delta t = T_p/10$.
3. Determine the elevation–storage relationship for the reservoir site and the elevation–outflow relationships for the outlet device selected.
4. Establish the storage–outflow relationship using the following table:

Elevation	Outflow (O)	Storage (S)	$2S/\Delta t$	$(2S/\Delta t) + O$

5. Graph the $[(2S/\Delta t) + O]$ versus (O) relationship.
6. Perform storage routing computations using a table with the following headings.

Time	Inflow (I_i)	Inflow (I_j)	$(2S/\Delta t) - O$	$(2S/\Delta t) + O$	Outflow (O)

The solution procedure is explained in more detail using the following example problem.

Example 11.9 (adapted from Normann and Houghtalen*)

A development is being planned in Albemarle County, Virginia, that encompasses a 250-acre watershed. Virginia’s stormwater-management ordinance requires that the 2-year peak discharge after development be no greater than the 2-year peak discharge under current conditions (173 cfs). A detention pond is proposed at the outlet of the watershed to meet the criterion. Determine the required storage and the outlet device size if the peak discharge after development is 241 cfs.

Solution

- (a) **Determine the appropriate inflow hydrograph for the reservoir.** SCS procedures were used to determine the 2-year, 24-hour hydrograph after development. The following table lists the design flows coming into the proposed pond.

Time	Flow (cfs)	Time	Flow (cfs)	Time	Flow (cfs)
11:30	8	12:15	72	13:00	156
11:35	9	12:20	119	13:05	126
11:40	11	12:25	164	13:10	114
11:45	12	12:30	210	13:15	100
11:50	13	12:35	240	13:20	85
11:55	15	12:40	241	13:25	79
12:00	18	12:45	227	13:30	67
12:05	27	12:50	202	13:35	59
12:10	43	12:55	181	13:40	52

* J. M. Normann and R. J. Houghtalen, *Basic Stormwater Management in Virginia* (Richmond, VA: Division of Soil and Water Conservation, Virginia Department of Conservation and Historic Resources, 1982).

- (b) **Select a routing interval (Δt).** It appears from the inflow hydrograph that it takes about an hour for the hydrograph to peak. Based on $\Delta t = T_p/10$, a 5-minute time increment is chosen.
- (c) **Determine the elevation–storage relationship for the reservoir site and the elevation–outflow relationships for the outlet device selected.** The potential pond site is depicted on the contour map in Figure 11.24. It is located at the outlet of the basin and is currently being used as a farm pond. An earthen dam will be built, and corrugated metal pipes will be used to release pond outflows. The elevation–storage relationship is determined using the average end-area method.

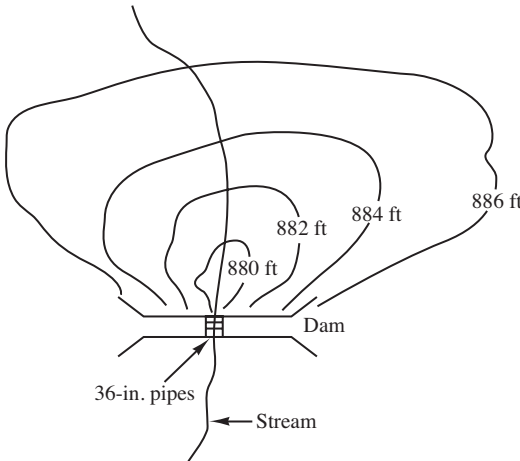


Figure 11.24 Contour map of potential pond site

Elevation (ft, MSL)	Area (acres)	Δ Storage (acre-ft)	Storage (acre-ft)
878	0.00		0.00
880	0.22	0.22	0.22
882	0.78	1.00	1.22
884	1.66	2.44	3.66
886	3.14	4.80	8.46

Two 36-inch corrugated metal pipes (CMPs) will be used for the outlet device. Remember, the outlet device (size and number of pipes) must be designed to meet Virginia’s stormwater-management criterion. At this point, they are trial sizes until the reservoir routing is completed to determine whether they meet design requirements. The CMPs will be placed on the existing stream bed (elevation 878 ft, MSL) with the earthen dam built over them. We will assume the CMPs operate as orifices (inlet control). Therefore, we will use Equation 8.19:

$$Q = \text{Outflow } (O) = C_d A (2gh)^{1/2}$$

where C_d is a discharge coefficient set equal to 0.6 (square edge), A is the flow area of the two pipes, and h is the driving head measured from the middle of the opening to the water surface.

The following table provides the information for the elevation–outflow relationship.

Elevation (ft, MSL)	Head (ft)	Outflow (cfs)
878	0.0	0
880	0.5	48
882	2.5	108
884	4.5	144
886	6.5	173

(d) Establish the storage–outflow relationship:

Elevation (ft, MSL)	Outflow (O) (cfs)	Storage (S) (acre-ft)	$2S/\Delta t$ (cfs)	$(2S/\Delta t) + O$ (cfs)
878	0	0	0	0
880	48	0.22	64	112
882	108	1.22	354	462
884	144	3.66	1,060	1,210
886	173	8.46	2,460	2,630

The first three columns in the table are obtained from the elevation–storage and the elevation–discharge information previously computed. The fourth column is obtained by doubling the storage in the third column and dividing by the routing interval we have chosen ($\Delta t = 5 \text{ minute} = 300 \text{ seconds}$). Of course, the storage will have to be converted from acre-feet to cubic feet to get the desirable units (cfs) for $2S/\Delta t$. Recall that there are 43,560 square feet in an acre. The last column is found by adding the second column (outflow, O) to the fourth column ($2S/\Delta t$). The second and third columns represent the relationship between outflow and storage that is needed to solve a mass–balance equation that contains two unknowns. However, it is more convenient computationally to use the relationship between outflow and $[(2S/\Delta t) + O]$, as will be seen.

(e) Graph the $[(2S/\Delta t) + O]$ versus (O) relationship. [See Figure 11.25 (a).]

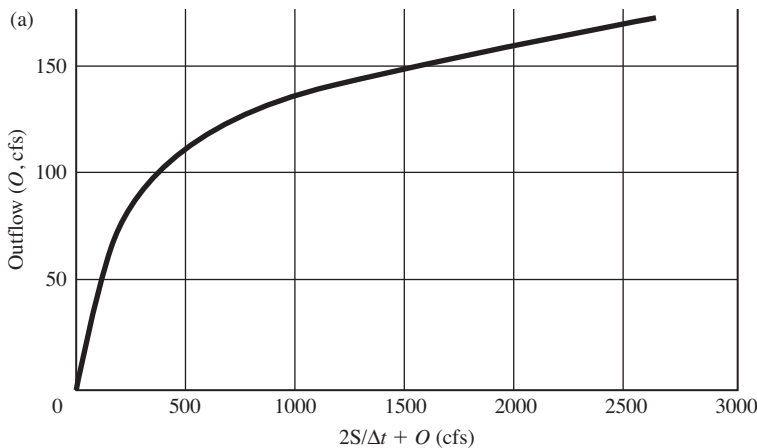


Figure 11.25 (a) Routing relationship between $[(2S/\Delta t) + O]$ and outflow (O)

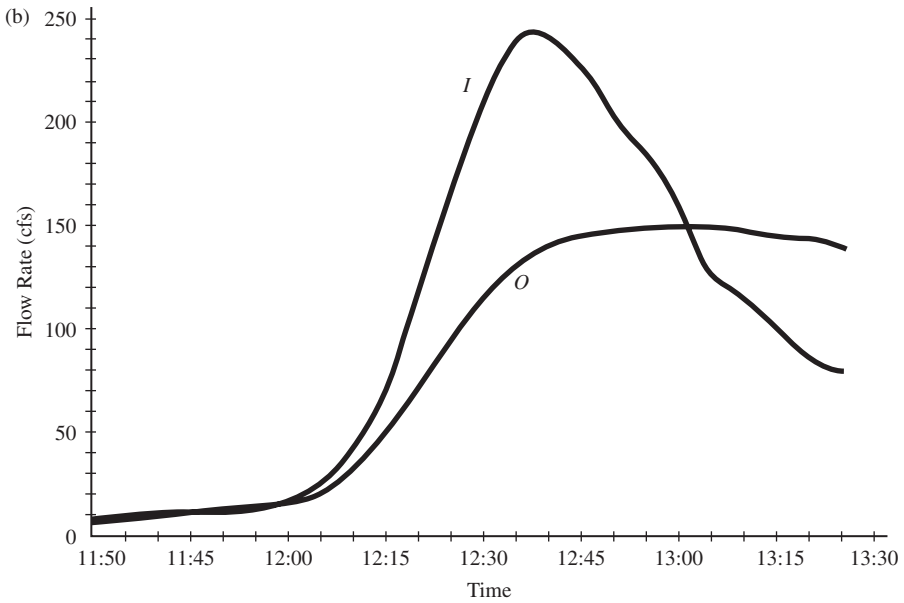


Figure 11.25 (b) Inflow (*I*) and outflow (*O*) hydrographs

(f) Perform the storage (modified Puls) routing computations:

Time	Inflow (I_i) (cfs)		Inflow (I_j) (cfs)		$(2S/\Delta t) - O$ (cfs)	$(2S/\Delta t) + O$ (cfs)	Outflow (O) (cfs)
11:30	8	+	9	+	0^1		6^2
11:35	9		11		3^5	17^3	7^4
11:40	11		12		5	23	9
11:45	12		13		4	28	12
11:50	13		15		5	29	12
11:55	15		18		5	33	14
12:00	18		27		4	38	17
12:05	27		43		9	49	20
12:10	43		72		13	79	33
12:15	72		119		26	128	51
12:20	119		164		71	217	73
12:25	164		210		166	354	94
12:30	210		240		308	540	116
12:35	240		241		496	758	131
12:40	241		227		699	977	139
12:45	227		202		879	1,167	144
12:50	202		181		1,014	1,308	147
12:55	181		156		1,101	1,397	148
13:00	156		126		1,140	1,438	149
13:05	126		114		1,126	1,422	148
13:10	114		100		1,072	1,366	147
13:15	100		85		996	1,286	145
13:20	85		79		897	1,181	142
13:25	79		67		783	1,061	139
13:30	67						

Comments on completing the storage (reservoir) routing table: Column 1 (time) and column 2 (inflow) represent the design inflow hydrograph. Column 3 is column 2 moved up one

time increment (i.e., I_j follows I_i by one time increment). The rest of the columns are unknown or blank when the routing process begins. The primary objective is to fill in the last column representing the outflow hydrograph or discharges from the detention pond. The footnotes that follow apply to the numbers in the table with superscripts.

1. To initiate the routing process when there is very little inflow, assume $(2S_i/\Delta t - O_i)$ equals zero because there is very little storage in the pond or outflow from the pond.
2. The first value of outflow is given as 6 cfs. If it was not given, then we could assume that the first value of outflow is equal to the first value of inflow.
3. The values in this column are found by applying the mass–balance equation stated in Equation 11.18 as

$$(I_i + I_j) + [(2S_i/\Delta t) - O_i] = [(2S_j/\Delta t) + O_j]$$

or, in this case,

$$(8 + 9) + [0] = [17] \text{ for time 11:35}$$

In words, we are determining the value of $[(2S_j/\Delta t) + O_j]$ at time 11:35. The equation states that we need to add the current inflow to the previous inflow and the previous value of $[(2S_i/\Delta t) - O_i]$. The plus signs in the table show which numbers are to be added.

4. The outflow values, except for the first one mentioned in footnote (2), are obtained from the $[(2S/\Delta t) + O]$ versus (O) relationship graphed in step e (Figure 11.25a). For time 11:35, using the $[(2S/\Delta t) + O] = 17$ cfs value, we read a value of outflow approximately equal to 7 cfs using a more detailed graph than could be displayed in Figure 11.25 (a). Note that you may interpolate the table in step d to obtain roughly the same answer.
5. The values of $[(2S/\Delta t) - O]$ are found using algebra. Simply double the outflow values in the last column and subtract it from $[(2S/\Delta t) + O]$; in this case, $17 - 2(7) = 3$ cfs. After completing the routing for time 11:35, move on to time 11:40. Repeat the preceding steps 3 through 5 by determining $2S/\Delta t + O$ from the mass balance equation, then outflow (O) from the graph in step e, and finally $(2S/\Delta t - O)$ using algebra. Repeat for the rest of the storm.

Note that the modified Puls routing procedure was stopped at time 13:25. At this point, the outflow has already begun to decrease. Because our primary concern was the peak outflow, the procedure was stopped. The inflow and the calculated outflow hydrographs are plotted in Figure 11.25 (b). The attenuation and the time lagging of the peak discharge represent typical effects of detention basins on flood waves. Larger detention basins or smaller outlet devices cause more pronounced effects. The area between the two hydrographs represents the volume of water stored in the detention basin. The maximum storage occurs at the time the two hydrographs intersect [at around 13:05 in Figure 11.25 (b)]. Before this time, the inflow rates are higher than the outflow rates, and therefore the detention basin is filling. Beyond this time, the outflow rates exceed the inflow rates, indicating that the detention basin is emptying. Also, it is not a coincidence that the peak outflow occurs when the two hydrographs intersect. There is a single-valued storage–outflow relationship in which the outflow increases with increasing storage. Therefore, the outflow rate is maximized at the same time the storage is maximized.

The outflow peak of 149 cfs is less than our target outflow of 173 cfs. Thus, the design storm is slightly “overcontrolled” with the outlet device chosen. The routing procedure can be repeated using slightly larger pipes. Larger pipes will cost more, but the pond will not get as deep during the design storm. This will leave more usable land and will probably offset the added pipe cost.

The elevation of the pond at peak outflow (or any outflow) can be obtained using the elevation–outflow relationship developed in step c. By interpolation using the peak discharge of 149 cfs, the peak elevation is 884.3 ft MSL. (The peak storage can be determined in the same manner.) The peak elevation would normally

establish the emergency spillway elevation. A different design storm would be used to size the emergency spillway (usually a weir) to keep the dam from overtopping during rare events (i.e., the 100-year storm).

This stormwater-management pond is considered a “dry pond” that has water in it during and shortly after a storm. Eventually, it drains completely because the pipes have been placed on the stream bed. Dry ponds are often placed in the corner of a park or a ball field to make use of the land when it is dry. “Wet ponds” have water in them all the time. Therefore, the volume required for storage must be reserved above the normal pool elevation.

11.8 Hydraulic Design: The Rational Method

Peak flows resulting from rainfall events represent the major design requirement for many hydraulic structures (e.g., drainage inlets, storm sewers, drainage channels, and culverts). The statistical techniques covered in Chapter 12 are effective tools for obtaining peak flows and their associated probabilities on gauged streams. Unfortunately, most small watersheds are not gauged. In addition, most hydraulic structures are installed in small watersheds. The rational method is one of the oldest and most widely used hydrologic methods used to size such structures.

The rational method is based on the equation

$$Q_p = C I A \quad (11.19)$$

where C is a dimensionless *runoff coefficient*, I is the average rainfall intensity in in./hr of probability (P), and A is the contributing drainage area in acres. Q_p is the peak discharge in acre-in./hr or cfs (because 1 ac-in./hr is approximately equal to 1 cfs).

The original determination of watershed runoff coefficients resulted from separate frequency analyses of rainfall and runoff (Chapter 12). In other words, rainfall and runoff with the same return periods were determined from gauge information, and the runoff coefficient was then found as the ratio of runoff to rainfall. The concept is now simplified, assuming that runoff always has the same return period as the rainfall that produces it. Thus the runoff coefficient, essentially a ratio of runoff to rainfall, varies from zero (no runoff) to 1.0 (complete runoff). In practice, it is determined from a table based on the land use, soil type, and land slope (Table 11.10). From Table 11.10, it is evident and intuitive that runoff increases with increasing slope and imperviousness or decreasing vegetative cover and soil permeability. An area-weighted average C is used for watersheds with mixed land uses.

The average rainfall intensity (I) is obtained from an IDF curve (Figure 11.26). As discussed in detail in Chapter 12, IDF curves are graphs of average rainfall intensity versus storm duration for a given geographic location. An array of different return intervals is typically plotted. Long-term rainfall records from the National Weather Service are used to develop these curves for most U.S. cities. Figure 11.26 suggests that the longer the storm, the less intense the rainfall, although the total rainfall amount increases. Again this matches our intuition: If it is raining “cats and dogs,” then it is not likely to last long.

To make use of an IDF curve for the rational method, we need to establish the design storm’s duration. It can be shown that the peak discharge occurs when the entire watershed is contributing runoff to the design point. Therefore, the storm duration is set equal to the time of concentration. With the storm duration and the return interval, the rainfall intensity is obtained from the IDF curve. The intensity is assumed to be constant throughout the storm and is used to solve Equation 11.19. Because of this assumption and many others in the rational method, it is only applicable to small watersheds (e.g., 200 acres is a commonly mentioned upper limit).

TABLE 11.10 Typical Range of Values for Runoff Coefficients

Land Use (Soils and Slopes)	Runoff Coefficient (<i>C</i>)
Parking lots, roofs	0.85–0.95
Commercial areas	0.75–0.95
Residential:	
single family	0.30–0.50
apartments	0.60–0.80
Industrial	0.50–0.90
Parks, open space	0.15–0.35
Forest, woodlands	0.20–0.40
Lawns:	
sandy soil, flat (<2%)	0.10–0.20
sandy soil, steep (>7%)	0.15–0.25
clay soil, flat (<2%)	0.25–0.35
clay soil, steep (>7%)	0.35–0.45
Crop lands:	
sandy soil	0.25–0.35
loam soil	0.35–0.45
clay soil	0.45–0.55

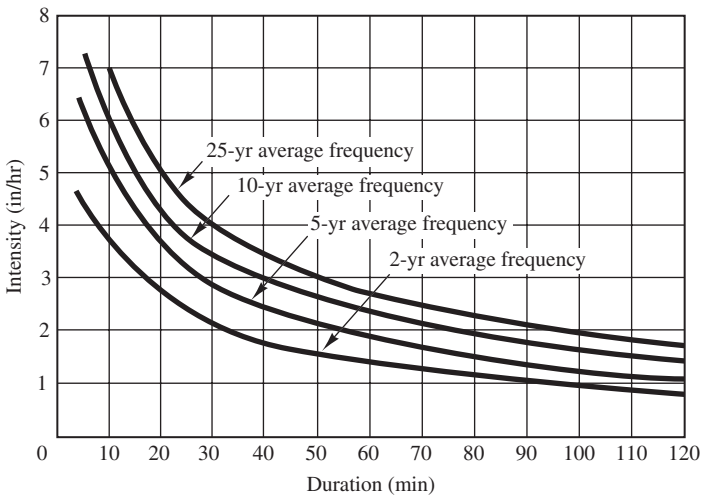


Figure 11.26 Typical intensity-duration-frequency (IDF) curve

Example 11.10

Estimate the 10-year peak discharge (Q_{10}) before and after development of the 250-acre watershed described in Example 11.8. Assume the IDF curve in Figure 11.26 is appropriate to the location of the watershed.

Solution

The rational equation may be used to estimate the peak discharge, even though the watershed is larger than 200 acres. Applying Equation 11.19 for predevelopment conditions yields

$$Q_p = CIA = (0.35)(2.8 \text{ in./hr})(250 \text{ acres}) = 245 \text{ cfs}$$

where C is found using Table 11.10 and I is obtained from Figure 11.26 (with a storm duration equal to the 45-min time of concentration). The C value for open space is used, and a value from the high end of the range is selected because the soil is clay loam and the land slope is steep.

For postdevelopment conditions, the new time of concentration must be computed. Applying Equation 11.9,

$$T_c = [L^{0.8}(S + 1)^{0.7}]/(1,140 Y^{0.5})$$

requires a determination of the maximum potential retention (S). Thus, from Equation 11.4,

$$S = (1,000/CN) - 10 = (1,000/80) - 10 = 2.50 \text{ in.}$$

where the curve number is found in Table 11.4 (residential areas, ½-acre lots, C soil). Substituting the hydraulic length and watershed slope into Equation 11.9 yields

$$T_c = [(4,500)^{0.8} (2.50 + 1)^{0.7}]/[1,140 (8)^{0.5}] = 0.624 \text{ hours (37.4 min)}$$

Applying Equation 11.19 again for postdevelopment conditions yields

$$Q_p = C I A = (0.45)(3.1 \text{ in./hr})(250 \text{ acres}) = 349 \text{ cfs}$$

where C is found using Table 11.10 and I is obtained from Figure 11.26 (with a storm duration equal to the 37.4-min time of concentration). A C value for residential areas is required. The 1/2-acre lots are large, indicating a C value on the low end of the range, but the clay soils and steep slopes increase the C value. Thus, a C value in the upper end of the range was selected.

Note that the development increased the 10-year peak discharge from 245 cfs to 349 cfs. This is the result of a combination of more runoff (higher C value) from the increased impervious area and a reduced time of concentration because runoff makes it to receiving streams more quickly (flowing through street gutters and storm pipes). These are typical consequences of urbanizing watersheds that have led to the establishment of stormwater-management ordinances and low-impact development initiatives to increase infiltration and decrease the speed of runoff.

11.8.1 Design of Stormwater-Collection Systems

Stormwater-collection and -transport systems represent one of the most costly and important components of our urban infrastructure. These systems collect stormwater runoff and convey it to a nearby stream, river, detention basin, lake, estuary, or the ocean in order to minimize the damage and inconvenience of urban flooding. Stormwater system components include street gutters, storm drainage inlets, stormwater pipes, manholes, and, in some cases, constructed channels, ponds, infiltration devices, and stormwater wetlands.

The cost of stormwater systems depends greatly on the storm frequency (or recurrence interval) they are designed to convey. The optimal storm frequency is based purely on economics: when the cost of additional capacity exceeds the benefits. This is rarely done in practice because the analysis is costly and difficult. Instead, local governmental agencies set a design standard, typically a 10-year storm. Consequently, on the average of once every ten years the system will be overloaded, resulting in some minor street or lowland flooding. Problem areas may require a more stringent design standard.

Storm drainage inlets remove water from the street. The most common types are grate, curb opening, and combinations (Figure 11.27). Locating inlets usually constitutes the initial phase of stormwater system design. The number of inlets, each of which must be served by a pipe, directly affects the cost of the system. To minimize cost, the flow in the street gutter must be maximized. In general, inlets are placed in the following locations:

- in all sumps where water collects with no other outlet,
- along the curb when the gutter capacity (curb height) is exceeded,
- along the curb when the water has *spread* out into the street far enough to hinder traffic flow or safety (*pavement encroachment criteria* are established by many local governments),
- upgrade of all bridges (to prevent bridge icing in cold weather), and
- along the curb before an intersection for traffic safety reasons.

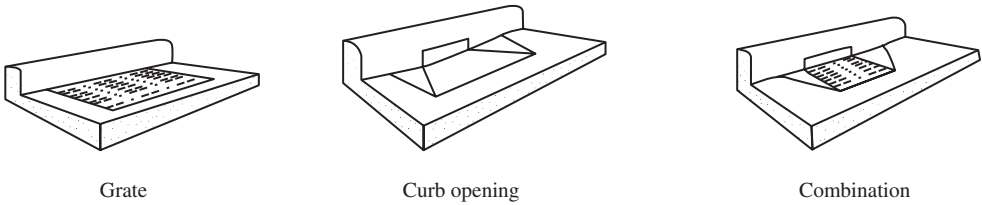


Figure 11.27 Typical storm drainage inlets

Inlet location design to limit street spread combines Manning’s equation for gutter capacity and the rational equation for the rate of surface runoff. Given the street geometry (cross-section information and street slope), pavement encroachment criteria (or gutter height depending on what limits the depth of flow at the curb), and Manning’s roughness coefficient, the capacity of the gutter is established using

$$Q = AV = (1.49/n) A R_h^{2/3} S_e^{1/2} \tag{11.20}$$

with the variables defined in Equation 11.8.

The drainage area contributing flow to the inlet is calculated by substituting the gutter flow found above into the rational equation, which is rearranged as

$$A = Q/(CI) \tag{11.21}$$

Using a topographic map, the proposed inlet location is moved up or down the street until the area contributing surface flow to the inlet matches the drainage area computed above. Although this seems easy, the rational equation requires a rainfall intensity, which depends on a time of concentration. If the inlet has not been located, then the time of concentration cannot be established and an iterative process is necessary. The following example problem will clarify the procedure.

Example 11.11

Curb opening inlets are needed on Barudi Street, which serves a single-family residential neighborhood ($C = 0.35$). A contour map of the area is shown in Figure 11.28 (scale of 1 in. = 100 ft) along with the street cross-section geometry. The asphalt street ($n = 0.015$) has a 1/4-inch per foot cross slope and a longitudinal street slope of 2.5 percent. The local government stipulates a 5-year design storm and allows 6 feet of pavement encroachment on each side of the 30 feet wide street. How far west from the drainage divide should the first inlet be placed to adequately drain the north side of the street? The 2-year, 24-hour rainfall is 3.2 inches and the IDF curve in Figure 11.26 applies.

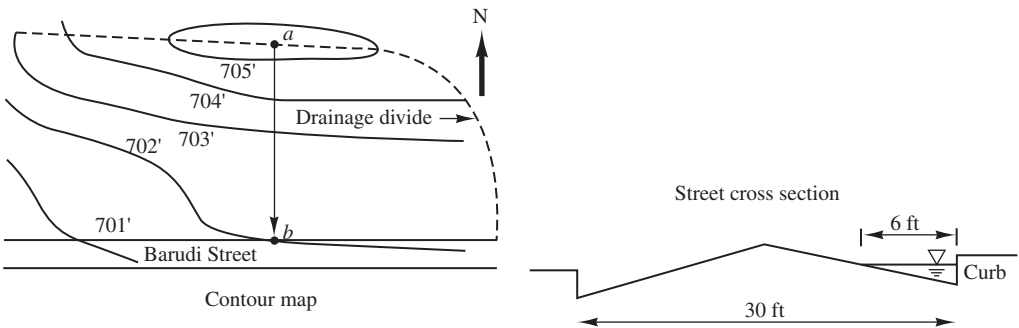


Figure 11.28 Inlet location example problem

Solution

Based upon the cross slope of 1/4-inch per foot and the limiting spread of 6 feet, the depth of flow at the curb is 1.5 inches. This results in a wetted perimeter of 6.13 feet. (Draw the triangular flow section and verify.) Applying Manning's equation (11.20) results in a gutter capacity of

$$Q = (1.49/n) A R_h^{2/3} S e^{1/2}$$

$$Q = (1.49/0.015) (0.375 \text{ ft}^2) (0.375/6.13)^{2/3} (0.025)^{1/2}$$

$$Q = 0.914 \text{ cfs}$$

Next we will apply the rational equation to determine how much surface area can be drained before the gutter has reached its capacity. From the contour map, it appears that the time of concentration will involve primarily overland flow time from the drainage boundary to the street with very little gutter flow time. The greatest overland flow distance is about 100 feet (flow line *a, b* on the contour map; $s \approx 3\%$). Applying Equation 11.5 for sheet flow yields

$$T_{t_1} = [0.007(nL)^{0.8}]/(P_2^{0.5} s^{0.4})$$

$$T_{t_1} = [0.007 \{(0.15)(100)\}^{0.8}]/[(3.2)^{0.5} (0.03)^{0.4}]$$

$$T_{t_1} = 0.139 \text{ hr} = 8.3 \text{ min}$$

Table 11.6 provided the *n* value (short grass), and the contour map supplied the land slope and distance in the preceding equation. Adding a little gutter flow time to the sheet flow time above gives us a time of concentration of roughly 10 min, which equals the storm duration for the rational method. It should be mentioned that some local governmental agencies will specify a time of concentration for stormwater system design (e.g., 5 or 10 min, depending on local spread criteria and topography). Using the IDF curve in Figure 11.26, a storm duration of 10 minutes produces an intensity of 5.2 in./hr for the 5-year storm. Substituting into the rational equation (Equation 11.21) results in

$$A = Q/(CI)$$

$$A = 0.914 \text{ cfs}/[(0.35)(5.2 \text{ in./hr})]$$

$$A = 0.502 \text{ acres (approximately } 22,000 \text{ ft}^2)$$

Because the drainage area on the north side of the street is roughly rectangular with a width of 100 feet, the inlet is placed about 220 ft down the street from the drainage divide.

Unless located in a sump, an inlet rarely captures all of the gutter flow. The percentage of flow captured depends on the street slope, cross slope, flow rate, and type of inlet. Inlet manufacturers provide information on the capture efficiency of their inlets.

A similar design procedure may be used to locate the second inlet.

- The gutter capacity at the second inlet location is computed with Manning's equation.
- The gutter capacity is reduced by an amount equal to the flow that passes by the first inlet.
- With the rational equation, the area contributing flow to the second inlet is determined using the reduced flow from inlet bypass. A new time of concentration must be calculated.

If all design factors remain the same (*C* value, street slope and geometry, pavement-encroachment criteria, time of concentration, and capture efficiency of the inlets) and the width to the drainage divide remains constant, then the inlet spacing from the first to the second inlet repeats itself down the street until a street intersection is reached.

11.8.2 Design of Stormwater Pipes

The next phase of the design is stormwater-pipe sizing. The analysis begins at the first (highest) inlet and proceeds downhill to the outlet point. The rational equation is used to compute the design flow rate for each pipe. Manning's equation is used to obtain a pipe size capable of conveying the peak discharge while flowing just full (not under pressure).

Previous calculations to determine the peak flow using the rational equation for inlet location cannot be used for stormwater-pipe sizing except at the first inlet. Inlet location design accounts for local surface water contributions only. Stormwater pipes must accommodate local surface water contributions at an inlet as well as flows from all upstream pipes. Therefore, the rational equation uses the entire upstream drainage area. In addition, an area-weighted runoff coefficient (C) may be necessary for the upstream drainage area. Finally, the time of concentration is computed using the longest combination of inlet flow time and pipe flow time to the design point (i.e., the entrance to the pipe being sized). Using this time of concentration, the rainfall intensity is obtained from the appropriate IDF curve.

Stormwater-pipe design is not difficult but does require a large data-collection effort. Most of the necessary data can be gleaned from contour maps of the area. (A contour interval of 2 ft or less is often required.) A well-designed table or spreadsheet helps to assimilate the data and clarify the necessary calculations. The rational equation (Equation 11.19) is used to determine the peak flow (Q_p , in cfs) that each stormwater pipe must convey. Manning's equation (Equation 11.20) is used to determine required pipe diameter (D_r , in ft) and can be written as

$$D_r = \left[\frac{nQ_p}{0.463 \sqrt{S_o}} \right]^{3/8} \quad (11.22)$$

where the pipe slope (S_o) is used in place of the energy grade line slope (S_e). Because the required pipe diameter is not likely to be commercially available, the next standard pipe size larger is selected for design purposes. (Standard pipe sizes are usually available in 3-in. intervals from 12 to 24 in., 6-in. intervals from 24 to 48 in., and 1-ft intervals thereafter.) Because the flow time in each pipe is required to size downstream pipes (i.e., time of concentration calculations for the rational equation), velocities in pipes flowing partially full will be needed. Hence, the selected pipe diameter (D) is used to determine the full flow area (A_f), the full flow hydraulic radius (R_f), and the full flow velocity (V_f). The appropriate equations are as follows:

$$A_f = \pi D^2/4 \quad (11.23)$$

$$R_f = D/4 \quad (11.24)$$

$$V_f = (1.49/n) R_f^{2/3} S_o^{1/2} \quad (11.25)$$

Once full flow conditions have been obtained, design aids like Figure 11.29 may be used to determine the actual flow depth (y), the flow velocity (V), and the flow time (t) for the design peak discharge (Q_p) as it passes through the selected pipe. The example problem that follows describes the design procedure in detail.

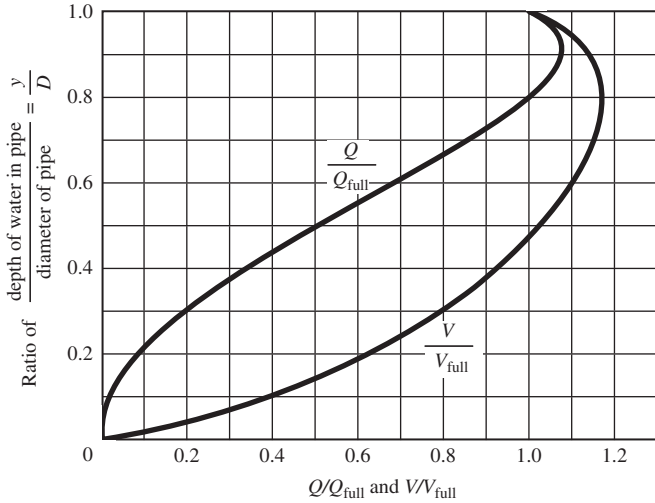


Figure 11.29 Hydraulic characteristics of pipes flowing partly full

Stormwater-pipe design is often subject to certain design standards established by convention or local governmental agencies. Typical standards are as follows:

- Because stormwater pipes are buried, a minimum cover over the crown of the pipe of 3 to 4 ft is required for structural and other reasons.
- Pipe slopes match overlying ground slopes when possible to minimize excavation costs.
- In flat topography, minimum slopes should produce velocities of 2 to 3 ft per second when flowing full to minimize sedimentation.

A minimum pipe diameter of 12 or 15 in. is required to reduce clogging problems.

- Pipe sizes are never reduced downstream even if increased slopes provide adequate flow capacity. Again, clogging is the concern.
- Manholes (or inlets) are provided at pipe junctions, changes of grade, and changes of alignment for constructability and maintenance reasons.
- Maximum manhole spacing in straight reaches may be specified (e.g., 400 ft) because of cleaning equipment limitations. Longer distances are permitted for large pipe sizes.
- Because flow through a manhole produces a small head loss, a drop in the invert elevation from the incoming to the outgoing pipe is recommended (say, 1 in. or 0.1 ft). Minor loss equations are available in some local design standards to evaluate this explicitly.

Example 11.12

Design the stormwater-collection pipes that service a portion of a small town depicted in Figure 11.30. Use the 5-year design storm (IDF curve in Figure 11.26) and a minimum pipe size of 15 in. Inlet times (in min), drainage areas (in acres), and runoff coefficients for each inlet are displayed. In addition, ground elevations at each manhole (in ft, MSL) are provided. Stormwater-pipe (concrete, $n = 0.013$) lengths are provided in the computation table.

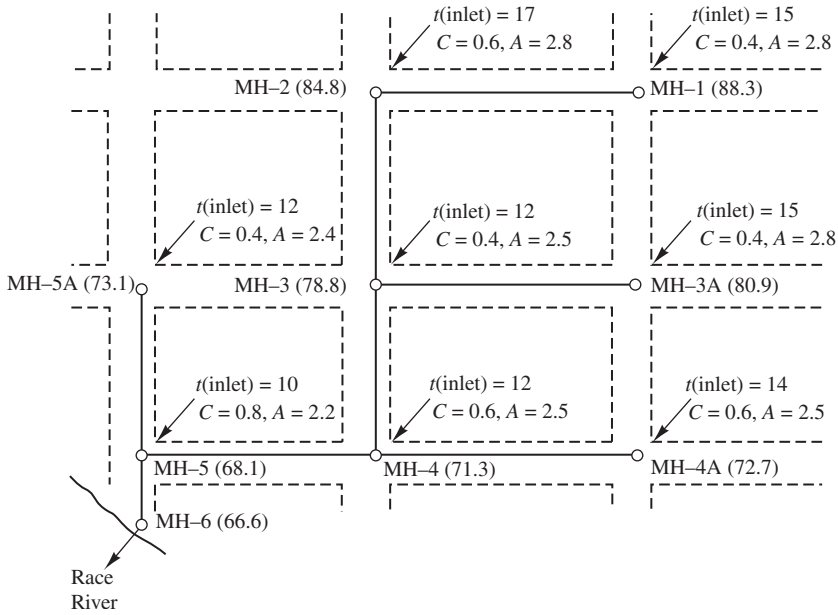


Figure 11.30 Stormwater pipe design example problem

Solution

The following table facilitates the design. The process begins at the highest manhole (which collects flow from two inlets) and proceeds to the outlet point. Each column represents the computations for one pipe. Proceed from one column to the next until each pipe is designed.

Computation Table: Stormwater Pipe Design

Stormwater Pipe	1-2	2-3	3A-3	3-4
Length (ft)	350	300	350	250
Inlet time (T_i) (min)	15	17	15	12
Time of concentration (T_c) (min)	15	17	15	17.5
Runoff coefficient (C)	0.4	0.5	0.4	0.45
R/F intensity (I) (in./hr)	4.3	4.1	4.3	4.0
Drainage area (A) (acres)	2.8	5.6	2.8	10.9
Peak discharge (Q_p) (cfs)	4.8	11.5	4.8	19.6
Slope (ft/ft)	0.01	0.02	0.006	0.03
Required pipe diameter (D_r) (in.)	13.4	16.3	14.8	18.5
Design pipe diameter (D) (in.)	15	18	15	24
Full pipe area (A_f) (ft ²)	1.23	1.77	1.23	3.14
Full pipe velocity (V_f) (ft/sec)	5.28	8.43	4.09	12.5
Full pipe flow (Q_f) (cfs)	6.48	14.9	5.02	39.3
Q_p/Q_f (or Q/Q_f)	0.74	0.77	0.96	0.50
y/D	0.63	0.65	0.78	0.50
V/V_f	1.11	1.13	1.17	1.02
Flow depth (y) (in.)	9.45	11.7	11.7	12.0
Pipe velocity (V) (ft/sec)	5.86	9.52	4.78	12.8
Pipe flow time (min)	1.0	0.5	1.2	0.3

Description of Parameters

- Stormwater pipe: Pipes are designated by manhole numbers.
- Length: Pipes lengths are obtained from appropriate maps.

Inlet time:	Inlet time of concentration includes sheet flow and gutter flow times.
Time of concentration:	The time of concentration is the longest flow time to the entrance of the current pipe (design point) through any flow path. If there are no upstream pipes, then the T_c is the inlet time. Otherwise, compare the local inlet time to all other T_c s to upstream manholes plus the pipe flow time from that manhole to the design point. The largest time governs. For pipe 2–3, the inlet time of 17 min exceeds the T_c of pipe 1–2 plus its travel time ($15 + 1 = 16$ min).
Runoff coefficient:	The area-weighted runoff coefficient is calculated for the entire upstream drainage area. For pipe 2–3: $C = [(2.8)(0.4) + (2.8)(0.6)]/5.6 = 0.5$.
Rainfall intensity:	The rainfall intensity is found from the IDF curve (Figure 11.26).
Drainage area:	The total drainage area contributing flow to the pipe is determined.
Peak discharge:	The rational peak discharge is computed using Equation 11.19: $Q_p = C I A$.
Slope:	The pipe slope is found by dividing the pipe length by the surface elevation difference at the pipe ends (manholes). The pipe will be buried deep enough to meet minimum cover requirements. If a full flow velocity of 2 ft/sec (or more stringent local criteria) is not met, then a greater slope will be required.
Required pipe diameter	Use Equation 11.22 to determine D_r and convert to inches.
Design pipe diameter:	Take the required pipe diameter D_r and use the next larger commercially available pipe size. (Minimum size = 15 in., 21 in. not available.)
Full pipe area:	Solve Equation 11.23 using the design pipe diameter, D (in ft).
Full velocity:	Solve Equation 11.25 $R_f = D/4$ (Equation 11.24 with D in ft).
Full pipe flow:	Solve the continuity equation: $Q = A_f V_f$.
Q_p/Q_f (or Q/Q_f):	Obtain the ratio of rational peak (design) flow divided by the full pipe flow.
y/D :	Using the ratio of Q/Q_f , obtain the ratio of depth of flow to pipe diameter from Figure 11.29.
V/V_f :	Using the y/D ratio and Figure 11.29, obtain the ratio of the actual (partially full) pipe flow velocity to the full pipe flow velocity.
Flow depth:	Using y/D and the design pipe diameter, determine the actual flow depth (y).
Pipe velocity:	Using V/V_f and the full pipe velocity, determine the actual pipe velocity (V).
Pipe flow time:	The pipe flow time is found by dividing the pipe length by the pipe velocity and converting the answer to minutes.

PROBLEMS

(SECTION 11.1)

11.1.1. Components of the hydrologic cycle may be classified as follows:

- (a) water-holding elements,
- (b) liquid transport phases. and
- (c) vapor transport phases.

Using Figure 11.1, apply one of these three descriptors to each component of the hydrologic cycle as depicted. Can you think of any other components of the hydrologic cycle that are not shown? Which of the three descriptors applies to them?

- 11.1.2. As water moves through the hydrologic cycle, water quality changes are common because of natural phenomena or anthropogenic pollution. Using Figure 11.1, describe how water-quality changes occur during each phase of the hydrologic cycle. For example, when lake water is evaporated, trace elements and salts are left behind, producing a water quality change.
- 11.1.3. Delineate a watershed using a topographic map and design point provided by your course instructor. Alternatively, obtain a sample topographic map from a USGS (or alternative) Web site, designate an arbitrary design point along a stream, and delineate the contributing watershed.
- 11.1.4. An in-ground swimming pool may have a leak. The 30-ft by 10-ft by 5-ft (depth) pool is filled on June 1. On June 13, a hose, flowing at a rate of 10 gallons per minute, is used to add water to the pool. The hose is turned off after 1 hr. Four inches of rain fell during the month. Evaporation from a nearby lake was 8 in. during June. It is estimated that 25 percent more will evaporate from the pool than the lake. On July 1, the pool is 5 inches below the top. Does the pool have a leak? If so, what is the rate of leakage in gallons per day? Are you confident enough in your assessment to testify as an expert witness in court?
- 11.1.5. A water-supply reservoir has been lined with clay to limit leakage (infiltration) through the bottom. An assessment of the effectiveness of the liner is required to fulfill the terms of the construction contract. The following data have been collected for a test week:

Parameter	Day						
	1	2	3	4	5	6	7
Stream inflow (m ³ /sec)	0.0	0.2	0.4	0.5	0.5	0.2	0.1
Stream outflow (m ³ /sec)	0.0	0.1	0.3	0.2	0.1	0.1	0.0
City use (m ³ /sec)	0.3	0.2	0.3	0.2	0.3	0.3	0.3
Precipitation (cm)	0.0	1.5	7.0	2.5	0.0	0.0	0.0

The surface area of the lake is 40 hectares. The reservoir surface elevation fell 15.5 cm during the week. If the evaporation for the week was estimated to be 3 cm, what was the amount of leakage in cubic meters for the week?

- 11.1.6. A rainfall event occurs over a 150-square-mile watershed. The following incremental depths of precipitation, interception, and infiltration are estimated during a 30-min storm in 5-min increments. Determine the total volume of rainfall (acre-feet) and the total volume of runoff (acre-feet) that will drain into a reservoir at the outlet of the watershed.

Parameter	Time Increment					
	1	2	3	4	5	6
Precipitation (in.)	0.05	0.10	0.45	0.30	0.15	0.05
Interception (in.)	0.05	0.03	0.02	0.00	0.00	0.00
Infiltration (in.)	0.00	0.07	0.23	0.20	0.10	0.05

- 11.1.7. An annual water budget for a critical water-supply basin is needed. During the year, the following data were collected for the 6,200-km² watershed: precipitation = 740 mm, evaporation and transpiration = 350 mm, average annual streamflow leaving the basin (outflow) = 75.5 m³/sec, groundwater outflow = 0.200 km³, and groundwater inflow (infiltration) = 0.560 km³. Determine the change of storage in the watershed (surface reservoirs) and the change in storage in the drainage basin (surface reservoirs and groundwater together) in km³. (*Hint: Sketch the control volumes; a different one is needed for each solution.*)

(SECTION 11.2)

- 11.2.1. What are the mechanisms by which air masses laden with water vapor are lifted, resulting in cooling and precipitation? What are the resulting categories (types) of precipitation?
- 11.2.2. Referring to Figure 11.4, answer the following question:
 - (a) Why does Virginia receive more rain than Kansas?
 - (b) Why does Maine receive less rain than Virginia?
 - (c) Why does northern California receive so much rain?
 - (d) Why does Nevada receive so little rain?
 - (e) Why does western North Carolina receive so much rain?
 - (f) Why do the Rocky Mountain states receive so little rain?
- 11.2.3. A large drainage basin can be divided into four sub-basins. The areas of the sub-basins are 52 km², 77 km², 35 km², and 68 km². The average annual precipitation in each is 124 mm, 114 mm, 126 mm, and 99 mm, respectively. Determine the average annual precipitation (in cm) for the whole drainage area.
- 11.2.4. A Kansas farmer wants to determine the average rainfall on his sunflower crop. The sunflower field takes up an entire section (a square of 1 mile per side, or 640 acres). The farmer has rain gauges located at the four corners and in the center. The rainfall depths are NE = 3.2 in., NW = 3.6 in., SE = 3.8 in., SW = 4.0, and center = 3.8 in. Determine the average depth of rainfall using the Thiessen polygon method. (*Note:* Plot on grid paper and count squares for the areas or use a planimeter.)
- 11.2.5. A Kansas farmer wants to determine the average rainfall on his sunflower crop. The sunflower field takes up an entire section (a square of 1 mile per side, or 640 acres). The farmer has rain gauges located at the four corners and in the center. The rainfall depths are NE = 3.2 in., NW = 3.6 in., SE = 3.8 in., SW = 4.0, and center = 3.8 in. Determine the average depth of rainfall using the isohyetal method. (*Note:* Use an isohyet interval of 0.2 in. Plot on grid paper and count squares for the areas or use a planimeter.)
- 11.2.6. Rainfall depths (in centimeters) on a commercial farm during a July storm are depicted at various gauge locations in Figure P11.2.6. (Note that the exact location of each gauge is signified by a dot adjacent to the rainfall depth.) Determine the average depth of rainfall on the field using the Thiessen polygon method. Estimate areas by counting squares; each of the 225 squares represents 100 hectares.

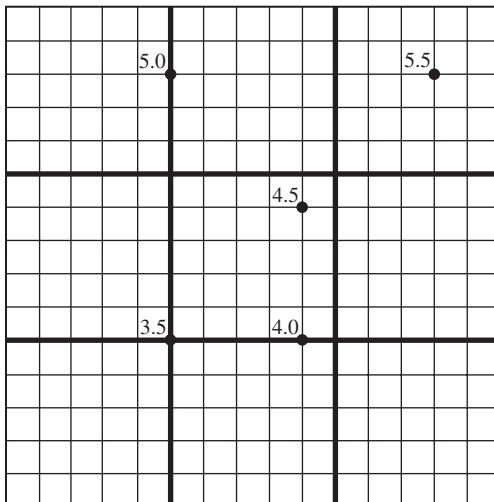


Figure P11.2.6

- 11.2.7. Rainfall depths (in cm) on a commercial farm during a July storm are depicted at various gauge locations in Figure P11.2.6. (*Note:* The exact location of each gauge is signified by a dot adjacent to the rainfall depth.) Determine the average depth of rainfall on the field using the isohyetal method. Estimate areas by counting squares; each of the 225 squares represents 100 hectares.
- 11.2.8. Rainfall depths (in cm) on a commercial farm during a July storm are depicted at various gauge locations in Figure P11.2.6. (*Note:* The exact location of each gauge is signified by a dot adjacent to the rainfall depth.) Determine the average depth of rainfall on the field using the inverse-distance weighting method. (*Note:* Each of the 225 squares represents 100 hectares, so each square is 1,000 m by 1,000 m in size).
- 11.2.9. Precipitation station X was inoperative for one month. A storm occurred during this time. The precipitation depths recorded during the storm at three surrounding stations (A, B, and C) were 6.02 in., 6.73 in., and 5.51 in., respectively. The normal annual precipitation amounts at stations X, A, B, and C, are 53.9 in., 61.3 in., 72.0 in., and 53.9 in., respectively. Discuss two different methods to estimate the precipitation depth at station X for the missing storm. Which method do you have more confidence in and why?

(SECTION 11.3)

- 11.3.1. Determine a 10-yr, 24-hr storm hyetograph for Virginia Beach, Virginia, if the review agency mandates the use of the Type III distribution instead of the Type II as was used in Example 11.3. Compare the peak intensity and time of this storm versus the peak intensity and time for the Type II storm.
- 11.3.2. Determine a 10-yr, 24-hr storm hyetograph for Miami, Florida. Plot a bar graph of the storm (time versus rainfall intensity).
- 11.3.3. The 10-yr, 24-hr storm hyetograph for Virginia Beach, Virginia, was determined in Example 11.3. From this data, estimate the 10-yr, 6-hr storm hyetograph at the same location.
- 11.3.4. The following incremental depths of rainfall (inches) are recorded during a 45-minute storm. Determine the total depth of rainfall for the 150-square-mile watershed, the maximum intensity (in./hr), and the total rainfall (acre-feet).

Storm Time (min)	0 to 6	6 to 18	18 to 21	21 to 30	30 to 36	36 to 45
Rainfall Depth (in.)	0.06	0.28	0.18	0.50	0.30	0.18

(SECTION 11.4)

- 11.4.1. The U.S. Geological Survey maintains a network of stream flow gauges on most of the major rivers and creeks around the country. Many of these gauges are located at bridge crossings. What are the advantages of locating a gauge at a bridge crossing? Are there any disadvantages?
- 11.4.2. The USGS publishes stream flow information by state and gauge every year in its annual water data reports. Search the service’s Internet site (www.usgs.gov) to find information at a particular gauge. What information is provided on individual gauge stations? How frequently are stream flows reported? What other information can be found in the report?
- 11.4.3. The following point velocities (m/sec) and depths (m) were collected in the river cross section shown in Figure 11.12 where the width of each vertical section is 1.0 m. Determine the discharge (m³/sec) of the river.

	Section Number (Left to Right)										
Velocity	1	2	3	4	5	6	7	8	9	10	11
@ 0.2 y	0.2 ^a	2.3	3.3	4.3	4.5	4.7	4.8	4.4	4.2	3.8	3.0
@ 0.8 y		1.4	2.3	3.3	3.7	3.9	3.8	3.6	3.4	2.0	1.2
Depth ^b	1.0	1.6	1.8	2.0	2.0	2.0	2.0	2.0	2.0	1.6	0.6

^aOne point measurement representing the average velocity.

^bThe depths (in meters) given are on the right side of the vertical sections.

11.4.4. The following point velocities (ft/sec) and depths (ft) were collected in the river cross section shown in Figure 11.12, where the width of each vertical section is 2.0 ft. Determine the discharge (ft³/sec) of the river.

	Section Number (Left to Right)										
Velocity	1	2	3	4	5	6	7	8	9	10	11
@ 0.2 y	0.1 ^a	0.3	0.4	0.5	0.6	0.7	0.7	0.6	0.5	0.4	0.3
@ 0.8 y		0.1	0.2	0.3	0.4	0.5	0.5	0.4	0.3	0.2	0.1
Depth ^b	1.8	3.6	4.2	4.8	4.8	4.8	4.8	4.8	4.8	3.6	1.0

^aOne point measurement representing the average velocity.

^bThe depths (in feet) given are on the right side of the vertical sections.

11.4.5. The following stream flow versus stage information was collected at a stream gauging station during a storm event.

Stage (ft)	1.2	3.6	6.8	10.4	12.1	14.5	18.0
Q (cfs)	8	28	64	98	161	254	356

- (a) Plot a rating curve with the discharge plotted on the abscissa.
- (b) During a storm event some time later, the following stages were recorded. Plot a hydrograph (discharge vs. time on the abscissa) of the storm event using the rating curve. What was the peak flow rate during the storm?

Hour	0500	0600	0700	0800	0900	1000	1100	1200	1300
Stage (ft)	2.2	6.6	12.8	16.4	14.1	11.5	9.2	6.8	4.2

(SECTION 11.5)

11.5.1. Refer to Example 11.4 to answer the following questions qualitatively.

- (a) Would you expect the stream flows measured in the watershed to change if the 12.3 cm of precipitation fell in a 6-hr instead of 24-hr period? If so, how?
- (b) Why might groundwater contribution to stream flow increase after a storm event?
- (c) Why is the sum of the flows in the direct runoff column multiplied by 6 hours when obtaining the runoff volume?
- (d) Would the unit hydrograph change if the stream flows were measured in 3-hr instead of 6-hr increments but the 6-hr values did not change?

11.5.2. The effective part of a 2.5 in. rain (total depth) lasts 4 hours on a 5-sq-mi watershed, producing the stream flows shown in the table below. Assuming a base flow that varies linearly from 20 cfs (at time 0) to 40 cfs (at time 20), determine the 4-hr unit hydrograph for the watershed.

Time (hrs)	0	2	4	6	8	10	12	14	16	18	20
Q (cfs)	20	90	370	760	610	380	200	130	90	60	40

11.5.3. The stream flow given in the following table was produced by a 1-cm rainfall event (total depth). The effective portion of the rainfall lasted 1 hr (from time 9 to time 10). If the watershed is 20 square kilometers, derive a 1-hr unit hydrograph for the watershed. Assume the groundwater contribution is linear from 1.4 m³/sec (at time 9) to 3.0 m³/sec (at time 17).

Time (hrs)	8	9	10	11	12	13	14	15	16	17	18	19
Q (m ³ /sec)	1.6	1.4	4.6	9.7	13.0	10.5	7.8	5.8	4.5	3.0	2.9	2.8

11.5.4. A storm occurs over a 27-square-mile watershed. The measured rainfall and stream flow are shown in the following tables. Derive the unit hydrograph from the data. Is the result a 1-hr, 2-hr, or 3-hr unit hydrograph? What fraction of the precipitation appears as runoff? Assume the groundwater contribution is linear from 100 cfs (at $t = 0600$) to 160 cfs (at $t = 0200$).

Time	5 to 6 A.M.	6 to 7 A.M.	7 to 8 A.M.	8 to 9 A.M.	9 to 10 A.M.
Rainfall (in.)	0.2	0.8	0.6	0.6	0.1

Time (hrs)	0400	0600	0800	1000	1200	1400	1600	1800	2000	2200	2400	0200
Q (cfs)	110	100	1,200	2,000	1,600	1,270	1,000	700	500	300	180	160

11.5.5. Measured stream flow data during a storm event on Judith Creek produces a roughly triangular hydrograph shape. The peak occurs 3 hours into the storm and measures 504 cfs. There is no base flow before the event, so the discharge starts at time zero, climbs to the peak in 3 hours, and recedes linearly back to zero in another 5 hours. The drainage area is 1,000 acres, and most of the runoff from the 2.5 inches of rainfall occurs during a 2-hour period. Sketch the storm hydrograph and determine the unit hydrograph. Is the unit hydrograph you developed a 1-hr, 2-hr, or 3-hr unit hydrograph?

11.5.6. Lost Creek’s 1-hr unit hydrograph at the Chamberlain Avenue crossing is shown in the following table. Determine the expected flow rate in the creek at that location if a 2-hr storm produces 2.0 cm of rain in the first hour and 2.5 cm of rain in the second hour. Assume losses to rainfall are constant at 0.5 cm/hr and base flow is constant at 5 m³/sec.

Time (hrs)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Q (m ³ /sec)	0	10	30	70	120	100	70	40	20	0

11.5.7. Referring to Example 11.5, determine the stream flow expected from a new storm that produces 6 cm of runoff in the first 24 hours of the storm and 4 cm of runoff in the next 12 hours. Assume the base flow is the same as in Example 11.4.

11.5.8. A 1-hr unit hydrograph for the No Name Creek watershed (307 acres) is given in the following table. Determine the stream flow that would result from a rainfall event on the same watershed if the excess rainfall (runoff) in a 2-hr storm in 30-min increments is 0.5 in., 0.5 in., 0.25 in., and 0.25 in. Assume a base flow of 10 cfs. Also determine the runoff volume in acre-feet.

Time (hrs)	0	1	2	3	4	5	6
Q (cfs)	0	60	100	80	50	20	0

11.5.9 In Example 11.4, a 12-hr unit hydrograph (UH_{12}) is computed. Using the UH_{12} , compute a 24-hr unit hydrograph (UH_{24}). (*Hint:* A 24-hr unit hydrograph would still produce 1 in. of runoff, half coming in the first 12 hrs and half coming in the second 12 hrs. Can the 6-hr unit hydrograph be computed in a similar way?)

11.5.10 A 2-hr storm (2 cm of runoff) of uniform intensity produces the flows on Wolf Creek shown in the following table. Determine the peak flow and time to peak of a 4-hr design storm that generates 1.5 cm of runoff in the first 2 hours and 3 cm of runoff in the subsequent 2 hrs. Assume a negligible base flow.

Time	0400	0500	0600	0700	0800	0900	1000	1100	1200	1300	1400	1500
Q (m^3/sec)	0	160	440	920	860	720	580	460	320	180	60	0

(SECTION 11.6)

11.6.1. A 200-acre watershed near Chicago, Illinois, includes 169 acres of open space with 90 percent grass cover and a 31-acre industrial development park. The existing soil has coarse to fine texture based on sieve tests. Determine the runoff depth (in.) and volume (acre-feet) for the 10-yr, 24-hr rainfall event.

11.6.2. A 100-hectare watershed is composed of three different land uses: 20 hectares of golf course (40% in Drexel soils and the rest in Bremer soils), 30 hectares of commercial area (Bremer soils), and 50 hectares of residential area (half-acre lots and Donica soils). Determine the runoff volume (cubic meters) from a 15-cm storm. (*Note:* Drexel soils are coarse to fine-textured loams, Bremers are fine textured, and Donicas are sandy soils.)

11.6.3. Determine the rainfall excess (i.e., runoff) resulting from the 10-yr, 24-hr rainfall for the watershed in Example 11.8. This amounts to completing steps 1, 2, and 3 of the design process described in Section 11.6.4 and elaborated in Example 11.6. Albemarle County, in western Virginia, receives 6.0 in. of rain in the 10-yr, 24-hr storm. Use a 2-hr time step. (*Hint:* Replace the i and ΔP columns in Table 11.5 with P_1/P_T and P_2/P_T for the type II storm in Table 11.1.)

11.6.4. The hydraulic length (longest flow path) for a watershed is 2,800 ft. Along this path, runoff initially travels over the land surface (dropping 4 ft in elevation over 200 ft in length through short grass) and then moves into shallow concentrated flow for a distance of 600 feet with an average velocity of 2 ft/sec. The remainder of the travel path is through a 2-ft diameter (concrete) pipe flowing half-full on a 1-percent slope. Determine the time of concentration in minutes. The 2-yr, 24-hr rainfall depth is 3.4 in.

11.6.5. A 100-acre watershed with type B soils has been commercially developed. The length of the longest flow path is 3,000 feet, and the average watershed slope is 2.5 percent. Determine the peak discharge and time to peak for an SCS dimensionless unit hydrograph.

11.6.6. Referring to Example 11.8, plot the unit hydrograph using a spreadsheet program. Determine the volume of runoff in acre-feet and the depth of runoff in inches.

- 11.6.7. Referring to Example 11.8, determine the SCS unit hydrograph after development if half the watershed is developed commercially and the rest with townhouses.
- 11.6.8. A 400-acre watershed just south of Chicago is currently farmed with small grain crops. The soils are primarily clay, the hydraulic length of the watershed is about 1 mile, and the average land slope is 2 percent. Determine the SCS synthetic unit hydrograph before development.
- 11.6.9. Determine the change in SCS synthetic unit hydrograph of Problem 11.6.8 if the farmland is developed into an industrial park.

(SECTION 11.7)

- 11.7.1. Build a $2S/\Delta t + O$ versus O relationship in 0.2-m increments for a pond that is expected to reach a depth of 1.0 m over the top of the service spillway (normal pond level). The elevation-storage relationship for the pond is described by $S = 600 h^{1.2}$, where S = storage in the pond in m^3 and h = water level in meters above the service spillway. The discharge over the spillway is described by $O = k_w L(2g)^{0.5} h^{1.5}$, where O = discharge in m^3/sec , $k_w = 0.45$ (discharge coefficient), $L = 0.3$ m (spillway crest length), and g = gravitational acceleration. Initially, $h = 0$ and $S = 0$, and the routing interval (Δt) is 8 minutes.
- 11.7.2. Build a $2S/\Delta t + O$ versus O relationship in half-foot increments for an underground cistern that is 2 ft deep, 4 ft long, and 3 ft wide. A 2-in.-diameter bottom drain acts as an orifice with a discharge coefficient of 0.6. Assume a routing interval of 10 seconds.
- 11.7.3. Referring to Example 11.9, continue computations in the routing table through time 13:45. Additional inflows are given in the following table. Graph the inflow and outflow hydrograph on the same set of axes. Determine the maximum elevation and storage attained in the pond during the design storm.

Time	13:30	13:35	13:40	13:45	13:50
Inflow (I_j), cfs	67	59	52	46	40

- 11.7.4. A flood occurs before the storage pool of a reservoir is empty. Given the information in the following tables, determine the peak outflow during the flood.

Stage (m, MSL)	Outflow (O) (m^3/sec)	Storage (S) ($\times 10^3 m^3$)	$(2S/\Delta t) + O(m^3/\text{sec})$
0.0	0.0	0	0
0.5	22.6	408	136
1.0	64.1	833	295
1.5	118.0	1,270	471
2.0	118.0	1,730	662

Time (hr)	Inflow (I_j) (m^3/sec)	Inflow (I_j) (m^3/sec)	$(2S/\Delta t) - O$ (m^3/sec)	$(2S/\Delta t) + O$ (m^3/sec)	Outflow, $O(m^3/\text{sec})$
0:00	5		40	50	5
2:00	8				
4:00	15				
6:00	30				
8:00	85				
10:00	160				
12:00	140				
14:00	95				
16:00	45				
18:00	15				

11.7.5. A portion of a reservoir routing table immediately follows (90 min into the flood). Fill in the blanks (?) in the table. Also determine the peak reservoir outflow, stage (H), and volume in the reservoir at 120 minutes. (*Note:* 1 acre = 43,560 ft².)

Stage (H) (ft)	Outflow (O) (cfs)	Storage (S) (acre-ft)	$(2S/\Delta t) + O$ (cfs)
0.0	0	0.0	?
0.5	3	1.0	?
1.0	8	?	298
1.5	17	?	453
2.0	30	?	611

Time (min)	Inflow (I_1) (cfs)	Inflow (I_2) (cfs)	$(2S/\Delta t) - O$ (cfs)	$(2S/\Delta t) + O$ (cfs)	Outflow, O (cfs)
90	50	45	?	514	23
100	45	?	?	563	?
110	?	30	?	599	?
120	30	26	554	614	30

11.7.6. Referring to Example 11.9, perform the storage routing computations again using a 10-min routing increment starting with the flow at time 11:30 and progressing to 11:40, then 11:50, and so on and dropping the flows in between. Will this require a new $(2S/\Delta t + O)$ versus O relationship? If so, revise this before performing the storage routing computations. Compare the new peak outflow with the peak outflow from Example 11.9 (149 cfs).

11.7.7. The following table shows the elevation–outflow–storage data for a flood-control reservoir. Determine the maximum pool elevation and peak outflow rate during a 6-day storm that produces the displayed inflow hydrograph values.

Elevation (ft, MSL)	Outflow (O) (cfs)	Storage (S) (acre-ft)	Day/Time	Inflow (cfs)	Day/Time	Inflow (cfs)
865	0	0	1/noon	2	4/noon	366
870	20	140	midnight	58	midnight	302
875	50	280	2/noon	118	5/noon	248
880	130	660	midnight	212	midnight	202
885	320	1,220	3/noon	312	6/noon	122
			midnight	466	midnight	68

11.7.8. The stage storage relationship at a pond site is described by $S = 600 h^{1.2}$, where S = storage in the pond in m³ and h = water level in meters above the spillway crest. The discharge over the spillway is described by $O = k_w L (2g)^{0.5} h^{1.5}$, where the discharge (O) is in m³/sec, $k_w = 0.45$ (discharge coefficient), $L = 0.3$ m (spillway crest length), and $g = 9.81$ m/sec² (gravitational acceleration). Initially, $h = 0$ and $S = 0$. Build a $(2S/\Delta t + O)$ versus O relationship for pond stages in 0.2-m increments up to 1.0 m over the spillway crest. Perform storage routing computations for the inflow hydrograph information in the following table to determine the peak outflow and stage.

Time (min)	Inflow (I_i) (m^3/sec)	Time (min)	Inflow (I_i) (m^3/sec)
0	0	40	0.58
8	0.15	48	0.46
16	0.38	56	0.35
24	0.79	64	0.21
32	0.71	72	0.1

11.7.9. An empty water storage tank (cylindrical shape) with a diameter of 20 m and a height of 5 m is being filled at a rate of $2.5 \text{ m}^3/\text{sec}$. However, it is also losing water (because of a hole in the bottom of the tank) at the rate of $O = 0.7(h)^{0.6}$, where h is the depth of water in meters and O is the outflow rate in m^3/sec . Determine the depth of water in the tank after 4 min. Use a depth interval of 1 m for the $(2S/\Delta t + O)$ versus O relationship and a routing interval of 30 sec. What would the depth of water be in the tank if there was no leak?

(SECTION 11.8)

- 11.8.1.** The peak discharge from a 20-acre watershed is required for a new culvert installation. The hilly watershed (average slope 7%) is composed of pastureland with dense grass and clay soils. The hydraulic length of the watershed is 1,200 feet. The watershed is slated for commercial development in the next two years. Using the IDF curve in Figure 11.26, determine the 10-yr peak discharge before and after development.
- 11.8.2.** A large circular parking lot with a diameter of 400 feet is being constructed as overflow parking for the state fairgrounds. All stormwater will drain as sheet flow to the center, flow into a drop inlet, and travel through a pipe system to the nearest stream. The parking lot will be mowed grass (clay soil) with a slope of 2 percent toward the center. The 2-yr, 24-hr rainfall is 2.4 inches. Using SCS procedures to determine the time of concentration and the IDF curve in Figure 11.26, determine the 10-yr peak discharge that will be required to design the pipe system.
- 11.8.3.** Determine the 10-year peak discharge using the rational equation for a concrete parking lot that is 300 ft wide by 600 ft long. The longest flow path is 300 ft of sheet flow across the lot on a slope of 0.5 percent and 600 ft of shallow concentrated flow along the parking lot's length in a paved channel on a slope of 1.5 percent. Use SCS procedures to determine the time of concentration using a 2-yr, 24-hr rainfall of 2.4 in. Use Figure 11.26 for the rainfall intensity in the rational equation.
- 11.8.4.** The paved parking lot depicted in Figure P11.8.4 drains as sheet flow into a concrete drainage channel. The rectangular channel discharges into a stormwater-management pond. Determine the 5-yr peak discharge. Assume the 2-yr, 24-hr rainfall is 2.8 in. Use Figure 11.26 for the rainfall intensity in the rational equation. (*Hint:* Assume the channel flows at a depth of 1 ft to determine the channel flow time.)

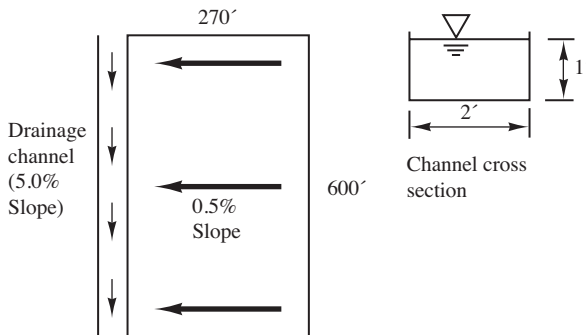


Figure P11.8.4

- 11.8.5.** A 150-acre forested watershed with a time of concentration of 90 min will soon be developed with the following land uses: 60 acres of single-family homes, 40 acres of apartments, and the rest in native forest. After development, the longest flow path is 5,600 feet. A 100-foot sheet flow segment passes through woods with light underbrush on a 2-percent slope. A 1,200-ft segment of shallow concentrated flow occurs next over an unpaved area on a 1-percent slope. Channel flow occurs over the remaining distance on a slope of 0.5 percent in a stony natural stream. A typical cross section of the stream has 18 ft² of flow area and 24 ft of wetted perimeter. The 2-yr, 24-hr precipitation is 3.8 in. Using the IDF curve in Figure 11.26, determine the 25-yr peak discharge before and after development using the rational equation.
- 11.8.6.** The paved parking lot depicted in Figure P11.8.4 drains as sheet flow into a concrete side channel that discharges into a stormwater-management pond. The travel time across the parking lot as sheet flow is 5 min, and the travel time through the entire drainage channel is 2 min. Thus, the time of concentration for the parking lot is 7 min. Determine the peak discharge resulting from a 5-yr, 5-min storm based on the rational equation using Figure 11.26 for the rainfall intensity.
- 11.8.7.** Referring to Example 11.11, describe subjectively (no analysis required) how the location of the inlet would change (one change at a time, not collectively) if:
- the 10-year design storm is the standard.
 - the pavement-encroachment criterion is 8 feet.
 - the lawns are Bermuda grass.
 - the longitudinal street slope is 3 percent.
 - the street cross slope is 3/8 inch per foot.
- Also, are inlets required for the south side of the street? Why or why not?
- 11.8.8.** Referring to Example 11.11, locate the first inlet if the following changes in the design conditions are taken collectively:
- the 10-year design storm is the standard,
 - the pavement-encroachment criteria is 8 feet,
 - the lawns are Bermuda grass, and
 - the street cross slope is 3/8 inch per foot.
- All other design conditions remain the same.
- 11.8.9.** Referring to Example 11.11, locate the next downstream inlet. Assume the street slope has increased from 2.5 percent to 3 percent and the first inlet intercepts 75 percent of the gutter flow. All other data can be obtained from Example 11.11.
- 11.8.10.** Referring to Example 11.11, determine the pipe size (concrete) that would be required to convey the design flow away from the first inlet. If the minimum pipe size specified by community standards is 12 in., what is the depth of flow in the pipe at peak flow (0.914 cfs)? What is the travel time per 100 feet of pipe?
- 11.8.11.** Referring to Example 11.12, size the pipes up to MH-4 given the following changes in the design conditions (taken collectively, not one at a time):
- the 10-year design storm is the standard,
 - all inlet times are 15 minutes, and
 - the runoff coefficient into MH-3A is 0.5.
- All other design conditions remain the same.
- 11.8.12.** Referring to Example 11.12, finish the problem by sizing the pipes from MH-4 to MH-6. Pipe lengths are as follows: MH4A to 4, 350 ft; MH4 to 5, 320 ft; MH5A to 5, 250 ft; MH-5 to 6, 100 ft.
- 11.8.13.** Design the stormwater-collection pipes (concrete; minimum size of 12 in.) for the housing subdivision in Figure P11.8.13. The data for each drainage area (basin) contributing flow to the manholes is provided in the following table, including the inlet time (T_i) and the runoff coefficient (C). In addition, stormwater-pipe information is given, including the ground elevation at each manhole.

The rainfall intensity-duration relationship for the design storm can be described by $i = 18/(t_d^{0.50})$, where i = intensity in in./hr, and t_d = storm duration = time of concentration in minutes.

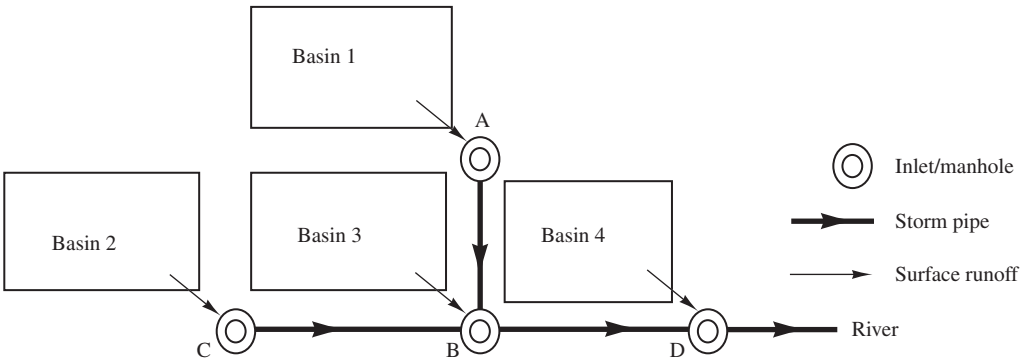


Figure P11.8.13

Basin	Area (acres)	T_i (min.)	C	Stormwater Pipe	Length (ft)	Upstream Elevation (ft)	Downstream Elevation (ft)
1	2.2	12	0.3	AB	200	24.9	22.9
2	1.8	10	0.4	CB	300	24.4	22.9
3	2.2	13	0.3	BD	300	22.9	22.0
4	1.2	10	0.5	DR	200	22.0	21.6

- 11.8.14.** Design the stormwater-collection pipes in Problem 11.8.13 given the following data changes. The upstream elevation for pipe AB is 23.9 ft instead of 24.9 ft, the inlet time for basin 1 is 14 min instead of 12 min, and the inlet time for basin 3 is 10 min instead of 13 min.
- 11.8.15.** Design the stormwater-collection pipes in Problem 11.8.13 given the following data changes. The upstream elevation for pipe CB is 23.5 ft instead of 24.4 ft, the inlet time for basin 2 is 14 min instead of 10 min, and the downstream elevation for pipe DR is 22.0 ft instead of 21.6 ft.



Statistical Methods in Hydrology

Statistical methods are indispensable tools in hydrology. Most hydrologic processes, such as rainfall, are not amenable to purely deterministic analysis because of inherent uncertainties. These uncertainties arise from the randomness of natural processes, lack of data in sufficient quantity and quality, and a lack of understanding of all the causal relationships in complex hydrologic processes. Statistical methods account for these uncertainties, and competent predictions made using these methods are always accompanied by some probability of occurrence or likelihood. In applying statistical methods, we presume that the natural processes are governed by some mathematical rules rather than the physical laws underlying these processes. In allowing this presumption, statistical methods can be used to analyze diverse hydrologic processes.

The coverage of statistical methods herein emphasizes frequency analysis techniques and the concepts used in these techniques. The purpose of a frequency analysis is to extract meaningful information from observed hydrologic data to make decisions concerning future events. As an example, suppose a hydrologic data series contains the instantaneous discharges at a stream section observed during the past 20 years and that a highway bridge is being planned on this stream. The bridge must be designed to pass a design discharge without being flooded. What discharge should be used in the design? What are the chances that the bridge will be flooded during its life span if a particular design discharge is used? A plot of the peak discharges from the past 20 years would look rather erratic, and it would provide no answers to these questions. Only after a frequency analysis of the streamflow data can we answer these types of questions intelligently.

The hydrologic data used in a frequency analysis must represent the situation being studied; that is, the data set must be *homogeneous*. For example, the future surface runoff from a developed, urbanized area cannot be determined using historical runoff data observed under undeveloped conditions. Other changes that affect the data set include relocated gauges, streamflow diversions, and construction of dams and reservoirs during the period the observations were made.

The available hydrologic data may contain more information than needed for frequency analysis. In that event, the data should be reduced into a useful form. For example, suppose we have a daily record of streamflow at a gauge site for the past N years. Such a record would be called a *complete duration series*. For the most part, however, we are interested in extremes, particularly the high streamflows for flood studies. We can form an *annual exceedence series* by considering the highest N values on the record. Alternatively, we can obtain an *annual maximum series* by using the largest N values occurring in each of the N years. Either the annual exceedence or the annual maximum series can be used in a frequency analysis. The results from the two approaches become very similar when extreme events of rare occurrence are investigated. Usually, however, we use the annual maximum series because the values included in this series are more likely to be statistically independent, as assumed in frequency analysis methods.

12.1 Concepts of Probability

Understanding probability concepts requires the definition of some key terms: *random variable*, *sample*, *population*, and *probability distribution*. A *random variable* is a numerical variable that cannot be precisely predicted. In probabilistic methods, we treat all hydrologic variables as random variables. These include, but are not limited to, rates of rainfall, streamflow, evaporation, wind velocity, and reservoir storage. A set of observations of any random variable is called a *sample*. For example, the annual maximum streamflows observed at a specific gauge site during the past N years form a sample. Likewise, the annual maximum streamflows that will occur over some specific period in the future form another sample. We assume that samples are drawn from an infinite hypothetical *population*, which is defined as the complete assemblage of all the values representing the random variable being investigated. In simpler terms, the population of annual maximum streamflows at a specified site would contain the maximum yearly values observed over an infinite number of years. A *probability distribution* is a mathematical expression that describes the probabilistic characteristics of a population. A probability distribution is useful in calculating the chances that a random variable drawn from this population will fall in a specified range of numerical values. For example, the probability distribution of annual maximum streamflows will enable us to estimate the chances that the maximum streamflow will exceed a specified magnitude in any one year in the future.

12.2 Statistical Parameters

Most theoretical probability distributions are expressed in terms of *statistical parameters* that characterize the population, such as the *mean*, *standard deviation*, and *skewness*. We cannot determine these parameters precisely because we do not know all the values included in the entire population. However, we can estimate these statistical parameters from a sample.

Let a sample contain N observed values of a random variable, x_i , with $i = 1, 2, \dots, N$. For an annual maximum streamflow series, x_i would denote the maximum streamflow observed during the i th year. The sample estimate of the mean (m) is

$$m = \frac{1}{N} \sum_{i=1}^N x_i \quad (12.1)$$

Simply stated, m is the average of all the observed values included in the sample.

The *variance* is a measure of the variability of the data. The square root of the variance is called the *standard deviation*. A sample estimate of the standard deviation (s) is

$$s = \left[\frac{1}{N-1} \sum_{i=1}^N (x_i - m)^2 \right]^{1/2} \quad (12.2)$$

Skewness, or *skew*, is a measure of the symmetry of a probability distribution about the mean. We can estimate the *skew coefficient* from the data as

$$G = \frac{N \sum_{i=1}^N (x_i - m)^3}{(N-1)(N-2)s^3} \quad (12.3)$$

When performing frequency analyses on data measured at a gauging station, G is frequently referred to as the *station skew* of the sample.

Experience indicates that logarithms of the observed values for many hydrologic variables are more apt to follow certain probability distributions. Hence, the aforementioned statistical parameters are calculated as

$$m_l = \frac{1}{N} \sum_{i=1}^N \log x_i \quad (12.4)$$

$$s_l = \left[\frac{1}{N-1} \sum_{i=1}^N (\log x_i - m_l)^2 \right]^{1/2} \quad (12.5)$$

$$G_l = \frac{N \sum_{i=1}^N (\log x_i - m_l)^3}{(N-1)(N-2)s_l^3} \quad (12.6)$$

where m_l , s_l , and G_l are the mean, standard deviation, and skew coefficient of the logarithms (base 10) of the observed data values. Throughout this text, *log* refers to the common logarithm (base 10) of the operand, whereas *ln* is used to refer to the natural logarithm (base $e = 2.718$).

Example 12.1

Annual peak discharges (Q_i) of the Meherrin River at Emporia, Virginia, are tabulated in column 2 of Table 12.1 for the years 1952 to 1990. Determine the mean, the standard deviation, and the skew coefficient of these data.

Solution

Equations 12.1, 12.2, and 12.3 will be used, substituting Q_i for x_i . The calculations are best performed in tabular (or spreadsheet) form as shown in Table 12.1. Column sums needed for the computations are given

TABLE 12.1 Mean, Standard Deviation, and Skew Calculations (Meherrin River Peak Flows)

Year	Q_i	$Q_i - m$	$(Q_i - m)^2$	$(Q_i - m)^3$	$\log Q_i$	$(\log Q_i - m_l)$	$(\log Q_i - m_l)^2$	$(\log Q_i - m_l)^3$
1952	9,410	-4.08E+02	1.66E+05	-6.78E+07	3.97E+00	2.59E-02	6.70E-04	1.73E-05
1953	11,200	1.38E+03	1.91E+06	2.64E+09	4.05E+00	1.02E-01	1.03E-02	1.05E-03
1954	5,860	-3.96E+03	1.57E+07	-6.20E+10	3.77E+00	-1.80E-01	3.23E-02	-5.81E-03
1955	12,600	2.78E+03	7.74E+06	2.15E+10	4.10E+00	1.53E-01	2.33E-02	3.56E-03
1956	7,520	-2.30E+03	5.28E+06	-1.21E+10	3.88E+00	-7.15E-02	5.11E-03	-3.65E-04
1957	7,580	-2.24E+03	5.01E+06	-1.12E+10	3.88E+00	-6.80E-02	4.63E-03	-3.15E-04
1958	12,100	2.28E+03	5.21E+06	1.19E+10	4.08E+00	1.35E-01	1.82E-02	2.46E-03
1959	9,400	-4.18E+02	1.74E+05	-7.29E+07	3.97E+00	2.54E-02	6.46E-04	1.64E-05
1960	8,710	-1.11E+03	1.23E+06	-1.36E+09	3.94E+00	-7.69E-03	5.91E-05	-4.54E-07
1961	6,700	-3.12E+03	9.72E+06	-3.03E+10	3.83E+00	-1.22E-01	1.48E-02	-1.80E-03
1962	12,900	3.08E+03	9.50E+06	2.93E+10	4.11E+00	1.63E-01	2.65E-02	4.32E-03
1963	8,450	-1.37E+03	1.87E+06	-2.56E+09	3.93E+00	-2.08E-02	4.35E-04	-9.06E-06
1964	4,210	-5.61E+03	3.14E+07	-1.76E+11	3.62E+00	-3.23E-01	1.05E-01	-3.38E-02
1965	7,030	-2.79E+03	7.77E+06	-2.17E+10	3.85E+00	-1.01E-01	1.02E-02	-1.02E-03
1966	7,470	-2.35E+03	5.51E+06	-1.29E+10	3.87E+00	-7.44E-02	5.53E-03	-4.12E-04
1967	5,200	-4.62E+03	2.13E+07	-9.85E+10	3.72E+00	-2.32E-01	5.37E-02	-1.24E-02
1968	6,200	-3.62E+03	1.31E+07	-4.73E+10	3.79E+00	-1.55E-01	2.41E-02	-3.75E-03
1969	5,800	-4.02E+03	1.61E+07	-6.49E+10	3.76E+00	-1.84E-01	3.40E-02	-6.26E-03
1970	5,400	-4.42E+03	1.95E+07	-8.62E+10	3.73E+00	-2.15E-01	4.64E-02	-9.98E-03
1971	7,800	-2.02E+03	4.07E+06	-8.21E+09	3.89E+00	-5.56E-02	3.09E-03	-1.72E-04
1972	19,400	9.58E+03	9.18E+07	8.80E+11	4.29E+00	3.40E-01	1.16E-01	3.93E-02
1973	21,100	1.13E+04	1.27E+08	1.44E+12	4.32E+00	3.77E-01	1.42E-01	5.34E-02
1974	10,000	1.82E+02	3.32E+04	6.06E+06	4.00E+00	5.23E-02	2.73E-03	1.43E-04
1975	16,200	6.38E+03	4.07E+07	2.60E+11	4.21E+00	2.62E-01	6.85E-02	1.79E-02

(continued)

TABLE 12.1 Mean, Standard Deviation, and Skew Calculations (Meherrin River Peak Flows) (*continued*)

Year	Q_i	$Q_i - m$	$(Q_i - m)^2$	$(Q_i - m)^3$	$\log Q_i$	$(\log Q_i - m_l)$	$(\log Q_i - m_l)^2$	$(\log Q_i - m_l)^3$
1976	8,100	-1.72E+03	2.95E+06	-5.07E+09	3.91E+00	-3.92E-02	1.54E-03	-6.03E-05
1977	5,640	-4.18E+03	1.75E+07	-7.29E+10	3.75E+00	-1.96E-01	3.86E-02	-7.58E-03
1978	19,400	9.58E+03	9.18E+07	8.80E+11	4.29E+00	3.40E-01	1.16E-01	3.93E-02
1979	16,600	6.78E+03	4.60E+07	3.12E+11	4.22E+00	2.72E-01	7.42E-02	2.02E-02
1980	11,100	1.28E+03	1.64E+06	2.11E+09	4.05E+00	9.76E-02	9.53E-03	9.30E-04
1981	4,790	-5.03E+03	2.53E+07	-1.27E+11	3.68E+00	-2.67E-01	7.15E-02	-1.91E-02
1982	4,940	-4.88E+03	2.38E+07	-1.16E+11	3.69E+00	-2.54E-01	6.45E-02	-1.64E-02
1983	9,360	-4.58E+02	2.09E+05	-9.59E+07	3.97E+00	2.36E-02	5.56E-04	1.31E-05
1984	13,800	3.98E+03	1.59E+07	6.32E+10	4.14E+00	1.92E-01	3.69E-02	7.10E-03
1985	8,570	-1.25E+03	1.56E+06	-1.94E+09	3.93E+00	-1.47E-02	2.17E-04	-3.19E-06
1986	17,500	7.68E+03	5.90E+07	4.53E+11	4.24E+00	2.95E-01	8.72E-02	2.58E-02
1987	16,600	6.78E+03	4.60E+07	3.12E+11	4.22E+00	2.72E-01	7.42E-02	2.02E-02
1988	3,800	-6.02E+03	3.62E+07	-2.18E+11	3.58E+00	-3.68E-01	1.35E-01	-4.98E-02
1989	7,390	-2.43E+03	5.89E+06	-1.43E+10	3.87E+00	-7.91E-02	6.25E-03	-4.94E-04
1990	7,060	-2.76E+03	7.60E+06	-2.10E+10	3.85E+00	-9.89E-02	9.78E-03	-9.67E-04
Sum	3.83E+05	-3.27E-11	8.24E+08	3.45E+12	1.54E+02	3.11E-14	1.47E+00	6.53E-02

in the bottom row of the table. All values are given in three significant figures to match the original flows. The mean is calculated from Equation 12.1 with $N = 39$:

$$m = \frac{1}{N} \sum_{i=1}^N Q_i = (3.83 \times 10^5)/39 = 9,820 \text{ cfs}$$

Next, the standard deviation is computed from Equation 12.2:

$$s = \left[\frac{1}{N-1} \sum_{i=1}^N (Q_i - m)^2 \right]^{1/2} = [(8.24 \times 10^8)/38]^{1/2} = 4,660 \text{ cfs}$$

Finally, the skew coefficient is calculated using Equation 12.3:

$$G = \frac{N \sum_{i=1}^N (Q_i - m)^3}{(N-1)(N-2)s^3} = (39)(3.45 \times 10^{12})/[(38)(37)(4,660)^3] = 0.946$$

Example 12.2

Consider the Meherrin River discharges given in Example 12.1. Compute the mean, standard deviation, and skew coefficient of the logarithms of the observed discharges.

Solution

First, the logarithms of the Q 's are obtained and tabulated in column 6 of Table 12.1. Then calculations are performed in tabular form as in Example 12.1. However, in this case, Equations 12.4, 12.5, and 12.6 are used where $\log Q_i$ has been substituted for the $\log x_i$ terms. Thus,

$$m_l = \frac{1}{N} \sum_{i=1}^N \log Q_i = (1.54 \times 10^2)/39 = 3.95$$

$$s_l = \left[\frac{1}{N-1} \sum_{i=1}^N (\log Q_i - m_l)^2 \right]^{1/2} = [(1.47)/38]^{1/2} = 0.197$$

and

$$G_l = \frac{N \sum_{i=1}^N (\log Q_i - m_l)^3}{(N-1)(N-2)s_l^3} = (39)(6.53 \times 10^{-2})/[(38)(37)(0.197)^3] = 0.237$$

Note that the mean flow for the log-transformed data set can be found by taking the antilog (sometimes referred to as the inverse log):

$$Q(\log \text{ mean}) = 10^{3.95} = 8,910 \text{ cfs}$$

which is much lower than the arithmetic mean.

12.3 Probability Distributions

Among the many theoretical probability distributions available, the normal, log-normal, Gumbel, and log-Pearson type III distributions are used most often in hydrology.

12.3.1 Normal Distribution

The *normal distribution*, also known as the *Gaussian distribution*, is probably the most common model of probability. However, it is rarely used in hydrology. The main limitation of the normal

distribution is that it allows random variables to assume values from $-\infty$ to ∞ , while most hydrologic variables, such as stream discharge, are non-negative. In other words, in reality there is no such thing as a “negative” discharge, as may be computed using the normal distribution.

The normal distribution is expressed in terms of a *probability density function*, $f_X(x)$, as

$$f_X(x) = \frac{1}{s \sqrt{2\pi}} \exp \left[-\frac{(x - m)^2}{2s^2} \right] \quad (12.7)$$

where “exp” is the base of Naperian logarithms (i.e., 2.71828 ...) raised to the power of the value in the brackets, and the mean (m) and standard deviation (s) of the sample are used as estimates of the mean and standard deviation of the population.

To explain the meaning of the probability density function, let the random variable X represent the annual maximum discharges of a stream. Suppose the parameters m and s have been obtained by analyzing the annual maximum discharge series of this stream. Further suppose we want to determine the probability that the maximum discharge in any one year in the future will be between two specified numerical values x_1 and x_2 . This probability can be computed as

$$P [x_1 \leq X \leq x_2] = \int_{x_1}^{x_2} f_X(x) dx \quad (12.8)$$

Or, in words, the probability that a particular value X will fall between two values, x_1 and x_2 , can be calculated as the definite integral from x_1 to x_2 of the probability density function.

12.3.2 Log-Normal Distribution

The log transformation of hydrologic random variables is more likely to follow the normal distribution than the original values. In such cases, the random variable is said to be log-normally distributed. The probability density function for *log-normal distribution* is

$$f_X(x) = \frac{1}{(x)s_l \sqrt{2\pi}} \exp \left[\frac{-(\log x - m_l)^2}{2s_l^2} \right] \quad (12.9)$$

where “exp” is the base of Naperian logarithms (i.e., 2.71828 ...) raised to the power of the value in the brackets. Note that the log function in the brackets is the common log (base 10) of x .

12.3.3 Gumbel Distribution

The *Gumbel distribution*, also known as the *extreme value type I* distribution, is commonly used for frequency analysis of floods and maximum rainfall. The probability density function for this distribution is

$$f_X(x) = (y) \{ \exp[-y(x - u) - \exp[-y(x - u)]] \} \quad (12.10)$$

in which y and u are intermediate parameters, defined as

$$y = \frac{\pi}{s\sqrt{6}} \quad (12.11)$$

and

$$u = m - 0.45s \quad (12.12)$$

where m and s represent the mean and standard deviation, respectively, of the sample, as defined previously.

12.3.4 Log-Pearson Type III Distribution

The *log-Pearson type III distribution* was recommended by the U.S. Water Resources Council* to model annual maximum streamflow series. The council’s duties on guidelines for flood-frequency studies were taken over by the Interagency Advisory Committee on Water Data of the U.S. Geological Survey in 1981. The probability density function for the log-Pearson type III distribution is

$$f_X(x) = \frac{\nu^b (\log x - r)^{b-1} \exp[-\nu (\log x - r)]}{x \Gamma(b)} \tag{12.13}$$

where Γ is the gamma function. Values of the gamma function can be found in standard mathematical tables. The parameters b , ν , and r are related to the sample statistical parameters through the expressions

$$b = \frac{4}{G_l^2} \tag{12.14}$$

$$\nu = \frac{s_l}{\sqrt{b}} \tag{12.15}$$

and

$$r = m_l - s_l \sqrt{b} \tag{12.16}$$

The sample statistical parameters m_l , s_l , and G_l are obtained from Equations 12.4, 12.5, and 12.6, respectively.

The skew coefficient used in the log-Pearson type III distribution is sensitive to the sample size. For samples having less than 100 data values, the Water Resources Council recommended using a weighted average of the skewness coefficient obtained from the sample and the generalized map skews given in Figure 12.1. Following the weighting procedure described by the Interagency Advisory Committee on Water Data,†† the *weighted skew coefficient* (for samples having less than 100 data points) is expressed as

$$g = \frac{0.3025 G_l + V_G G_m}{0.3025 + V_G} \tag{12.17}$$

In this expression G_l is the skew coefficient obtained from the sample using Equation 12.6, G_m is the generalized map skew obtained from Figure 12.1 depending on the geographical location, and V_G is the mean square error of the sample skew. Based on the studies of Wallis et al.,† V_G can be approximated as

$$V_G \approx 10^{A-B \log(N/10)} \tag{12.18}$$

* Water Resources Council, “A uniform technique for determining flood flow frequencies.” Water Resources Council Bulletin 15. Washington, DC: U.S. Water Resources Council, 1967.

† Interagency Advisory Committee on Water Data, “Guidelines for determining flood flow frequency.” Bulletin 17B. Reston, VA: U.S. Department of the Interior, U.S. Geological Survey, Office of Water Data Coordination, 1982.

†† Wallis, J. R., Matalas, N. C., and Slack, J. R., “Just a moment!” *Water Resources Research*, 10(2): 211–221, 1974.

Then using Equation 12.18 with $N = 39$,

$$V_G \approx 10^{-0.311 - 0.878 \log(39/10)} = 10^{-0.830} = 0.148$$

and finally, from Equation 12.17, the weighted skew coefficient is

$$g = [0.3025(0.237) + 0.148(0.7)] / (0.3025 + 0.148) = 0.389$$

12.4 Return Period and Hydrologic Risk

The probability density functions for different probability distributions were summarized in Section 12.3. The cumulative density functions are more useful from a practical viewpoint. Given the probability density function $f_X(x)$, the cumulative density function for any distribution can be expressed as

$$F_X(x) = \int_{-\infty}^x f_X(u) du \tag{12.20}$$

where u is a dummy variable of integration. The lower limit of integration should be changed to zero if the distribution allows only positive values.

The numerical value of $F_X(x)$ represents the probability that the random variable being modeled will take a value smaller than x . Suppose we hypothesize that one of the probability distributions discussed in Section 12.3 can be used to describe an annual maximum discharge series of a stream. Using the sample mean, standard deviation, skew coefficient, and the chosen probability density function, we evaluate Equation 12.20 with an upper limit of integration of $x = 3,000$ cfs. Suppose further that the resultant cumulative density, using Equation 12.20, computes to be 0.80. Then we can say that the annual maximum discharge of the stream being studied will be smaller than 3,000 cfs with a probability of 0.80, or 80 percent, in any single year in the future. It should be noted that the numerical value of $F_X(x)$ will always be between zero and unity. Occasionally, $F_X(x)$ is referred to as the *non-exceedence probability*.

Hydrologists who deal with flood studies are usually more interested in the *exceedence probability* (p), which is expressed as

$$p = 1 - F_X(x) \tag{12.21}$$

Obviously, p will take values between zero and 1. In the previous example, $p = 1 - 0.80 = 0.20$. This means that in any given year in the future, the maximum discharge will exceed $x = 3,000$ cfs with a probability of 0.20, or 20 percent. Sometimes hydrologists state that 3,000 cfs has an exceedence probability of 0.20, or 20 percent, to express the same outcome.

The *return period*, also called the *recurrence interval*, is defined as the average number of years between occurrences of a hydrologic event with a certain magnitude or greater. Denoting the return period by T ,

$$T = \frac{1}{p} \tag{12.22}$$

by definition. For example, in the previous example, the return period of 3,000 cfs will be $1/0.20 = 5$ years. In other words, the annual maximum discharge of the stream being considered will exceed 3,000 cfs once every 5 years on average. We can express the same outcome by stating that the 5-year discharge (or 5-year flood) is 3,000 cfs.

Hydraulic structures are usually sized to accommodate, at full capacity, a *design discharge* having a specified return period. Generally, the structure will fail to function as intended if the design discharge is exceeded. The *hydrologic risk* is the probability that the design discharge will be exceeded one time or more during the service life of the project. Denoting the risk by R and the service life of the project in years by n ,

$$R = 1 - (1 - p)^n \quad (12.23)$$

Example 12.4

A highway culvert is required to accommodate a design discharge that has a return period of 50 years. The service life of the culvert is 25 years. Determine the hydrologic risk associated with this design. In other words, what is the probability that the culvert's capacity will be exceeded during its 25-year service life?

Solution

From Equation 12.22, $p = 1/50 = 0.02$. Then, from Equation 12.23,

$$R = 1 - (1 - 0.02)^{25} = 0.397 = 39.7\%$$

12.5 Frequency Analysis

The purpose of a frequency analysis of a series of observed values of a hydrologic variable is to determine the future values of this variable corresponding to different return periods of interest. To achieve this, we need to determine the probability distribution that fits the available hydrologic data using statistical means. Only after we identify a probability distribution that adequately represents the data series can we interpolate and extrapolate from the observed data values in an intelligent manner. Frequency factors as well as special probability graph papers are useful for this purpose.

12.5.1 Frequency Factors

For most theoretical distributions used in hydrology, closed form analytical expressions are not available for cumulative density functions. However, Chow* showed that Equation 12.21 can be written in a more convenient form as

$$x_T = m + K_T s \quad (12.24)$$

where m and s are the sample mean and standard deviation, respectively; x_T is the magnitude of a hydrologic variable corresponding to a specified return period T ; and K_T is the frequency factor for that return period. When log-transformed variables are used, as in the case of log-normal and log-Pearson type III distributions,

$$\log x_T = m_l + K_T s_l \quad (12.25)$$

The value of the frequency factor depends on the probability distribution being considered.

An explicit analytical expression for K_T is available only for the Gumbel distribution:

$$K_T = \frac{-\sqrt{6}}{\pi} \left(0.5772 + \ln \left[\ln \left(\frac{T}{T-1} \right) \right] \right) \quad (12.26)$$

The Gumbel frequency factors for various return periods are tabulated in Table 12.2.

* Chow, V. T., "A general formula for hydrologic frequency analysis." *Transactions of the American Geophysical Union*, 32(2): 231-237, 1952.

TABLE 12.2 Frequency Factors for Gumbel, Normal, and log-Normal Distributions

T (years)	p	K_T (Gumbel)	K_T (Normal)	K_T (log-Normal)
1.11	0.9	-1.100	-1.282	-1.282
1.25	0.8	-0.821	-0.841	-0.841
1.67	0.6	-0.382	-0.253	-0.253
2	0.5	-0.164	0	0
2.5	0.4	0.074	0.253	0.253
4	0.25	0.521	0.674	0.674
5	0.2	0.719	0.841	0.841
10	0.1	1.305	1.282	1.282
20	0.05	1.866	1.645	1.645
25	0.04	2.044	1.751	1.751
40	0.025	2.416	1.960	1.960
50	0.02	2.592	2.054	2.054
100	0.01	3.137	2.327	2.327
200	0.005	3.679	2.576	2.576

For normal and log-normal distributions, following Abramowitz and Stegun,* we can approximate the frequency factor by

$$K_T = z \tag{12.27}$$

where

$$z = w - \frac{2.515517 + 0.802853w + 0.010328 w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \tag{12.28}$$

and

$$w = [\ln(T^2)]^{1/2} \tag{12.29a}$$

or

$$w = [\ln(1/p^2)]^{1/2} \tag{12.29b}$$

Equations 12.27 to 12.29 are valid for p values of 0.5 or smaller (that is, T values of 2 years or greater). For $p > 0.5$, $1 - p$ is substituted for p in Equation 12.29b, and a negative sign is inserted in front of z in Equation 12.28. The frequency factors listed for various return periods in Table 12.2 were obtained by using these equations.

We can approximate the log-Pearson type III frequency factors, for return periods between 2 and 200 years, using a relationship developed by Kite† and expressed as

$$K_T = z + (z^2 - 1)k + (z^3 - 6z)\frac{k^2}{3} - (z^2 - 1)k^3 + zk^4 + \frac{k^5}{3} \tag{12.30}$$

where

$$k = \frac{G_I}{6} \tag{12.31a}$$

and z is as obtained from Equation 12.28. If the Water Resources Council's weighted skew coefficient concept is used,

$$k = \frac{g}{6} \tag{12.31b}$$

* Abramowitz, M., and Stegun, I. A., *Handbook of Mathematical Functions*. New York: Dover Publications, 1965.

† Kite, G. W., *Frequency and Risk Analysis in Hydrology*. Fort Collins, CO: Water Resources Publications, 1977.

TABLE 12.3 Frequency Factors (K_T) for log-Pearson Type III Distribution

Skew Coefficient (g)	Exceedence Probability, p				
	0.5	0.1	0.04	0.02	0.01
	Return Period, T (years)				
	2	10	25	50	100
2.0	-0.307	1.302	2.219	2.912	3.605
1.5	-0.240	1.333	2.146	2.743	3.330
1.0	-0.164	1.340	2.043	2.542	3.022
0.8	-0.132	1.336	1.993	2.453	2.891
0.6	-0.099	1.328	1.939	2.359	2.755
0.4	-0.066	1.317	1.880	2.261	2.615
0.2	-0.033	1.301	1.818	2.159	2.472
0.1	-0.017	1.292	1.785	2.107	2.400
0.0	0.000	1.282	1.751	2.054	2.326
-0.1	0.017	1.270	1.716	2.000	2.252
-0.2	0.033	1.258	1.680	1.945	2.178
-0.4	0.066	1.231	1.606	1.834	2.029
-0.6	0.099	1.200	1.528	1.720	1.880
-0.8	0.132	1.166	1.448	1.606	1.733
-1.0	0.164	1.128	1.366	1.492	1.588
-1.5	0.240	1.018	1.157	1.218	1.257
-2.0	0.307	0.895	0.959	0.980	0.990

The values of K_T for various values of g are given in Table 12.3. Linear interpolation of these values for g 's not listed in the table is not recommended. Instead, Equations 12.27 through 12.31 should be used. Note that for $g = 0$, the frequency factors for the log-Pearson type III distribution are the same as those for the log-normal distributions. It is important to note that the frequency factors presented in these sections are useful to estimate the magnitudes of future events only if the probability distribution is specified. The methods for testing the goodness of fit of data to a probability distribution will be discussed in the next section.

Example 12.5

The statistical parameters of the annual maximum discharge series of the Meherrin River have been computed as $m = 9,820$ cfs, $s = 4,660$ cfs, $m_l = 3.95$, and $s_l = 0.197$ in Examples 12.1 and 12.2. Also, we determined the weighted skew coefficient to be $g = 0.389$ in Example 12.3. Determine the magnitude of the 25-year discharge on the Meherrin River if the data fits (a) the normal distribution, (b) the log-normal distribution, (c) the Gumbel distribution, and (d) the log-Pearson type III distribution.

Solution

- (a) To solve part (a), we first obtain $K_{25} = 1.751$ from Table 12.2 for $p = 0.04$ (i.e., $T = 25$ years). Then, from Equation 12.24,

$$Q_{25} = m + K_{25}(s) = 9,820 + 1.751(4,660) = 18,000 \text{ cfs (normal)}$$

- (b) We can use the same frequency factor, $K_{25} = 1.751$, to solve part (b) of the problem. From Equation 12.25,

$$\log Q_{25} = m_l + K_T(s_l) = 3.95 + 1.751(0.197) = 4.29$$

Then, taking the antilog of 4.29, we obtain $Q_{25} = 19,500$ cfs (log normal).

- (c) For the Gumbel distribution, we first obtain $K_{25} = 2.044$ from Table 12.2 for $p = 0.04$ (i.e., $T = 25$ years). Then, from Equation 12.24,

$$Q_{25} = m + K_{25}(s) = 9,820 + 2.044(4,660) = 19,300 \text{ cfs (Gumbel)}$$

- (d) We will use the Equations 12.28 through 12.31 to solve part (d), the log-Pearson type III distribution. The sequence is Equations 12.31b, 12.29a, 12.28, and 12.30:

$$k = g/6 = 0.389/6 = 0.0648$$

$$w = [\ln T^2]^{1/2} = [\ln (25)^2]^{1/2} = 2.54$$

$$z = w - \frac{2.515517 + 0.802853w + 0.010328 w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}$$

$$z = 2.54 - \frac{2.515517 + 0.802853(2.54) + 0.010328(2.54)^2}{1 + 1.432788(2.54) + 0.189269(2.54)^2 + 0.001308(2.54)^3} = 1.75$$

$$K_T = z + (z^2 - 1)k + (z^3 - 6z)(k^2/3) - (z^2 - 1)k^3 + zk^4 + k^5/3$$

$$K_T = 1.75 + (1.75^2 - 1) 0.0648 + [1.75^3 - 6(1.75)] (0.0648^2/3) - (1.75^2 - 1)0.0648^3 + 1.75(0.0648)^4 + 0.0648^5/3$$

$$K_T = 1.877$$

Now, from Equation 12.25,

$$\log Q_{25} = m_l + K_T (s_l) = 3.95 + 1.877(0.197) = 4.32$$

Taking the antilog of 4.32, we obtain $Q_{25} = 20,900$ cfs (log-Pearson type III).

12.5.2 Testing Goodness of Fit

The chi-square test is a statistical procedure to determine the goodness of fit of data to a probability distribution. In this test, we divide the entire range of possible values of the hydrologic variable into k class intervals. We then compare the actual number of data values falling in these intervals to the number of data values expected according to the probability distribution being tested. The number of the class intervals, k , is selected so that the expected number of data values in each class interval is at least 3. The limits of the class intervals are determined so that the number of expected data values is the same in each interval. The frequency factors discussed in Section 12.5.1 can be used to determine the class limits.

To perform a chi-square test, it is necessary to first choose a *significance level*, α . Commonly, $\alpha = 0.10$ is used in hydrology. The meaning of α can be explained as follows: If we use $\alpha = 0.10$, and as a result of the chi-square test we reject the probability distribution being considered, then there is a 10-percent chance that we have rejected a satisfactory distribution.

The test statistic is calculated using

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \tag{12.32}$$

where O_i and E_i are the observed and expected number of data values in the i th interval. Then we accept the distribution being tested if

$$\chi^2 < \chi_\alpha^2$$

TABLE 12.4 Values of χ^2_α

ν	Significance Level		
	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.50$
1	3.84	2.71	0.455
2	5.99	4.61	1.39
3	7.81	6.25	2.37
4	9.49	7.78	3.36
5	11.1	9.24	4.35
6	12.6	10.6	5.35
7	14.1	12.0	6.35
8	15.5	13.4	7.34
9	16.9	14.7	8.34
10	18.3	16.0	9.34
15	25.0	22.3	14.3
20	31.4	28.4	19.3
25	37.7	34.4	24.3
30	43.8	40.3	29.3
40	55.8	51.8	39.3
50	67.5	63.2	49.3
60	79.1	74.4	59.3
70	90.5	85.5	69.3
80	101.9	96.6	79.3
90	113.1	107.6	89.3
100	124.3	118.5	99.3

and reject it otherwise, where χ^2_α is the critical value of χ^2 at significance level α . The values of χ^2_α for $\alpha = 0.05, 0.10,$ and 0.50 are given in Table 12.4 as a function of ν , where

$$\nu = k - kk - 1 \tag{12.33}$$

and $kk =$ the number of sample statistics, such as the mean, standard deviation, and skew coefficient used to describe the probability distribution being tested. For the normal, log-normal, and Gumbel distributions, $kk = 2$; and for the log-Pearson type III distribution, $kk = 3$. Once again, k is the number of the class intervals used in the test. It should be noted that the result of a chi-square test is sensitive to the value of k being used. Therefore, this test must be used cautiously.

Example 12.6

Consider the annual maximum discharges (log transformed) on the Meherrin River given in Example 12.2 with sample statistics $m_l = 3.95$ and $s_l = 0.197$. Test the goodness of fit of these data to the log-normal distribution at a significance level of $\alpha = 0.10$. Use five class intervals—that is, $k = 5$.

Solution

Because the probability ranges from 0 to 1.0, the exceedence probability increment is $(1.0 - 0.0)/5 = 0.20$ for each class interval. Accordingly, the upper and lower probability limits of the five class intervals are determined as listed in Table 12.5. The exceedence probabilities of $p = 0.8, 0.6, 0.4,$ and 0.2 correspond to return periods of $T = 1.25, 1.67, 2.50,$ and 5.00 years. The frequency factors for these return periods for the log-normal distribution are obtained from Table 12.2 as $K_T = -0.841, -0.253, 0.253,$ and 0.841 . The corresponding discharges are determined, by using Equation 12.25 with $m_l = 3.95$ and $s_l = 0.197$, as being $Q = 6,090, 7,940, 9,990,$ and $13,100$ cfs, respectively. The upper and lower discharge limits listed in Table 12.5 are based

TABLE 12.5 Chi-Square Test for Example 12.6

Class Interval	Exceedence Probability Limits		Discharge Limits (cfs)		E_i	O_i	$(O_i - E_i)^2/E_i$
	Higher	Lower	Lower	Upper			
i							
1	1.0	0.8	0	6,090	7.8	9	0.185
2	0.8	0.6	6,090	7,940	7.8	9	0.185
3	0.6	0.4	7,940	9,990	7.8	7	0.082
4	0.4	0.2	9,990	13,100	7.8	6	0.415
5	0.2	0.0	13,100	Infinity	7.8	8	0.005
				Totals	39	39	0.872

on these values. Note that the upper limit in terms of the exceedence probability corresponds to the lower limit in terms of the discharge for each class interval.

The expected number of data values is $E_i = N/5 = 39/5 = 7.8$ for each interval. The observed number of values (O_i) is obtained from Table 12.1. For instance, Table 12.1 contains only nine values between 0 and 6,090 cfs for class 1. The value in column 8 for class 1 is computed as

$$\chi^2 = \frac{(9 - 7.8)^2}{7.8} = 0.185$$

Other entries in column 8 are computed likewise.

The test statistic is calculated by summing the values in column 8 of Table 12.5 to be 0.872. In this example, $k = 5$ and $kk = 2$ (because we are using the log-normal distribution). Therefore $\nu = 5 - 2 - 1 = 2$. Then, from Table 12.4 for $\alpha = 0.10$, we obtain $\chi^2_\alpha = 4.61$. Because $\chi^2 < \chi^2_\alpha$ (i.e., because $0.872 < 4.61$), we conclude that the log-normal distribution does indeed adequately fit the annual maximum discharge data series (log transformed) of the Meherrin River.

12.5.3 Confidence Limits

There are uncertainties associated with the estimates made using frequency factors. Usually, we present these estimates within a range called a *confidence interval*. The upper and lower limits of a confidence interval are called the *confidence limits*. The width of the confidence interval depends on the size of the sample and the *confidence level*. An interval is said to have a confidence level of 90 percent if the true value of the estimated hydrologic variable is expected to fall in this range with a probability of 0.90, or 90 percent. The upper and lower confidence limits are expressed, respectively, as

$$U_T = m + K_{TU}(s) \tag{12.34}$$

$$L_T = m + K_{TL}(s) \tag{12.35}$$

where K_{TU} and K_{TL} are modified frequency factors developed by Chow et al.* For log-transformed samples, the corresponding equations are

$$\log U_T = m_l + K_{TU}(s_l) \tag{12.36}$$

$$\log L_T = m_l + K_{TL}(s_l) \tag{12.37}$$

* Chow, V. T., Maidment, D. R., and Mays, L. W., *Applied Hydrology*. New York: McGraw-Hill, 1988.

The approximate expressions for the modified frequency factors are

$$K_{TU} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} \quad (12.38)$$

$$K_{TL} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} \quad (12.39)$$

where K_T is the frequency factor appearing in Equations 12.24 and 12.25, and for a sample size of N :

$$a = 1 - \frac{z^2}{2(N-1)} \quad (12.40)$$

$$b = K_T^2 - \frac{z^2}{N} \quad (12.41)$$

The value of the parameter z depends on the confidence level. In practice, a 90-percent confidence level is most commonly used; at this level, $z = 1.645$. For other confidence levels, Equation 12.28 can be used to obtain z with

$$w = \left[\ln \left(\frac{2}{1-\beta} \right) \right]^{1/2} \quad (12.42)$$

where β is the confidence level expressed as a fraction.

Example 12.7

The statistical parameters of the annual maximum discharge series of the Meherrin River have been computed as $m = 9,820$ cfs, $s = 4,660$ cfs, $m_l = 3.95$, and $s_l = 0.197$ in Examples 12.1 and 12.2. Also, we determined the weighted skew coefficient to be $g = 0.389$ in Example 12.3. Determine the 90-percent confidence limits for the 25-year discharge, assuming the data fit (a) the normal distribution, (b) the log-normal distribution, (c) the Gumbel distribution, and (d) the log-Pearson type III distribution.

Solution

In Example 12.5, we determined that $K_{25} = 1.751$ for the normal and log-normal distributions, $K_{25} = 2.044$ for the Gumbel distribution, and $K_{25} = 1.877$ for the log-Pearson type III distribution. Also, when a 90-percent confidence level is used, $z = 1.645$ for $\beta = 0.90$. From Equation 12.40,

$$a = 1 - \frac{z^2}{2(N-1)} = 1 - \frac{(1.645)^2}{2(39-1)} = 0.9644$$

For parts (a) and (b) of the problem, we first find the parameter b using Equation 12.41 as

$$b = K_T^2 - \frac{z^2}{N} = (1.751)^2 - \frac{(1.645)^2}{39} = 2.997$$

Next, from Equations 12.38 and 12.39,

$$K_{25U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.751 + \sqrt{(1.751)^2 - (0.9644)(2.997)}}{0.9644} = 2.25$$

$$K_{25L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.751 - \sqrt{(1.751)^2 - (0.9644)(2.997)}}{0.9644} = 1.38$$

Then, for the normal distribution using Equations 12.34 and 12.35, the confidence limits are

$$U_{25} = m + K_{25U}(s) = 9,820 + 2.25(4,660) = 20,300 \text{ cfs}$$

$$L_{25} = m + K_{25L}(s) = 9,820 + 1.38(4,660) = 16,300 \text{ cfs}$$

For the log-normal distribution, the confidence limits using Equations 12.36 and 12.37 are

$$\log U_{25} = m_l + K_{25U}(s_l) = 3.95 + 2.25(0.197) = 4.39 \quad (U_{25} = 24,500 \text{ cfs})$$

$$\log L_{25} = m_l + K_{25L}(s_l) = 3.95 + 1.38(0.197) = 4.22 \quad (L_{25} = 16,600 \text{ cfs})$$

Parts (c) and (d) can be solved in a similar manner. For part (c), from Equation 12.41,

$$b = K_T^2 - \frac{z^2}{N} = (2.044)^2 - \frac{(1.645)^2}{39} = 4.109$$

and from Equations 12.38 and 12.39,

$$K_{25U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{2.044 + \sqrt{(2.044)^2 - (0.9644)(4.109)}}{0.9644} = 2.60$$

$$K_{25L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{2.044 - \sqrt{(2.044)^2 - (0.9644)(4.109)}}{0.9644} = 1.64$$

Then, for the Gumbel distribution using Equations 12.34 and 12.35, the confidence limits are

$$U_{25} = m + K_{25U}(s) = 9,820 + 2.60(4,660) = 21,900 \text{ cfs}$$

$$L_{25} = m + K_{25L}(s) = 9,820 + 1.64(4,660) = 17,500 \text{ cfs}$$

To solve part (d) of this problem, with $K_{25} = 1.877$ for the log-Pearson type III distribution, from Equation 12.41,

$$b = K_T^2 - \frac{z^2}{N} = (1.877)^2 - \frac{(1.645)^2}{39} = 3.454$$

and from Equations 12.38 and 12.39,

$$K_{25U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.877 + \sqrt{(1.877)^2 - (0.9644)(3.454)}}{0.9644} = 2.40$$

$$K_{25L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.877 - \sqrt{(1.877)^2 - (0.9644)(3.454)}}{0.9644} = 1.49$$

For the log-Pearson type III distribution, the confidence limits using Equations 12.36 and 12.37 are

$$\log U_{25} = m_l + K_{25U}(s_l) = 3.95 + 2.40(0.197) = 4.42 \quad (U_{25} = 26,300 \text{ cfs})$$

$$\log L_{25} = m_l + K_{25L}(s_l) = 3.95 + 1.49(0.197) = 4.24 \quad (L_{25} = 17,400 \text{ cfs})$$

Example 12.8

The statistical parameters of the annual maximum discharge series of the Meherrin River have been computed as $m = 9,820$ cfs, $s = 4,660$ cfs, $m_l = 3.95$, and $s_l = 0.197$ in Examples 12.1 and 12.2. Also, in Example 12.6 we determined that the Meherrin River data fit the log-normal distribution. Determine the 1.25-, 2-, 10-, 25-, 50-, 100-, and 200-year peak discharges and the 90-percent confidence limits for the Meherrin River.

Solution

The solution is presented in Table 12.6. The values in column 2 are obtained from Table 12.2, and Equation 12.25 is used to determine the entries in column 3. The antilogs of the values in column 3 become the discharges listed in column 4. Equations 12.40 and 12.41 are used to calculate the values in columns 5 and 6, respectively.

TABLE 12.6 Peak Discharges and 90-Percent Confidence Limits for the Meherrin River

1	2	3	4	5	6	7	8	9	10	11	12
T	K_T	$\log Q_T$	Q_T	a	b	K_{TU}	K_{TL}	$\log U_T$	$\log L_T$	U_T	L_T
1.25	-0.841	3.78	6.09E+03	0.964	0.638	-0.557	-1.187	3.840	3.716	6.92E+03	5.20E+03
2	0	3.95	8.91E+03	0.964	-0.069	0.268	-0.268	4.003	3.897	1.01E+04	7.89E+03
10	1.282	4.20	1.59E+04	0.964	1.574	1.697	0.962	4.284	4.140	1.92E+04	1.38E+04
25	1.751	4.29	1.97E+04	0.964	2.997	2.251	1.381	4.393	4.222	2.47E+04	1.67E+04
50	2.054	4.35	2.26E+04	0.964	4.150	2.613	1.647	4.465	4.274	2.92E+04	1.88E+04
100	2.327	4.41	2.56E+04	0.964	5.346	2.941	1.884	4.529	4.321	3.38E+04	2.10E+04
200	2.576	4.46	2.87E+04	0.964	6.566	3.242	2.100	4.589	4.364	3.88E+04	2.31E+04

Likewise, Equations 12.38 and 12.39 are used to determine the entries in columns 7 and 8. The logarithms of the upper and lower confidence limits listed in columns 9 and 10 are obtained by using Equations 12.36 and 12.37, respectively. The antilogs of these yield the upper and lower limits listed in columns 11 and 12, respectively.

12.6 Frequency Analysis Using Probability Graphs

12.6.1 Probability Graphs

Graphical representation of hydrologic data is an important tool for statistical analysis. Usually, we plot the data on specially designed probability paper. The ordinate usually represents the value of the hydrologic variable, and the abscissa represents the return period (T) or the exceedence probability (p). The ordinate scale can be linear or logarithmic, depending on the probability distribution being used. The abscissa scale is designed such that Equation 12.24, or Equation 12.25, will plot as a theoretical straight line. When plotted, the data points should fall on or near this straight line if the probability distribution being used represents the data series adequately. With this linear relationship, we can easily interpolate and extrapolate the plotted data.

Normal, log-normal, and Gumbel distribution graph papers are available commercially. Figure 12.2 provides an example of normal distribution (probability) paper. For the log-Pearson type III distribution, there would have to be a different graph paper for each different value of the skew coefficient. Commercial log-Pearson type III papers are not available because of this impracticality. A log-normal probability paper can be used for log-Pearson type III distribution, but Equation 12.25 will plot as a smooth theoretical curve rather than a straight line, and extrapolation of the plotted data will be relatively difficult.

12.6.2 Plotting Positions

Plotting position refers to the return period T (or the exceedence probability $p = 1/T$) assigned to each data value that will be plotted on a probability paper. Among the many methods available in the literature, most of which are empirical, the Weibull method is adopted herein. In this method, the data values are listed in an order of decreasing magnitude and a rank (r) is assigned to each data value. In other words, if there are N data values in the series, $r = 1$ for the largest

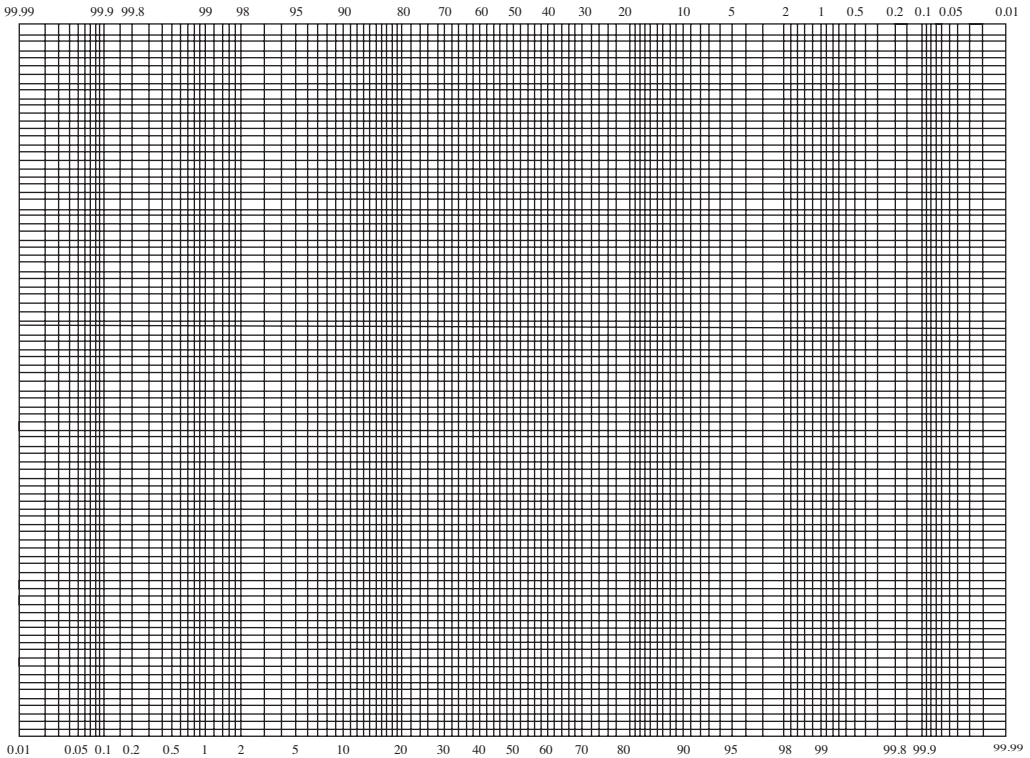


Figure 12.2 Probability (normal distribution) graph paper

value in the series and $r = N$ for the smallest. Then the exceedence probability assigned to each data value for plotting purposes is found as

$$p = \frac{r}{N + 1} \tag{12.43}$$

which is equivalent to the plotting position formula adopted by the Water Resources Council,* and

$$T = \frac{N + 1}{r} \tag{12.44}$$

where T is the assigned return period for plotting purposes.

Example 12.9

The annual maximum discharge series of the Meherrin River were tabulated in chronological order in Table 12.1. Determine the return periods assigned to these data values for plotting purposes.

Solution

This problem can be solved in tabular form. As shown in Table 12.7, the observed values of annual peak discharges are listed in decreasing order. Then a rank, $r = 1$ to 39, is entered in column 1. Subsequently,

* Interagency Advisory Committee on Water Data, "Guidelines for Determining Flood Flow Frequency." Bulletin 17B. Reston, VA: U.S. Department of the Interior, U.S. Geological Survey, Office of Water Data Coordination, 1982.

TABLE 12.7 Plotting Positions for Example 12.9

Rank (r)	Q (cfs)	Plotting Position (p)	Plotting Position (T) years
1	21,100	0.025	40.00
2	19,400	0.050	20.00
3	19,400	0.075	13.33
4	17,500	0.100	10.00
5	16,600	0.125	8.00
6	16,600	0.150	6.67
7	16,200	0.175	5.71
8	13,800	0.200	5.00
9	12,900	0.225	4.44
10	12,600	0.250	4.00
11	12,100	0.275	3.64
12	11,200	0.300	3.33
13	11,100	0.325	3.08
14	10,000	0.350	2.86
15	9,410	0.375	2.67
16	9,400	0.400	2.50
17	9,360	0.425	2.35
18	8,710	0.450	2.22
19	8,570	0.475	2.11
20	8,450	0.500	2.00
21	8,100	0.525	1.90
22	7,800	0.550	1.82
23	7,580	0.575	1.74
24	7,520	0.600	1.67
25	7,470	0.625	1.60
26	7,390	0.650	1.54
27	7,060	0.675	1.48
28	7,030	0.700	1.43
29	6,700	0.725	1.38
30	6,200	0.750	1.33
31	5,860	0.775	1.29
32	5,800	0.800	1.25
33	5,640	0.825	1.21
34	5,400	0.850	1.18
35	5,200	0.875	1.14
36	4,940	0.900	1.11
37	4,790	0.925	1.08
38	4,210	0.950	1.05
39	3,800	0.975	1.03

the exceedence probability (p) and the return period (T) for each discharge are calculated using Equations 12.43 and 12.44 and are tabulated in columns 3 and 4, respectively.

12.6.3 Data Plotting and Theoretical Distributions

As previously mentioned, a graphical representation of a hydrologic data series can be obtained by plotting the data points on specially designed probability paper. The type of probability paper used depends on the probability distribution appropriate to the data or one that is being tested. The data are plotted using the plotting positions discussed in the preceding section.

A theoretical straight line representing the probability distribution may be plotted using the frequency factors discussed in Section 12.5.1. Although two points are adequate to draw a straight line, it is good practice to use at least three points to detect any computational errors. For a perfect fit, all the data points must fall on the straight line. We never see a perfect fit in actual applications; if the data points are close enough to the theoretical straight line, then the probability distribution being tested is acceptable. It is possible to quantify and test the suitability of the distribution being applied to the given data using a *goodness-of-fit* statistical test, as described in Section 12.5.2. In this case, the upper and lower limits of the class intervals corresponding to the selected probability limits can be determined directly from the graph using the theoretical (straight-line) probability distribution.

As noted in Section 12.5.3, uncertainties are associated with the estimates made using statistical methods, and we usually present those estimates with a range called the *confidence band*. A confidence band is obtained by plotting the upper and lower confidence limits on probability paper. We obtain the lower boundary of the confidence band by drawing a line through the calculated lower confidence limits. The line defining the upper boundary passes through the calculated upper confidence limits. Obviously, the theoretical probability distribution must fall within this band. The width of the band will depend on the confidence level discussed in Section 12.5.3. A confidence level of 90 percent is common in hydrology.

Example 12.10

Prepare a log-normal plot of the annual maximum discharge series of the Meherrin River. Draw the theoretical (straight-line) probability distribution and the 90-percent confidence band.

Solution

Figure 12.3 displays the Meherrin River data on log-normal probability paper. The observed data points were plotted using the plotting positions calculated in Example 12.9 and presented in Table 12.7.

In Example 12.8 the discharges corresponding to various return periods were calculated for a log-normal distribution. These are theoretical values and are used to plot the theoretical probability distribution. Only two points are required to draw the straight line, but three points are used in this example.

As to the confidence band, the lower and upper confidence limits were also calculated in Example 12.8 for various return periods. By plotting U_T versus T on the probability paper, we obtain the upper boundary of the confidence band. Likewise, a plot of L_T versus T will give the lower boundary, as shown in Figure 12.3.

12.6.4 Estimating Future Magnitudes

When a statistical distribution is fit to a data series and the confidence band is developed, the future expected magnitudes of the hydrologic variable being considered may be estimated quite easily. For instance, in Example 12.6 we have shown that the annual maximum discharge series of the Meherrin River fits a log-normal distribution with the corresponding theoretical straight line and the confidence band displayed in Figure 12.3. We can now use Figure 12.3 to estimate the discharge for virtually any return period even though the original data series only contains 39 years of data.

Suppose we want to estimate the discharge that has a return period of 100 years. Using $T = 100$ years and the theoretical straight line in Figure 12.3, we could read Q_{100} directly off the log-normal graph or, as computed in Example 12.8, $Q_{100} = 25,600$ cfs. Likewise, from the 90-percent confidence band, $U_{100} = 33,800$ cfs and $L_{100} = 21,000$ cfs, as computed in Example 12.8, shown in Table 12.6, and plotted in Figure 12.3. We can now interpret these results as follows: There is only a 5-percent chance that the actual 100-year discharge will be greater than

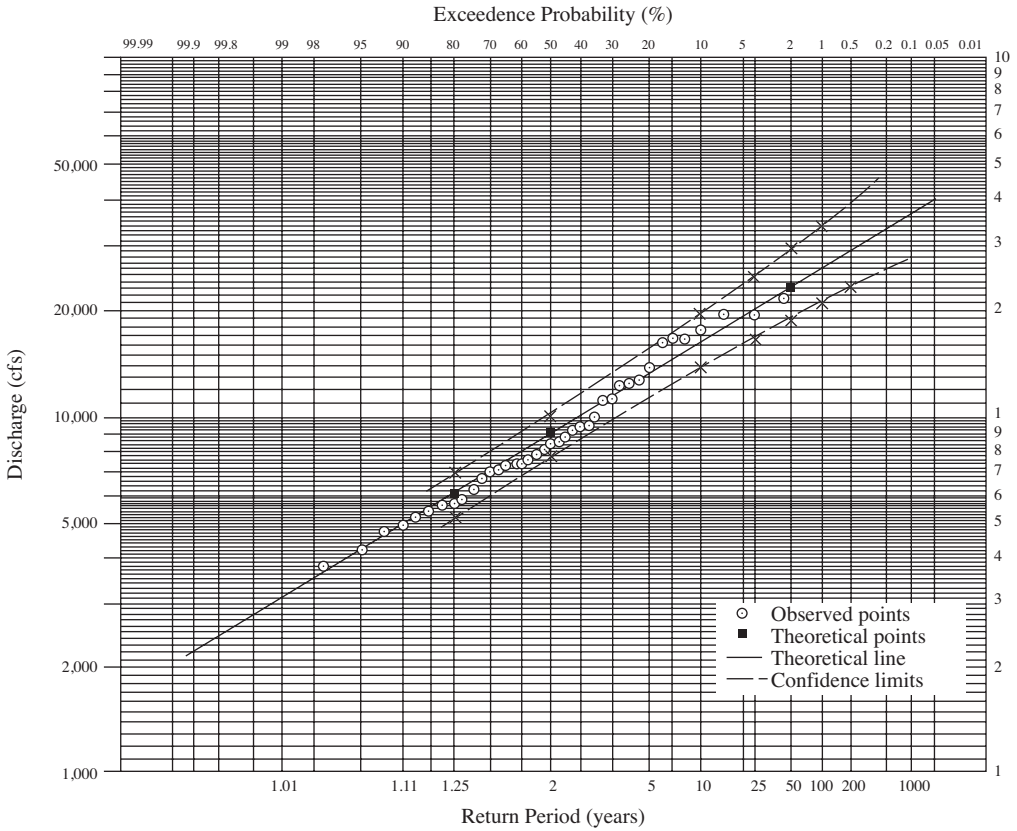


Figure 12.3 Log-normal probability plot of the Meherrin River data

33,800 cfs. Likewise, the probability that the actual value will be less than 21,000 cfs is 5 percent. The most likely value of the 100-year discharge is 25,600 cfs.

The theoretical line of a probability graph can also be used to estimate the return period of a given magnitude of discharge at a given location. For example, the return period of 20,000 cfs for as elsewhere Meherrin River can be read directly off Figure 12.2 as being 25 years.

12.7 Rainfall Intensity-Duration-Frequency Relationships

Frequency analysis techniques can be used to develop relationships between the average rainfall intensity, duration, and return period. These relationships are often presented in chart form as *intensity-duration-frequency (IDF) curves*. IDF curves are used in engineering practice for designing a variety of urban hydraulic structures.

To develop IDF curves for a given location, we first extract from rainfall records the annual maximum rainfall depths corresponding to selected rainfall durations. Then the data series for each duration are fit to a probability distribution. Next, the return periods are determined for various rainfall depths using this distribution. In the end, we divide these depths by the duration being considered to find the IDF relationships. Normally, the Gumbel distribution is used for rainfall frequency analysis. The following example problem will help clarify the procedure.

TABLE 12.8 Mean and Standard Deviation of Rainfall Depths for Example 12.11

<i>j</i>	<i>t_d</i> = 15 Minutes		<i>t_d</i> = 30 Minutes		<i>t_d</i> = 60 Minutes		<i>t_d</i> = 120 Minutes	
	<i>P_j</i> (in.)	(<i>P_j</i> - <i>m</i>) ² (in. ²)	<i>P_j</i> (in.)	(<i>P_j</i> - <i>m</i>) ² (in. ²)	<i>P_j</i> (in.)	(<i>P_j</i> - <i>m</i>) ² (in. ²)	<i>P_j</i> (in.)	(<i>P_j</i> - <i>m</i>) ² (in. ²)
1	1.55	0.436	2.20	0.985	2.80	1.775	3.20	2.027
2	1.40	0.260	2.00	0.628	2.55	1.172	2.80	1.048
3	1.35	0.212	1.85	0.413	2.20	0.536	2.60	0.678
4	1.26	0.137	1.72	0.263	2.00	0.283	2.47	0.481
5	1.20	0.096	1.60	0.154	1.90	0.187	2.40	0.389
6	1.16	0.073	1.53	0.104	1.80	0.110	2.29	0.264
7	1.10	0.044	1.47	0.069	1.70	0.054	2.18	0.163
8	1.05	0.026	1.40	0.037	1.60	0.018	2.07	0.086
9	1.01	0.014	1.34	0.018	1.52	0.003	2.00	0.050
10	0.97	0.006	1.28	0.005	1.48	0.000	1.90	0.015
11	0.92	0.001	1.24	0.001	1.43	0.001	1.81	0.001
12	0.88	0.000	1.20	0.000	1.40	0.005	1.71	0.004
13	0.86	0.001	1.14	0.005	1.35	0.014	1.64	0.019
14	0.82	0.005	1.09	0.014	1.29	0.032	1.60	0.031
15	0.80	0.008	1.04	0.028	1.25	0.047	1.53	0.061
16	0.75	0.020	1.00	0.043	1.21	0.066	1.46	0.100
17	0.71	0.032	0.95	0.066	1.18	0.083	1.40	0.142
18	0.68	0.044	0.90	0.095	1.16	0.095	1.35	0.182
19	0.65	0.058	0.86	0.121	1.12	0.121	1.29	0.237
20	0.60	0.084	0.82	0.150	1.08	0.150	1.22	0.310
21	0.56	0.109	0.78	0.183	1.05	0.174	1.16	0.380
22	0.53	0.130	0.74	0.219	1.00	0.219	1.11	0.444
23	0.50	0.152	0.71	0.248	0.93	0.289	1.09	0.471
24	0.48	0.168	0.68	0.278	0.86	0.369	1.07	0.499
25	0.46	0.185	0.65	0.311	0.83	0.407	1.06	0.513
Σ	22.25	2.300	30.19	4.436	36.69	6.210	44.41	8.594
<i>m</i>	0.890		1.208		1.468		1.776	
<i>s</i>		0.310		0.430		0.509		0.598

Example 12.11

The annual maximum rainfall depth (P) series for a period of 25 years are given in columns 2, 4, 6, and 8 of Table 12.8 for storm durations (t_d) of 15, 30, 60, and 120 minutes, respectively. Develop IDF curves, assuming the data fit the Gumbel distribution. (A similar example was previously presented by Akan and Houghtalen.*)

Solution

The calculations can be performed in tabular form. The mean and standard deviation of rainfall depths (for each duration) are calculated in Table 12.8 using Equations 12.1 and 12.2. The frequency analyses of rainfall depths for the four storm durations are performed separately. Table 12.9 summarizes the calculations. The K_T values shown in column 2 of Table 12.9 are obtained from Table 12.2 for Gumbel distribution. The corresponding precipitation depths are computed using Equation 12.24, as shown in columns 3, 5, 7, and 9 of Table 12.9. The average rainfall intensity, i_{avg} , corresponding to the each precipitation depth is computed simply by dividing P by the duration, t_d , in hours. The results are plotted in Figure 12.4.

TABLE 12.9 Rainfall Frequency Calculations for Example 12.11

T	K_T	$t_d = 15 \text{ min.}$ $m = 0.890 \text{ in.}$ $s = 0.310 \text{ in.}$		$t_d = 30 \text{ min.}$ $m = 1.208 \text{ in.}$ $s = 0.430 \text{ in.}$		$t_d = 60 \text{ min.}$ $m = 1.468 \text{ in.}$ $s = 0.509 \text{ in.}$		$t_d = 120 \text{ min.}$ $m = 1.776 \text{ in.}$ $s = 0.598 \text{ in.}$	
		P	i_{avg}	P	i_{avg}	P	i_{avg}	P	i_{avg}
5	0.719	1.113	4.450	1.517	3.033	1.833	1.833	2.207	1.103
10	1.305	1.294	5.176	1.769	3.537	2.131	2.131	2.557	1.279
25	2.044	1.523	6.091	2.086	4.173	2.507	2.507	3.000	1.500
50	2.592	1.692	6.770	2.322	4.644	2.786	2.786	3.327	1.664
100	3.137	1.861	7.444	2.556	5.112	3.063	3.063	3.654	1.827

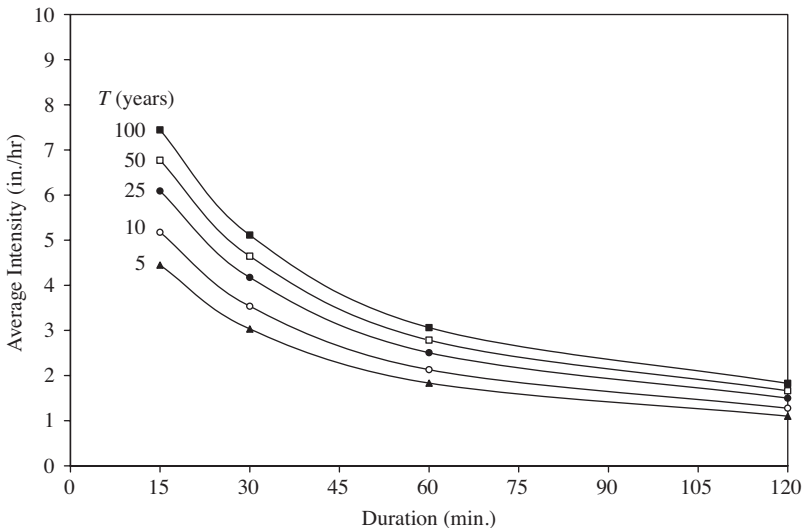


Figure 12.4 IDF Curves for Example 12.11

* Akan, A. O., and Houghtalen, R. J., *Urban Hydrology, Hydraulics and Stormwater Quality*. Hoboken, NJ: John Wiley and Sons, 2003.

12.8 Applicability of Statistical Methods

The statistical methods presented in this chapter, in particular the frequency analysis procedures, are applicable to a wide range of hydrologic problems. Much of this chapter examined the application of frequency analysis to discharges on a river. However, these techniques can be used to predict flood stage elevations, flood storage volumes, precipitation depths, pollutant loadings, and many other hydrologically related phenomena.

For the techniques in this chapter to be used, an annual maximum or annual exceedence series must be given. We cannot, for example, take 27 discharge measurements from 13 years of streamflow data and use these 27 values as if we had 27 years of data. Furthermore, the data to be analyzed must represent *hydrologically independent events*. In other words, the magnitude of one event cannot be dependent or related to the magnitude of another event, nor can it be part of another event. For example, high flood stage elevations associated with a large rainfall event that occurs at the end of a year should only be used to represent the peak elevation in one of the two years.

As long as the frequency distribution selected is appropriate for the type of analysis being performed, and the above conditions are satisfied, the statistical results should be valid. The log-Pearson type III and Gumbel distributions were developed to predict streamflows associated with rainstorms. They are not particularly adept at predicting drought conditions. In any case, we can and should test our statistical results using a goodness-of-fit test such as the chi-square test.

PROBLEMS (SECTION 12.2)

12.2.1. The annual precipitation measurements (P_i in inches) for Mythical City, Indiana, over a 20-year period are shown in the following table. Determine the mean, standard deviation, and skew coefficient for this series.

Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
P_i	44.2	47.6	38.5	35.8	40.2	41.2	38.8	39.7	40.5	42.5
Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
P_i	39.2	38.3	46.1	33.1	35.0	39.3	42.0	41.7	37.7	38.6

12.2.2. Determine the mean, standard deviation, and skew coefficient for the log values of annual rainfall for Mythical City, Indiana, given in Problem 12.2.1. Also determine the mean precipitation (in inches) of the log-transformed data.

12.2.3. A forensic engineer is studying the past history of flooding in Wisconsin over 20-year periods. The annual maximum flood discharges (Q_i in m^3/sec) for the Wolf River (tributary to Lake Michigan) at New London, Wisconsin, from 1950 to 1969 are given in the following table. Determine the mean, standard deviation, and skew coefficient for this series.

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
Q_i	114	198	297	430	294	113	165	211	94.0	91.0
Year	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
Q_i	222	376	215	250	218	98.0	283	147	289	175

- 12.2.4.** Determine the mean, standard deviation, and skew coefficient for the log values of annual maximum flood discharges for the Wolf River given in Problem 12.2.3. Also determine the mean annual flood (in m^3/sec) of the log-transformed data set.

(SECTION 12.3)

- 12.3.1.** Using the results of Examples 12.1 and 12.2, write the probability density function assuming a normal distribution and a log-normal distribution for the annual peak discharges (Q_i) of the Meherrin River at Emporia, Virginia.
- 12.3.2.** Using the results of Example 12.1, write the probability density function assuming a Gumbel distribution for the annual peak discharges (Q_i) of the Meherrin River.
- 12.3.3.** Using the results of Examples 12.2 and 12.3, write the probability density function assuming a log-Pearson type III distribution for the annual peak discharges (Q_i) of the Meherrin River at Emporia, Virginia.

(SECTION 12.4)

- 12.4.1.** A riparian property owner has been informed that the first floor of her garage is just a little lower than the 5-year flood elevation. Although she has insurance, she would like to determine the risk of leaving some belongings (which she hopes to sell within three years) in the garage. What is the probability that the garage will be flooded next year? What is the probability that the garage will be flooded at least once in the next three years?
- 12.4.2.** Referring to Example 12.4, it is evident that there is roughly a 40-percent chance of the culvert's capacity being exceeded in the 25-year service life. Determine the return period that the culvert would need to be designed for in order to reduce the risk of capacity exceedence during the service life to 20 percent.
- 12.4.3.** A city depends on the flow of a nearby river for its water supply. If the discharge drops below a base level of $30 \text{ m}^3/\text{sec}$, various emergency actions begin. After two consecutive days of sub-base flow, a small reservoir is brought online; after 10 consecutive days of sub-base flow, water must be piped in from a nearby town. River discharge records have been assessed to determine the exceedence probability of the two-day drought (70 percent) and the 10-day drought (20 percent). Determine the following:
- the probability of having to utilize the reservoir at least once in the next two years;
 - the probability of not having to rely on the reservoir in the next two years;
 - the probability of having to rely on the reservoir during each of the next two years; and
 - the probability of having to rely on the reservoir exactly once in the next two years.
- 12.4.4.** A city depends on the flow of a nearby river for its water supply. If the discharge drops below a base level of $30 \text{ m}^3/\text{sec}$, various emergency actions begin. After two consecutive days of sub-base flow, a small reservoir is brought online; after 10 consecutive days of sub-base flow, water must be piped in from a nearby town. River discharge records have been assessed to determine the exceedence probability of the two-day drought (70 percent) and the 10-day drought (20 percent). Determine the following:
- the probability of having to pipe in water at least once in the next two years;
 - the probability of not having to pipe in water in the next two years;
 - the probability of having to pipe in water during each of the next two years; and
 - the probability of having to pipe in water exactly once in the next two years.
- 12.4.5.** A riparian power plant is scheduled for construction. A temporary cofferdam is being built to protect the construction site from the 10-year flood. What is the probability of the construction site being flooded during the first year of the four-year construction period? What is the risk of being

flooded during the four-year construction period? What is the probability of not being flooded during the construction period? If the owners of the power plant want to reduce the risk of flooding during the construction period to 25 percent, how quickly (in years) could the construction be completed?

(SECTION 12.5)

- 12.5.1.** Annual precipitation measurements (P_i in inches) for Mythical City, Indiana, were given in Problem 12.2.1. The statistical parameters for this annual series were computed to be $m = 40.0$ in., $s = 3.50$ in., and $G = 0.296$. Determine the magnitude of the 10-year precipitation depth if the data fit (a) the normal distribution and (b) the Gumbel distribution. How many times was the P_{10} (normal) exceeded in the 20-year annual precipitation record given in Problem 12.2.1?
- 12.5.2.** Annual precipitation measurements (P_i in inches) for Mythical City, Indiana, were given in Problem 12.2.1. In Problem 12.2.2, the statistical parameters for this annual series were computed to be $m_l = 1.60$, $s_l = 0.0379$, and $G_l = 0.0144$. Determine the magnitude of the 10-year precipitation depth if the data fit (a) the log-normal distribution and (b) the log-Pearson type III distribution. How many times was the P_{10} (log-normal) exceeded in the 20-year annual precipitation record given in Problem 12.2.1?
- 12.5.3.** Annual maximum flood discharges (Q_i in m^3/sec) for the Wolf River (tributary to Lake Michigan) at New London, Wisconsin, from 1950 to 1969 were given in Problem 12.2.3. The statistical parameters for this annual series were computed to be $m = 214$ m^3/sec , $s = 94.6$ m^3/sec , and $G = 0.591$. Determine the return interval of the 1953 flood (430 m^3/sec) if the data fit (a) the normal distribution and (b) the Gumbel distribution.
- 12.5.4.** Annual maximum flood discharges (Q_i in m^3/sec) for the Wolf River (tributary to Lake Michigan) at New London, Wisconsin, from 1950 to 1969 were given in Problem 12.2.3. In Problem 12.2.4, the statistical parameters for this annual series were computed to be $m_l = 2.29$, $s_l = 0.202$, and $G_l = -0.227$. Determine the return interval of the 1953 flood (430 m^3/sec) if the data fit (a) the log-normal distribution and (b) the log-Pearson type III distribution.
- 12.5.5.** In Example 12.6 we determined that the log-transformed Meherrin River data fit the log-normal distribution at a significance level of $\alpha = 0.10$. Would our conclusion be different for $\alpha = 0.05$ or $\alpha = 0.50$?
- 12.5.6.** In Example 12.6 we determined that the log-transformed Meherrin River data fit the log-normal distribution at a significance level of $\alpha = 0.10$. How many additional flows would need to be in the first class interval (lowest flows, transferred from the middle interval) for the test to fail at the same significance level? How many additional flows would need to be in the first class interval (lowest flows, transferred from the fifth interval or largest flows) for the test to fail at the same significance level?
- 12.5.7.** Consider the annual maximum discharges on the Meherrin River given in Example 12.1 with sample statistics $m = 9,820$ cfs and $s = 4,660$ cfs. Test the goodness of fit of these data to the normal distribution at a significance level of $\alpha = 0.10$. Use five class intervals ($k = 5$).
- 12.5.8.** Consider the annual maximum discharges on the Meherrin River given in Example 12.1 with sample statistics $m = 9,820$ cfs and $s = 4,660$ cfs. Test the goodness of fit of these data to the Gumbel distribution at a significance level of $\alpha = 0.50$. Use five class intervals ($k = 5$).
- 12.5.9.** Annual precipitation measurements (P_i in inches) for the 20-year record at Mythical City, Indiana, were given in Problem 12.2.1. The statistical parameters for this annual series were computed to be $m = 40.0$ in., $s = 3.50$ in., and $G = 0.296$. Determine the 90-percent confidence limits for the 10-year precipitation depth assuming the data fit (a) the normal distribution and (b) the Gumbel distribution.

- 12.5.10.** Annual precipitation measurements (P_i in inches) for the 20-year record at Mythical City, Indiana, were given in Problem 12.2.1. In Problem 12.2.2, the statistical parameters for this annual series were computed to be $m_l = 1.60$, $s_l = 0.0379$, and $G_l = 0.0144$. Determine the 90-percent confidence limits for the 10-year precipitation depth assuming the data fit (a) the log-normal distribution and (b) the log-Pearson type III distribution.
- 12.5.11.** Consider the annual maximum discharges on the Meherrin River given in Example 12.1 with sample statistics $m = 9,820$ cfs and $s = 4,660$ cfs. It was shown in Problem 12.5.7 that the Meherrin River data fit the normal distribution. Determine the 1.25-, 2-, 10-, 25-, 50-, 100-, and 200-year peak discharges and the 90-percent confidence limits for the Meherrin River.
- 12.5.12.** Consider the annual maximum discharges on the Meherrin River given in Example 12.1 with sample statistics $m = 9,820$ cfs and $s = 4,660$ cfs. It was shown in Problem 12.5.8 that the Meherrin River data fit the Gumbel distribution. Determine the 1.25-, 2-, 10-, 25-, 50-, 100-, and 200-year peak discharges and the 90-percent confidence limits for the Meherrin River.

(SECTION 12.6)

- 12.6.1.** The largest Meherrin River discharge recorded at the gauge site (Example 12.10) during the 39-year period was 21,100 cfs. Based on a log-normal distribution, determine the return period for this discharge in two different ways, noting that $m_l = 3.95$ and $s_l = 0.197$.
- 12.6.2.** A bridge will be built over the Meherrin River near the site that the discharge measurements were taken in Example 12.10. The service life of the proposed bridge is 50 years. If a design discharge of 25,500 cfs is used for the bridge, based on the log-normal distribution: (a) what is the probability that the bridge will be flooded in any one year? (b) what is the probability that the bridge will be flooded at least once over its service life?
- 12.6.3.** Annual precipitation measurements (P_i in inches) for the 20-year record at Mythical City, Indiana, were given in Problem 12.2.1. Determine the return periods assigned to these data values for plotting purposes. Prepare a probability plot (normal distribution) of the annual maximum precipitation series. Use normal distribution (probability) paper found on the Internet or use Figure 12.2. In addition, draw the theoretical (straight line) probability distribution on the plot. The statistical parameters for this annual series were computed to be $m = 40.0$ in., $s = 3.50$ in., and $G = 0.296$.
- 12.6.4.** Obtain annual precipitation depths from the Internet (e.g., from the National Weather Service) for the full gauge record at a location of interest. Using this data, determine the following:
- the mean, standard deviation, and skew coefficient for this data series;
 - the mean, standard deviation, and skew coefficient for the log-transformed data;
 - the goodness of fit of these data to the normal distribution ($\alpha = 0.10$, $k = 5$);
 - the goodness of fit of these data to the log-normal distribution ($\alpha = 0.10$, $k = 5$);
 - the 2-, 10-, 25-, 50-, and 100-year precipitation depths and the 90-percent confidence limits for the normal distribution and the log-normal distribution;
 - a normal probability plot of the precipitation series with the theoretical (straight-line) probability distribution and the 90-percent confidence band; and
 - a log-normal probability plot of the precipitation series with the theoretical (straight-line) probability distribution and the 90-percent confidence band.
- Note:* Special probability paper is available on the Internet at various locations.
- 12.6.5.** Obtain annual peak discharge values from the Internet (e.g., from the U.S. Geological Survey) for the full gauge record at a location of interest. Using this data, determine the following:
- the mean, standard deviation, and skew coefficient for this data series;
 - the mean, standard deviation, and skew coefficient for the log-transformed data;
 - the goodness of fit of these data to the Gumbel distribution ($\alpha = 0.10$, $k = 5$);

- (d) the goodness of fit of these data to the log-Pearson type III distribution ($\alpha = 0.10$, $k = 5$);
- (e) the 2-, 10-, 25-, 50-, and 100-year peak discharge values and the 90-percent confidence limits for the Gumbel and log-Pearson type III distributions;
- (f) a Gumbel probability plot of the annual peak discharge series with the theoretical (straight-line) probability distribution and the 90-percent confidence band; and
- (g) a log-Pearson type III probability plot of the annual peak discharge series with the theoretical (straight-line) probability distribution and the 90-percent confidence band.

Note: Special probability paper is available on the Internet at various locations.

This page intentionally left blank

Symbols

A	cross-sectional area, watershed area	e	roughness height, efficiency, pipe wall thickness
b	channel width, confined aquifer depth	e_p, e_m	pump and motor efficiency
BHP	brake horse power	F	force, channel freeboard
C	Chezy's coefficient, celerity (surface wave speed), weir coefficient, runoff coefficient (rational method)	F_s	specific force
C_d	discharge coefficient	f	friction factor
C_{HW}	Hazen-Williams coefficient	G	skew coefficient
CN	SCS curve number	g	gravitational acceleration
$^{\circ}C$	degree Celsius	H	total head
D	diameter, hydraulic depth	H_a	approaching head
d	water depth (occasionally), soil interstitial space dimension	H_p	pump head
E	energy per unit weight of water (energy head), water surface elevation, evaporation	H_s	static pressure head, elevation rise
E_b	modulus of elasticity (bulk)	H'_s	net positive suction head
E_c	composite modulus of elasticity	H_{SH}	system head
E_p	modulus of elasticity (pipe material)	H_v	velocity head
E_s	specific energy	HGL	hydraulic grade line
EGL	energy grade line	h	elevation (position) head, depth of water in an aquifer
		h_b	bend loss
		h_c	contraction loss

h_d	discharge (exit) loss	S	slope, shape number, storage, SCS maximum potential retention
h_e	entrance loss	S_c	critical slope, aquifer storage constant
h_E	enlargement loss	S_e	slope of the energy grade line
h_L	head loss	S_f	energy (friction) slope
h_f	friction loss	S_o	channel slope
Σh_{fc}	friction loss in clockwise direction	S_w	water surface slope
$h_v =$	valve loss	S_y	unconfined storage coefficient
Σh_{fcc}	friction loss in counterclockwise direction	sp. gr.	specific gravity
I	moment of inertia, linear impulse, infiltration, rainfall intensity, reservoir inflow	s	aquifer drawdown, standard deviation
I_o	moment of inertia about the neutral (centroidal) axis	s_l	standard deviation of logs
K	coefficient of permeability, coefficient of energy loss	T	temperature, top width of a channel, aquifer transmissibility, transpiration, torque, return period or recurrence interval
K_p	SCS unit hydrograph constant	T_c	time of concentration
K_T	frequency factor	T_L	hydrograph lag time
k	number of class intervals	T_p	time to peak
kk	number of sample statistics	T_{t1}	sheet flow travel time
L	hydraulic length	t	time
L_T	lower confidence limit	U_T	upper confidence limit
L	liters	u, v, w	velocity in x, y, z direction
M	moment, momentum, total mass	u_i, u_o	impeller speed (inside and outside)
m	meter	v_i, v_o	radial velocity (inside and outside)
m	mass, model, mean, side slope, number of flow channels (flow net)	$v_{i, v_{o}}$	tangential velocity (inside, outside)
M, N	exponents of the gradually varied flow function	V	mean velocity
N	newtons	V_G	mean square error of sample skew
N	number of values in sample	V_i, V_o	inlet, outlet velocity
N_f	Froude number	V_s	seepage velocity
N_R	Reynolds number	Vol	volume
N_r	pump rotational speed	W	weight, work
N_S	specific speed of pump	$W(u)$	well function
N_w	Weber number	x, y, z	coordinate axes
NPSH	net positive suction head	y, y_P	water depth, depth to center of pressure
n	Manning's coefficient, number of equipotential drops (flow net)	y_I, y_2	initial, sequent depth
O	reservoir outflow	y_n, y_c	normal depth, critical depth
P	wetted perimeter, pressure, precipitation, SCS cumulative rainfall depth	Y	average watershed slope
P_i, P_o	input, output power	z	elevation, slope (run per unit rise)
p	pressure, probability	α	porosity, energy coefficient, angle, statistical significance level
Q	volumetric flow rate (discharge)	β	vane angle, momentum coefficient, confidence level (as a fraction)
Q_i, Q_o	inflow and outflow	ΔD	effective storm duration
q	discharge per unit width	ε	Poisson's ratio
q_p	SCS unit hydrograph peak discharge	γ	specific weight
R	radius, risk (hydrologic), SCS cumulative runoff depth	μ	absolute viscosity
R_h	hydraulic radius	ν	kinematic viscosity
r	variable radius, homologous ratio, rank of numbers in a listing	θ	angle
r_i, r_o	inside, outside radius	ρ	density
r_o, r_w	radius of influence, well radius	σ	surface tension, cavitation parameter
		τ	shear stress
		τ_o	wall shear stress
		χ^2	chi-square test statistic
		ω	angular velocity

Answers to Selected Problems

Chapter One

- 1.2.1 $E_{\text{total}} = 2.72 \times 10^7$ calories
1.2.3 $E_{\text{total}} = 7.57 \times 10^4$ calories
1.2.5 Time = 62.3 minutes
1.3.1 $Vol = 8.32 \times 10^{-2} \text{ m}^3$
1.3.3 $\gamma = 133 \text{ kN/m}^3$; sp. gr. = 13.6
1.3.5 $m = 800 \text{ kg}$, W (moon) = 1,310 N
1.3.7 $Vol_2 = 104.4 \text{ m}^3$ (4.40%)
1.3.9 $7.376 \times 10^{-1} \text{ ft} \cdot \text{lb}$
1.4.1 $[\mu_{\text{air}} / \mu_{\text{water}}]_{20^\circ\text{C}} = 1.813 \times 10^{-2}$
 $[\nu_{\text{air}} / \nu_{\text{water}}]_{20^\circ\text{C}} = 15.04$, etc.
1.4.3 1 poise = $2.088 \times 10^{-3} \text{ lb} \cdot \text{sec/ft}^2$
1 stoke = $1.076 \times 10^{-3} \text{ ft}^2/\text{sec}$
1.4.5 at $y = 0 \text{ ft}$, $\tau = -9.00 \text{ N/m}^2$
at $y = 1/3 \text{ ft}$, $\tau = 27.0 \text{ N/m}^2$, etc.
1.4.7 $\nu = 4.57 \times 10^{-3} \text{ ft/sec}$
1.4.9 $\mu = 3.65 \times 10^{-1} \text{ N} \cdot \text{sec/m}^2$

- 1.5.1 for $h = 3.0 \text{ cm}$, $D = 0.0971 \text{ cm}$
for $h = 2.0 \text{ cm}$, $D = 0.146 \text{ cm}$
for $h = 1.0 \text{ cm}$, $D = 0.291 \text{ cm}$
1.5.3 $\sigma = 1.61 \times 10^{-3} \text{ lb/ft}$
1.5.5 $h_2 = 1.204(h_1)$, a 20% increase!
1.6.1 $E_b = 9.09 \times 10^9 \text{ N/m}^2$
1.6.3 $W = 7,500 \text{ lbs}$, $\rho = 1.95 \text{ slug/ft}^3$

Chapter Two

- 2.2.1 $P = 7.37 \times 10^6 \text{ N/m}^2$
 $= 1,070 \text{ psi}$ (gauge)
2.2.3 $P_{\text{atm}} = 99.9 \text{ kN/m}^2$; error = 4.20%
2.2.5 $F = 2,110 \text{ N}$, $F = 1,060 \text{ N}$ (side)
2.2.7 $P_{\text{bottom}} = 5.60 \times 10^4 \text{ N/m}^2$,
 $h = 6.75 \text{ m}$
2.2.9 $P_{\text{air}} = 20.0 \text{ psi}$ (gauge),
 $P_{\text{abs}} = 34.7 \text{ psi}$
2.4.1 $h_{\text{oil}} = 61.5 \text{ cm}$

- 2.4.3 $h = 74.2$ cm
 2.4.5 $h = 3.00$ ft (manometer is correct)
 2.4.7 $P = 11.5$ psi
 2.4.9 $P_A - P_B = 7.85$ psi
 2.4.11 $h = 0$
 2.5.1 $F = 58.7$ kN, $y_p = 1.50$ m
 2.5.3 $F = 7.69$ kN, $y_p = 1.46$ m
 2.5.5 $h = 1.25$ m
 2.5.7 Locate 0–0' 4.07 ft above gate bottom.
 2.5.9 $F = 149$ kN, $y_p = 2.76$ m
 2.5.11 $d = 8.77$ ft; any less makes it close
 2.5.13 $h_A = [(\gamma_B) / (\gamma_A)](h_B)$
 2.6.1 $F = 1,470$ kN; $\theta = 48.1^\circ \searrow$
 2.6.3 $W = 3,520$ lb
 2.6.5 $F = 3,650$ kN; $\theta = 14.4^\circ \searrow$, through point O
 2.6.7 $F = 7,050$ lb (on cylinder end)
 $F = 22,700$ lb; $\theta = 8.93^\circ \searrow$ passing through center of tank
 2.6.9 $F_H = 5,980$ lb; $y_p = 9.50$ ft
 $F_{V_{\text{Tri}}} = 1,000$ lb; 1.33 ft from wall
 $F_{V_{\text{quad}}} = 780$ lb; 1.70 ft from wall
 2.6.11 $\gamma_{\text{cone}} = 19,100$ N/m³; sp. gr. = 1.95
 2.8.1 $\gamma_{\text{metal}} = 6.14 \times 10^4$ N/m³;
 sp. gr. = 6.27
 2.8.3 $h = 1.45$ m
 2.8.5 $T = 260$ lb
 2.8.7 $\theta = 40.6^\circ$
 2.8.9 $h = 1.77$ m; $h_b = 1.37$ m;
 $h_g = 1.52$ m; $GM = 0.151$ m
 2.8.11 $GM = 5.93$ ft;
 $M = 1.12 \times 10^6$ ft · lb

Chapter Three

- 3.3.1 $F_x = 371$ N \rightarrow
 3.3.3 $V = 105$ ft/sec
 3.3.5 $P = 2.83 \times 10^5$ N/m² (pascals)
 3.3.7 $F = 43.6$ kN; $\theta = 44.4^\circ \searrow$
 3.5.1 $f = 0.011$; turbulent–transitional zone
 3.5.3 $\Delta P = 24.9$ kN/m²
 3.5.5 $h = 10.1$ m
 3.5.7 $Q = 78.8$ m³/sec
 3.5.9 $L = 1,340$ ft
 3.5.11 $D = 1.52$ ft ≈ 1.5 ft
 3.5.15 $Q_{\text{leak}} = 8$ L/sec
 3.7.1 $h_f = 408$ m (DW), 337 m (HW), 469 (M)
 3.7.3 $Q = 0.309$ m³/sec (DW), 0.325 m³/sec (HW), 0.288 m³/sec (M)
 3.7.5 $Q = 80.6$ m³/sec (HW), 61.8 m³/sec (M)
 3.7.7 $Q_{30} = 0.136$ m³/sec; $Q_{20s} = 0.0934$ m³/sec
 3.7.9 $C_{HW} = 99.9$
 3.11.1 $h_c = 0.606$ m; $h_E = 1.03$ m
 3.11.3 $K_V = 3.16$
 3.11.5 $Q = 0.430$ m³/sec
 3.11.7 $Q = 2.49$ ft³/sec
 3.11.9 $t = 485$ sec
 3.11.11 The expansion coefficient is always larger.
 3.12.1 $[(D_E^{5.33}) / (n_E^2 L_E)]^{1/2} = \Sigma[(D_i^{5.33}) / (n_i^2 L_i)]^{1/2}$
 3.12.3 $h_{f_{AF}} = 54.0$ ft;
 $Q_1 = 87.5$ cfs; $Q_2 = 32.5$ cfs
 3.12.5 $h_{f_{AF}} = 73.8$ m;
 $Q_1 = 20.4$ m³/sec; $Q_2 = 59.6$ m³/sec
 3.12.7 $L_E = 2,344$ m

Chapter Four

- 4.1.3 $V = 1$ m/s and $D = 1$ m; for $V = 2$ m/s and $D = 0.5$ m; etc.
 4.1.5 $h_A = 786$ m
 4.1.7 $Q = 10.2$ L/sec
 4.1.9 $Q_8 = 3.75$ cfs;
 $Q_{16} = 16.1$ cfs (+429%)
 4.1.11 $D = 0.82$ ft
 4.1.13 $P_0 = 43.9$ kPa
 4.1.15 $Q_{AB} = Q_{CD} = 18.0$ cfs,
 $h_B = 209.7$ ft; $h_C = 117.1$ ft;
 $Q_{BC_1} = 6.84$ cfs; and $Q_{BC_2} = 19.2$ cfs
 4.2.1 $P_s/\gamma = -6.10$ m
 (> -10.1 m; no concern)
 4.2.3 $P_s/\gamma = -34.9$ ft (cavitation will occur)
 4.2.5 $D_B = 0.174$ m = 17.4 cm
 4.2.7 $L = 56.0$ m
 4.2.9 $H_p = 75.1$ m
 4.2.11 $P_0 = 28.6$ kPa

4.3.3 $Q_1 = 2.055 \text{ m}^3/\text{sec}$,
 $Q_2 = 0.002 \text{ m}^3/\text{sec}$,
 $Q_3 = 2.056 \text{ m}^3/\text{sec}$, $(P/\gamma)_J = 10 \text{ m}$

4.3.5 $Q_1 = 168 \text{ ft}^3/\text{sec}$,
 $Q_2 = 52 \text{ ft}^3/\text{sec}$,
 $Q_3 = 221 \text{ ft}^3/\text{sec}$,
 $\text{Elev}(J) = 5,111.2 \text{ ft}$

4.3.7 $h_3 = 3,143.9 \text{ ft} \approx 3,144 \text{ ft}$

4.4.1 (a) $P_F = 167.4 \text{ kPa}$, (b) node F

4.4.3 (a) $Q_1 = 4.49 \text{ L/sec}$, $Q_2 = 7.51 \text{ L/sec}$,
 $h_f = 13.7 \text{ m}$;

(b) $Q_1 = 4.49 \text{ L/sec}$, $Q_1 = 7.51 \text{ L/sec}$,
 $h_f = 13.4 \text{ m}$

4.4.5 $Q_{AB} = 506 \text{ L/sec}$, $Q_{AC} = 494 \text{ L/sec}$
 $Q_{BD} = 517 \text{ L/sec}$, $Q_{CE} = 483 \text{ L/sec}$
 $Q_{CB} = 11 \text{ L/sec}$, $Q_{ED} = 33 \text{ L/sec}$

4.4.7 $Q_{AB} = 10.9 \text{ cfs}$, $Q_{AC} = 14.1 \text{ cfs}$
 $Q_{BD} = 7.06 \text{ cfs}$, $Q_{CE} = 7.13 \text{ cfs}$
 $Q_{BF} = 3.81 \text{ cfs}$, $Q_{CF} = 1.00 \text{ cfs}$
 $Q_{FG} = 4.81 \text{ cfs}$, $Q_{GD} = 0.94 \text{ cfs}$
 $Q_{GE} = 3.87 \text{ cfs}$; pressures okay

4.4.11 $Q_{AB} = 150 \text{ L/sec}$, $Q_{FA} = 50 \text{ L/sec}$
 $Q_{BC} = 15 \text{ L/sec}$, $Q_{BD} = 135 \text{ L/sec}$
 $Q_{DE} = 38 \text{ L/sec}$, $Q_{EC} = 15 \text{ L/sec}$
 $Q_{FG} = 250 \text{ L/sec}$, $Q_{GD} = 153 \text{ L/sec}$
 $Q_{GH} = 97 \text{ L/sec}$, $Q_{EH} = 23 \text{ L/sec}$

4.4.13 $Q_1 = 8.367 \text{ cfs}$, $Q_2 = 0.515 \text{ cfs}$
 $Q_3 = 1.852 \text{ cfs}$, $Q_4 = 0.877 \text{ cfs}$
 $Q_5 = 1.779 \text{ cfs}$, $Q_6 = 1.983 \text{ cfs}$
 $Q_7 = 6.387 \text{ cfs}$, $Q_8 = 8.859 \text{ cfs}$

4.5.3 $\Delta P = 574 \text{ psi}$; $\Delta P = 383 \text{ psi}$

4.5.5 $\Delta P = 10.1 \text{ MPa}$

4.5.7 $e = 15.6 \text{ mm} \approx 16 \text{ mm}$

4.5.9 $e = 1.63 \text{ cm}$

4.6.3 $y_{\max} \approx 9.80 \text{ m}$

4.6.5 $D_s = 3.21 \text{ m}$

Chapter Five

5.1.1 $P_m = 62.3 \text{ kW}$

5.1.3 $e = 0.588$ (58.8%)

5.1.5 $T = 298 \text{ kN}\cdot\text{m}$

5.1.7 $\omega = 185 \text{ rpm}$, $P_i = 420 \text{ hp}$

5.5.1 $h_v = 77.6 \text{ ft}$, not efficient system

5.5.3 $Q = 0.595 \text{ m}^3/\text{sec}$, $V = 3.00 \text{ m/sec}$

5.5.5 $Q = 1.08 \text{ L/sec}$, $H_p = 15.0 \text{ m}$

5.6.1 (d) two pumps in parallel

(e) four pumps, two parallel pipes with two pumps in series in each pipe

5.6.3 $Q \approx 37.5 \text{ cfs}$; $H_p \approx 235 \text{ ft}$;
 $V = 11.9 \text{ ft/sec}$; minor losses would not change the answer significantly.

5.7.1 $Q_{\text{sys}} = 4.3 \text{ m}^3/\text{sec}$; $H_p = 20.8 \text{ m}$
 $Q_1 = 3.0 \text{ m}^3/\text{sec}$; $Q_2 = 1.3 \text{ m}^3/\text{sec}$

5.7.3 $Q_1 \approx 22.5 \text{ cfs}$; $Q_2 \approx 11.5 \text{ cfs}$
 $Q_3 \approx 34.0 \text{ cfs}$; $H_{p_A} \approx 90 \text{ ft}$

5.9.1 $h_p \leq 9.58 \text{ ft}$

5.9.3 $h_p \leq -0.27 \text{ m}$

5.9.5 $h_p \leq 4.97 \text{ m}$

5.10.3 $e = 0.80$

5.10.5 $Q = 9.56 \text{ m}^3/\text{sec}$, $e = 0.891$

5.11.1 $H_{\max} = 41.5 \text{ m}$ when $Q = 0$; the match point is $Q = 69 \text{ L/sec}$; $H_p = 39.5 \text{ m}$

5.11.3 The match point is
 $\omega = 4,350 \text{ rpm}$, $e \approx 61\%$,
 $Q \approx 120 \text{ L/sec}$, $H_p \approx 45 \text{ m}$

5.11.5 $Q \approx 800 \text{ gpm}$ (1.78 cfs)
 $H_p \approx 102 \text{ ft}$, $e_p \approx 82\%$, $P_i \approx 26 \text{ hp}$
 $P_o = \gamma Q H_p = 20.6 \text{ hp}$
 $e_p = P_o/P_i = 79\% \approx 82\%$

5.11.7 Best: Use two pumps (IV) in parallel
 $\omega = 3,550 \text{ rpm}$, $e \approx 61\%$,
 $Q \approx 150 \text{ L/sec}$, $H_p \approx 30 \text{ m}$

Chapter Six

6.1.1 (a) unsteady, varied; (b) steady, varied

6.2.1 $n = 0.030$; $0.0275 < n < 0.0330$

6.2.3 $Q = 22.8 \text{ m}^3/\text{sec}$

6.2.5 $y_n = 0.875 \text{ ft}$

6.2.7 $m = 3.34 \text{ m/m}$ (with $n = 0.022$)

6.2.9 $y_2 = 4.37 \text{ m}$

6.3.1 $y = 7.60 \text{ m}$; $b = 8.74 \text{ m}$

6.3.3 $d_o = 0.952 \text{ m}$

6.3.5 $m = 1$; $\theta = 45^\circ$

6.4.1 $N_f = 0.233$ (subcritical); $y_c = 1.37 \text{ m}$ because $1.37 \text{ m} < 3.6 \text{ m}$, subcritical flow

6.4.3 $V = 7.53 \text{ ft/sec}$;
 $N_f = 0.755$ (subcritical)

6.4.5 $N_f = 1.00$ (critical flow); $E = 9.00 \text{ ft}$
 $S = 0.0143$

6.4.7 $y_n = 6.92 \text{ ft}$; $y_c = 3.90 \text{ ft}$

6.4.9 $\Delta z = 0.39 \text{ m}$

- 6.5.3 $\Delta E = 1.23$ m, $Q = 48.2$ m³/sec,
 $N_{F_1} = 3.07$, $N_{F_2} = 0.403$
- 6.8.1 (a) S-2; (b) M-2; (c) M-1;
 (d) H-2 and M-2
- 6.8.3 M-2
- 6.8.5 M-2, 0.75 m
- 6.8.7 2.51 m, 2.42 m, within 1% of y_h
- 6.8.9 M-1, 5.16 m, 4.20 m, 2.77 m,
 1.10 m
- 6.8.11 M-1, 4.84 ft, 4.60 ft, 4.23 ft
- 6.8.13 S-1, 3.19 m, 2.98 m, 2.76 m
- 6.9.1 $b = 3.53$ ft and $y = 4.47$ ft
- 6.9.3 $y = 4.95$ ft, $b = 3.00$ ft

Chapter Seven

- 7.1.1 $\alpha = 0.378$
- 7.1.3 $\alpha = 0.338$
- 7.1.5 $t = 4,760$ sec = 79.3 min
 ≈ 80 min/test
 $t = 18.7$ hrs (tracer test time)
- 7.1.7 $t = 19.1$ hours
- 7.1.9 $Q = 0.315$ cfs (ft³/sec)
- 7.2.3 $r_o = 1,790$ ft
- 7.2.5 $s = 9.60$ m (30 m from well)
- 7.2.7 $T = 1,050$ m²/day
- 7.2.9 $r_o = 453$ m
- 7.2.11 $Q = 4.36 \times 10^{-3}$ m³/sec
- 7.3.1 $s = 5.19$ m, 6.71 m, and 7.39 m
- 7.3.3 $r_2 = 392$ m
- 7.3.5 $s = 9.83$ m
- 7.4.1 $T = 4.39 \times 10^{-3}$ m²/s; $r_o = 785$ m
- 7.4.3 $T = 39.0$ m²/hr; $r = 381$ m
- 7.4.7 $T = 2.11$ m²/hr;
 $S = 8.04 \times 10^{-5}$; $s = 1.97$ m
- 7.4.9 $T = 1.53$ m²/hr, $S = 1.01 \times 10^{-4}$,
 $s = 5.16$ m
- 7.5.3 $s_{30} = 19.2$ m, $s_w = 23.9$ m
- 7.5.5 $Q = 1.30$ cfs
- 7.5.7 $s = 47.4$ ft; $t = 279$ hr
- 7.8.1 $q = 41.2$ m³/day per meter
- 7.8.3 $q = 0.0171$ m³/hour per meter
 $V_S = 4.53 \times 10^{-6}$ m/sec
- 7.8.5 $q = 3.56 \times 10^{-6}$ m³/sec per meter,
 20% seepage reduction

- 7.9.1 $Q = 0.509$ m³/day
 $V_S = 4.09 \times 10^{-8}$ m/sec
- 7.9.3 $q = 0.994$ m³/day-m
- 7.9.5 $Q = 98.8$ ft³/day
 $V_S = 1.99 \times 10^{-6}$ ft/sec

Chapter Eight

- 8.3.1 $FR_{\text{slide}} = 1.21$ (not safe)
- 8.3.3 $FR_{\text{slide}} = 2.11$ (safe)
 $FR_{\text{over}} = 3.71$ (safe)
- 8.3.5 $P_T = 25.0$ kN/m²
 $P_H = 638$ kN/m²
- 8.3.7 *Hint:* Let $e = B/6$ in Equation 8.4.
- 8.3.9 At crest: $\sigma = 0$
 At mid: $\sigma = 3.46 \times 10^3$ kN/m²
 At base: $\sigma = 4.83 \times 10^3$ kN/m²
- 8.5.1 $y_c = 0.313$ m, $Q = 2.19$ m³/sec
- 8.5.3 $Q = 3.66$ m³/sec, $V = 2.28$ m/sec
- 8.5.5 $Q = 9.20$ m³/sec, $Q = 10.1$ m³/sec
- 8.5.7 $x = 2.10$ ft
- 8.6.1 (a) $Q = 625$ cfs; (b) $Q = 899$ cfs;
 (c) $Q = 902$ cfs
- 8.6.3 $V_a = 0.921H_s^{1/2}$
- 8.6.5 $L = 38.0$ m; $x_{P.T.} = 1.99$ m;
 $y_{P.T.} = -1.12$ m
- 8.7.1 $y = 7.45$ ft
- 8.7.3 $y = 3.87$ m
- 8.7.5 $y = 11.5$ m, 12.2 m, and 12.4 m
- 8.8.1 $h_c - h_1 = 5.65$ m
- 8.8.3 $Q = 686$ cfs, $P_c/\gamma = -24.5$ ft
- 8.8.5 $P_c/\gamma = -37.5$ ft; yes,
 cavitation danger
- 8.8.7 $A = 1.65$ m²
- 8.9.1 (a) $h_L = 3.10$ m $>$ 2.07 m
 (will not work)
 (b) $h_L = 3.10$ m $>$ 2.35 m
 (will not work)
- 8.9.3 $Q = 10.0$ m³/sec
- 8.9.5 (a) $D = 6.67$ ft; (b) $D = 5.00$ ft
- 8.9.7 $D = 0.99$ m (or 1.0 m)
- 8.10.1 $d_2 = 4.1$ ft; $L = 11.1$ ft; $\Delta E = 9.9$ ft
 Efficiency = 0.31 or 31%
- 8.10.3 Type II; $d_2 = 5.2$ m; $L = 22.4$ m;
 $\Delta E = 23.3$ m; Efficiency = 18%

Chapter Nine

- 9.1.1 sp. gr. (oil) = 0.732
- 9.1.3 sp. gr. (oil) = 0.850
- 9.2.1 $P = 31.3 \text{ kPa}$;
 $V = 1.38 \text{ m/sec}$ (not average)
- 9.2.3 $V = 20.3 \text{ ft/sec}$; $Q = 399 \text{ cfs}$
- 9.2.5 $V = 10.0 \text{ ft/sec}$; $V = 38.9 \text{ ft/sec}$
- 9.3.1 $Q = 0.513 \text{ m}^3/\text{sec}$
- 9.3.3 $\Delta P = 52.1 \times 10^3 \text{ Pa} = 52.1 \text{ kPa}$
- 9.3.5 $\Delta h = 0.661 \text{ ft} = 7.93 \text{ in.}$
- 9.3.7 $\Delta h = 0.0526 \text{ m} = 5.26 \text{ cm}$
- 9.4.1 $Q = 13.3 \text{ m}^3/\text{sec}$; $Q = 5.93 \text{ m}^3/\text{sec}$
- 9.4.3 $L = 3.24 \text{ m}$
- 9.4.5 $Q = 11.0 \text{ m}^3/\text{sec}$
- 9.4.7 $\Delta h = 1.77 \text{ ft}$
- 9.4.9 $p = 2.08 \text{ m}$
- 9.4.11 BG: $q = 3.47LH^{3/2}$ and
 $q = 2.46LH^{3/2}$
- 9.4.13 $Q = 0.503H^{2.61}$;
BG: $Q = 0.810H^{2.58}$

Chapter Ten

- 10.2.1 $y_m = 0.400 \text{ m}$ and $b_m = 0.800 \text{ m}$
- 10.2.3 $L_p/L_m = 241+$ (use $L_r = 250$); thus,
 $L_m = 67.6 \text{ ft}$ and
 $Q_m = 1.70 \times 10^{-3} \text{ cfs}$
- 10.2.5 $Q_m = 0.0240 \text{ m}^3/\text{sec}$; $H_m = 10.8 \text{ m}$
- 10.2.7 $F_r = 2.40 \times 10^4$;
 $Q_p = 3.29 \times 10^4 \text{ cfs}$
- 10.2.9 $F_p/L_p = 510 \text{ lbs/ft}$
- 10.3.1 $V_m = 50.0 \text{ m/sec}$
- 10.3.3 $Q_m = 2.63 \text{ cfs}$
- 10.3.5 $Q_m = 1.45 \text{ m}^3/\text{sec}$
- 10.4.1 $T_m = 0.0316 \text{ day}$ (45.5 min)
- 10.4.3 $Q_m = 44.6 \text{ cfs}$, $V_p = 54.2 \text{ ft/sec}$
 $V_m = 17.1 \text{ ft/sec}$
- 10.4.5 $Q_m = 0.848 \text{ cfs}$, $V_m = 6.56 \text{ ft/sec}$
 $N_f = 5.82$, $TW = 7.88 \text{ ft} = y_2$
- 10.5.1 $Q_r = 0.0894$, $F_r = 0.200$
- 10.5.3 $Q_r = 1,000$, $E_r = 10,000$,
 $p_r = 0.01$, $P_r = 10$
- 10.7.1 $\mu_m = 9.01 \times 10^{-7} \text{ N} \cdot \text{sec/m}^2$
- 10.7.3 $V_m = 0.15 \text{ m/sec}$
- 10.8.1 $n_m = 0.021$, $V_m = 0.394 \text{ ft/sec}$

- 10.8.3 $n_m = 0.013$, $V_m = 0.213 \text{ m/sec}$
 $Q_m = 2.65 \times 10^{-4} \text{ m}^3/\text{sec}$
 $= 0.266 \text{ L/sec}$, $N_r = 1,940$
(no longer turbulent)
- 10.8.5 $X_r = 116$, $T_r = 14.4$, $Q_m = 0.174 \text{ cfs}$
- 10.9.1 $q = H^{3/2} g^{1/2} \phi'(h/H)$
- 10.9.3 $\Delta P_1 = (V^2 \rho / D) \phi'(DV \rho / \mu)$
- 10.9.5 $V = (\mu / D \rho) \phi'$
 $[\mu \{ \rho (D^3 g)^{1/2} \}] \phi'' [D \rho \sigma / \mu^2]$
or $V = (\mu / D \rho) \phi''' [\sigma / \{ \rho g D^2 \}]$

Chapter Eleven

- 11.1.1 Clouds—water (vapor) holding element;
precipitation—liquid transport;
interception/depression storage/
snowpack—water (or ice) holding, etc.
- 11.1.5 $I = 25,000 \text{ m}^3$ (leakage)
- 11.1.7 $\Delta S = -0.52 \text{ km}^3$; $\Delta S = -0.16 \text{ km}^3$
- 11.2.3 $P_{\text{avg}} = 11.4 \text{ cm}$
- 11.2.5 3.73 in.
- 11.2.7 4.27 cm
- 11.2.9 $P_x = 6.09 \text{ in.}$ (average); $P_x = 5.28 \text{ in.}$
(weighted average based on annual
precipitation)
- 11.3.1 $i_{(\text{peak})} = 3.624 \text{ in./hr}$ (vs. 4.56 in./hr);
at the same time (11.5 to 12.0 hrs)
- 11.4.3 $Q = 62.8 \text{ m}^3/\text{sec}$
- 11.4.5 $Q_{(\text{peak})} \approx 310 \text{ cfs}$
- 11.5.3 Flows (m^3/sec) in 1-hour increments;
0, 4.1, 10.8, 15.1, 11.4, 7.4, 4.4, 2.3, 0
- 11.5.5 The 2-hr unit hydrograph developed has
a peak of 252 cfs at a time of 3 hrs.
- 11.5.7 $Q_{(\text{peak})} = 1,021 \text{ m}^3/\text{sec}$ (at hour 36)
- 11.5.9 $Q_{(\text{peak})} = 101.5 \text{ m}^3/\text{sec}$ (at hour 30)
- 11.6.1 $R = 1.0 \text{ in.}$; Vol. = 16.7 ac-ft
- 11.6.5 $T_p = 0.35 \text{ hr}$; $q_p = 216 \text{ cfs}$
- 11.6.7 $Q_{(\text{peak})} = 703 \text{ cfs}$ (at time 16 min)
- 11.6.9 $Q_{(\text{peak})} = 516 \text{ cfs}$ (at time 35 min)
- 11.7.1 At $h = 1.0 \text{ m}$; outflow = $0.60 \text{ m}^3/\text{sec}$
and storage = 600 m^3
- 11.7.3 $S_p = 4.49 \text{ ac-ft}$;
Elev_p = 884.3 ft, MSL
- 11.7.5 $Q_p = 30 \text{ cfs}$; $S_p = 4.0 \text{ ac-ft}$;
 $H_p = 2.0 \text{ ft}$
- 11.7.7 $Q_p = 262 \text{ cfs}$; Elev_p = 883.5 ft, MSL

- 11.7.9 $h \approx 1.55$ m; $h = 1.91$ m (without leak)
- 11.8.1 $Q_{10} = 36.4$ cfs; $Q_{10} = 122$ cfs (post)
- 11.8.3 $Q_{10} = 22.3$ cfs
- 11.8.5 $Q_{25} = 94.5$ cfs; $Q_{25} = 182$ cfs (post)
- 11.8.9 The second inlet is placed about 185 feet down the street from the first inlet.
- 11.8.11 Pipe sizes (in.): 15, 18, 18, and 21
- 11.8.13 Pipes sizes (in.): AB = 12, CB = 15, BD = 24, and DR = 30.
- 11.8.15 Pipes sizes (in.): AB = 12, CB = 18, BD = 24, and DR = 36.
- Chapter Twelve**
- 12.2.1 $m = 40.0$ in.; $s = 3.50$ in.;
 $G = 0.296$
- 12.2.3 $m = 214$ m³/sec; $s = 94.6$ m³/sec,
 $G = 0.591$
- 12.4.1 $p = 0.20 = 20.0\%$;
 $R = 0.488 = 48.8\%$
- 12.4.3 (a) $R = 91.0\%$; (b) $p = 9.0\%$
- 12.4.5 (a) $p = 10\%$; (b) $R = 34.4\%$
- 12.5.1 (a) $P_{10} = 44.5$ in.;
(b) $P_{10} = 44.6$ in.
- 12.5.3 (a) $T \approx 90$ years;
(b) $T \approx 35$ years
- 12.5.7 Because $\chi^2 < \chi^2_\alpha$ (i.e., because $4.462 < 4.61$), the normal distribution does adequately fit the annual maximum discharge data.
- 12.5.9 $U_{10} = 46.7$ in.; $L_{10} = 43.0$ in. (normal)
 $U_{10} = 46.8$ in.; $L_{10} = 43.0$ in. (Gumbel)
- 12.5.11 $Q_{10} = 15,800$ cfs; $Q_{100} = 20,700$ cfs
- 12.6.1 $T = 33$ years and $T \approx 35$ years.

Index

Absolute humidity, 376

Absolute pressure, 14–16
Absolute viscosity, 5, 52
Abutment reaction, 288, 289
Acre-feet, 375
Adverse channel, 209–211
Alternate depths, 199
Angular acceleration, 349
 deformation, 6
 displacement, 349
 momentum, 144–146, 347
 velocity, 146, 169, 347
Annual exceedence series, 431
Annual maximum series, 431
Antiseep collar, 291
Apparent velocity, 234
Aquifer, 232, 372
 coastal, 264
 confined, 232, 233
 drawdown, 237
 homogeneous, 237
 isotropic, 237
 recovery, 247
 storage coefficient, 242
 transmissivity, 238
 unconfined, 232, 233
Archimedes' principle, 30
Area of influence (well), 237
Artesian spring, 232
Artesian well, 232
Artificial recharge, 265
Atmospheric composition, 2
Atmospheric pressure,
 2, 14, 15
Axial flow pump, 149

Backwater curve, 213, 216

Base flow, 387
Bend meter, 330
Bernoulli's equation, 57
Best hydraulic section, 194
Body force, 9
Boiling point, 3
Booster pump, 152, 153
Bourdon tube gauge, 322
Brake horsepower, 153
Branching pipe systems, 97
Bubble collapse, 3, 93, 166, 304
Buckingham Pi-theorem, 363
Buoyancy force, 30

Calibration (meter), 326, 329

Calorie, 2
Capillary action, 7, 242
Catchment, 374
Cavitation, 3, 93, 165, 166, 304
Celerity, 121
Center of buoyancy, 31
Center of gravity, 31, 33
Center of pressure, 23, 28
Centrifugal force, 144, 197, 330
Centrifugal pump, 143
Centroid of plane areas, 23–25
Channel classification, 209
Channel liners, 222
Channel roughness, 190
Characteristic curves (pump), 153
Chezy's formula, 189
Chi-square test, 443
Cipolletti weir, 333
Closed system, 372
Closure error, 106
Coastal aquifer, 263
Coefficients (discharge)
 bend meters, 330
 nozzle meters, 328
 orifice, 307, 406
 orifice meters, 328
 venturi meters, 331
 weirs, 331–335
Coefficients (energy loss)
 bend (pipe), 76
 contraction, 72
 confusor, 72
 diffuser, 74
 entrance loss, 72
 exit, 75
 valve, 76
Colebrook equation, 63
Compressibility of water,
 8, 119
Condensation, 3, 372
Condensation nuclei, 376
Cone of depression, 237
Confidence limits, 445
Confined aquifer, 232, 233
Confusor, 72
Conservation of
 angular momentum, 144
 energy, 58, 80, 81, 89–92, 95, 97,
 103–106

 mass, 54, 97–98, 104–107, 372,
 403, 404
 momentum, 55, 144, 149, 204,
 301, 334
Continuity equation, 55
Control section, 209, 211, 212
Control volume, 54–57, 149, 150,
 188, 204, 372
Convective precipitation, 376
Critical channel, 209, 211
 depth, 199–201, 208–211, 291,
 292, 335
 flow, 199
Culvert, 305
Current meters, 324–326
Curve number, 393–395
Cutoff wall, 268, 285
Cyclonic precipitation, 376

Darcy's law, 234

Darcy-Weisbach equation, 60
Dams, arch, 282, 283, 288
 buttress, 283
 earth, 283, 289–291
 gravity, 282–288
 simple embankment, 291
 zoned embankment, 291
Datum, reference, 51, 197
Demour's centrifugal pump, 143
Density, 3, 4
Depth, 185
 alternate, 199
 critical, 199–201, 208–211,
 291–292, 335
 flow, 185
 hydraulic, 186, 199
 initial, 203
 normal, 191, 192, 207
 sequent, 203
 uniform, 191, 207
Depression storage, 372, 385
Design event (storm), 380, 381
Design flow rate, 371, 381
Dew point, 376
Diffusor, 74
Dimensional analysis, 345, 362
Direct runoff, 385, 387, 393
Direct step method, 214
Discharge head loss, 75
Dominant force, 350

- Drag, 358
- Drainage area, 374
- Drainage basin, 374
- Drainage blanket, 271
- Drawdown, 237
- Drop inlet, 291
- Dynamic similarity, 350
- Earthquake force (dams), 286**
- Efficiency (pump), 147, 153
 - (channel), 194
- Elasticity, modulus of, 8
 - composite-water/pipe, 121
 - pipe materials, 121
 - water, 8, 119
- Electrical resistivity method, 261
- Elevation head, 57
- Embankment, 289–291
- Emergency spillway, 291, 296
- Energy,
 - coefficient, 197
 - dissipators, 310
 - equation, 58
 - grade line (EGL), 59, 67, 184
 - head, 57
 - kinetic, 56
 - latent, 2
 - open channels, 184, 197
 - pipe flow, 56
 - potential, 56
 - pressure, 56
 - specific, 198
- Equilibrium test (aquifers), 248
- Equipotential line, 267
- Evaporation, 2, 3
- Exceedence probability, 439
- Expansion joint, 121
- Extradados, 288
- Eye of the pump, 144
- Flow net, 267**
- Flotation stability, 31
- Flumes, 335
 - Parshall, 335
 - Venturi, 335
- Force, 9
 - body, 9
 - buoyancy, 30
 - centrifugal, 144, 197, 330
 - earthquake, 286
 - drag, 358
 - hydrostatic, 22–27, 284
 - line, 9, 10
 - pressure, 23
 - resistance, 5, 188
 - sedimentation, 284
 - shear, 5
 - specific, 205
 - surface, 9, 10
 - uplift, 284
 - viscous, 60
 - weight (dam), 284
- Force ratio against sliding, 286
- Force ratio against overturning, 286
- Form drag, 358, 359
- Freeboard, 195, 222
- Free-body diagram, 27, 188, 285
- Friction factor, 60
- Frictionless weir, 292
- Froude number, 200, 310, 355
- Gauge pressure, 14–16, 93**
- Gauging station, 385
- Geometric similarity, 347
- Ghyben-Herzberg relation, 264
- Goodness of fit, 443
- Gradually varied flow, 186, 190, 206
- Gravitational acceleration, 4
- Groundwater runoff, 385
- Gumbel distribution, 436, 440
- Hagen-Poiseuille law, 61**
- Hardy Cross method, 105
- Hazen-Williams equation, 67
- Head, 57
 - elevation, 57
 - energy, 56, 157
 - kinetic, 56
 - loss, 58
 - net positive suction, 166
 - pressure, 16, 57
 - pump, 146, 153
 - suction, 166
 - system, 155
 - velocity, 57
- Head loss, 58
 - bend (pipe), 75
 - contraction, 71
 - entrance, 72
 - exit (discharge), 75
 - expansion, 73
 - friction, 60
 - major, 60
 - minor, 60
 - valve, 76
- Heat (energy), 2
 - fusion, 2
 - latent, 2
 - specific, 2
 - vaporization, 2
- Heel of dam, 285
- Horizontal channel, 209
- Humidity, 376
- Hydraulically rough pipe, 63
- Hydraulically smooth pipe, 62
- Hydraulic, 1
 - conductivity, 234
 - depth, 186, 199
 - efficiency, 194
 - grade line (HGL), 59, 184
 - jack, 16
 - jump, 203, 204
 - radius, 67, 186
- Hydrograph, 387
- Hydrologic budget, 372
- Hydrologic cycle, 372
- Hydrologic risk, 440
- Hydrology, 371
- Hydrostatic force, 22–27, 284
- Hydrostatic pressure, 14, 15, 22
- Hyetograph, 383, 387
- IDF Curve, 410, 452**
- Impulse-momentum, 55, 144, 149
- Infiltration, 372
- Interception, 372, 385
- Interflow, 385
- Jacob's method, 243**
- Jet pump, 152
- Joint (expansion), 121
- Junction (branching pipes), 97
- Junction equation, 104
- Keyway, 286, 290**
- Kinematic similarity, 348
- Kinematic viscosity, 6
- Kinetic energy, 56, 197
- Lag time, 398**
- Lake overturn, 1
- Laminar flow, 52, 60
- Laminar sublayer, 62
- Latent energy, 2
- Linear impulse, 149
- Linear momentum, 144, 149
- Linearity, 391
- Line force, 9, 10
- Logarithmic method (surge tanks), 129
- log-normal distribution, 436, 441
- log-Pearson type III distribution, 437, 441
- Loop equation, 104
- Low level outlet, 290, 291

Major (head) loss, 60

Manning's equation, 69, 190
 Manning's "n" values, 69, 190
 Manometer, 18
 open, 18, 321
 differential, 18, 21
 inclined, 322
 single-reading, 20
 Match point, 155
 Mean, 431
 Mean sea level, 51
 Metacenter, 31
 Metacentric height, 31
 Method of images, 256
 Mild channel, 209–212
 Minor (head) loss, 60
 Model, distorted, 361
 fixed-bed, 360
 movable, 361
 scaled, 345
 undistorted, 360
 Modulus of elasticity, 8, 119
 Moment, righting, 34
 Moment of inertia, 23–25, 33
 Momentum, 55, 144, 149
 Moody diagram, 64
 Multistage propeller pump, 151

Nappe, 293
 Negative pressure, 92, 123, 165,
 294, 297, 302–304
 Net positive suction head, 166
 Newton, 4
 Newton method (pipe networks), 116
 Newtonian fluid, 5
 Newton's law of viscosity, 5, 353
 Newton's second law of motion, 55,
 119, 123, 128, 353, 363, 364
 Nonequilibrium test (aquifers), 252
 Non-point source pollution, 377
 Normal depth, 191, 192, 207
 Normal distribution, 435
 Nozzle meter, 328

Open channel flow, 184
 Open systems, 372
 Orifice equation, 307, 404, 406
 Orifice meter, 328
 Orographic precipitation, 376
 Overturning (dams), 285–287
 Overturn of lake, 1

Parshall flume, 335
 Pascal, 2
 Pascal's law, 16
 Pavement encroachment (spread),
 412

Pearson type III distribution, 437
 Performance curves (pump), 153
 Peripheral velocity, 147, 352
 Permeability coefficient, 234
 Phases (of water), 2
 Phreatic line and surface, 270
 Piezometer, 97, 321
 Piezometric surface, 232–237
 Pipe, 51
 elasticity, 121
 equivalent, 80–82
 flow, 51
 friction, 53, 60
 network, 88, 104
 systems (branching), 97
 wall roughness, 53, 60–63
 Pipeline, 88
 Pipeline anchoring, 121
 Pipes in parallel, 81
 Pipes in series, 80
 Piping (through earth dams), 270
 Pitot tube, 324
 Plotting position, 448
 Poise, 5
 Poisson's ratio, 121
 Population, 431
 Porosity, 233
 Porous media, 234–235
 Positive displacement pump, 143
 Potential energy, 56, 197
 Power (pump), 144, 146–147
 Prandtl-type pitot tube, 325
 Precipitation, 372, 376, 385
 Pressure, 14
 absolute, 14–16
 atmospheric, 2, 14
 cell, 323
 center of, 23, 28
 force, 23, 27
 energy, 56, 197
 gauge, 14–16, 93
 head, 16, 57
 hydrostatic, 14, 15, 22
 measurement, 321
 negative, 92, 165, 294, 297,
 302–304
 pipe flow, 51
 stagnation, 324
 surface of equal, 17, 18
 vapor, 2, 93
 water hammer, 119–124
 wave, 119–124
 zero reference, 185
 Prismatic channels, 186, 214
 Probability distribution, 431
 Prototype, 345

Pump, 143
 axial, 149
 booster, 152
 brake horsepower, 153
 centrifugal, 143
 characteristic curves, 153
 discharge side, 95
 eye of, 147
 efficiency, 147, 153
 head, 146, 153
 housing, 144
 impeller, 144
 jet, 152
 multistage propeller, 151
 performance curves, 153
 peripheral velocity, 147
 positive displacement, 143
 power, 144, 146, 147
 propeller, 149
 rated capacity, 153
 similarity, 169
 suction side, 95
 tangential velocity, 147
 turbo-hydraulic, 143
 Pumping trough, 266

Radial flow (aquifer), 238

Radial flow equation, 238, 240
 Radius of influence, 238
 Rainfall excess, 393
 Random variable, 431
 Rapid closure (valves), 122, 127
 Rapid flow, 199
 Rapidly varied flow, 186, 207
 Rating curve, 385, 386
 Rational method, 410
 Recharge well, 257
 Recurrence interval, 439
 Reference datum, 51, 197
 Relative roughness, 62
 Resistance force, 5, 188
 Return period, 380, 439
 Reynolds number, 52, 352
 Rib (arch dam), 288
 Righting moment, 34
 Riser and barrel assembly, 291
 Risk (hydrologic), 440
 Roughness coefficient
 (Manning's *n*), 69, 190
 Roughness height (pipe), 62
 Runoff coefficient, 410

Sample, 431

Scaled model, 345
 Seawater intrusion, 263
 Secondary current, 76

- Sedimentation, 286, 346, 361, 416
 Seepage, 267
 Seepage velocity, 234
 Seismic wave propagation, 262
 Sequent depth, 203
 Shallow concentrated flow, 397
 Shallow water wave, 358
 Shape number, 169
 Sharp-crested weir, 293, 331
 Shear stress, 5
 Sheet flow, 396
 Shutoff head, 153
 Significance level, 443
 Skew coefficient, 432
 Skewness, 431
 Sliding (dams), 285, 286
 Sloughing, 270
 Slow closure (valves), 123, 127
 Slug, 4
 Soil piping, 270
 Specific energy, 198
 Specific force, 205
 Specific gravity, 4
 of alcohol, 18
 of concrete, 40, 314
 of mercury, 18
 of Meriam oil, 18
 of seawater, 34
 of water, 4
 Specific heat, 2
 Specific speed, 169
 Specific weight, 3, 4
 Spillway, 291, 296, 299, 302
 Spiral flow, 76
 Stagnation pressure, 324
 Standard deviation, 431, 432
 Steady flow, 54, 55, 186
 Steep channel, 209
 Stoke, 6
 Standard deviation, 432
 Standard step method, 212
 Storage coefficient (aquifer), 242
 Storativity, 242
 Storm drainage inlets, 412
 Strainer, 166
 Streamline, 52, 71, 267
 Subcritical flow, 199, 200
 Submerged weir, 294
 Subsurface barrier, 267
 Suction head, 166
 Supercritical flow, 199, 200, 291
 Superposition, 239, 257, 391
 Surface force, 9, 10
 pitting, 166, 297
 runoff, 372, 385
 of saturation, 270
 tension, 7, 8, 357
 wave, 200, 346, 357
 Surge tanks, 127
 Swamee-Jain equation, 65
 Synthetic unit hydrograph, 393
 System head curve, 155
- Thisis equation (well flow), 242**
 Thiessen method, 377
 Three-reservoir problem, 97
 Tilt angle, 32–34
 Time of concentration, 396, 398
 Time-to-peak, 398
 Toe of dam, 285
 Top width, 185
 Transitional zone (pipe flow), 65
 Transmissivity, 238
 Transpiration, 372
 Turbo-hydraulic pump, 143
 Turbulent flow, 52, 53, 60, 62–65
- Unconfined aquifer, 232, 233**
 Uniform depth, 191, 192, 207
 Uniform flow, 186, 188
 Unit hydrograph, 387
 Unsteady flow, 186
 Uplifting force, 284
- Vapor pressure, 2, 3, 93**
 Varied flow, 186
 Varied flow equation, 208
 Velocity, 51
 actual, 235
 apparent, 234
 distribution (pipes), 51–53
 head, 57, 184, 197
 mean, 51
 seepage, 234
 Vena contracta, 71, 328
 Venturi flume, 335
 Venturi meter, 326
 Vessel (pressure), 14, 18, 19
 Virtual mass, 359
 Viscosity, 5, 6
 absolute, 5, 52
 kinematic, 6, 52
 Viscous dissipation, 53, 60
 Viscous force, 60, 352
 Volume modulus of elasticity, 8
 Von Karman equation, 62, 63
- Wall friction, 53, 60**
 Wall shear stress, 189
 Water-bearing formation, 232
- Water, 2
 compressibility, 8
 density, 1, 3
 elasticity, 8, 119–123
 hammer, 119–127
 specific gravity, 4
 specific weight, 4
 surface profile, 207, 212
 table, 232
 viscosity, 5
 Watershed, 372
 Waterways Experiment
 Station, 360
 Wave, 188
 disturbance, 200, 211
 flood, 188, 346
 forces, 358, 359
 front, 119–124, 127
 gravity, 200, 358, 359
 negative pressure, 123
 period, 118
 pressure, 119–124,
 127
 seismic, 262, 263
 shallow water, 358
 shock, 262
 speed, 121, 211, 262
 surface, 200, 346, 357
 Weber number, 357
 Weight, specific, 3, 4
 Weir, 291, 331
 broad-crested, 334
 Cipolletti, 333
 contracted, 332
 frictionless, 292
 sharp-crested, 293, 331
 submerged, 294
 trapezoidal, 333
 uncontracted, 332
 V-notched, 333
 Well, 232, 237
 area of influence, 237
 cone of depression, 237
 discharge, 238
 drawdown, 237
 function, 243
 radial flow, 238
 radius of influence,
 238
 recharge, 257
 Wetted perimeter, 67, 186
 Wide rectangular
 channel, 207
- Zero pressure reference, 185**